# Strength Analysis and Design of Bicycle Frame

MSE221: Bicycle Frame Project
Att: [redacted] Professor of Mechatronic Systems Engineering
November 29, 2018

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# Table of Contents

# Executive Summary

This report documents findings from the research conducted into stress analysis of the Allegro bicycle frame. It consists of an introduction into the various tests and sample calculations performed on the bike frame. The purpose of these tests was to see how the choice of material and design has an effect on how the bike frame will handle different stresses. Test results were used to propose a new design for the bicycle frame. Solidworks was used to help model the bicycle frame and calculate the angles and perform other measurements. Finite Element Analysis(FEA) program was used to see how the bicycle frame would react under certain loading conditions. The FEA was used to see how the bicycle frame would fair against mechanical stresses. Results from FEA were used to propose a new design for the bicycle frame and then compared the stress results of the original design. The new design was simply based on what would reduce the stress the most and maybe make it more affordable.

## Introduction

Bicycles were introduced in the 1800s and still remain as one of the most widely used means of transportation. Over time bicycle designs have changed and become much more advanced. Bicycles have recently begun to regain popularity once again and this is proof of how efficient and reliable they can be. The recent increase in the eco-friendly mindset has commuters searching for their bicycles that were collecting dirt for years. In addition, the increase in automobile traffic in urban areas which has made bicycle transport far more efficient.

Taking this into account, we were given the project to analyze the stress encountered by a bicycle frame under a common load. Furthermore, we will find the safety factor of each component of the frame and learn which parts are most prone to failure under a realistic load. From our findings, we will create a new design which will be better suited for similar conditions.

## Measurements and Materials:

We began the analysis by first performing measurements on a bicycle frame we owned. Appendix XIV shows the bike used for this analysis. The measurements were performed by a standard 30cm ruler. A table of measured values can be found in Appendix II. The material for the bike was approximated to steel 4130 from a label on the frame. The modulus of elasticity of steel 4130 is 208 GPa (see Appendix XIII). Using these measurements and assuming that all the bars on the frame are straight, we constructed a sketch on SolidWorks. A sketch with the measurements can be seen in Appendix VI. To calculate cross sectional area of each bar we measured the circumference of each bar and assume that they are all approximately round. Recorded circumference values are outlined in Appendix III. From the circumference values we were able to calculate the radius of the pipes.

# Analysis

## Loading Conditions:

The analysis assumes a stationary bike. This means the tires are statically balanced and there are no forces acting on the frame. The weight of the bike relative to that of the rider is also assumed to be negligible. It is then assumed that a 180lb rider jumps onto the seat. The applied load is assumed to be strictly vertical. As a result of the rider jumping into the seat, it is approximated that two times their weight, 360 lb, is applied to the seat, and one fourth their weight is applied to the handle bars as they tries to balance themselves. Two reaction forces are considered, one at each wheel. Refer to Appendix IX for a loading diagram.

## Failure Criteria:

Since a bicycle is intended for long term use, we consider fatigue loading to determine the failure criteria. The fatigue stress of a material is given as $S_{fl} = S_f(NFL)^b$ where $S_f$ = intercept, b = slope of the stress-cycle curve, NFL = loading cycles ($10^6$ for steel alloys). Using the provided failure criteria software [3], $S_{fl}$ for 4130 steel was approximated by considering hand forged as well as Q/T steel with varying Ultimate Strengths and Brinell Hardness numbers (a measure of a materials hardness). By averaging the loading fatigue for 4 different types of 4130 steel, the fatigue loading stress was found to be 442.5 MPa. This calculation is outlined in Appendix XIII. To find the safety factor of we use the equation SF = $\frac{Fatigue\ stress}{Allowable\ stress}$ , where allowable, stress, $\sigma_{al} = \frac{F}{A}$ . As per the project description [2], a SF > 3 is considered a safe design. The safety factor for the whole bike is considered to be the lowest safety factor for individual sections.

# Calculations

## Part 1: Pin joint, truss structure

Appendix IV contains detailed calculations, as well as a table of stresses and safety factors for individual trusses.

We can observe that the seat stays are subjected to the highest stress. Even then, a safety factor of 21 was calculated, far higher than the 3 needed to consider the design safe. This implies that the system can handle a much higher loading than what would typically be applied to it.

Part 2: Truss/Beam, top tube removed

Appendix V contains detailed calculations, as well as a table of stresses and safety factors for the truss/beam system.

Based on these calculations, it would appear the frame is not safe if the top tube is removed under the current loading conditions. The lack of such a large and vital structural member causes too much bending stress in the front section of the bicycle. It is apparent that the top tube is an important part of the bikes design.

Part 3: All beams, using MatLab (actual case)

*Please refer to Appendix IX and X for Finite Element Analysis (FEA) of all Beam structures.*

As is seen in the FEA of the all Beam frame, the stresses in the frame are far lower than the design allows for. Considering anything above a safety factor of 3 is deemed a safe design, the analysed bike is a safe design. Analysis of the visual representation of the loaded frame also shows a reasonably even stress distribution across different sections of the frame.

Comparison of parts 1-3:

The combined results of hand calculations and Matlab simulations would appear to show that the current design of the bike is safe under the proposed loading conditions. The assumptions made in part 1 do not reflect the true nature of the stresses in the bike frame. Bending stress is the predominant factor in the stress on the frame. For this reason, calculations from part 3 should primarily be considered when analysing the design. The FEA shows that the frame is able to easily handle the loading conditions.

## Current Frame Analysis

Is the Frame Safe?

Given a safety factor of 3, the analysed frame can safely withstand a stress of nearly 150 MPa, but the highest stress experienced by the frame is only subjected to a maximum stress of 113 MPa in one member, with the others being a full order of magnitude lower. This implies the frame is safe to use. Additionally, the loading conditions assume ¼ of the riders weight being on the handlebars. This will not always be the case, which means the head tube and fork will not experience such high bending stress all the time.

Which bar is least Safe?

As seen in the MatLab FEA results, the seat stay is subjected to highest axial stress of all members, and the head tube/fork experience the highest bending stress. This makes sense since most of the riders weight rests on the seat stay, and the rider leans on the head tube. The geometry of the frame is such that the seat and handlebars are approximately at the same height from the ground, meaning the rider has to lean farther forward and therefore apply a larger load to the from section of the bike.

Suggestions to improve the current design, if materials are changed

Stress is an inverse function of the cross-sectional area of a member, so by decreasing the hollow diameter of the tubes, we can lower stress without having to increase the physical dimensions of the frame. This would, of course, add weight. Given the ability to change materials, a less dense alloy of steel could be chosen, or perhaps a different metal all together. The stress in a member has no relation to the materials weight/density, so decreasing weight would have no negative effect. If however, the same material is used and weight is reduced by decreasing the amount of material used, a good place to start would be to shorten the seat stay. This would have the double effect of lowering the rider's position, and would therefore put less stress on the head tube/fork. A lighter bike would be easier to ride for longer periods of time and would require less energy to begin moving, making it a good way of improving rideability.

## Design Change

After conducting these various tests and performing all these calculations, we had to come up with a new and improved design for the bike. We started by selecting a new material. After looking at various factors like strength and weight we decided on using 6061 aluminium as the material for our new design. While not as strong, it is much lighter and allows us to use smaller but thicker pipes while also decreasing weight. By decreasing the external and internal diameters and adding more material, we are able to achieve a larger cross-sectional area and moment of inertia while making the frame slightly thinner and lighter.

Another thing we decided to do was move the seat stay around, we realized if we moved the seat stay closer to the seat tube by making the chain stay smaller, theoretically this would help reduce the amount of stress on just the one single bar and help it spread out better throughout the bike. All this allowed us to design a new bike that would be lighter and smaller but still maintain the same rigidity.

### Analysis/Comparison

*Please refer to Appendix XI and XII for the analysis of the new frame design.*

After performing the FEA on our new proposed design, we saw that the new proposed design was better. We observed that the stresses were spread out better through the entire structure, this in turn reduced the net stress on the frame. If we had to propose another design change we would most likely try another material or maybe try to move the seat lower than it currently is. Throughout this project we learned the value of stress analysis and how to perform stress analysis on truss structures. We learned that although current bike designs are great and work well, there is always  something to improve upon.

## Conclusion

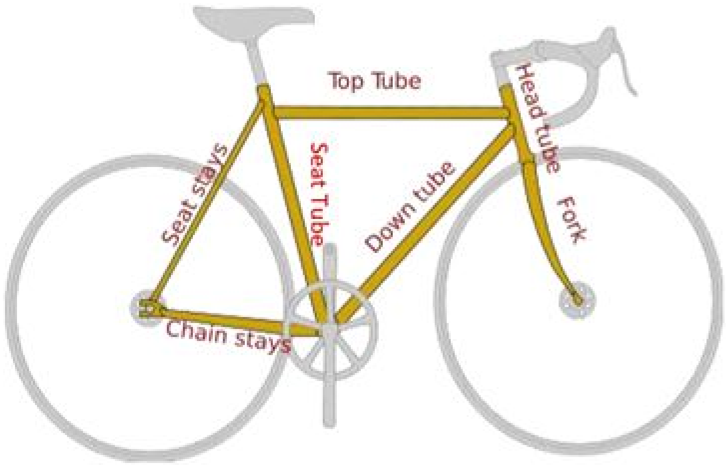
Throughout this project we learned the value of stress analysis and how to perform stress analysis on truss structures. We gained a deeper understanding of the technologies used in industry to perform engineering work, such as SolidWorks and Matlab. We saw different ways in which a frame structure can be analysed, either by simplifications and assumptions, or by considering the real life scenario for a more accurate result. By implementing design changes, we saw how small modifications to a design can greatly benefit the performance of a product. We also learned that although current bike designs are great and work well, there is always something to improve upon.

## References

1. Wikipedia, "6061 aluminum alloy," [Online]. Available: https://en.wikipedia.org/wiki/6061_aluminium_alloy. [Accessed 28 November 2017].
2. G. Wang, *MSE 221 Statics and Strength of Materials Course Project Description,* Surrey: Simon Fraser University , 2017.
3. eFatigue, "Material Property Finder: Material property estimator," [Online]. Available: https://www.efatigue.com/fem/stresslife/materials/#a. [Accessed 28 November 2017].
4. AISI 4130," *All Metals & Forge Group*, 28-Feb-2018. [Online]. Available: https://www.steelforge.com/alloy-steel-4130-aisi-4130/. [Accessed: 30-Nov-2018].

# Appendix

## Appendix I:




*Appendix 1 - Components of a bicycle frame*

## Appendix II:

| Frame Part | Dimensions (cm) |
|---|---|
| Top Tube | 53.0 |
| Seat Stays | 47.0 |
| Chain Stays | 44.0 |
| Seat Tube | 48.0 |
| Down Tube | 54.5 |
| Head Tube | 11.5 |
| Fork | 37.0 |

*Appendix 2 – Length Measurements taken of the real/current bike frame*

## Appendix III:

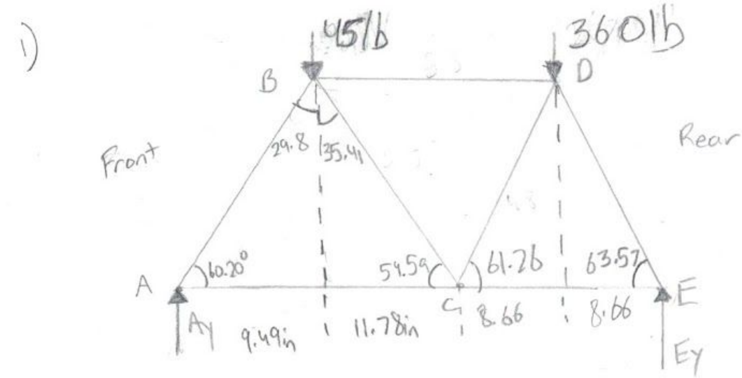
| Frame Part | Circumference (cm) |
|---|---|
| Top Tube | 9.5 |

| | |
|---|---|
| Seat Stays | 5 (x2) |
| Chain Stays | 5.5 (x2) |
| Seat Tube | 9.5 |
| Down Tube | 9.5 |
| Head Tube | 11.0 |
| Fork | 6.5 (x2) |

*Appendix 3- Circumference measurements taken of the real/current bike frame*

## Appendix IV:

1)




$\Sigma F_y = 0: \; -45 - 360 + A_y + E_y = 0$

$\circlearrowleft^{+} \Sigma M_A = 0: \; -(45)(9.49) - (360)(29.93) + E_y(38.59) = 0$

$E_y(38.59) = 11201.85$

$E_y = 290.28 \text{lb}$

$A_y = 405 - 290.28 = 114.721$

Joint A:

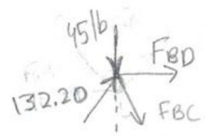

$\Sigma F_y: 114.72 + F_{AB}\sin(60.20°)$

$F_{AB} = \frac{-114.72}{\sin 60.20} = -132.20 \text{lb}$ comp. ↙

$\Sigma F_x: -132.20\cos(60.20°) + F_{AC}$

$F_{AC} = 65.70 \text{lb}$ tens. →

Joint B:




$\Sigma F_y: -45 + 132.20\cos(29.8) - F_{BC}\cos(35.41) = 0$

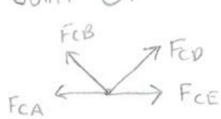
$F_{BC}\cos(35.41) = 69.72$

$F_{BC} = 85.54 \text{lb}$ tens. ↘

$\Sigma F_x: F_{BD} + 132.20\sin(29.8) + 85.54\sin(35.41) = 0$

$F_{BD} = -115.26 \text{lb}$ comp. ←

Joint C:



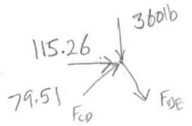

$\Sigma F_y: 85.54\sin(54.59) + F_{CD}\sin(61.26°) = 0$

$F_{CD} = -79.51 \text{lb}$ comp. ↙

$\Sigma F_x: -65.70 - 85.54\cos(54.59) - 79.51\cos(61.26) + F_{CE} = 0$

$F_{CE} = 153.50 \text{lb}$ tens →

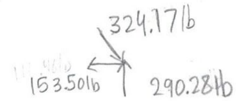
Joint D:




$\Sigma F_y$: $-360 + 79.51 \sin(61.28) - F_{DE} \sin(63.57) = 0$

$F_{DE} = -324.17$ lb comp.

(Check) Joint E:



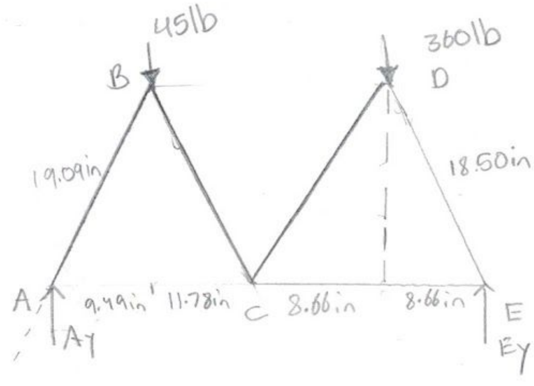

$\Sigma F_y$: $290.28 - 324.17 \sin(63.57) = 0$ ✓

$\Sigma F_x$: $-153.50 + 324.17 \cos(63.57) = 0$ ✓

| | Force | Cross-sectional Area | Stress (MPa) | SF |
|---|---|---|---|---|
| Top Tube | 512.70N | 69.34 $mm^2$ | 7.39 | 59.88 |
| Seat Stays | 1441.98N | 69.34 $mm^2$ | 20.80 | 21.27 |
| Chain stays | 682.80N | 69.34 $mm^2$ | 9.85 | 44.92 |
| Seat tube | 353.68N | 69.34 $mm^2$ | 5.10 | 86.76 |
| Down tube | 380.50N | 64.34 $mm^2$ | 5.49 | 80.60 |
| Head tube | 588.05N | 132.54 $mm^2$ | 4.44 | 99.66 |
| Fork | 588.05N | 132.54 $mm^2$ | 4.44 | 99.66 |

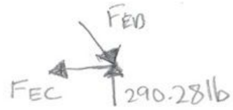
*Appendix 4: Hand calculations of part 1, table of stresses and safety factors*

<u>Appendix V:</u>



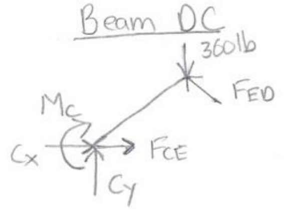

$A_y = 114.72\text{lb}$
$E_y = 290.28\text{lb}$

Joint E: Values taken from part A




$F_{ED} = 324.16\text{lb} = 1441.94\text{N}$
$F_{EC} = 144.28\text{lb} = 641.79\text{N}$

Beam DC

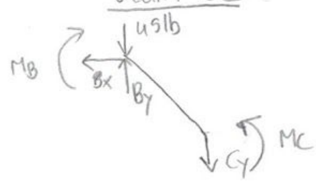



$\Sigma F_y:\ C_y - 360 + 324.16\sin(63.57) = 0$
$C_y = 69.72\text{lb}$

$\Sigma F_x:\ C_x + 144.28 - 324.16\cos(63.57) = 0$
$C_x = 0$

$\circlearrowleft + \ \Sigma M_C: -M_C - (360)(8.66) + (290.29)(8.66) + (144.28)(16.35) = 0$
$M_C = 1755.6\text{lb}\cdot\text{in} = 198.36\text{Nm}$

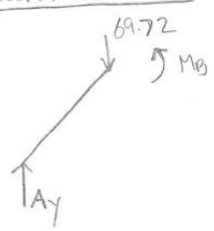
Beam BC




$\Sigma F_x:\ B_x = 0$
$\Sigma F_y:\ -45 + B_y - 69.72 = 0 \Rightarrow B_y = 114.72\text{lb}$

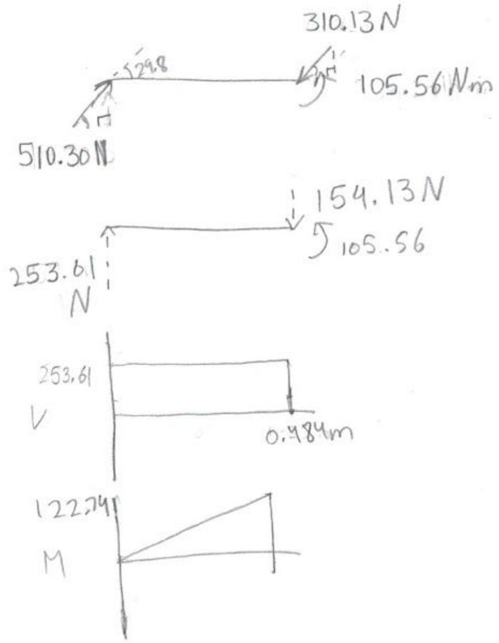
$\circlearrowleft + \ \Sigma M_C:\ 1755.6 - M_B - (69.72)(11.78) = 0$
$M_B = 934.29\text{lb}\cdot\text{in} = 105.56\text{Nm}$

Check: Beam AB




$\circlearrowleft + \ \Sigma M_A = 934.29 - (69.72)(9.49) = 0$
Difference $= 272.6\text{lb}\cdot\text{in} = 30.8\text{Nm}$

Beam AB




Stress from Axial loading

$F = 510.30 \times \cos(29.8) = 442.82\,N$

$\sigma = \dfrac{442.82N}{132.54mm^2} = 3.34MPa$ comp.

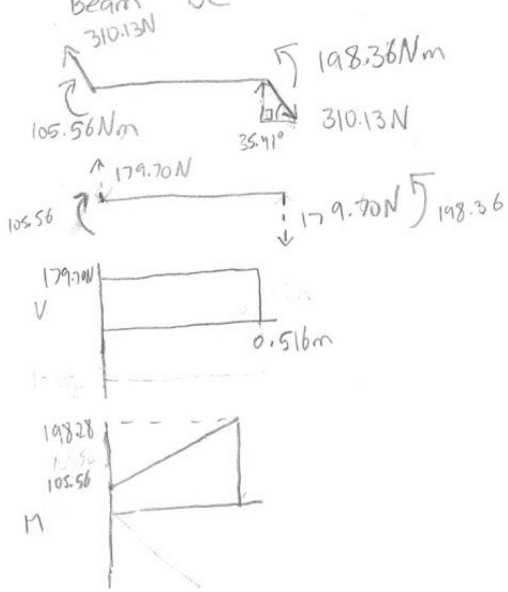
Max stress from bending $\left(S = \dfrac{\pi(D^4-d^4)}{32D}\right)$

$\sigma = \dfrac{M}{S} = \dfrac{122.74}{1.08\times10^{-6}} = 113.65MPa$

Total Stress: $113.65 + 3.34 = 116.99\ MPa$

$SF = \dfrac{442.5}{116.99} = 3.78$

Beam BC




Stress from Axial loading

$\sigma = \dfrac{252.76\,N}{69.34mm^2} = 3.65MPa$ tens.

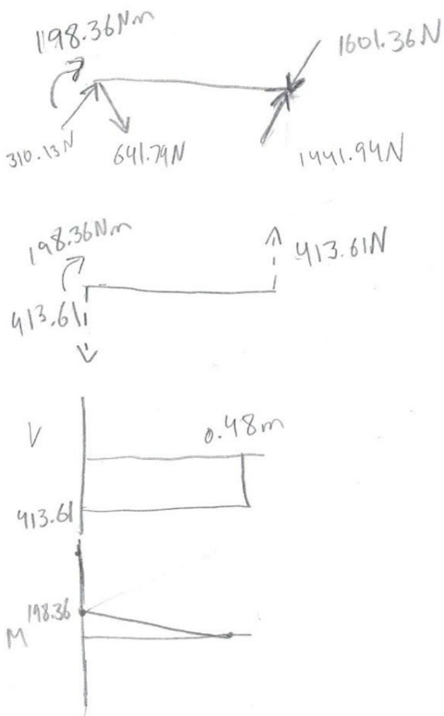
Max stress from bending

$\sigma = \dfrac{198.28}{4.985\times10^{-7}} = 397.75MPa$

Total stress: $3.65 + 397.75 = 401.39MPa$

$SF = \dfrac{442.5}{401.39} = 1.10$

Beam DC




Stress from Axial loading

$F = 580.53\,N$ comp.

$$\sigma = \frac{580.53\,N}{69.34\,mm^2} = 8.37\,MPa$$

Max stress from bending

$$\sigma = \frac{198.36}{4.985 \times 10^{-7}} = 397.91\,MPa$$

Total stress: $397.91 + 8.37 = 406.28\,MPa$

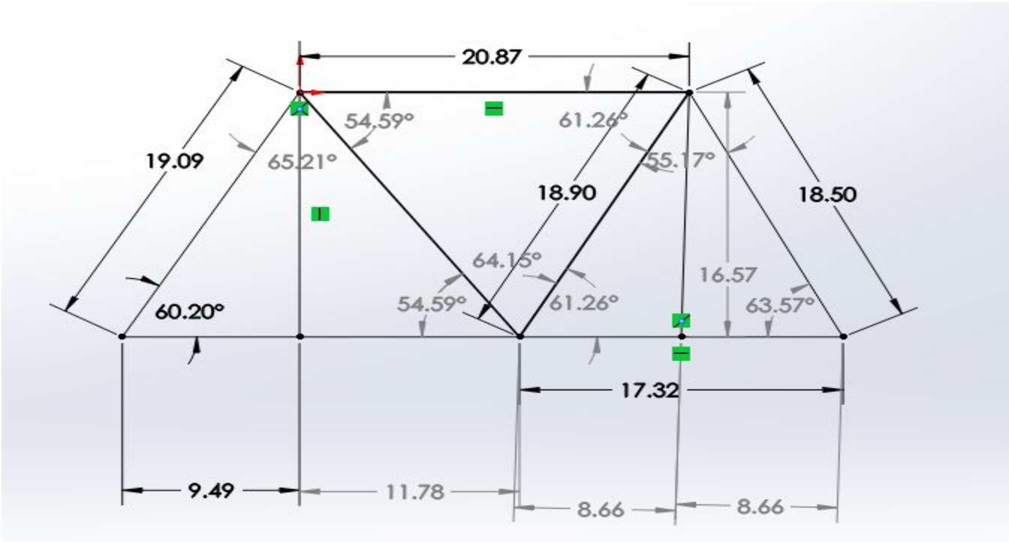
$$SF = \frac{442.5}{406.28} = 1.09$$

*Appendix 5-1: Hand calculations of part 2*

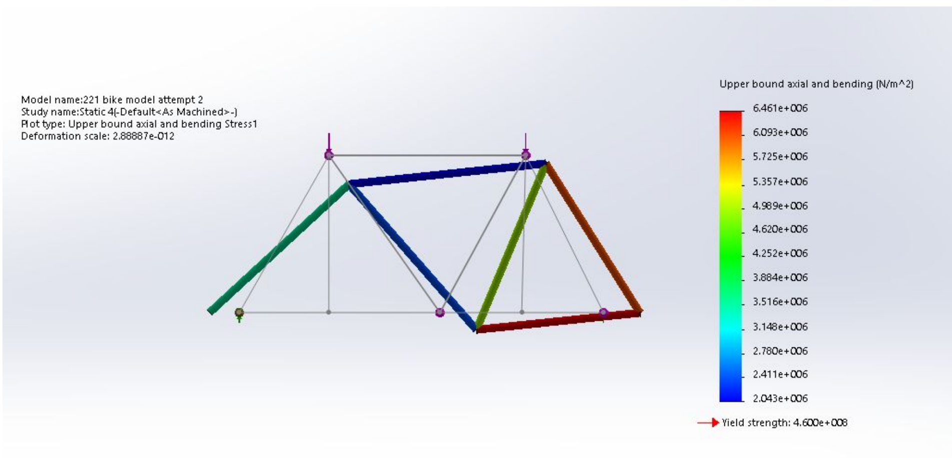
| **Member** | **Force (N)** | **Cross-sectional Area (mm^2)** | **Stress (MPa)** | **Safety Factor** |
|---|---|---|---|---|
| Seat Stays | 1441.98 | 69.34 | 20.80 | 21.27 |
| Chain Stays | 682.80 | 69.34 | 9.85 | 49.42 |
| Seat Tube | 580.53 | 69.34 | 406.28 | 1.09 |
| Down Tube | 252.76 | 69.34 | 401.39 | 1.10 |
| Head Tube/Fork | 442.82 | 132.54 | 116.99 | 3.78 |

*Appendix 5-2: Table of stresses and strains for truss-beam hand analysis.*

## Appendix VI:

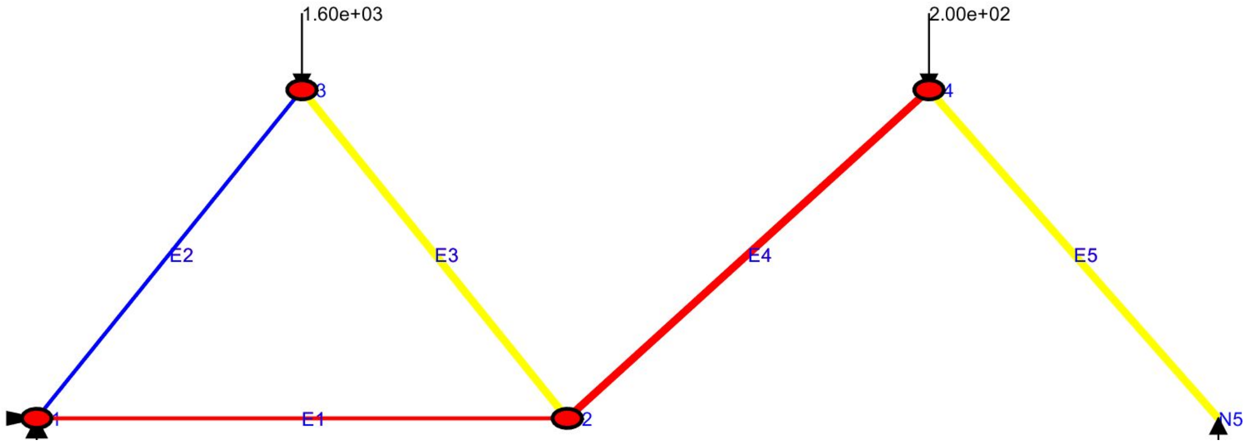



*Appendix 6-1: Bike Sketch with Measurements*

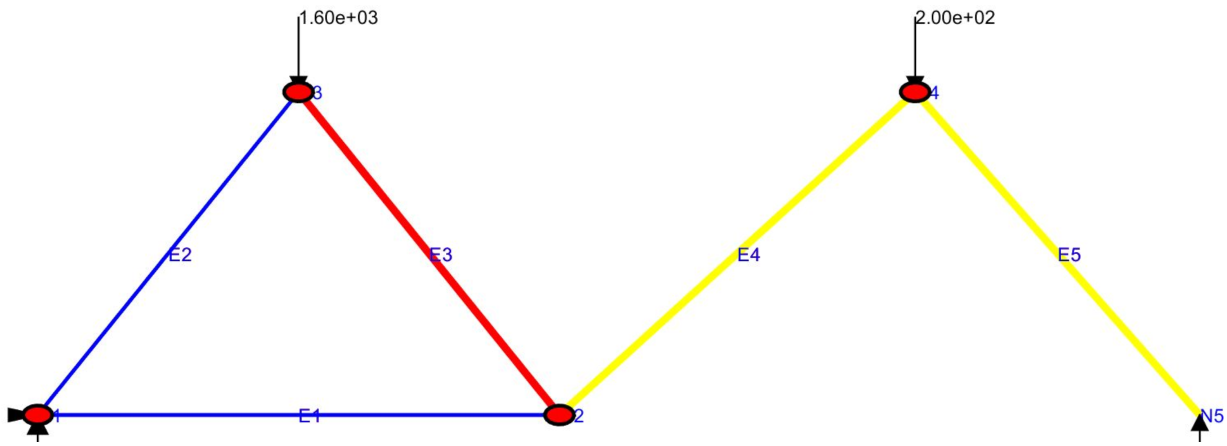



*Appendix 6-2: Stress Analysis in SolidWorks*

## Appendix VII:




*Appendix 7-1: MatLab diagram of axial loading on truss-beam structure (top tube removed)*




*Appendix 7-2: MatLab diagram of bending stress in truss-beam structure (top tube removed)*

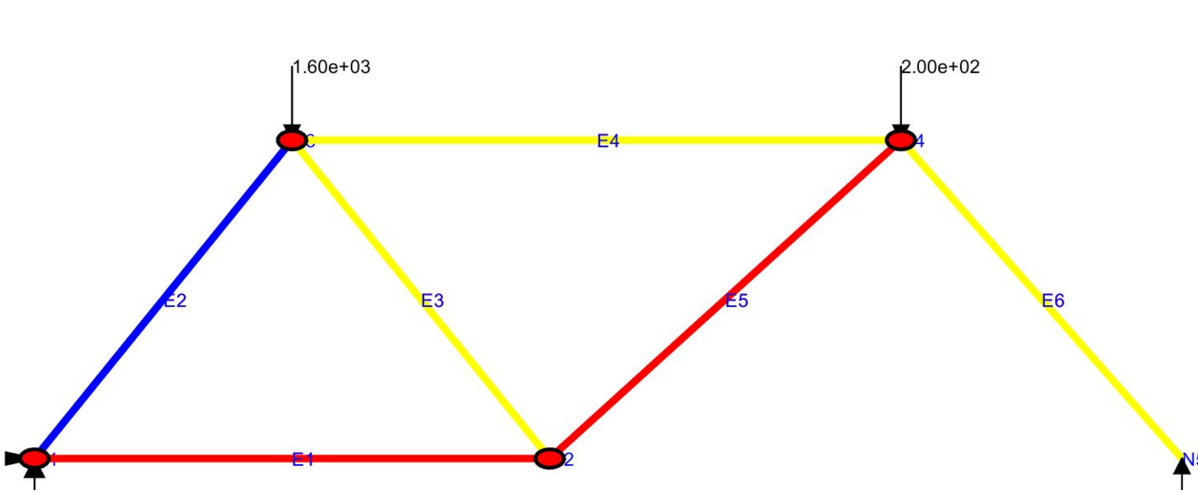
## Appendix VIII:

****Element Result******

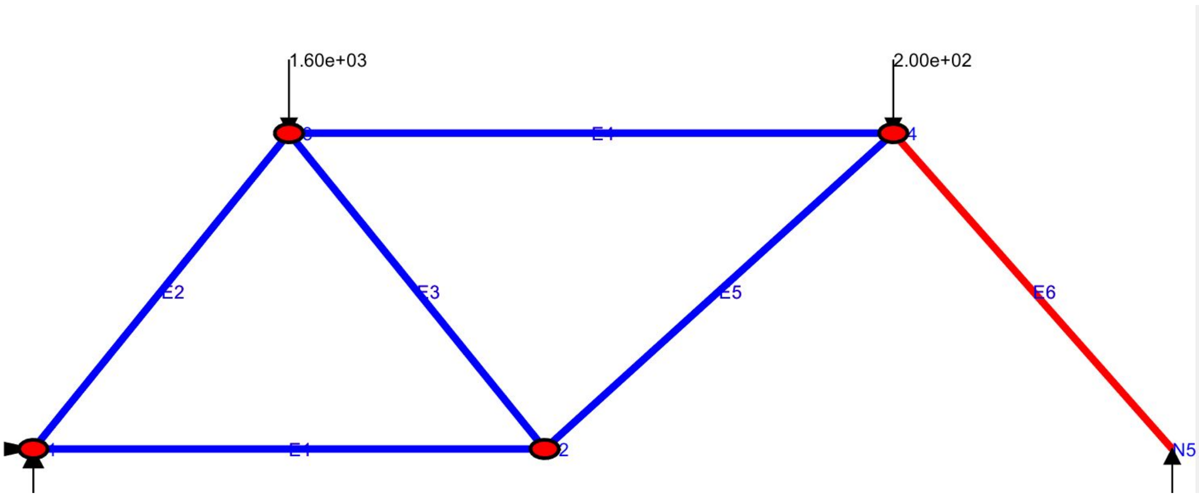
| Elem | AXFORCE | SHEARING | BENDMOMENT | AXSTRESS | MAXBENDSTRESS |
|---|---|---|---|---|---|
| 1 | 6.762e+02 | 0.000e+00 | 0.000e+00 | 9.758e+06 | 0.000e+00 |
| 2 | -1.457e+03 | 0.000e+00 | 0.000e+00 | -2.103e+07 | 0.000e+00 |
| 3 | -5.884e+02 | -4.552e+02 | 1.079e+02 | -8.490e+06 | 2.173e+08 |
| 4 | 2.523e+02 | 1.802e+02 | 4.650e+01 | 3.640e+06 | 9.362e+07 |
| 5 | -4.428e+02 | 2.530e+02 | 1.224e+02 | -3.329e+06 | 1.133e+08 |

*Appendix 8: Results of FEA on all truss-beam structure (top tube removed)*

Appendix IX:




*Appendix 9-1: MatLab diagram of axial forces on original all beam structure*




*Appendix 9-2: MatLab diagram of bending stress on original all beam structure*

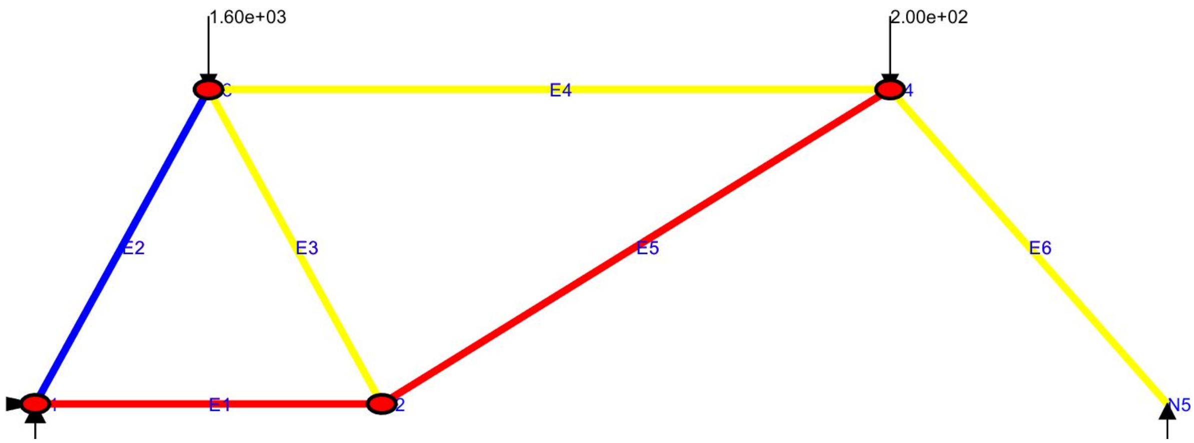
Appendix X:

****Element Result******

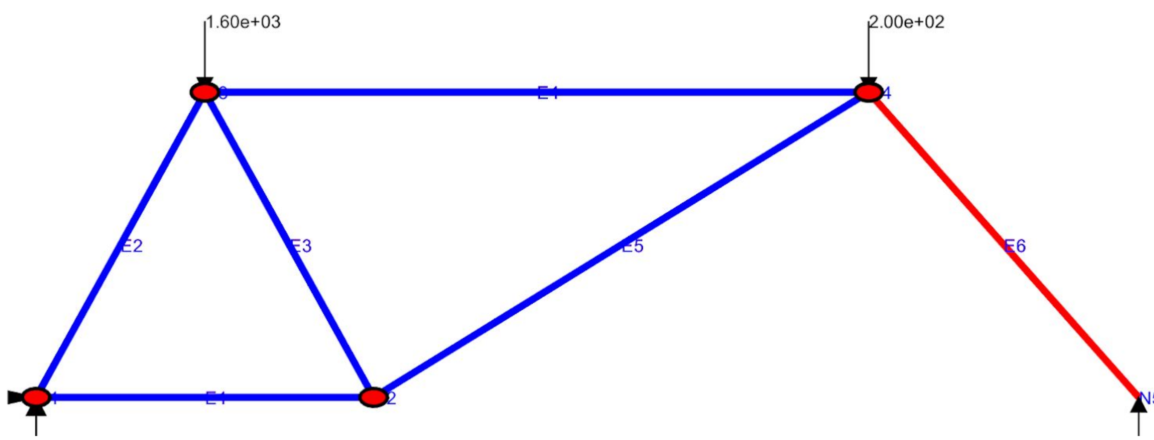
| Elem | AXFORCE | SHEARING | BENDMOMENT | AXSTRESS | MAXBENDSTRESS |
|---|---|---|---|---|---|
| 1 | 5.727e+02 | -6.931e+01 | 1.525e+01 | 8.265e+06 | 3.070e+07 |
| 2 | -1.348e+03 | -5.952e+01 | 1.411e+01 | -1.945e+07 | 2.841e+07 |
| 3 | -4.965e+02 | -5.048e+01 | 1.197e+01 | -7.164e+06 | 2.409e+07 |
| 4 | -2.977e+02 | -3.707e+01 | 9.638e+00 | -4.295e+06 | 1.940e+07 |
| 5 | 4.554e+02 | -4.048e+01 | 1.045e+01 | 6.572e+06 | 2.103e+07 |
| 6 | -4.428e+02 | 2.530e+02 | 1.224e+02 | -3.329e+06 | 1.133e+08 |

*Appendix 10: Results of FEA on original all beam structure*

# Appendix XI:




*Appendix 11-1: MatLab diagram of axial forces on new design*




*Appendix 11-2: MatLab diagram of bending stress on new design*

## Appendix XII:

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****Element Result******
Elem    AXFORCE    SHEARING  BENDMOMENT   AXSTRESS   MAXBENDSTRESS
  1   4.444e+02   -2.544e+01   3.816e+00   2.831e+06   4.155e+06
  2   -1.341e+03   -7.136e+00   1.591e+00   -8.543e+06   1.732e+06
  3   -3.555e+02   -2.736e+00   6.101e-01   -2.264e+06   6.642e+05
  4   -3.223e+02   1.601e+00   4.722e-01   -2.053e+06   5.141e+05
  5   4.461e+02   5.608e-01   1.705e-01   2.841e+06   1.857e+05
  6   -4.428e+02   2.530e+02   1.224e+02   -3.075e+06   1.166e+08
```

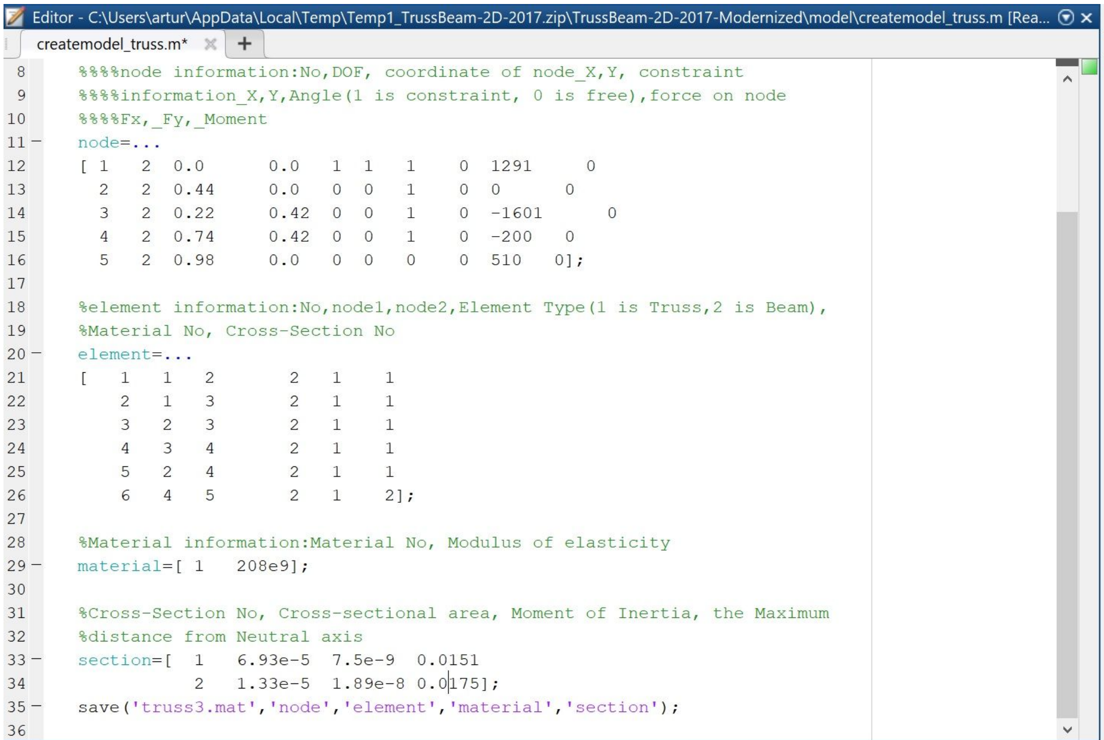
*Appendix 12: Results of FEA on new design*

## Appendix XIII:

| Material Name | Young's Modulus (GPa) | Ultimate Strength (MPa) | Fatigue Stress (MPa) |
|---|---|---|---|
| 4130 Steel Sheet (high) | 206.8 | 1241 | 443 |
| 4130 Steel Sheet (low) | 206.8 | 806 | 366 |
| 4130 Steel BHN=259 | 200 | 778 | 412 |
| 4130 Steel Quenched and Tempered | 221 | 1321 | 549 |
| *4130 Average* | 208 | 957 | 442 |
| | | | |
| Aluminum 6061-T6 Forged | 69 | 384 | 176 |
| Aluminum 6061-T6 Hand Forged | 72.7 | 340 | 158 |
| Aluminum 6061-T6 Sheet | 69.6 | 314 | 163 |
| Aluminum 6061-T6 Base | 72.4 | 310 | 138 |
| *6061-T6 Average* | 70.9 | 338 | 159 |

## Appendix XIV:

## Appendix XV:



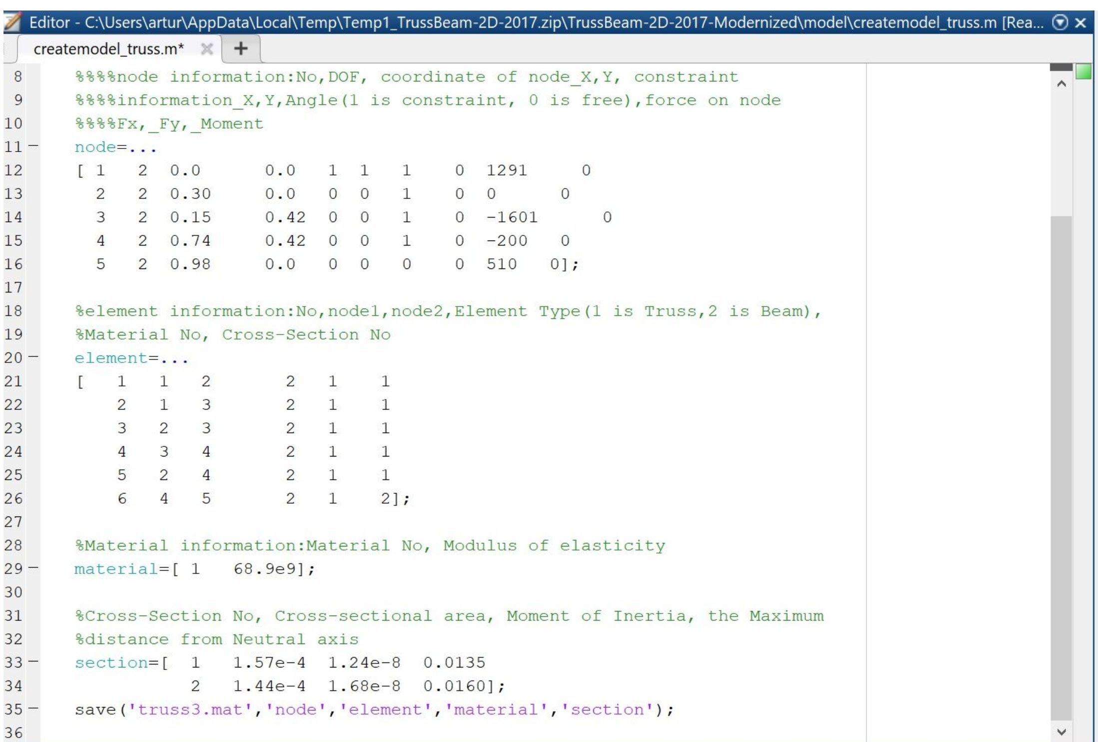

```
%%%%node information:No,DOF, coordinate of node_X,Y, constraint
%%%%information_X,Y,Angle(1 is constraint, 0 is free),force on node
%%%%Fx,_Fy,_Moment
node=...
[ 1   2  0.0      0.0   1  1   1    0  1291     0
  2   2  0.44     0.0   0  0   1    0  0        0
  3   2  0.22     0.42  0  0   1    0  -1601       0
  4   2  0.74     0.42  0  0   1    0  -200    0
  5   2  0.98     0.0   0  0   0    0  510    0];

%element information:No,node1,node2,Element Type(1 is Truss,2 is Beam),
%Material No, Cross-Section No
element=...
[   1   1   2       2   1    1
    2   1   3       2   1    1
    3   2   3       2   1    1
    4   3   4       2   1    1
    5   2   4       2   1    1
    6   4   5       2   1    2];

%Material information:Material No, Modulus of elasticity
material=[ 1   208e9];

%Cross-Section No, Cross-sectional area, Moment of Inertia, the Maximum
%distance from Neutral axis
section=[  1   6.93e-5  7.5e-9  0.0151
           2   1.33e-5  1.89e-8 0.0175];
save('truss3.mat','node','element','material','section');
```

*Appendix 15-1: Matlab code for original all-beam structure (units: N, m)*




```
%%%%node information:No,DOF, coordinate of node_X,Y, constraint
%%%%information_X,Y,Angle(1 is constraint, 0 is free),force on node
%%%%Fx,_Fy,_Moment
node=...
[ 1   2  0.0      0.0   1  1   1    0  1291     0
  2   2  0.30     0.0   0  0   1    0  0        0
  3   2  0.15     0.42  0  0   1    0  -1601       0
  4   2  0.74     0.42  0  0   1    0  -200    0
  5   2  0.98     0.0   0  0   0    0  510    0];

%element information:No,node1,node2,Element Type(1 is Truss,2 is Beam),
%Material No, Cross-Section No
element=...
[   1   1   2       2   1    1
    2   1   3       2   1    1
    3   2   3       2   1    1
    4   3   4       2   1    1
    5   2   4       2   1    1
    6   4   5       2   1    2];

%Material information:Material No, Modulus of elasticity
material=[ 1   68.9e9];

%Cross-Section No, Cross-sectional area, Moment of Inertia, the Maximum
%distance from Neutral axis
section=[  1   1.57e-4  1.24e-8  0.0135
           2   1.44e-4  1.68e-8  0.0160];
save('truss3.mat','node','element','material','section');
```

*Appendix 15-2: Matlab code for redesigned all-beam structure (units: N, m)*