MSE 222 Final Project:
Optimizing the Design of a Ball Track using Calculations & Experiments

for Dr. [redacted]

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Group 10

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KINEMATICS & DYNAMICS OF RIGID BODIES & MECHANISMS

MECHATRONICS SYSTEM ENGINEERING

SCHOOL OF ENGINEERING SCIENCE

SIMON FRASER UNIVERSITY

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Introduction

The purpose of this project was to design, simulate, build and test a dynamic system for transporting a glass marble within a set period of time. The objective was for the marble to travel from the top left to the bottom right of a 60 x $60cm^2$ board, along a predetermined path. This had to be done in exactly fifteen seconds and with no external inputs to set the ball in motion. Although the design of the path was largely left up to our team, several constraints were put in place; the system was to include at least one of each of the following: a sloped, curved and flat path, a rotating element, an impact and a minimum of two changes in direction (top to bottom/ left to right). The starting and ending positions of the ball were set at one centimeter from their respective edges. Additionally, the system was to be passive, meaning no electronic or magnetic devices could be used. Lastly, the ball's initial velocity was to be zero, and the motion must have been initiated by gravity. At the bottom, the ball was to come to a complete stop unassisted in order for the timer would be stopped. We lastly note that the evaluation of the project was based on the aforementioned factors in conjunction with other project requirements.

First, all of the design criteria of the project had to be met - failing to include all the necessary components would result in penalties. Next, the performance of the final system was judged. Three timed trials were allowed in order to get a time as close as possible to fifteen seconds. The most points were awarded to the team with the closest time, with all others receiving a percentage proportional to their relative performance. The majority of the evaluation was based on the simulation using MATLAB. Models of all individual rigid bodies in the system were created. Then, an algorithm was constructed to simulate a ball traveling along a variety of paths and interacting with other rigid bodies. These simulations were performed in order to optimize the layout of the track and grading of this section was based on the functionality and accuracy of the MATLAB code. Lastly, the construction of the system was graded based on materials choices. As conscious engineers, we faced the challenge of building this system using as few new materials as possible and doing our best to employ sustainable materials and techniques. In the short time ahead, we intend to outline a number of various factors and areas surrounding the design and functionality of project and we look forward to sharing our findings.

Part I: Analysis of Each Component of the System

Part 1 proved to be the most challenging of the entire project as it required a vast amount of prior MATLAB knowledge, innovative ideas, as well as a strong kinematics knowledge. Our code evaluates all the parameters outlined within the project's scope and it can be referenced in the attached project files.

Our code is able to accomplish several important tasks, namely;

- It can parametrize the geometry of the ramps, slopes, and curves so that our geometry can be easily modified to match the time requirements,
- It is able to output the time, position, velocity and the acceleration of the ball during the motion and at the end of the motion,
- Furthermore, it's able to take various inputs such as velocity, position, time, and acceleration as starting conditions and is able to produce multiple outputs.

With these outputs listed above, we note that our MATLAB code did adhere to the projects stipulated requirements and fulfilled the required obligations that were set out for us. As you'll come to see if the attached code files and in Part II below - our code is innovative in nature, like our final design - and it encompasses multiple important foundations of kinematics, MATLAB fundamentals, and teamwork. It also contains descriptive comments that analyses and locates each component of the system in a brief manner and contains multiple functions, such as our main function, ramp functions, impact function, drop functions, and curve functions. These aforementioned functions and comments have then been fused together which creates our overall system. The comments will be given above the relevant code, highlighted in green colored text.

Part II: Design, Simulation and Study of the Dynamic System

Our final system design was completed by slope manipulation in conjunction with a few other areas.

Below in Figure 1, we recognize that our plot of position of X vs. Y was relatively in line with our simulated and practiced findings. After computing this through Matlab, this led us into our preliminary design phases where we were able to develop the slopes and lengths accordingly.

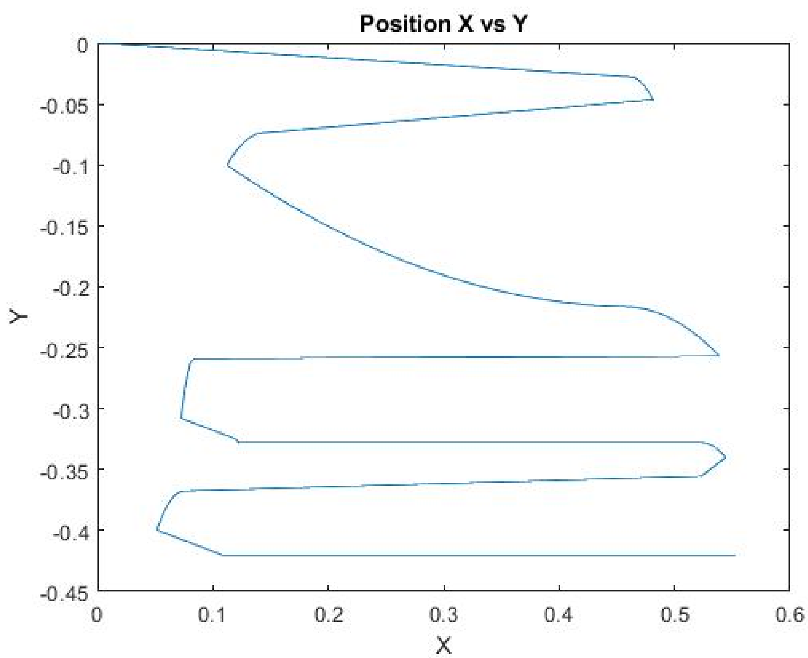


Figure 1: Plot Position of X vs. Y

Below in Figures 2 through 5, we can see the harvested data of our simulated Matlab code. It encompasses plots relating to linear velocity vs. time, linear acceleration vs. time, angular velocity vs. time, and angular acceleration vs. time. The establishment of this code has allowed us to accurately visually depict the given project requirements.

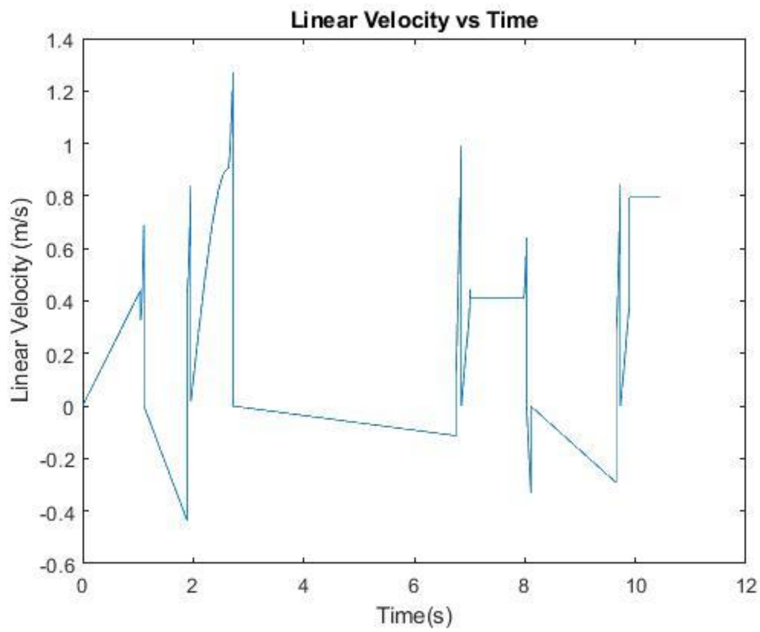


Figure 2: Plot of Linear Velocity vs. Time

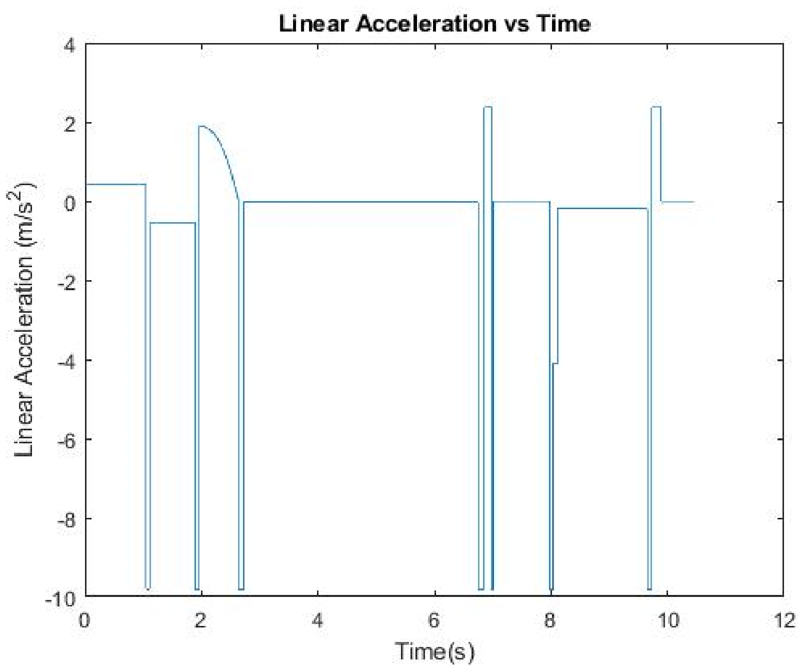


Figure 3: Plot of Linear Acceleration vs. Time

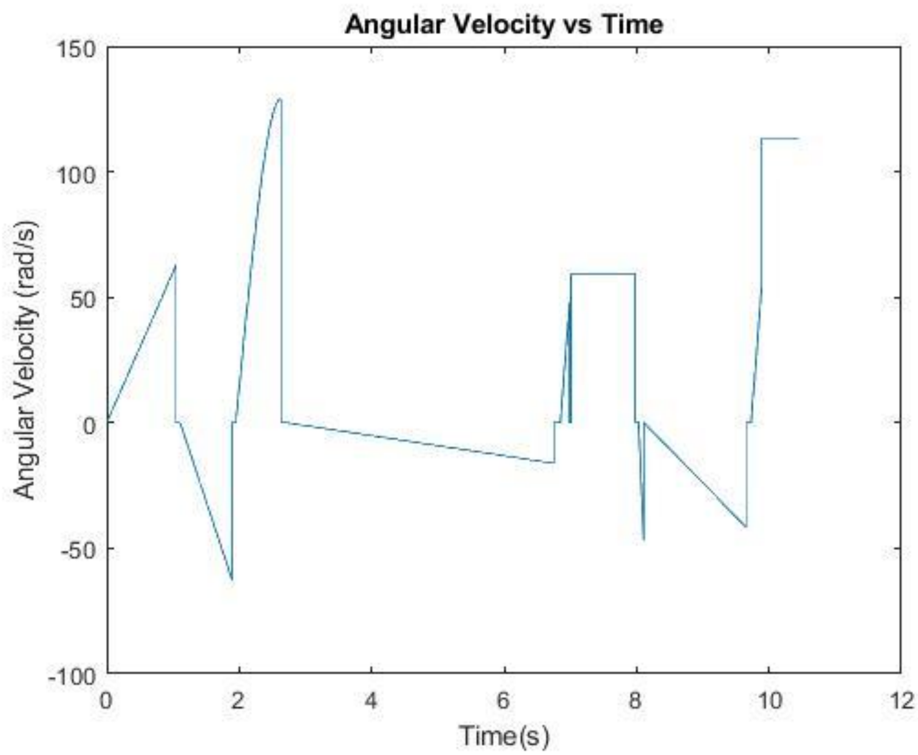


Figure 4: Plot of Angular Velocity vs. Time

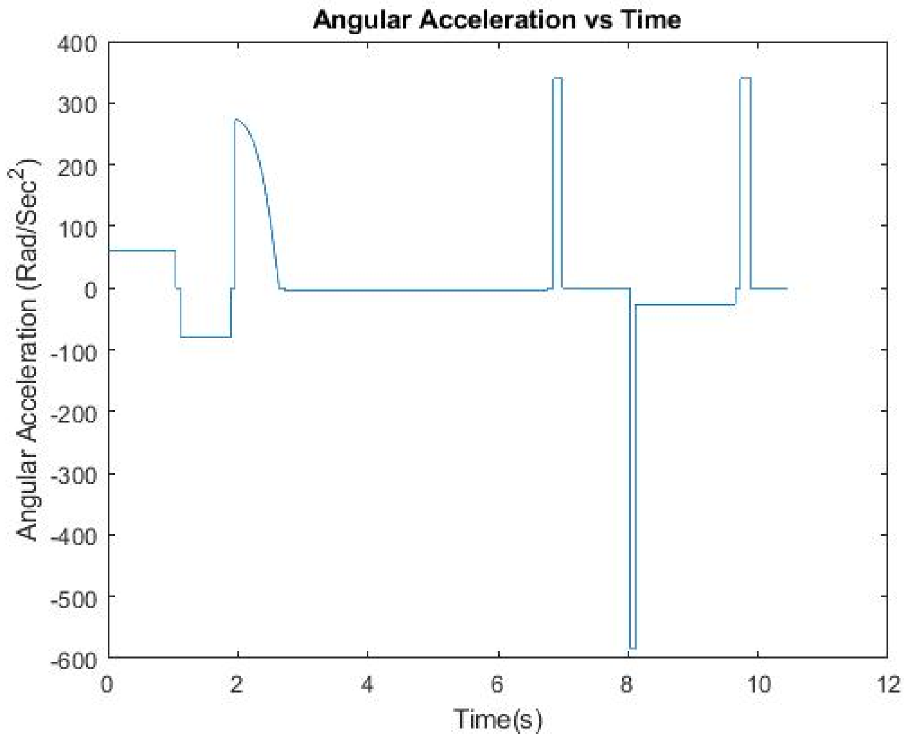


Figure 5: Plot of Angular Acceleration vs. Time

Our final design was determined after extensive sets of revisions, slope analysis and manipulation as well as efficient prototyping and dimensioning. After our design was completed in Matlab, we were able to take these dimensions within Solidworks and our prototyping ensued.

Sensitivity Study

Part of the analysis required an impact between the marble and a rotating element. In our case, a tin gate was attached to an axle, allowed to rotate freely. Neglecting the friction between the gate and the axle, the conservation of angular momentum was observed with respect to the fixed axle.

$$e = \frac{V_{B2} - V_{A2}}{V_{B1} - V_{A1}} = \frac{V_{Gate(2)} - V_{Marble(2)}}{V_{Gate(1)} - V_{Marble(1)}}$$

Equation 1: Coefficient of Restitution

Part of the analysis described above required a coefficient of restitution (CoR). The coefficient of restitution is defined as the ratio of the final velocity to the initial velocity between two objects after their collision. Another way of saying this is that the coefficient of restitution is the ratio of the velocity components perpendicular to the point of contact after and before the collision.

Due, to the unique nature of our experiment, a predetermined CoR was not available and therefore we needed to determine one experimentally. The process behind this goes as follows, a video would be recorded of the ball in two stages. One rolling immediately before impacting the gate and one after the gate. The values of the velocity of the ball before and after were determined using the frame rate of the video recording and the displacement traveled in a determined time interval. Also, the velocity of the gate was recorded in a similar fashion. Time trials of this test were completed, and the results can be found in the Table below along with other assumed variables.

Trial	**Velocity of the gate after impact: ($\boldsymbol{V_{Gate(2)}}$)**	**Velocity of the marble before impact: ($\boldsymbol{V_{Marble(1)}}$)**	**Velocity of the marble after impact: ($\boldsymbol{V_{Marble(2)}}$)**
1	0.42	0.28	0.34
2	0.47	0.29	0.37
3	0.46	0.31	0.32
4	0.43	0.27	0.39
5	0.41	0.31	0.36
6	0.49	0.24	0.32

7	0.42	0.29	0.33
8	0.41	0.26	0.34
9	0.38	0.28	0.35
10	0.42	0.30	0.37
Average	**0.431**	**0.283**	**0.349**

Table 1: Tabulated Velocities

Using the results from the experiment and the equation described above, a coefficient of restitution of 0.424 was ultimately determined and will be used in the MATLAB analysis. Time trials were completed varying the calculated coefficient of restitution by +/- 10% as seen in the table below. Further analysis is completed below using the experimentally approximated values.

	Coefficient of Restitution 'e'	**Time(s)**	**Radius of Marble (m)**	**Time(s)**	**Mass of Marble (kg)**	**Time(s)**
-10%	0.3816	10.47	0.0063	10.45	0.036	10.45
Experimental Value	0.4240	10.45	0.007	10.45	0.040	10.45
+10%	0.4664	10.44	0.0077	10.45	0.044	10.45
Results		***μ* = 10.453**		***μ* =10.45 *s* = 0**		***μ* = 10.45 *s* = 0**

		***s* = 0.01527**				

Table 2: Sensitivity Study for Coefficient of Restitution

As seen in the tables above, a decrease in the coefficient of restitution will increase the overall time taken and an increase will reduce the overall time. The relationship can be justified in the software prediction in the Appendix as the CoR directly affects the initial velocity passed into the drop function. The drop function simulates a classic trajectory problem onto an inclined surface with respect to the global coordinate system. Thus, a lower velocity would result in a shorter trajectory onto the next rigid body structure, reducing the overall displacement travelled in the track and therefore reducing the overall time.

Varying the mass in the simulation had no effect on the final time of the simulation as it is cancelled out in the initial ramp function call. This supports our calculations as it takes the conservation of momentum where both sides are dependent on mass. Varying the radius also had marginal effects on the output of the time, to a neglected digit. This could be justified, and as for the calculations, the moment of inertia was used to calculate the angular velocity from the conservation of momentum. However, for each ramp the global velocity was inputted as zero since for each level, there was a change in the horizontal direction.

Part III: Component Sourcing, Characterization, & Environmental Assessment

Procedure

As per the outlined requirements, the project stipulated that each component we used in our design must be either reused or recycled. Recognizing these parameters prior to beginning the fabrication of our project led us to consider the environmental impacts, the life-cycle of the products to be used, as well as the disposal plan of each component in the process. As we'll come to see in this section, the aforementioned topics will be covered and our thought processes surrounding these areas will be developed and explored.

Component Sourcing

As soon as the project parameters were taken into consideration and we have reviewed the elements required to complete our design, it was incumbent on our group members to be cost-efficient and to consider the environmental ramifications of our material selection. Our group was able to source together multiple components that resulted in no money out of pocket - in addition with the cost savings, it also assisted with time-saving measures. There was no need to make any quick trips to the hardware store to further source components as all of our material was allocated from old home building supplies. As we'll come to see in subsequent sections, these materials can easily be recycled and reused for further various purposes.

Materials & Components Utilized in Final Design

As previously stated, these materials were sourced with minimal cost implications in mind as well as potential environmental impacts. To minimize both of these factors, we utilized various materials from a local and existing building site, denoted in Table 1 below;

Quantity	Material	Purpose Prior to Project	Approx. Duration Prior Usage	Weight

1	⅝" Plywood Sheet, 60cm x 60cm	Use in home building construction	4 years	2.4 kg
1	Softwood (along the grain)	Various purposes at building site	4 years	3.7 kg
1	PVC Home Water piping lines	City water line distribution, existing pieces	4 years	890 g
1	Tin alloy (non-ferrous)	Various	1 year	210 g

Table 3: Material & Component Selection

Disposal Plan

For the disposal plan as it related to our project materials, we decided to utilize CES Edupack to more accurately outline and understand the environmental impacts and the life-cycle reporting as it pertained to each respective material.

Following the conclusion of our project, we intend to be as environmentally conscious as possible - this includes recycling as many materials as possible (as per the local City guidelines and regulations), so the tin and the PVC would be then recycled accordingly. As it relates to the softwood and the plywood, we note here that the materials were treated with an environmentally sensitive aerosol to protect the granular structure of the wood as well as reducing the friction between the various surfaces and the marble. As such, this makes it increasingly more difficult to recycle - as treated materials are not accepted for recycling purposes, as per local City regulations. Thankfully, once disassembled, a relative of mine decided that the wood segments of our project could re-utilized for a bird feeder in their backyard; this prevented the mass and majority of the wood from returning to the landfill. Further information can be referenced in the Appendix for detailed reporting on life cycle analysis (end-of-life potential) as well as environmental impact and footprint.

Part IV: System Design and Fabrication

The system design and fabrication of our model posed a series of challenges and underscored the need to review our course materials and readings to properly address, design, and fabricate our system accurately; the subsequent sections outline these variables accordingly.

Constraints

There were five constraints were considered for the initial design of our project, and it was vital to ensure adherence and compliance with them, namely;

1. The ball's initial position must be top left of the board, and its final must be bottom right (looking from the front view)..
2. Ball must move both from right to left and left to right (at least once) during its movement from start to finish.
3. Ball must move down (directing of gravity force) and up (at least once) during its movement from start to finish.
4. There must exist at least one curved path in the design.

5. Most importantly, duration of ball movement from initial to final position on the board must be as close as possible to 15 seconds.

Construction & Associated Timeline

Construction began relatively quickly as soon as the project constraints were laid out and a team meeting occurred. From here, decisions were made surrounding the impact and cost associated with the materials we intended to use. Furthermore, since one of our team members had a variety of tools and building materials, the project construction ended up occurring at that residence.

The most difficult decisions surrounded the orientation that the materials should make, and how long we would dedicate to implementing our design. We set aside roughly four hours to fabricate and test our project, and this timeline proved to be adequate for our needs; Figure X below denotes the construction phase.

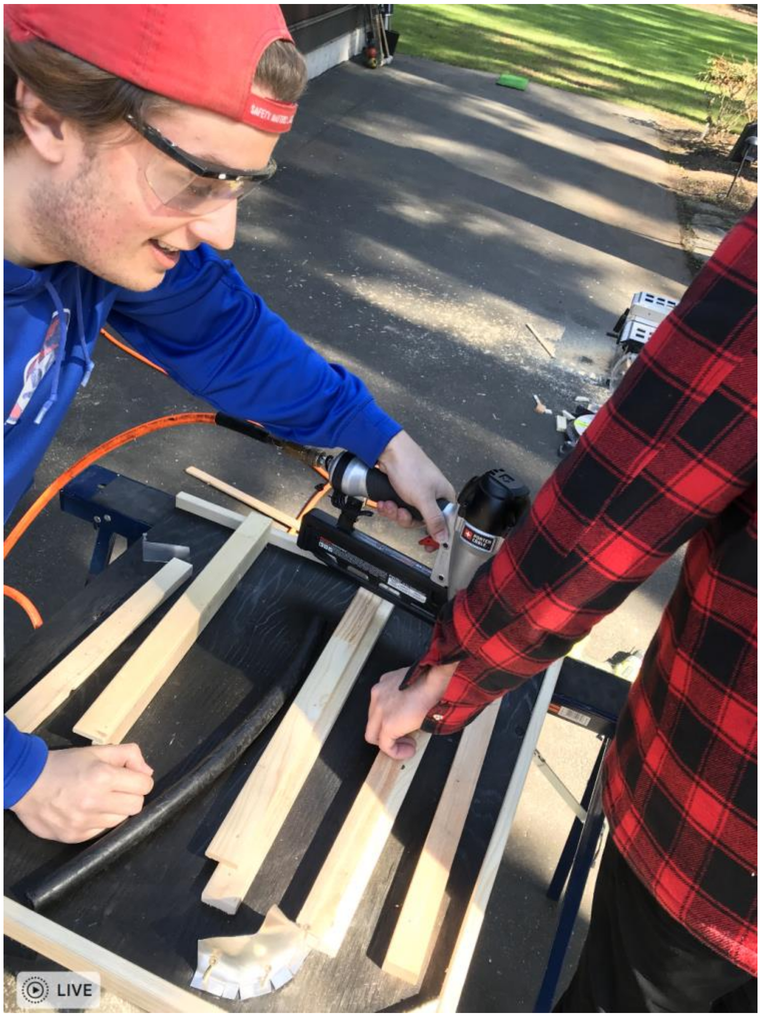


Figure 6: Construction Phase

Initial Solidworks Design

The initial design of our project in Solidworks was completed before the physical project was built. Since we only knew the theory behind the project and how the ball is supposed to move through our kinematics course learnings and readings, a rough sketch was drawn to establish some of the constraints associated with the project. Subsequently, the initial Solidworks was then designed made based on the drawing, denoted in the preliminary sketch below.

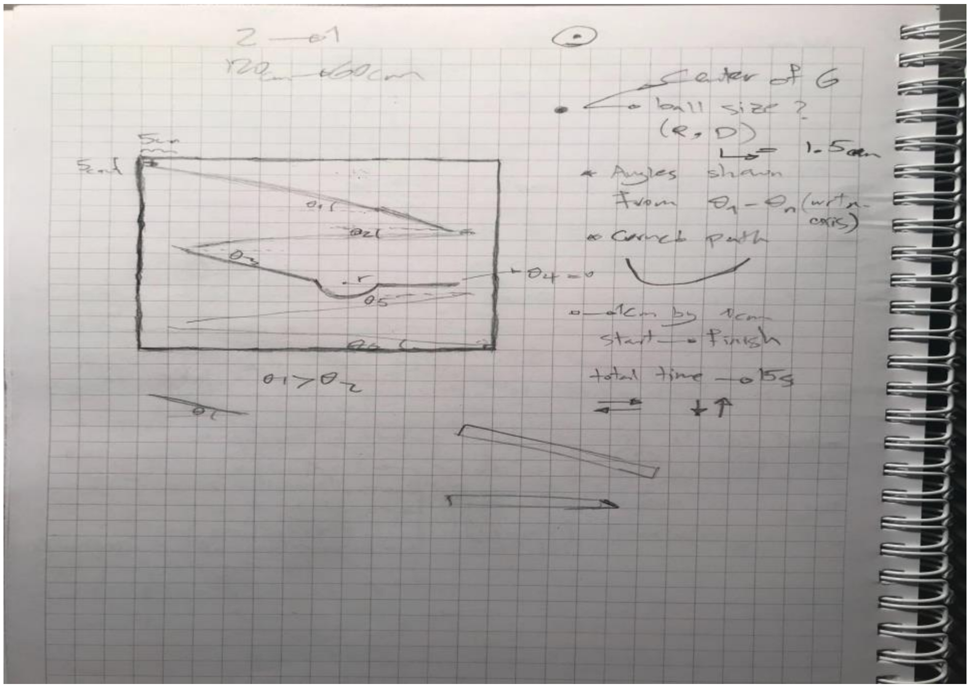


Figure 7: Initial Sketch

Final Solidworks Design

The initial design was instrumental in giving us an idea of what the project should look like. However, as the physical project was being built and after performing multiple time trials with the marble, we realized that initial design will be much quicker than the required fifteen seconds and it would violate constraint five.

Therefore, another similar system was proposed by our group where all construction constraints were observed but the design was altered in order to meet the time constraint of fifteen seconds. Some of these include, neglecting the friction of rolling between the marble and the track, rolling without slipping, neglecting bouncing between vertical drops, and environmental factors such as air resistance.

As mentioned, basic concepts of ball movement were similar between both designs, however, there are a few areas of revision:

1. Flatter paths were more prevalent in the final design compared to the initial design; the goal here was to simply increase the distance travelled by the ball to make the duration longer and closer to fifteen seconds (Concept #5).
2. Sloped paths have added guards. The addition of these guards alleviated the risk of the ball coming off the track if a sudden bump or movement to the structure was to influence it.
3. The curved section was changed out for a more rigid alternative, namely, a PVC pipe with a bend radius; the addition of this provided ease of construction during the fabrication process.
4. We decided to utilize tin as some of our transition pieces - this helped the marble gain velocity and ensured a consistent and safe travelling path for the marble to travel on.

As our system was now fabricated in accordance with the given project parameters, we were now able to fully design, dimension, and construct our final rendering in Solidworks - the following Figures represent our final design from Solidworks.

Figure 8: System Design Front View

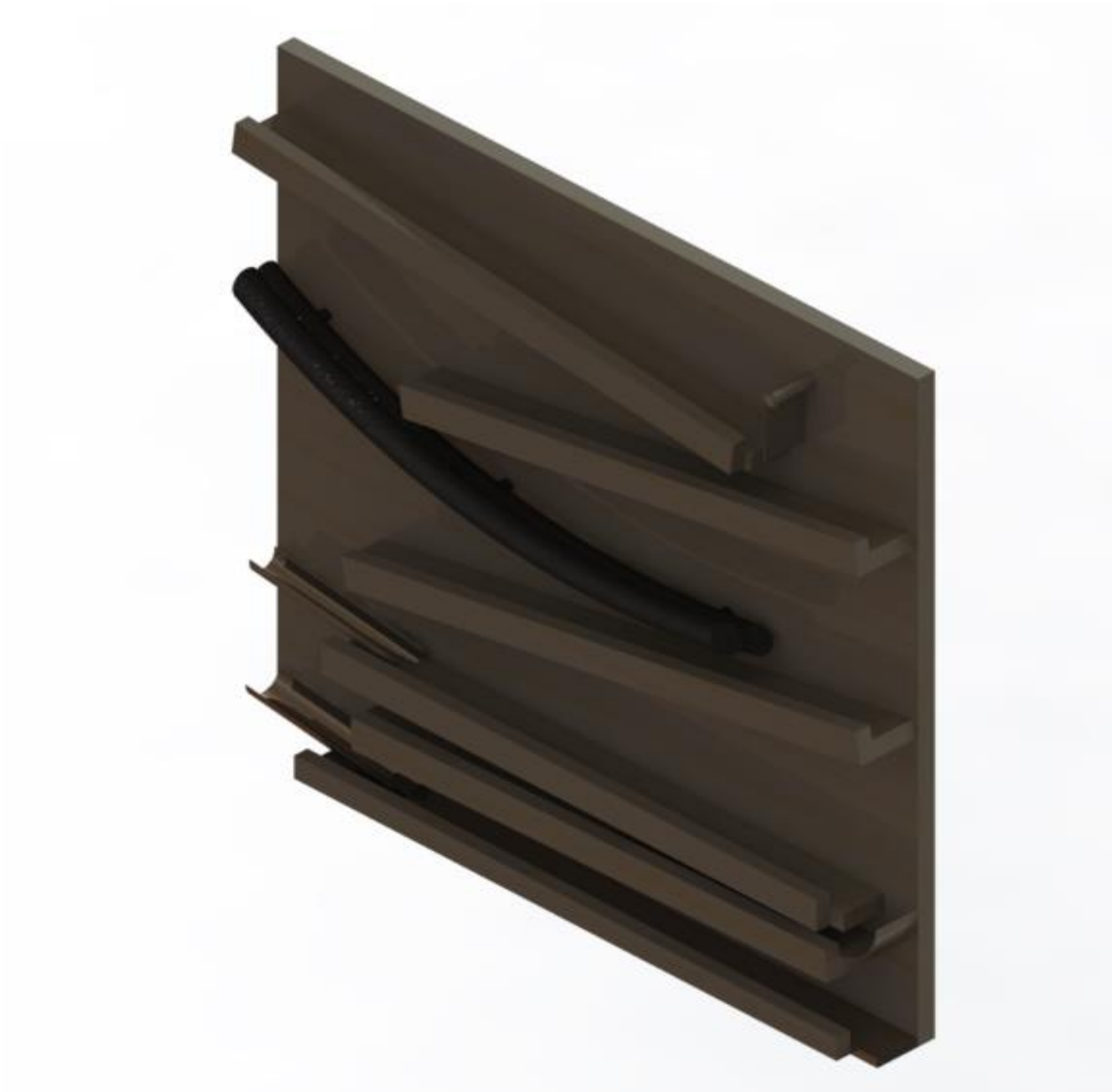


Figure 9: System Design Isometric View, RHS

Figure 10: Isometric View, LHS

Figure 11: System Designed Prototype

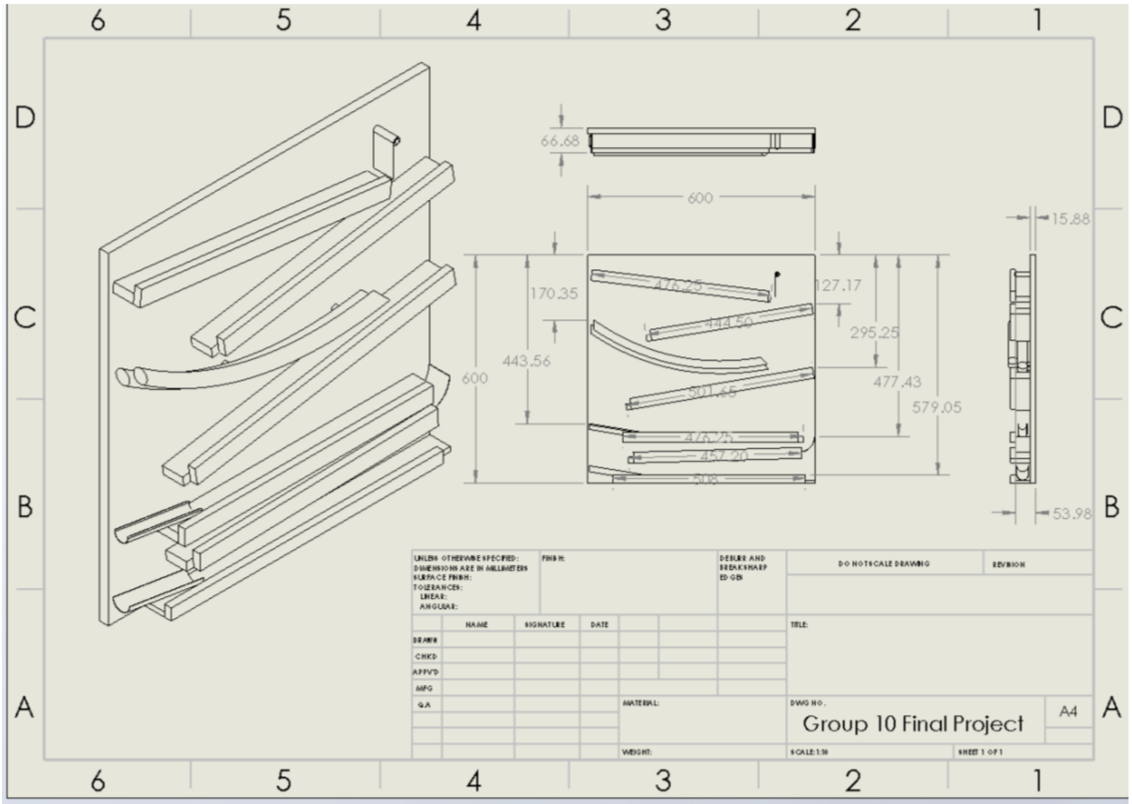


Figure 12: Dimensioned System Design

The revisions that were adhered to in this section allowed for more accurate forms of testing and permitted our final design to adhere to the outlined requirements. Furthermore, the added components ensure a higher level of marble stability, flow, and trajectory positioning.

Part V: Performance

The goal of the project was for the marble to traverse the path in a time of fifteen seconds. In MATLAB simulations, we achieved a time of 10.45 seconds and based on this result, we began construction of a prototype. Initial testing of the prototype resulted in times in the range of 14.9-15.1 seconds. However, as guardrails were installed to improve the reliability and consistency of the track, negative effects on performance were observed; consistent times of 13 seconds were achieved. Testing on different surfaces and at variable tilts proved effective. Prior to the demonstration, times of around 14.6-15.2 seconds were observed. During testing, the track was placed on a table in the front of a classroom. The first two trials yielded times that were as much as 2 seconds off target. For the last attempt, we placed the track on the floor rather than the table. This had a very positive impact on performance, and for the final run, a time of 14.54 seconds was achieved. Unfortunately, this wasn't an ideal performance, but it did provide us with great insight into possible areas of improvement for the design. As a result, our track times were noted as such: 12.54s, 13.78s, 14.48s. We notice that these times are a little slower than our initial time of 10.45s and we can attribute this to several factors, namely; neglecting the rolling friction between the marble and the track, and the moment loss from neglecting bouncing (loss of potential energy).

Per observations during testing and demonstration, the performance of the track is greatly affected by the type of surface and the incline/tilt at which its placed. The track is designed with ramps which are much wider than the marble, causing a variation in the exact path of travel. The MATLAB simulation assumes a straight path, with no side-to-side movement. Due to this variable path, some inconsistencies were observed. This also explains why changing tilt had such a notable effect on performance. Placing the track on a smooth and level surface and starting the ball approximately halfway between the guardrail and wall would yield a time much closer to the theoretical values seen in MATLAB simulation. The design can be improved by making thinner ramps, hence reducing the amount of side-to-side travel along the track. This will tighten up the travel path, resulting in more consistent results, which are also closer to the theoretically calculated results.

Conclusion & Reflection

With this project, our team aimed to design, simulate and construct a track with the goal of guiding a marble along a path in exactly 15 seconds. By making the vital assumption of rolling without slipping, we were able to simulate the system using MATLAB. Separate functions were written to model each individual rigid body, such as a ramp or a rotating component. Once MATLAB simulations were performed and the desired results were achieved, dimensions from the MATLAB code were taken and a track was designed using SolidWorks. Based on this a prototype was built out of wood. Initial testing was promising, but the addition of reliability components, and a variation in incline and surface used for testing had a negative impact on results. During demonstration, an acceptable time of 14.54 seconds was achieved. Future improvements to the system would include thinner ramps to reduce side-to-side travel of the ball, improving consistency and accuracy of results.

This project was challenging in numerous ways and posed multiple challenges for our team to overcome together. It required clear communication, innovative and out of the box thinking, as well as a solid kinematics foundation, from our teachings throughout the semester. Drawing on all of our experiences together as a group and from the semester, we're proud of the way our team performed, how we were innovative in our procedures surrounding MATLAB, how we were able to be cost-efficient, and work together for the betterment of the group. We look forward to applying our kinematics foundations in our next levels of learning and throughout our professional careers. We would like to thank Dr. Sparrey and the Teaching Assistants for an enjoyable semester of learning.

Team Contributions

Names	Contributions
A[redacted]	- Report writing, various
C[redacted]	- MATLAB programming, building, report writing
M[redacted]	- MATLAB programming, report writing
P[redacted]	- Prototype building & design, report

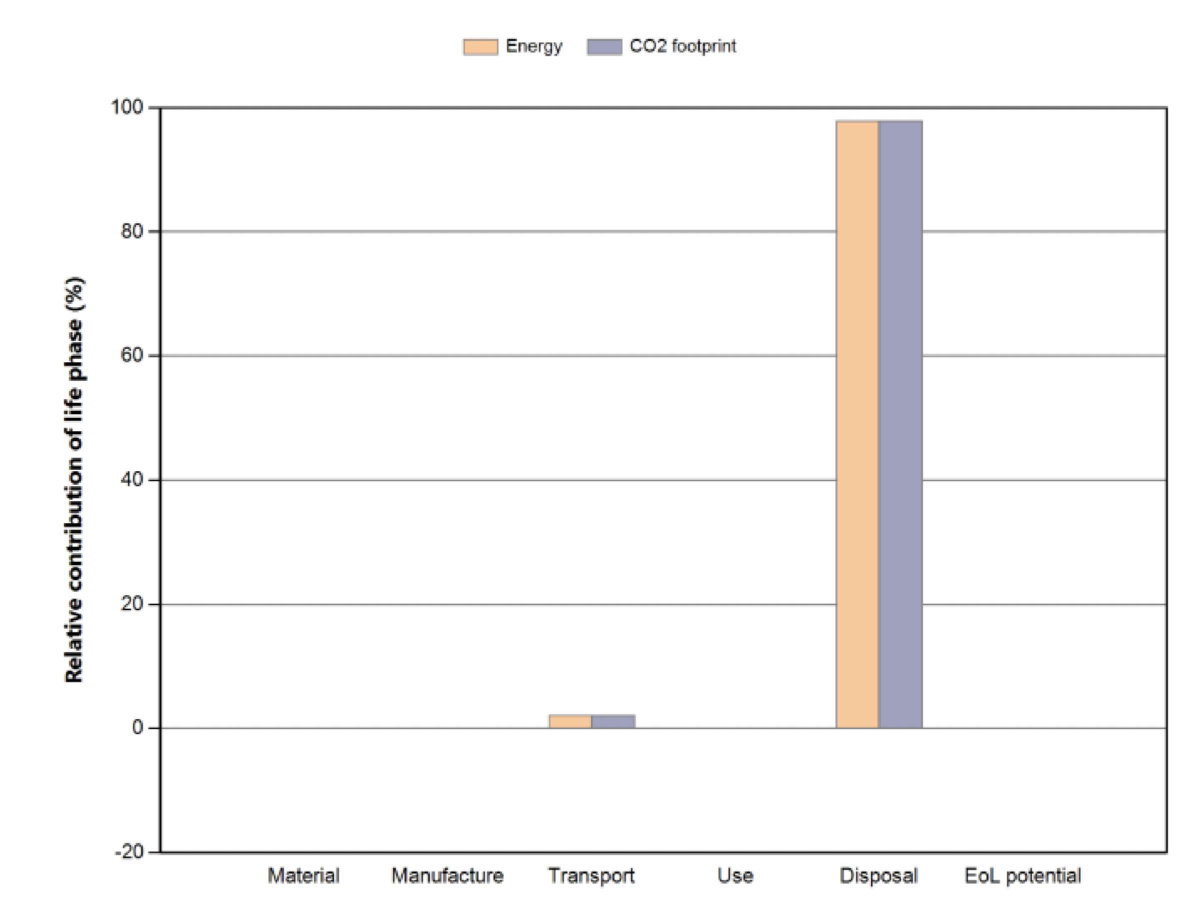
	writing, various
Seper Rezvani	- Solidworks development, various

Table 4: Team Contributions

References

- Hibbeler, R.C. (2016). *Engineering Mechanics: Statics & Dynamics.* Fourteenth Edition.
- Sparrey, C. (2019). *Lectures 3 through 21*. Retrieved from: https://canvas.sfu.ca/courses/43600/files/folder/Lecture%20Notes
- Sparrey, C. (2019). *Formula Sheet V1*. Retrieved from: https://canvas.sfu.ca/courses/43600/files/folder/Study%20Guides?preview=9891598
- MATLAB, r2017B Operating System.
- Solidworks, 2016 Operating System.

Appendix



Energy details   CO2 footprint details

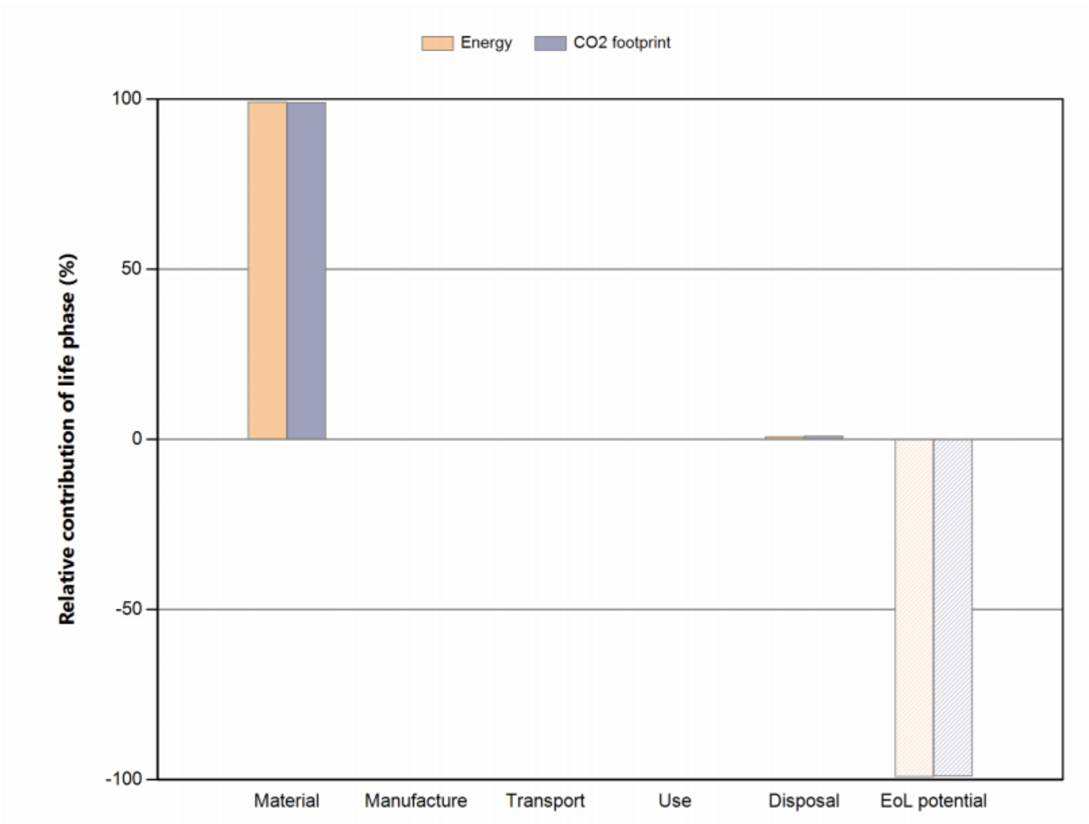
Phase	Energy (MJ)	Energy (%)	CO2 footprint (kg)	CO2 footprint (%)
Material	0	0.0	0	0.0
Manufacture	0	0.0	0	0.0
Transport	0.0157	2.1	0.00113	2.1
Use	0	0.0	0	0.0
Disposal	0.74	97.9	0.0518	97.9
Total (for first life)	**0.756**	**100**	**0.0529**	**100**
End of life potential	0		0	

Appendix 1: Plywood Analysis & End of Life Potential

Disposal:

Component	End of life option	Energy (MJ)	%
Plywood Sheet (Backing)	Reuse	0.48	100.0
Total		**0.48**	**100**

Appendix 2: Plywood Disposal Analysis



Energy details CO2 footprint details

Phase	Energy (MJ)	Energy (%)	CO2 footprint (kg)	CO2 footprint (%)
Material	62.2	99.2	3.39	99.0
Manufacture	0	0.0	0	0.0
Transport	0.0316	0.1	0.00227	0.1
Use	0	0.0	0	0.0
Disposal	0.48	0.8	0.0336	1.0
Total (for first life)	**62.7**	**100**	**3.43**	**100**
End of life potential	-62.2		-3.39	

Appendix 3: Softwood Analysis & End of Life Potential

Disposal:

Component	End of life option	Energy (MJ)	%
Softwood Eco Audit	Reuse	0.74	100.0
Total		**0.74**	**100**

Appendix 4: Softwood Disposal Analysis

Eco properties

Embodied energy, primary production	217	-	239	MJ/kg
CO2 footprint, primary production	15	-	16.6	kg/kg
Recycle	✓			

Appendix 5: Eco Properties of Tin

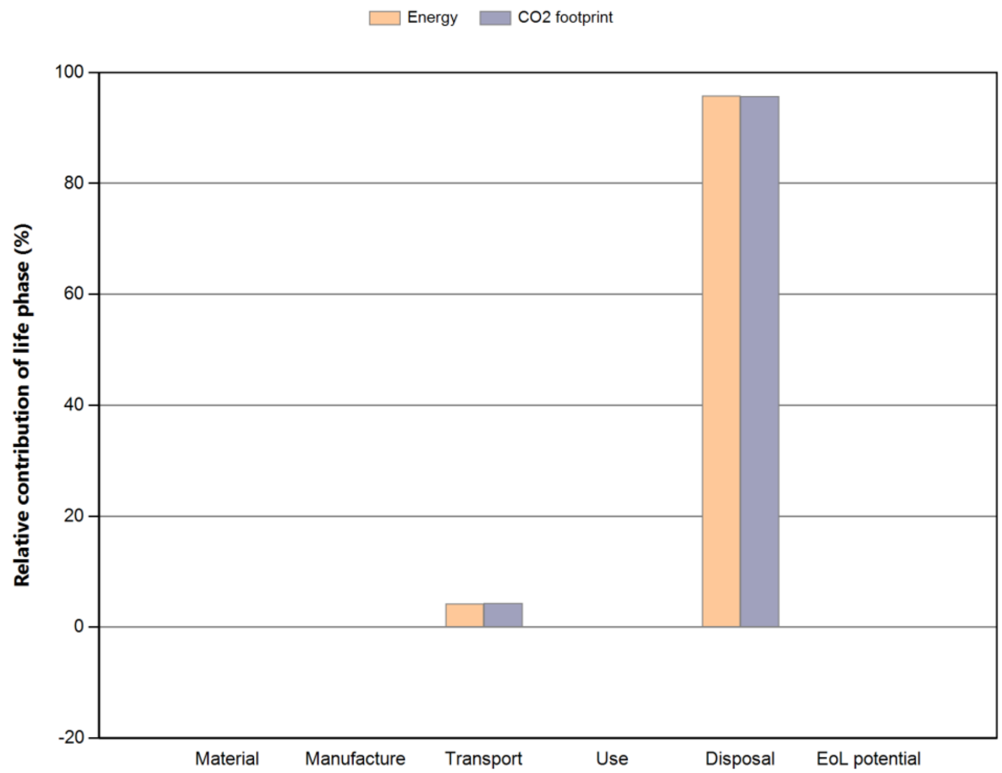
Eco properties

Embodied energy, primary production	53.7	-	59.2	MJ/kg
CO2 footprint, primary production	2.13	-	2.34	kg/kg
Recycle	✓			

Recycle mark

PVC

Appendix 6: Eco Properties of PVC



Energy details CO2 footprint details

Phase	Energy (MJ)	Energy (%)	CO2 footprint (kg)	CO2 footprint (%)
Material	0	0.0	0	0.0
Manufacture	0	0.0	0	0.0
Transport	0.0274	4.2	0.00197	4.3
Use	0	0.0	0	0.0
Disposal	0.623	95.8	0.0436	95.7
Total (for first life)	**0.65**	**100**	**0.0456**	**100**
End of life potential	0		0	

Appendix 7: PVC Analysis & End of Life Potential

Disposal:

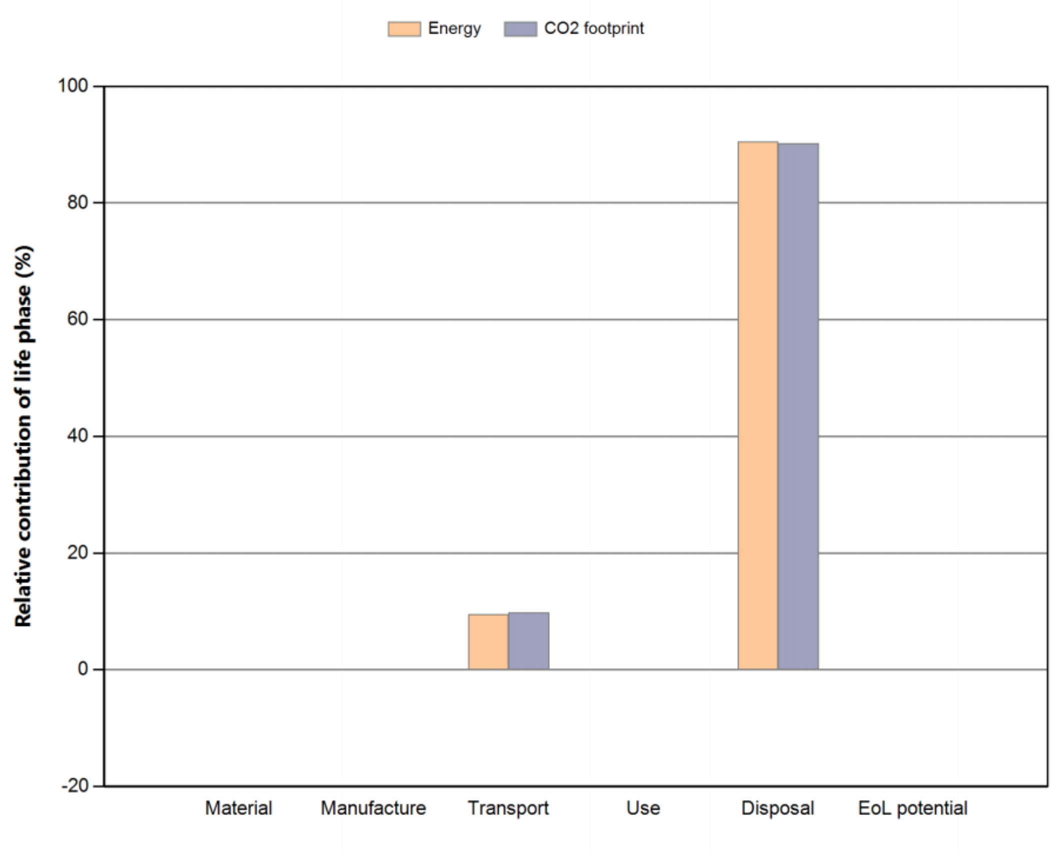
Component	End of life option	Energy (MJ)	%
Curved Section	Recycle	0.62	100.0
Total		**0.62**	**100**

Appendix 8: PVC Disposal Analysis

Disposal:

Component	End of life option	CO2 footprint (kg)	%
Impact Gate	Landfill	0.00035	11.9
Curve	Landfill	0.0026	88.1
Total		**0.0029**	**100**

Appendix 9: Tin Disposal Analysis



Energy details CO2 footprint details

Phase	Energy (MJ)	Energy (%)	CO2 footprint (kg)	CO2 footprint (%)
Material	0	0.0	0	0.0
Manufacture	0	0.0	0	0.0
Transport	0.00441	9.5	0.000318	9.7
Use	0	0.0	0	0.0
Disposal	0.042	90.5	0.00294	90.3
Total (for first life)	**0.0464**	**100**	**0.00326**	**100**
End of life potential	0		0	

Appendix 10: Tin Analysis & End of Life Potential