

Homework:
Working with Z-scores

MATH 150

Due: Feb 7, 2024

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Problem 1

Take a look at the faithful data set, which is pre-loaded in R. Find the mean and standard deviation of eruption lengths. What is the z-score of a five-minute eruption? Interpret your answer in ordinary human language.

Answer

Using R:

```
View(faithful)

x <- 5
mx <- mean(faithful$eruptions) # 3.487783
s <- sd(faithful$eruptions) # 1.141371

(x - mx) / s # 1.324912
```

—

The z-score of approx. 1.3 suggests that a five-minute eruption is relatively, but not boldly unusual, longer than the average eruption length in the sample.

Problem 2

A certain kids' fun run has two age categories: 8-11 and 12-14. Finishing times in the younger group have mean 33 minutes and standard deviation 4 minutes, while finishing times in the older group have mean 29 minutes and standard deviation 5 minutes.

- (a) Find and interpret the z-score of an 8-11 year old who finishes in 24 minutes.
- (b) Find and interpret the z-score of a 12-14 year old who finishes in 24 minutes.
- (c) Which is the more unusual of these two?

Answer

$$z = \frac{x - \bar{x}}{\sigma}$$

(a) Using R:

```
(24 - 33) / 4 # -2.25
```

—

The z-score of -2.25 suggests that the finishing time of this sample is 2.25 standard deviations below the mean. This suggests that the 8-11 year old finished the race much faster than the typical participant in their age category, making this sample a potentially unusual one.

(b) Using R:

```
(24 - 29) / 5 # -1
```

—

The z-score of -1 suggests that the finishing time of this sample is 1 standard deviations below the mean. This suggests that the 12-14 year old finished the race faster than the typical participant in their age category. However, the difference is not as significant as the younger age-group.

(c) As the z-score of the younger-age sample has an absolute value higher than 2, and is larger than the second sample's z-score, it is more unusual.

It is also important to recognize that the z-score of the 12-14 year-olds, although signifying they are faster than the average in their age group, but cannot be considered unusual, as it is not high enough.