SFU SCHOOL OF MECHATRONIC SYSTEMS ENGINEERING

MSE 380 - Systems Modeling and Simulation

Project Report: Water Wheel & Air Compressor System Powering a Nail Gun

Report Due Date: November 26, 2019

Members:

Sepehr Rezvani - 301291960

[redacted]

Abstract

The following report demonstrates our various findings after designing, modelling, simulating, and documenting both an energy storage and energy harvesting system with the purpose of further developing our knowledge through practical application. The following document contains an introduction to our group's final project, our design objectives, alternative concepts, selection, modelling breakdown, linear graph processing, design parameters and simulations with both an energy storage and energy consumption system.

Our project began with important design specifications being defined through research. The different iterations were done with the help of a layout within the SolidWorks program. All the elements within a proposed system was approximated using shapes with the appropriate lengths. With a general layout of the system elements, we can then make linear graphs and state vectors to effectively model the system.

As a group we were able to graph a plot on MATLAB that resembled what the subsystems will look like, with the subsystem and state vectors, coding was required on MATLAB to plot out a graph. With the MATLAB code and plots, discussion about the simulations proceeded, and thoughts about how to implement the modelling better. We introduced a nonlinear component into the system, repeated the simulation, and discuss it further.

Lastly, we documented some of the limitations of our model, and suggested improvements to the modelling and simulation process. Alongside with a reflection about how the simulation went.

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Introduction

The purpose of this project was to improve the quality of life in an underdeveloped village in Asia. Advancement of technology has made possible to develop efficient systems to boost living standards for people across the world, specifically areas that are financially struggling and not funded by their government. Our team had an original design that was introduced and analyzed in Project Proposal, but to be more creative and following more research, we upgraded our design and modeling to resolve the same problem. This report includes our Design Conceptualization, Modeling and Simulation that improves standards of living for people in that region.

Yangtze River, Yellow River, Brahmaputra River and Huai River were chosen as possible candidates to be the main focus of our research. These rivers were selected for two reasons. First, they are located in Asia and there are countless underdeveloped villages located along rivers that could benefit from this design. Second, they had higher average water flow rate during a year compared to other rivers, which would allow us to recover more energy. Among these rivers, the Yangtze has the highest average flow rate ($30166\,\frac{m^3}{s}$). Additionally, runs thru many regions, and connects to the ocean (high probability of being affected by typhoons and flooding). Therefore, the Yangtze river is the river of choice.

Yangtze River is located on the east side of Badong County, Hubei, China. Farming is one of the most popular occupations in Hubei, and our product is designed to convert a natural energy source (Yangtze River) to electricity that powers an air compressor. In farming, machines including water pumps, crop sprayers, tractors, conveyors, pneumatic handling material machines and even dairy machines require compressed air. This makes a reliable air compressor a vital component to a successful farm.

The goal of this project was to design a system that converts a renewable energy (hydropower) to another form of energy that can improve life standards. This project will explain how systems (energy recovery, storage and consumption) were modeled using various techniques and how the system response was simulated using MATLAB.

Design Objective

The main objective of this design is to improve life standards for people in Badong County. This design, Air Compressor, is very efficient and converts natural energy source (water flowing in Yangtze River) to mechanical rotation that runs the air compressor. As mentioned in the introduction, based on our research, people in this area are involved in farming activities and Air Compressor is a popular device. Based on our discussion with Dr. Firmani, our original design that was in Project Proposal was not creative. So, we did more research and came up with a new design, that could be very useful for people in that region. Design alternatives are stated in the next section.

We want our system to be able to operate 24/7 and produce a steady supply of readily available energy for use in areas where the people of the given village need it most. The system should not rely on any extra energy inputs from the operators and should not be dependant on factors such as weather and time of year.

Alternative Concepts

There are three alternative design concepts.

1. **First alternative**
   Supplemental electricity to villages, using flow of rivers.
    - Using small tidal barrage or water wheels to convert flow of water to supplemental electricity. The issue with this method is that blocking off huge areas with cement and altering the flow of water can be very harmful to the native ecosystem and cause more damage than good.
    - Stored in large capacitor banks and used when needed: supply is low or demand is high. This is an issue because even supercapacitors dissipate a lot quicker than normal batteries so it’s not convincing enough for the natives to use a whole new system that is less useful in practise.
    - As the waters beside the villages are usually dirty, there are no boats or people swimming, making building it there a great utilization of the unoccupied space.

The benefits of this will be reduced clutter caused by long-distance power lines, as the village’s homes are near water and can directly be connected to the dam-capacitor system.

will surely be helpful to change the mechanical energy to usable electrical energy from the water that is accessible.

2. **Second alternative**

Harnessing typhoons. Between July and September, China has a typhoon season. This brings extremely windy conditions almost on a weekly basis. Although typhoons can be very destructive, they can also benefit us. We can create sustainable energy from the powerful winds. A typical typhoon has approximately 600 TW of energy. A small rural house could be powered by as little as 500 kW for a year. If even a small portion of the typhoons power is harnessed, the village will have its electricity needs met for a year.

- Using windmills to drive a mechanical shaft we can produce electrical energy.
- The energy will be stored in a safe location in large capacitors, which is connected to the local grid.
- Can be used when needed to power many appliances in homes.

3. Third alternative

Multiple capacitors charged by bike. Most people in rural China ride bikes to get around. If all people in a village connect a small capacitor to their bike every morning and allow it to charge during their daily commutes, at the end of the day the capacitors can be brought to a central bank which would then power the village for the night.

Design Selection

An air compressor has been selected for this system. The primary advantage of this is that it is very valuable to have in a village as hand tools help there day to day life. Secondly, it will not experience dramatic seasonal fluctuations in output from the hand tool, as the pressure head is constantly being provided by the compressor. Nominal Output condition will happen as long as the river provide sufficient flow rate. Due to the terrain and weather condition of the village selected, during the times where typhoon season his, in the month of July , August, September, October, the tank will be able to storage more as off typhoon season months. This can be helpful as they can do repair work on their homes to secure and make enforcements on frames.

Data Collection

Location

For the energy collection system to be effective, sufficient energy should be present in the environment. As our source of energy is water flow, a river with a higher flow rate, and therefore higher water velocity, is desirable. We looked at 4 good candidates in China and East Asia. The findings are outlined in table 1 below. Note - “Relevance” refers to how suitable the river is for this project given factors such as landscape, average level of wealth near the river as well as frequency and severity of natural disasters which increase the need renewable energy. The numerical rating doesn’t represent any physical parameters, but rather is a subjective evaluation of the river.

River	Min flow rate (m^3/s)	Max flow rate (m^3/s)	Avg flow rate (m^3/s)	Relevance (1-4)
Yangtze	2000	110000	30166	4
Yellow river	1030	58000	2571	4
Brahmapura River	1050	100000	19800	1
Huai River	150	1500	1110	2

Table 1: River Flow Rate

The exact location along the Yangtze river is Badong County, which is near the Three Gorges Dam. Badong is one of the poorest counties in the region, and is surrounded by heavily forested mountain terrain, making accessing remote villages a huge challenge.

Tools/equipment

We intend to power 2 main categories of tools: light power tools such as air chizels, nail guns and drills for maintenance and construction, and larger tools such as farm equipment, small vehicles and larger construction equipment. The pressure requirements for them are outlined below in table 2. PSI is the pressure needed to power the tools, and CFM is the volume of air which needs to be delivered (cubic feet per minute)

Air powered tools	Typical PSI	CFM Most Typical
Most hand tools	90	3
Bigger manufacturing tools	120	10

Table 2: Hand Tools

Based on these findings, we believe it to be more appropriate to use a compressor rated at 120 PSI and adjust the pressure in the storage tanks as needed.

Single Stage	Below 100 psi is usually more common
2 Stage	Above 120 psi "vehicle construction, pneumatic tools and machines save untold sums of energy at assembly plants"

Table 3: Different Compressor Rating

Table 3 above shows the 2 possible configurations of compressor. For our purposes, a 2-stage compressor is more appropriate. Most 120 PSI compressors available on the market are powered either by a 2-4 HP combustion engine, or by electric motors outputting around 2000W. Therefore, any power output >2000W at the compressor input will be acceptable. Typically, this power is delivered at around 1500rpm, so this is the angular speed we will be aiming to produce.

RPM variation throughout the year

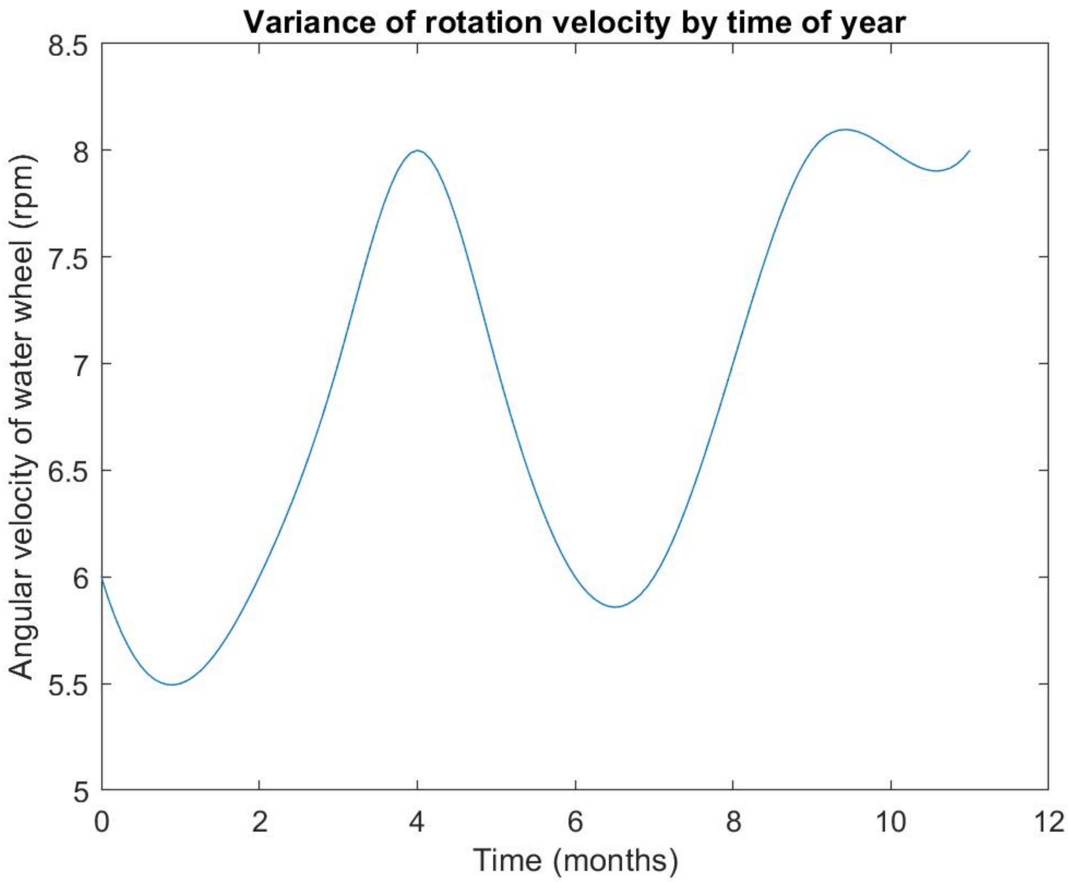


Figure 1: Varience of Rotation Velocity by Time of Year

The graph above shows the variance in angular velocity of the water wheel with time. This will be the input data and was obtained by a 1D interpolation of discrete data found from research. This data is directly proportional to the flow rate of the river. The graph begins at January (t = 0), where we see a lower angular velocity due to the low temperature and therefore higher dynamic viscosity of water. As temperatures rise in spring, we see an increased flow rate, with another decrease in the summer, when temperatures rise further and cause some of the water to evaporate. With fall, we once again see a rise in velocity due to rain bringing the water levels of the river back to where they were before the summer.

Mathematical Model

Energy Harvesting Modelling

An Overview schematic of the Energy Harvesting Systems is Presented Below:

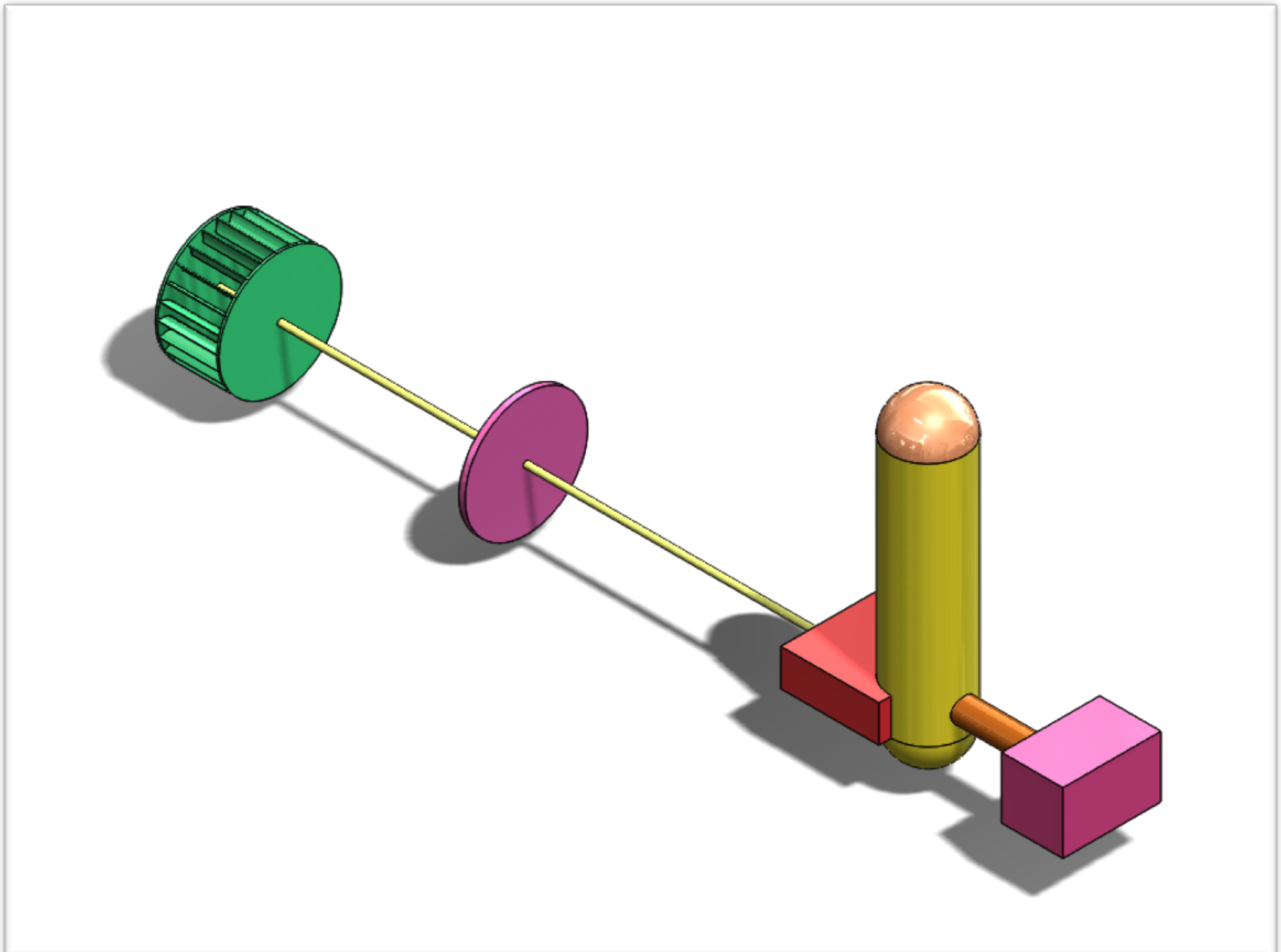


Figure 2: Solidworks

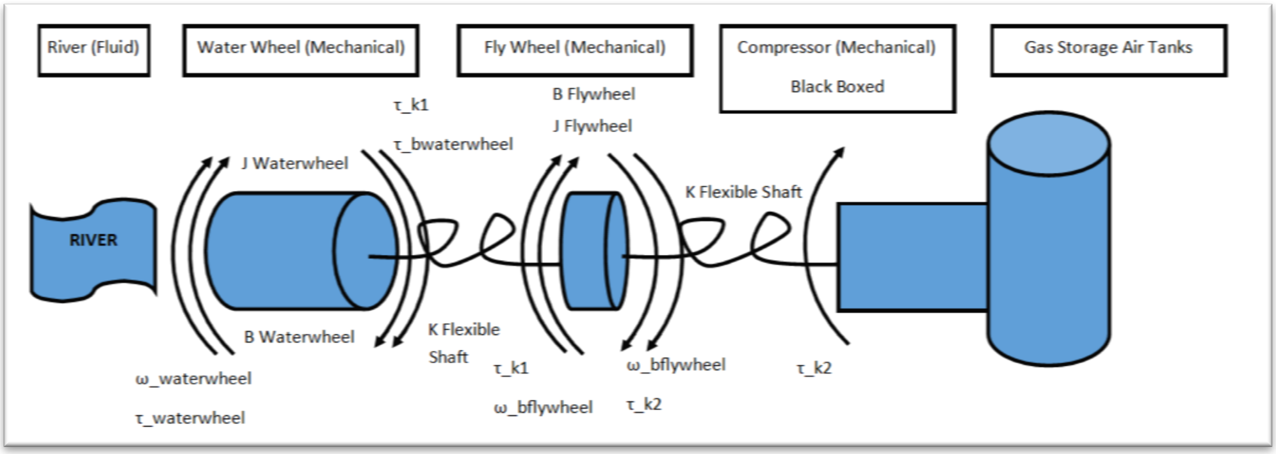


Figure 3:System Modelling

Looking at the above image, the Energy Harvesting System consists of

- Water wheel(green)
    - The water wheel is given an input torque from the flow of the river.
    - It's rotation causes friction and hence has a rotational frictional component element
    - It also has a moment of inertia JT which contributes to the state space model

- Flexible shaft(gold)
    - The water wheel is connected to the compressor with the flexible shaft
    - It stores potential energy (much like a spring) in the flex of the shaft during rotation.

- Compressor(red)
    - Black boxed element
    - We consider the ideal hydraulic turbine as a reference to how the system functions. Namely the following relationships shown below:

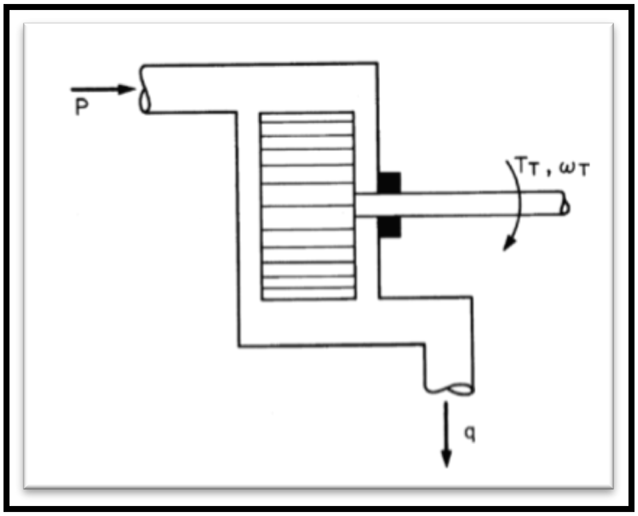


Figure 4:Compressor

$$P = K_1 \tau_T$$
$$\omega_t = K_2 Q$$

From these relations, we can substitute for elements that we want to serve as our inputs and outputs. In addition we ended making the assumption that the pump displacement parameter for both $K_1 = K_2$, there the same value.

- The relationship between the input pressure is a function of the turbine torque
- The flow rate out of the turbine is a function of the angular velocity of the torque
- It's important to note that these equations represent a constant value for P and w respectively. Therefore, we decided to use the fluid capacitor equation and blackbox the internal elements surrounding the compressor element.
- The output of the compressor is the increase in pressure in the air tanks.

Air tanks(yellow)

- The pressure rise in the gas tank would be the end of our simulation for energy storage
- Anything beyond the air tanks is treated as if the pressure was a finite value and would drop every time you use it.
- The assumption that is made is that we charge the gas tank with pressure from the river and it is taken away from the river, transported to the place of use and used as a finite input for our air powered nail gun

- Stores the energy in the form of compressed air for later use for an output in the form of a nail gun (pink).

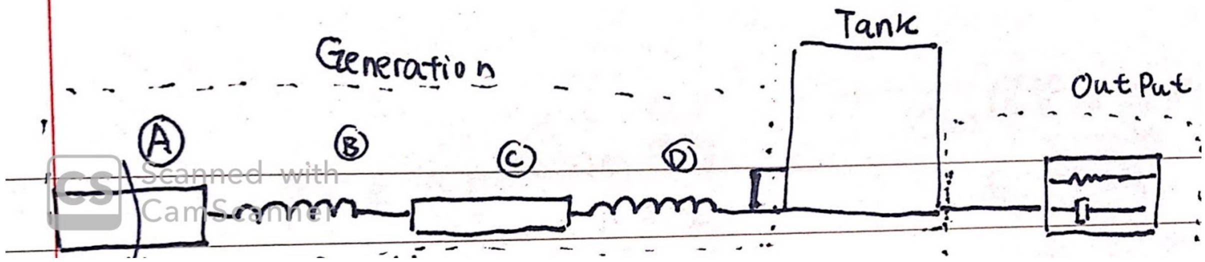
Nail gun (pink)

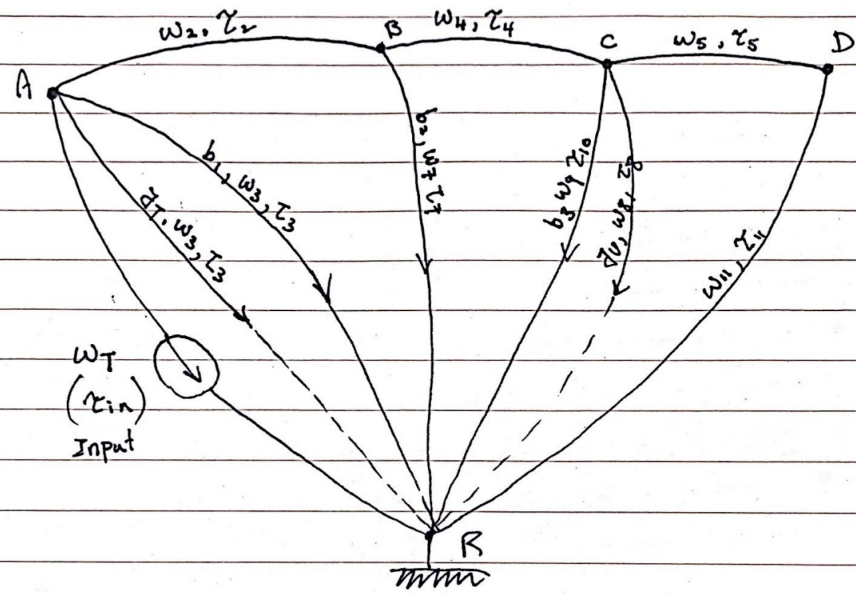
- The nail gun takes input flow and taking into account fluid inertance, resistance, and flexible container capacitance, it releases the pressure of the system in pulses which in turn sends pulses of force onto the nail inside the air gun.

Figure 5:Nail Gun

Linear Graphs

To confirm our State equations a quick linear graph was implemented and drawn to check our graphs so we can ensure proper state equations for simulation.





No of branches = $\underline{10}$

of nodes = 5

of sources = 1

of unknowns $\Rightarrow 2b - s = 2(10) - 1 = 19$

Constitutive Eqn's

$b - s = 10 - 1 = 9.$

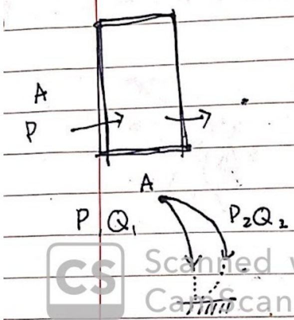
Node Eqn's.

$n - 1 = 4 - 1 = 3$

Loop Eqn's $= \underline{6}$

total 18 eqn's

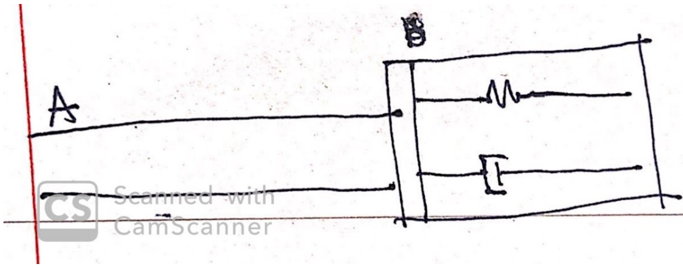
Storage Tank.

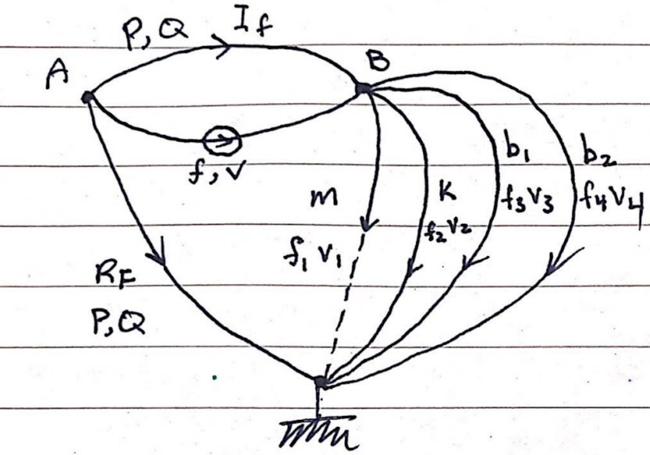


No of branches = b = 2
of nodes = n = 2
of sources = s = 0
of unknowns => 2b - s = 4

Constitutive Eqn's b - s = 2 - 0 = 2
Node Eqn's n - 1 = 2 - 1 = 1
Loop eqn's = 2

Total 5 eqn's





\# of branches => b = 7
of nodes => n = 3
of sources => s = 1
of unknown => 2b - s = 13

Constitutive Eqn's
b - s = 7 - 1 = 6
Node Eqn's
n - 1 = 3 - 1 = 2
Loop Eqn's = 5

Total # 13

By doing the Linear Graph, and cancelling out the redundancy values, we were able to reach a similar state matrix.

Given in the ẋ=Ax+B format.

$$
\begin{bmatrix} \dot{\omega}_T \\ \dot{\tau}_{k\theta 1} \\ \dot{\omega}_u \\ \dot{\tau}_{k\theta 2} \\ \dot{P}_T \\ \dot{\omega}_c \\ \dot{Q}_1 \\ \dot{P}_w \\ \dot{V} \\ \dot{F} \end{bmatrix}
=
\begin{pmatrix}
-\frac{b_{\theta 1}}{J_T} & -\frac{1}{J_T} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{\theta 1} & 0 & -k_{\theta 1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{J_u} & -\frac{b_{\theta 2}}{J_u} & -\frac{1}{J_u} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{\theta 2} & 0 & 0 & -k_{\theta 2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{B}{\vartheta}\left(\frac{1}{k_0}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{J_c} & -\frac{1}{J_c k_0} & -\frac{b_{\theta 3}}{J_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{R_f}{I_f} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_f} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{A}{m} & -\frac{(b_1+b_2)}{m} & -\frac{1}{m} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_1 & 0
\end{pmatrix}
*
\begin{bmatrix} \omega_T \\ \tau_{k\theta 1} \\ \omega_u \\ \tau_{k\theta 2} \\ P_T \\ \omega_c \\ Q_1 \\ P_w \\ V \\ F \end{bmatrix}
+
\begin{bmatrix} \frac{1}{J_T} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tau_{in}
$$

Figure 6: State Vecotr Matrix

State equations

$$\dot{\omega_T} = \frac{1}{J_T}(\tau_{IN} - b_{\theta 1}\omega_T - \tau_{k\theta 1})$$

This state equation is based on the constitutive equation of the torque on the water wheel. It also takes into account the friction from our rotational damper/torsional element, and the torque on the flexible shaft between the water wheel and the flywheel.

$$\dot{\tau_{k\theta 1}} = k_{\theta 1}(\omega_T - \omega_u)$$

This state equation is based on the constitutive equation of the rotational spring element of the flexible shaft between the waterwheel and the flywheel. It is related to the difference in angular velocity between the water wheel and the flywheel.

$$\dot{\omega_u} = \frac{1}{J_u}(\tau_{k\theta 1} - b_{\theta 2}\omega_u - \tau_{k\theta 2})$$

This state equation is based on the constitutive equation of the torque on the flywheel. It takes into account the friction from our rotational damper/torsional element, and the torque on the flexible shaft between the flywheel and the air compressor ("blackbox").

$$\dot{\tau_{k\theta 2}} = k_{\theta 2}(\omega_u - \omega_c)$$

This state equation is based on the constitutive equation of the rotational spring element of the flexible shaft between the flywheel and the "blackboxed" air compressor. It is related to the difference in angular velocity between the flywheel and what is actually turning the shaft of the air compressor.

$$\dot{P_T} = \frac{B}{\vartheta}\left(\frac{1}{k_0}\right)\omega_c$$

This state equation is based on fluid capacitance. It's important to note that the fluid capacitance is calculated based on the bulk modulus and that we assume that the fluid compression system is modeled as a control volume. In addition, we should also refer to the ideal hydraulic turbine relation and replace Q, or flow rate to form a direct relationship between the change in pressure in the tank and the rotation of the shaft of the air compressor ("blackboxed"). (Refer to mathematical model for more information on this). The following equation is the one that replaced Q in our state vector:

$$Q = \frac{\omega}{k}$$

$$\dot{\omega_c} = \frac{1}{J_c}\left(\tau_{k2} - b_{\theta 3}\omega_c - \frac{P_T}{k_0}\right)$$

This state equation is based on the constitutive equation of the torque on the shaft of the air compressor. It takes into account the inherent friction that comes from a damper/torsional element, and also the resistance exerted by the compressor itself as well as the tank. We approximated this again with the ideal hydraulic turbine relation and replace torque. The following is the equation that replaced torque in our equation:

$$\tau = \frac{\omega}{k}$$

$$\dot{Q}_1 = \frac{1}{I_f} R_f Q_1 = \frac{1}{I_f}(P_t - P_1)$$

This state vector is based on the constitutive equation of a fluid inerter. Namely, it is related to a difference in pressure. This difference in pressure in replaced in the vector as a simplification by way of a fluid resistor element. Note that the fluid inertance is related to the velocity distribution of flow of air and should be experimentally confirmed to get the right value.

$$\dot{P_w} = \frac{1}{C_{elastic} + C_{bulk}} Q_1$$

This state vector is based on the constitutive equation of a fluid capacitor element: flexible container. Its purpose is to relate the pressure that builds up when the compressed fluid is used as a form of energy and relate that pressure to the mechanical elements. The rate itself is related to flow and many assumptions were made to find the fluid capacitance. It's important to refer to the “limitations of the model” to get a full explanation on some of the more concerning elements of those assumptions.

$$\dot{V} = \frac{1}{m}(P_w A - F_1 - b_1 v_1 - b_2 v_1)$$

This constitutive equation is related to the mass element and takes the pressure from the flexible container state vector as an input. This component is where the output of the system is being analyzed as it also contains elements of friction with the enclosure, as well as a damper

$$\dot{F} = K_1 v_1$$

This constitutive equation is related to the energy stored in the spring. It is simply related to the speed of the spring via a spring constant.

State variables

ω_T - Angular velocity of the water wheel

$\tau_{k\theta 1}$ - Torque on the shaft between the water wheel and the flywheel

ω_u - Angular velocity of the flywheel

$\tau_{k\theta 2}$ - Torque on the shaft between the flywheel and the air compressor (blackboxed)

P_T - Pressure in the tank as a result of the applied torque (s)

ω_c - Angular velocity of the air compressor shaft

Q_1 - Flow of the air from the tank to the pipe (Inertance)

P_w - Pressure developed on the fluid face of the mass

V - Velocity of the mass

F - Force on the spring attached to the mass

Constants used

J_T - Moment of Inertia of the water wheel

$b_{\theta 1}$ - Friction coefficient (Rotational damper of the water wheel)

$k_{\theta 1}$ - Torsional spring coefficient (Rotational spring element related to the flexible shaft between water wheel and flywheel)

J_u - Moment of Inertia of the flywheel

$b_{\theta 2}$ - Friction coefficient (Rotational damper of the flywheel)

$k_{\theta 2}$ - Torsional spring coefficient (Rotational spring element related to the flexible shaft between the flywheel and the compressor)

β - Bulk Modulus of air

ν - Volume of the intake volume (air compressor)

k_0 - Pump displacement (CIPR - equivalent volumetric parameter m^3/rev)

J_c - Moment of inertia of the shaft of the air compressor

$b_{\theta 3}$ - Friction coefficient (Rotational damper of the compressor shaft)

R_f - Effective resistance of the pipe

I_f - Fluid Inertance (related to velocity distribution, density of the fluid, surface area, and length in question)

C_f -Fluid Capacitance (related to the bulk modulus, volume of the intake compressor, spring element, area of contact) - Both the elastic and bulk capacitance elements

A - Area of contact

m - Mass of the block (flexible container) that stores energy

b_1 - friction coefficient

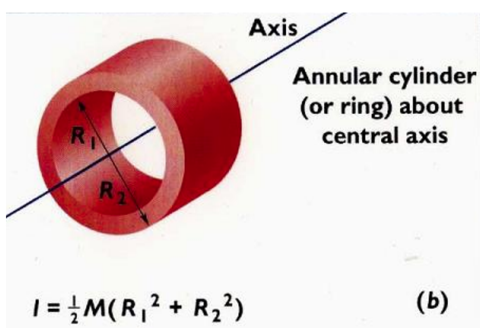
b_2 - damping coefficient

k_1 - Spring coefficient

Justification of Parameters:

1. J_T- Moment of Inertia of the water wheel

The moment of inertia of the water wheel is based on solidworks rendering. Based on the equation:



This is what was suggested by some of the research papers as an approximation. After some deliberation, we decided on the value: 2791 kg*m^2.

2. $b_{\theta 1}$- Friction coefficient (Rotational damper of the water wheel)

After some references of previously used values in problems and simulation testing, we decided on the value of 33.

3. $k_{\theta 1}$- Torsional spring coefficient (Rotational spring element related to the flexible shaft between water wheel and flywheel)

After some references of previously used values in problems and simulation testing, we decided on the value of 24.65.

4. J_u- Moment of Inertia of the flywheel

The moment of inertia of the water wheel is based on SolidWorks rendering. Based on the equation:

$$J = \frac{1}{2} * m * r^2$$

This is what was suggested by some of the research papers as an approximation. After some deliberation, we decided on the value: 3780kg*m^2.

5. $b_{\theta 2}$- Friction coefficient (Rotational damper of the flywheel)

After some references of previously used values in problems and simulation testing, we decided on the value of 334.

6. $k_{\theta 2}$- Torsional spring coefficient (Rotational spring element related to the flexible shaft between the flywheel and the compressor)

After some references of previously used values in problems and simulation testing, we decided on the value of 32.07.

7. β- Bulk Modulus of air

Bulk modulus of air is inherently nonlinear, but we have made the following assumptions:

$$\beta = \frac{\Delta P}{\frac{\Delta V}{V_0}}$$

As we have an instantaneous change in pressure resulting in an instantaneous change in volume, we estimated a relationship to be: β = 9

8. V - Volume of the intake volume (air compressor)

Volume of the intake valve of the air compressor is found from some research in being the following: 2.265 m^3.

9. k_o - Pump displacement (CIPR - equivalent volumetric parameter m^3/rev)

From research, we know that the average rpm for the wheel is 7 rpm. With this and the expected flow rate, we can calculate the pump displacement with the following equation:

$$CIPR = Q \cdot \frac{231}{RPM}$$

From this equation, we found that the value 0.00000237021

10. J_c - Moment of inertia of the shaft of the air compressor

The moment of inertia of the water wheel is based on solidworks rendering. Based on the equation:

$$J = \frac{1}{2} * m * r^2$$

This is what was suggested by some of the research papers as an approximation. After some deliberation, we decided on the value: 157.5kg$* m^2$.

11. $b_{\theta 3}$- Friction coefficient (Rotational damper of the compressor shaft)

After some references of previously used values in problems and simulation testing, we decided on the value of 470.

12. R_f - Effective resistance of the pipe

After some references of previously used values in problems and simulation testing, we decided on the value of 2.

13. I_f- Fluid Inertance (related to velocity distribution, density of the fluid, surface area, and length in question)

$$I_f = \alpha\rho\frac{\Delta x}{A}$$

Given that the air is flowing through a pipe, α=2, the air has a parabolic velocity distribution, and density of the air at a specific pressure. In addition, the length and surface area elements we considered in calculating the inertance. From these calculations, we found the value to be 63534.

14. C_f-Fluid Capacitance (related to the bulk modulus, volume of the intake compressor, spring element, area of contact) - Both the elastic and bulk capacitance elements.

$$C_f = C_{elast} + C_{bulk}$$

$$C_{elast} = \frac{A^2}{k}\ C_{bulk} = \frac{V}{\beta}$$

From these equations, we see that the volume, bulk modulus, spring constant, and surface areas are used to calculate the fluid capacitance of the energy storage component. We ended up with the following value: 0.0000023168+2.26535/B.

15. A- Area of contact

The area of contact is calculated with

$$A = \pi r^2$$

We ended up with the following value: 0.6283.

16. m- Mass of the block (flexible container) that stores energy

After some references of previously used values in problems and simulation testing, we decided on the value of 2.

17. b_1- friction coefficient

After some references of previously used values in problems and simulation testing, we decided on the value of 470.

18. b_2- damping coefficient

After some references of previously used values in problems and simulation testing, we decided on the value of 470.

19. k_1- Spring coefficient

After some references of previously used values in problems and simulation testing, we decided on the value of 1.2345.

Linearization

Based on Figure 2, the fluid resistor element was chosen as our non-linear model. There are two different formulas to find the resistance equation depending on its linearity. The fluid resistor was assumed to be nonlinear and the linearization process is as follows.

Figure 4 is the main model that included time dependent variables. In that matrix, row 7 on the left-hand side (Q1)' is what we are interested in because it includes the desirable nonlinear variable. There are three other equations that are included in the linearization matrix and are presented in rows, 8-10.

$$\dot{Q}_1 = \frac{1}{I_f} R_f Q_1 = \frac{1}{I_f}(P_t - P_1)$$

$$\dot{P_w} = \frac{1}{C_{elastic} + C_{bulk}} Q_1$$

$$\dot{V} = \frac{1}{m}(P_w A - F_1 - b_1 v_1 - b_2 v_1)$$

$$\dot{F} = K_1 v_1$$

Now, rewrite row 7:

$Q_1' = \frac{1}{I_f} \times (R_f \times Q_1) = \frac{1}{I_f} \times \Delta P \rightarrow$ This is the linear equation for the resistor

$Q_1' = \frac{1}{I_f} \times (K_R \times Q_1{}^n) = \frac{1}{I_f} \times \Delta P \rightarrow$ This is the non-linear equation we are interested in

Next step is to equate all the time derivatives equal to zero:

$$Q_1' = P_w' = V' = F' = 0$$

$Q_1' = 0 = \frac{1}{I_f} \times (K_R \times Q_1{}^n) = g \rightarrow$ we set this to 'g' for PDE calculations

Now rewrite these 4 equations in 'bar' format:

1. $\widehat{Q_1}' = \frac{\partial g}{\partial Q_1}$ ($\widehat{Q_1}$ at operating point) = $[\frac{1}{I_f} \times (K_R)]\, \widehat{Q_1}$
2. $\widehat{P_w}' = (\frac{1}{C_{elastic} + C_{Bulk}})\, \widehat{Q_1}' \rightarrow C_{elastic} + C_{Bulk} = C_f$
3. $\hat{V}' = \widehat{P_w}\,(\frac{A}{m}) + \widehat{V_1}\,(\frac{-(b1 + b_2)}{m}) + \hat{F}\,(\frac{-1}{m})$
4. $\hat{F}' = \widehat{V_1}\,(k_1)$

⇒ Insert 2 in 3 :

5. $\hat{V}' = [\,(\frac{1}{C_{elastic} + C_{Bulk}})\, \widehat{Q_1}' \times (\frac{A}{m})\,] + \widehat{V_1}\,(\frac{-(b1 + b_2)}{m}) + \hat{F}\,(\frac{-1}{m})$

Simulation

For easier simulation and to allow us to more closely analyse the system response, the state model was divided into 3 sub-models, which were then simulated using the MATLAB "lsim" function. The systems were simulated one by one, with output from the previous being used as an input to the next system.

First, the mechanical system was analyzed, which included the water wheel, flywheel and 2 flexible shafts connecting them together. The system behaved as expected, with a somewhat damped transient response, as is characteristic of a system with springs and no dampers. After about 50% of the sample time had elapsed, the system came to mostly steady state, with angular displacement of the water wheel having a higher amplitude, but otherwise being in phase with the flywheel. Torque on the second shaft was taken to be the input to the next system. The response of this parameter was a little less steady, but this reflects the input to the system, which is a function of fluctuating flow rate of the river.

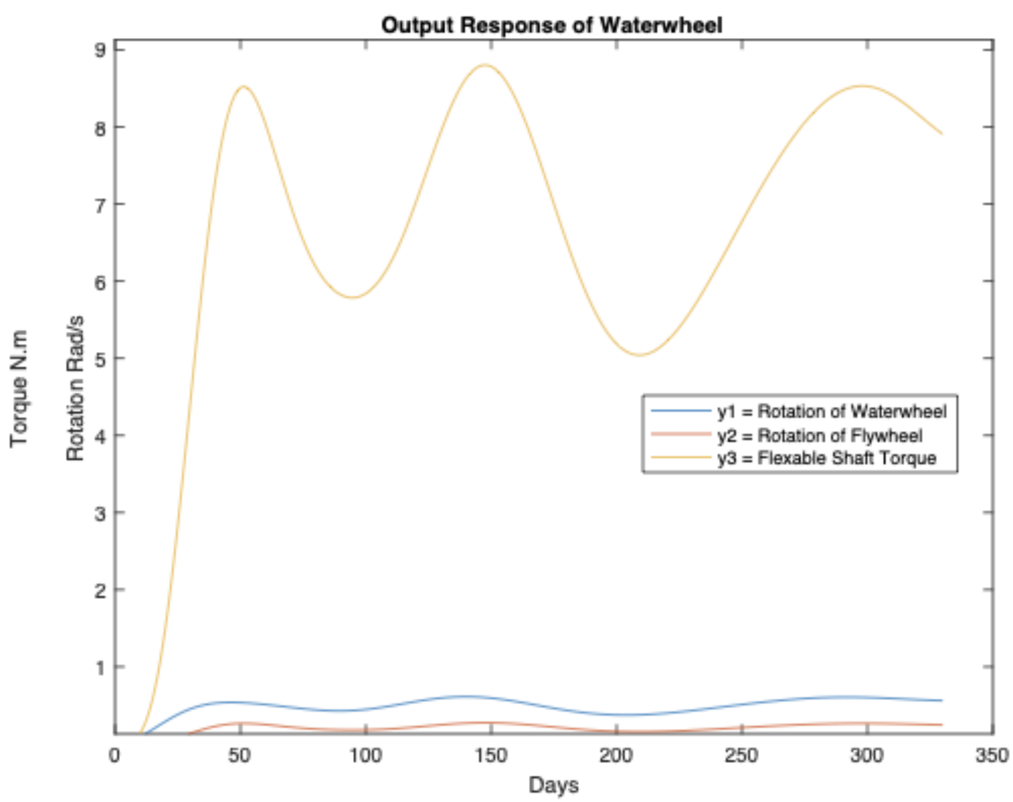


Figure 7:Output Response of Waterwheel

Next, the shaft torque was used as an input to simulate the "blackboxed" compressor system, along with the air tank holding the compressed air. The behaviour of the system was consistent with expected results. Starting at 0 (equivalent to atmospheric pressure), the tank pressure rises gradually at first, but as the system begins to stabilize and the torque on the flywheel and shaft become steadier, a greater pressure increase rate is observed. As the tank begins to fill up, the pressure increase tapers off and eventually the tank pressure hits a maximum value, based on the volume of the tank and the bulk modulus of air.

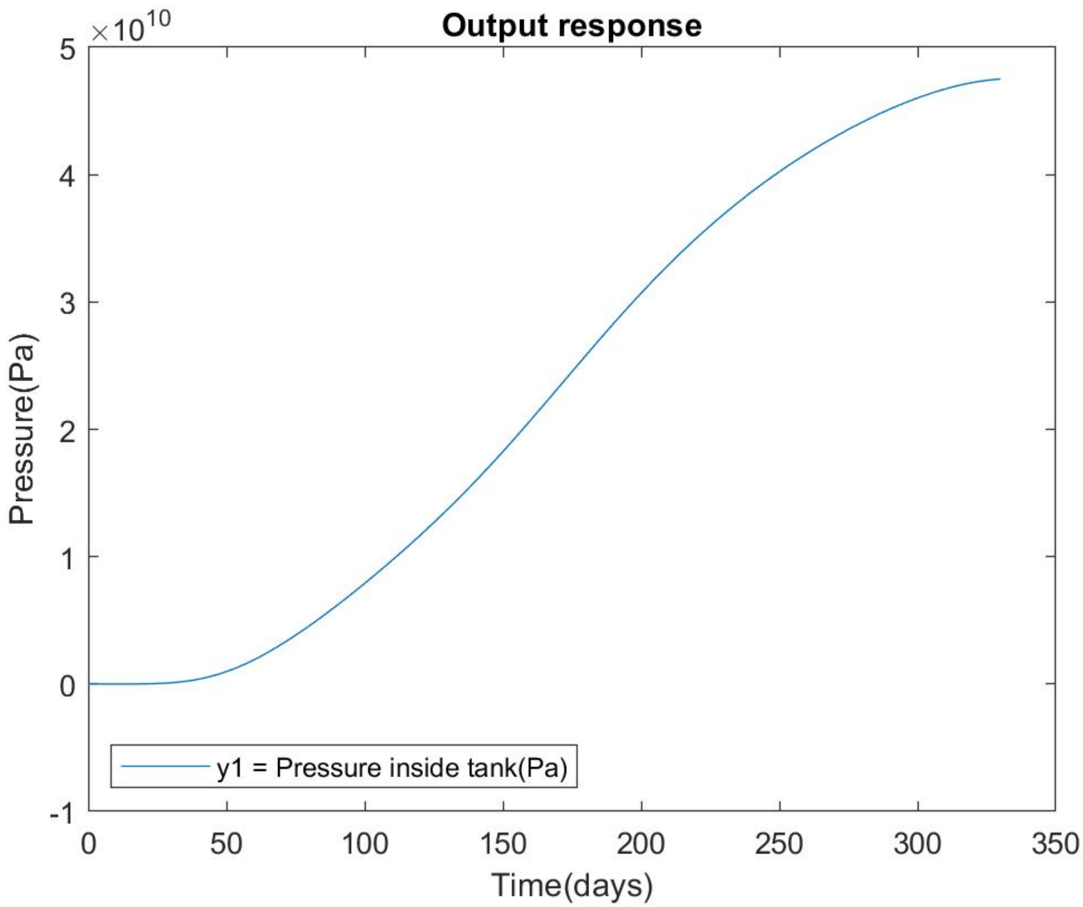


Figure 8: Response of Pressure Tank

Once we had a satisfactory response, the energy consumption system was analysed. The following tank was simulated when the tank was at its fullest and steady state. The simulation was meant to demonstrate a nail gun being used, with the trigger being pulled for brief periods of time. With each use of the nail gun, we can see pressure dropping by a predetermined amount, based on the desired force of impact hitting the nail, thus creating a step function, that is being lowered for each and every hit.

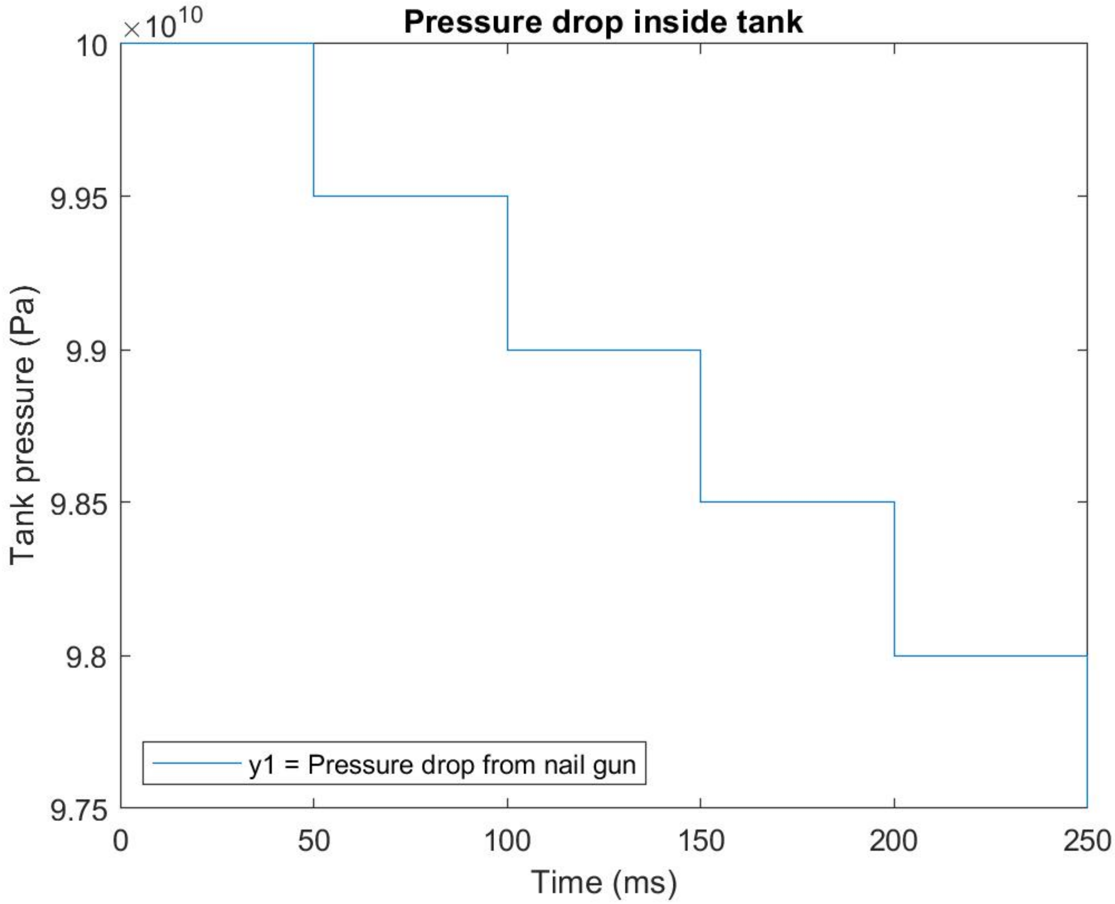


Figure 9: Response of Pressure Output

Lastly, we examined the force output of the nail gun. The input to this system will bursts out some pressure as seen in the previous simulation, which confirms an impulse wave, as it requires a short time to assert a force. The figure below represents the impulse when the nail gun is triggered, when triggered the system will output an impulse like function because the nail gun requires 100N of Force each time when it needs to drive a nail to the wall. This is accurate as a convection pneumatic nail gun, we here the loud bang almost instantly, and this accurately displays this, as each ‘bang’ we hear is a spike in the graph. We decided to make the output, to be happen every 0.5s.

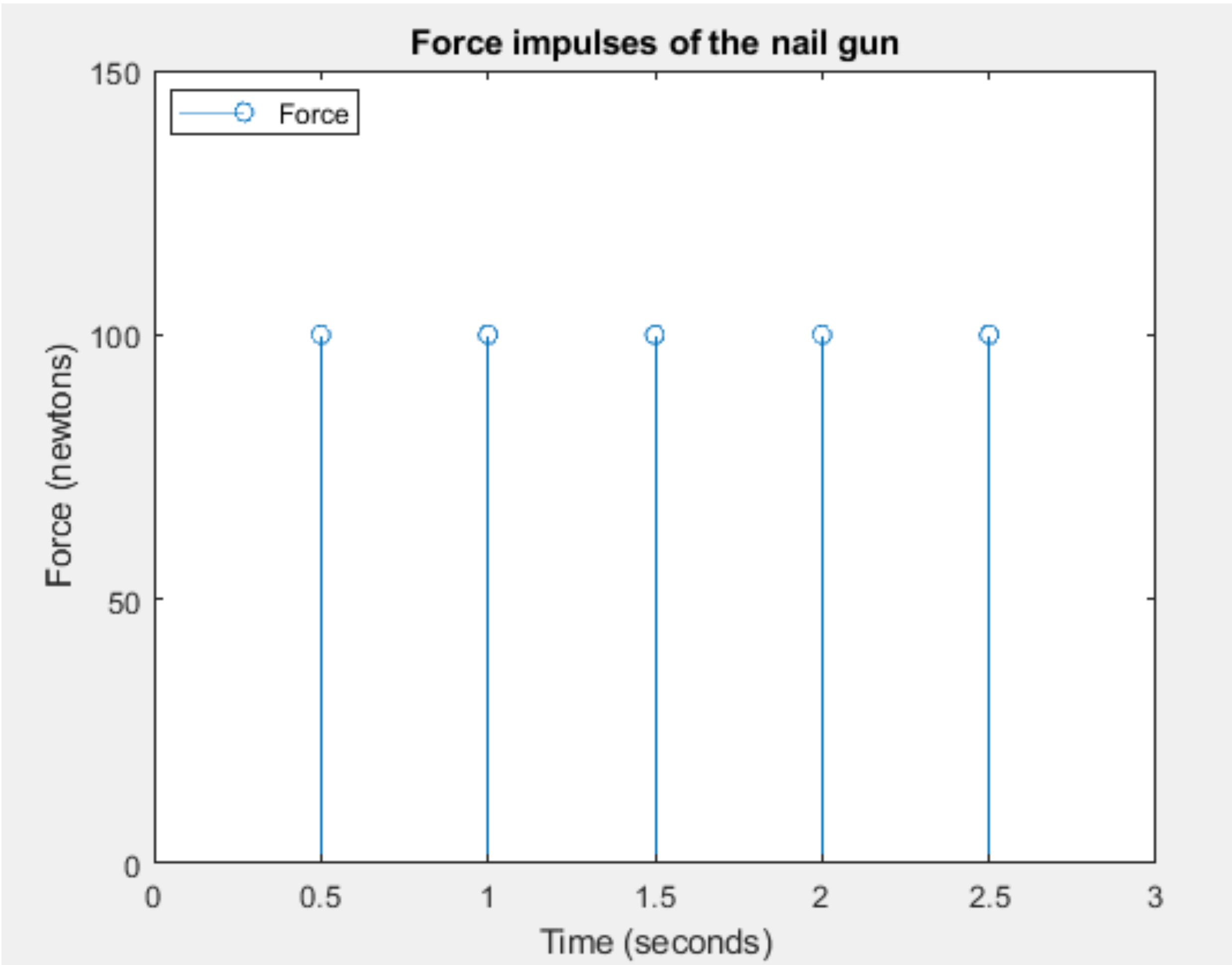


Figure 10: Output of Trigger from Nail Gun

Nonlinear Simulation:

The below graph demonstrates our output Flow Rate. We determine it to be nonlinear, as there is non-linear fluid resistance in the steel pipe, due to the change from laminar to turbulent flow as flow rate increases. As this general shape makes sense it is because the Pressure, we assumed is capped off in our energy storage system, and fully charged. In addition to the output flow rate, it only happens when the trigger on the nail gun is pulled. When triggered, from natural properties, higher pressure elements will like to go to lower pressure elements, in this case the tank pressure will like to go to the atmospheric pressure.

Thus, making the flow rate be instant, and have a spike similar to an impulse like shape. As shown below the graph describes an impulse like shape and this is correct, because the following flow rate gradually increases until it reaches a critical point where the force is great enough to push the wall of the nail gun and output a great flow rate, hence the near making a vertical line.

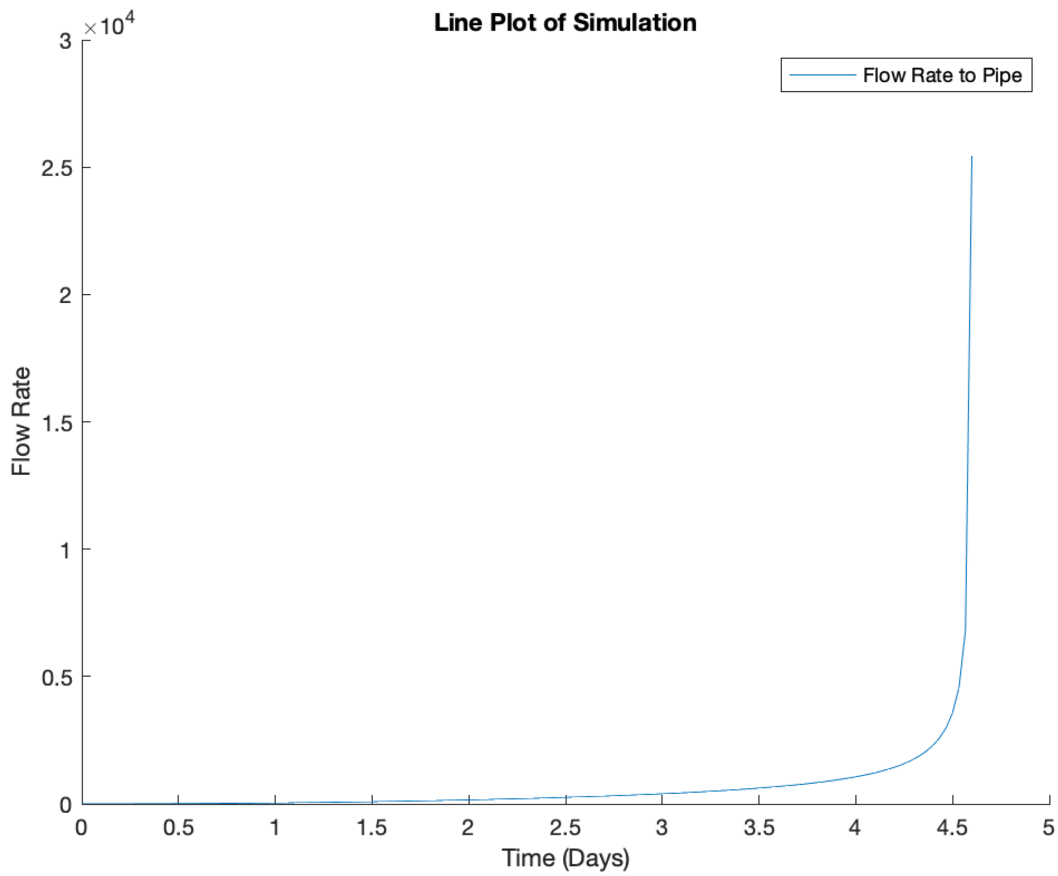


Figure 11: Flow Rate at Pipe

The above graph shows an ode45 simulation of the non-linear resistor model. Aside from the scaling, which is due to some incorrect constants, the general shape of the response is satisfactory, as it shows no/negligible flow while the tank is charging, and then a huge spike of fluid flow when the nozzle is opened to use a tool. This spike corresponds to the drops in pressure we observed in the tank pressure response.

Once we obtained a linearized model of our nonlinear system, we ran an "lsim" test to verify the results were consistent. The resulting graph is shown below. As we can see below, the general response matches that of the nonlinear graph, except the "build up". This is due to the fact that in the linear system, the flow rate simply grows exponentially with input, but when we simulate the system accounting for its nonlinear resistor element, we see the effect that the variable resistance has on the early response of the model.

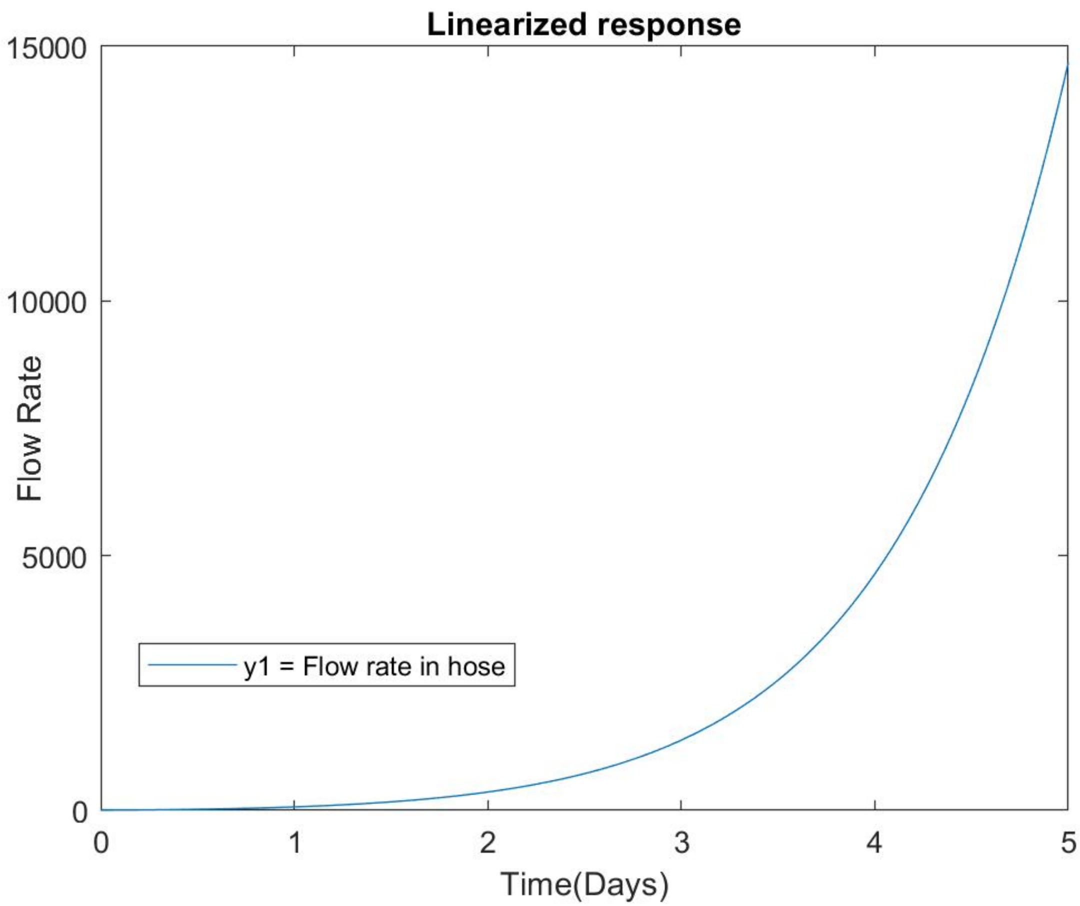


Figure 12: Linearized Response

Limitations to the model and simulation

The current limitations of the model are numerous, and there are many assumptions that we made that may not be advantageous to someone that is using this report as a reference. They are the following:

- The values for the friction coefficients are essentially arbitrary and are not tailored to our system. Most of these were admittedly found while looking at examples in questions that have very different system layouts.

- The nonlinearities we introduced exists only in parts of the systems and while it does work as intended, the model is by no means accurate. Given that most elements exhibit nonlinear behaviour when we go beyond a certain range, we haven't gone through and taken into account of those behaviours for simplicity of the model.
- Our moment of inertia of the waterwheel was calculated using essentially a hollow cylinder. This was something suggested by the website "The Physics of a Water Wheel", but we aren't sure of the validity or accuracy of this particular way of approximating it this way.

- Given that our flywheel is used as an energy storage device, we didn't take into account the standard sizes of a flywheel, so while the moment of inertia is correct in theory, it wouldn't be an accurate estimate of something you could buy off the shelf.

- The bulk modulus of the fluid is a nonlinear element and changes depending on the changes of pressure, instantaneous change in volume, as well as the original volume. We linearized this in our linear model but did not address this in the nonlinear model. Therefore, it is suggested to the reader to take direct your attention to this particular nonlinear element in particular for any inaccuracies or issues.

- From the SolidWorks drawing, we see that the model has one continuous shaft running throughout many of the elements. However, in reality, there would be losses in the penstock. Namely, the elbow sections, pipe joints that would in reality be essentially in putting this system into action. These, however, were not considered and should be in a future modelling endeavour.

- Some of the considerations that should be made clear is in the output of the system. Namely, the impulse forces that were generated by the drop in pressure do not take into account the continuous input from the water wheel mechanism. It's important to reaffirm that the output system assumes that the user has taken a full tank of the charged air tank and is using the pressure every 50ms. These assumptions are important to take note of when using this model.

- The fluid inertance coefficient of the model is an approximation based on the velocity profile of the pipe. We haven't done any rigorous experimentation into the flow of the system. For further improvements, we would have incorporated techniques such as analysing the fluid flow itself (laminar and turbulent) and see if that has an effect on the overall energy storage.

- Another assumption we made that limits the accuracy of the model is that heat transfer is never taken into account. This could make it so the absolute limit of the gas chamber is actually much lower in that pumping that much mass into a confined space inevitably causes an increase of temperature.

- Intrinsic to using mathematical modelling, approximations of natural behaviour are used for simplicity. Therefore, slight inaccuracies should be acceptable given they are explained.

Having said that, these limitations do not have enough bearing on the output results that makes it invalid since the graphs generated in MATLAB have matched what we expected out of the model. Some improvements could definitely be made. Namely, many of the friction coefficients, fluid properties at needed temperatures, should be confirmed experimentally. In addition, as mentioned before, some of the nonlinearities should be scrutinized with more rigor in order to ensure that the ones that are considered negligible are experimentally confirmed to be unimportant.

Reflection

Throughout this project, we encountered many challenges. While some of these were inherent with a complicated process such as system modelling, other challenges could have easily been avoided with more careful planning. For instance, the mechanical system design could be improved by tuning the flywheel and flexible shafts, as well as implementing dampers in order to improve the response of the system and maximize efficiency. We also found that the way our system was modelled presented some complications when running simulations and required us to break the model into smaller subsystems. Had we anticipated this and modelled accordingly from the beginning; we could have saved some time as well been able to monitor inputs more carefully. As shown above, the simulations were great, as they allowed us to see the difference between linear and nonlinear systems, and how the energy storage systems work. If more time was available to us, we would tune parameters more accurately. This would allow us to have more detailed and accurate graphs. Another way to have more accurate graphs is to simulate a system that allows us to see the variable changes in the system through a more detailed analysis of the system. One thing we found very helpful was monitoring multiple outputs of a single system, to be able to better troubleshoot issues. By using our intuition, we were able to spot inconsistencies between the theoretical model and our output, which helped us spot problems more easily and adjust our model accordingly.

Conclusion

As we were to simulate our final design. We chose this design as, there was a river near the village that we can use the flow rate, making it into mechanical work, to be converted to an air compressor, then later to be used in hand tools. We decided to use hand tools as our output as the villages probably required maintenance and a more efficient way of building structure. This was especially useful in areas that were high in maintenance, due to typhoon seasons in the months of August. This design was a mesh of all three designs as this was implemented all three designs in a way. As there is a typhoon, there is a higher flow rate for the water, thus increasing the speed of the water, which alternately increases the of work input to our system, therefore a potentially higher output for the hand tools.

This was a more accurate model to simulate, as this took the input from an everyday supply of water flow. This will then be more accurate as the river will never stop flowing, which helped the situation as there is always a constant input to the compressor tank, thus allowed a 24hour access to the tank, and its output. Also being able to have the water nearby, this was the most efficient design to implement mechanical work into air flow rate. As it required a compressor to change the mechanical input to an airflow, which then was stored in a tank. As this requires the least amount of components, this was the more efficient system as this doesn't lose as much efficiency through each process.

The model that we model, is a relatively efficient model, as electricity over dramatic temperatures can fluctuate, thus proving not great, in such extreme temperatures, as the village is commonly hot, and cold during the night. By having compressed air, not only is it 7 to 8 times more expensive than electricity, it doesn't fluctuate as greatly compared to electricity capacitors such as batteries, which will expand and contract in extreme temperature differences, thus it increases the reliability of the system.

Looking at the models, we can understand the following, and it backs up our depiction of the model logically. As the general shape makes sense to us, but some independent variables, such as time, is not correct, this could be simply and error in MATLAB coding as our group members are not quite proficient in MATLAB coding at the moment. Having more time to run tests and run more laboratory work we are able to get more accurate constants, therefore creating more accurate displays and graphs through MATLAB. Also, by having more time, we can ask more questions regarding our lacking area of expertise. Which will allow us to revise the code more and provide more accurate timelines in our graph.

References

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[2] E. Cheever and Swarthmore College, "Elements of Rotating Mechanical Systems," Elements of Translating Mechanical System. [Online]. Available: https://lpsa.swarthmore.edu/Systems/MechRotating/RotMechSysElem.html. [Accessed: 1-Nov-2019].

[3] C. Frewin, "Renewable Energy," Renewable Energy | Student Energy. [Online]. Available: https://www.studentenergy.org/topics/renewable-energy. [Accessed: 30-Nov-2019].

[4] "How the Agriculture Industry Can Use Air Compressors," Quincy Compressor. [Online]. Available: https://www.quincycompressor.com/industries/agriculture/. [Accessed: 30-Nov-2019].

[5] S. Muise, "Hydro Power," Hydro Power | Student Energy. [Online]. Available: https://www.studentenergy.org/topics/hydro-power. [Accessed: 30-Nov-2019].

Appendix

MATLAB code

```
%Contains all constants needed to model the whole system and subsystems.
%Additionally contains and interpolation of input data that is then passed
%to the simulation function as input.
%NOTE: Some constants may not be 100% accurate. While some were calculated
%via physical relations, some are simply ballpark estimates based on
%literature/past course assignments

J_t=2791;
b_o=33;
k_1=24.65;
k_2=32.07;
P_atm=1;
P_1 = 1;
B_1=9;
V=2.265; %m^3
J_c=157.49;
b_2=470;
I_f=63534;
R_f=2;
m_max = ro_a * V;
m=2;%kg
A_1=0.6283;%m2
b_11=470;
b_21=470;
C_f=0.0000023168+2.26535/B_1;
y = [0:11];
y2 = [0:1/30:11];

%interpolation of discreet input to a continuous function 330 days long
om_disc = 1.204e5.*[6, 5.5, 6, 7, 8, 7 ,6, 6, 7, 8, 8, 8];
om_cont = interp1 (y, om_disc, y2, 'spline');
t = 0:(((length(om_disc)-1)*30))-1;
plot(y2, om_cont);
```

```
%Mechanical Model

A = [-b_o/J_t, -1/J_t, 0, 0;          %water wheel
     k_1, 0, -k_1, 0;                 %flex shaft
     0, 1/J_v, -b_1/J_v, -1/J_v;      %flywheel
     0, 0, k_2, 0];                   %flex shaft
 B = [1/J_t; 0; 0; 0];
 C = [0, 1/k_1, 0, 1/k_2; % 3 outputs to better analyse response
        0, 0, 0, 1/k_2;
        0, 0, 0, 1;
        0, 0, 1, 0];
 D = [0;0;0;0];
```

constants.m | wheel_sim.m | tool_sim.m | submodel3.m | submodel2.m | submodel1.m | NLs

```
%Simulation of water wheel mechanical system
%inputs: space model of mechanical system, interpolated time, torque on
%wheel
%outputs: displacements of inertial elements, torque of shaft
clc;
close all;

t = 0:(((length(om_disc)-1)*30));
in = om_cont;
sys = ss(A,B,C,D);
x0 = [0 0 0 0]; %assume an equilibrium at the start

[y_1,t,x] = lsim(sys, in, t, x0);
figure(1);
plot(t, y_1);
title("Output response");
legend ({'y1 = theta1', 'y2 = theta2', 'y3 = torque'}, 'location', 'Southwest');
 figure(2);
 plot(t, x);
 title("State response");
 in2 = y_1(1:end, 3);
```

constants.m | submodel3.m | submodel2.m | submodel1.m | NLsim.m | model.m

```
% Compressor and tool

A = [R_f/I_f, 0, 0, 0;                    %flow rate
     1/C_f, 0, 0, 0;                       %tank pressure
     0, A_1/m, (-(b_11+b_21)/m), -1/m;    %internal velocity in nail gun
     0, 0, k_5, 0];                        %internal force in nail gun
```

+3 | tank_sim.m | constants.m | wheel_sim.m | tool_sim.m | submodel3.m | submodel2.m

```
%Simulation of compressed air tank
%input: torque
%output: pressure in tank

clc;
close all;

t = 0:(((length(om_disc)-1)*30));
in = om_cont;
sys = ss(A,B,C,D);
x0 = [0 0 0 0 1 0]; %Assume P = Patm

[y_2,t,x] = lsim(sys, in2, t, x0);
figure(1);
plot(t, y_2);
title("Output response");
legend ({'y1 = Tank Pressure'}, 'location', 'Southwest');
```

constants.m | submodel2.m | submodel1.m | NLsim.m | model.m

```
%Fluid Model

A = [-b_o/J_t, -1/J_t, 0, 0, 0, 0;
     k_1, 0, -k_1, 0, 0, 0;
     0, 1/J_v, -b_1/J_v, -1/J_v, 0, 0;
     0, 0, k_2, 0, 0, 0;
     0, 0, 0, 0, 0, B_1/(V*k_3);
     0, 0, 0, 1/J_c, 1/(J_c*k_4), -b_2/J_c];
B = [1/J_t; 0;0;0;0;0];
C = [0,0,0,0,1,0; %takes tank pressure as output
     0,0,0,0,0,0
     0, 0, 0, 0, 0, 0];
D = [0;0;0];
```

constants.m | tool_sim.m | submodel3.m | submodel2.m | submodel1.m | NLsim.m | model.m

```matlab
%simulation of output response
%input is output of tank_sim.m
%output: response of tool

clc;
close all;

t = 0:(((length(om_disc)-1)*30));
in3 = in2; %output from tank_sim.m
sys = ss(A,B,C,D);
x0 = [0 0 1 0]; %assume all initial values zero, except pressure, which is at Patm

[y_3,t,x] = lsim(sys, in3, t, x0);
figure(1);
plot(t, y_3);
title("Output response");
legend ({'y1 = Pressure inside tank(Pa)'}, 'location', 'Southwest');
```

```matlab
%Model to go with nonlinear ode45 simulation
%simple 2 equation representation of the subsystem
%If, k and n are nonlinearity constants

function dx= model(t,x)

If= 63534;
k=2;
n=1.23;

dx(1)=x(2);
dx(2)=[(k*x(2)^n/If)]*x(1)+1/12;


dx = dx';
end
```

```
%ode45 simulation of nonlinear fluid resistor element

clc; close all

T_amb_discrete = [6,5.5,6,7,8,7,6,6,7,8,8,8] ;
days = 0:length(T_amb_discrete)-1;
Time = 0:(length(T_amb_discrete)-1)*30-1;
T_amb = interp1 (days, T_amb_discrete, Time/30, 'spline');

X_i= [0; 0]; %initial condition of time flow rate

[t, y] = ode45(@model, Time, X_i);

figure;
hold on;
title("Line Plot of Simulation")
xlabel ("Time (Days)");
ylabel("Temperature (^oC)");

plot(t/30, y(:,1), 'DisplayName', "Flow Rate to Pipe");
legend;
```

```
%Model and simulation of single nonlinear element.
%Was done quickly to confirm ode45 results
A = R_f/I_f;%
B = 1;      % State
C = 1;      % Model
D = 0;      %
z = 0:5/(length(t)-1):5;
x0 = 5;
t = 0:(((length(om_disc)-1)*30));
in_nl = 100*(exp(z)/R_f).*sqrt(z);
sys = ss(A,B,C,D);

[y_4,z,x] = lsim(sys,in_nl , z, x0);
figure(1);
plot(z, y_4);
title("Linearized response");
legend ({'y1 = Flow rate in hose'}, 'location', 'Southwest');
```

constants.m | NLsim.m | fullmodel.m

```
%Model of entire system
%can be used buy may produce results inconsistent with the theoretical
%model. Has been replaced by muliple subsystems for easier troubleshooting
%and modulation

A = [-b_o/J_t, -1/J_t, 0, 0, 0, 0, 0, 0, 0, 0;
     k_1, 0, -k_1, 0, 0, 0, 0, 0, 0, 0;
     0, 1/J_v, -b_1/J_v, -1/J_v, 0, 0, 0, 0, 0, 0;
     0, 0, k_2, 0, 0, 0, 0, 0, 0, 0;
     0, 0, B_1/(V*k_3), 0, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 1/J_c, 1/(J_c*k_4), -b_2/J_c, 0, 0, 0, 0
     0, 0, 0, 0, 0, 0, R_f/I_f, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1/C_f, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 0, A/m, -(b_11+b_21)/m, -1/m;
     0, 0, 0, 0, 0, 0, 0, 0, k_5, 0];
 B = [1/J_t; 0; 0; 0; 0; 0; 0; 0; 0; 0];
 C = [0 0 0 0 0 0 0 0  0 1];
 D = 0;
```