HW7 – linear Algebra

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Q1) $W = \{(x,y,z) \mid x-(y+1)+2(z+1)=1;\ x,y,z \in \mathbb{R}\}$

firstly, $x,y,z \in \mathbb{R}$ so $(x,y,z) = W \subseteq \mathbb{R}^3$, and we know $\mathbb{R}^3$ is a V.S.

① $(0,0,0)$ is the additive id of $\mathbb{R}^3$.

$0 \cdot (x,y,z) = (0,0,0)$

and $0-(0+1)+2(0+1) = -1+2 = 1$

so $(0,0,0) \in W$

② let $w, \tilde{w} \in W$

$w+\tilde{w} = (x+\tilde{x}, y+\tilde{y}, z+\tilde{z})$

observe $(x+\tilde{x}) - ((y+\tilde{y})+1) + 2((z+\tilde{z})+1)$

$= (x+\tilde{x}) - y - \tilde{y} - 1 + 2z + 2\tilde{z} + 2$

we know $x-y-1+2z+2=1 \Rightarrow x-y+2z=0$

and that is $(x+\tilde{x}) - (y+\tilde{y}) + 2(z+\tilde{z})$

$= x+\tilde{x}-y-\tilde{y}+2z+2\tilde{z} = (x-y+2z) + (\tilde{x}-\tilde{y}+2\tilde{z}) = 0+0 = 0$

So $w+\tilde{w} \in W$

③ let $w \in W$, $\lambda \in \mathbb{R}$

$\lambda w = \lambda(x,y,z) = (\lambda x, \lambda y, \lambda z)$

observe $\lambda x - \lambda y + 2\lambda z = \lambda(x-y+2z) = \lambda(0) = 0$

so $\lambda w \in W$

hence W is a V.S ☒

Q2) let $W = \mathbb{R}^2$, with $W_1 = \{(a,0) \mid a \in \mathbb{R}\}$, $W_2 = \{(0,b) \mid b \in \mathbb{R}\}$

in this case $w_1 + w_2 = (a,b)$ with $a, b \in \mathbb{R}$

So $\forall (x,y) \in \mathbb{R}^2$, $\exists (a,b) \in W_1 + W_2$ with $a, b \in \mathbb{R}$

So, $W_1 + W_2 = W$. $\boxtimes$

Q3) let $V_1 + V_2 = (x,y,0,0) + (0,0,z,w) = (x,y,z,w) = S$

So $S = \{(x,y,z,w) \mid x,y,z,w \in \mathbb{R}\}$.

① additive id of $\mathbb{R}^4$ is $(0,0,0,0)$

$0 \cdot (x,y,z,w) = (0,0,0,0)$, since $0 \in \mathbb{R}$, $(0,0,0,0) \in S$.

② let $s_1, s_2 \in S$

$s_1 + s_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2, w_1 + w_2)$

since $x_1 + x_2$, if $x_1, x_2 \in \mathbb{R}$ is in $\mathbb{R}$, then $s_1 + s_2 \in \mathbb{R}^4 = V$

③ let $s \in S$, $\lambda \in \mathbb{R}$

$\lambda s = \lambda(x,y,z,w) = (\lambda x, \lambda y, \lambda z, \lambda w)$

since $\lambda x, \lambda y, \lambda z, \lambda w \in \mathbb{R}$, $\lambda s \in \mathbb{R}^4 = V$

hence $V_1 + V_2$ is a subspace of V. $\boxtimes$

Q4) let $(a,b,c,d) \in \mathbb{R}^4 = V$, $(x,y,z,w) \in V_1 + V_2$

$V_1 + V_2 = V$ means $\forall (a,b,c,d), \exists (x,y,z,w)$ s.t $V_1 + V_2 = V$.

in this case we can simply see if $a = x, b = y, \ldots, d = w$. then

$(a,b,c,d) = (x,y,z,w)$ for all (a,b,c,d).

So, $V_1 + V_2 = V$ $\boxtimes$

Q5) $V_1 = \{(-2w_2, 2w_2) \mid w_2 \in \mathbb{R}\}$

$V_2 = \{(v_1, -v_1) \mid v_1 \in \mathbb{R}\}$

so, $V_1 + V_2 = (v_1 - 2w_2, 2w_2 - v_1)$

but observe $v_1 - 2w_2 = -(2w_2 - v_1)$

so, $V_1 + V_2$ can be written as $(x, -x)$

with $x = v_1 - 2w_2$.

now let $(4,3) \in V = \mathbb{R}^2$

now $(4,3) \notin V_1 + V_2 = (x, -x)$

cause if $x = 4$, $-x$ has to be $-4 \neq 3$. ($\Rightarrow\Leftarrow$)

So $V_1 + V_2 \neq V$ $\boxtimes$