Homework:
Introduction to Probability

MATH 150

Due: Feb 16, 2024

Sepehr Akbari

Sepehr Akbari

Problem 1

A probability experiment consists of flipping a coin and then rolling a standard 6-sided die.

(a) Identify the sample space.

(b) Write the event, "the coin is a tail and the die roll isn't a 5" as a set.

(c) What is the probability of the event in part (b)? Assume all outcomes are equally likely.

(d) What is the probability that the coin is a head and the die roll is odd?

Answer

(a) The sample space (S):

$$S = \{(H,1), (H,2), ..., (H,6), (T,1), (T,2), ..., (T,6)\}$$

$$|S| = 12$$

(b) The event (E):

$$E = \{(T,1), (T,2), (T,3), (T,4), (T,6)\}$$

(c) The probability of the event:
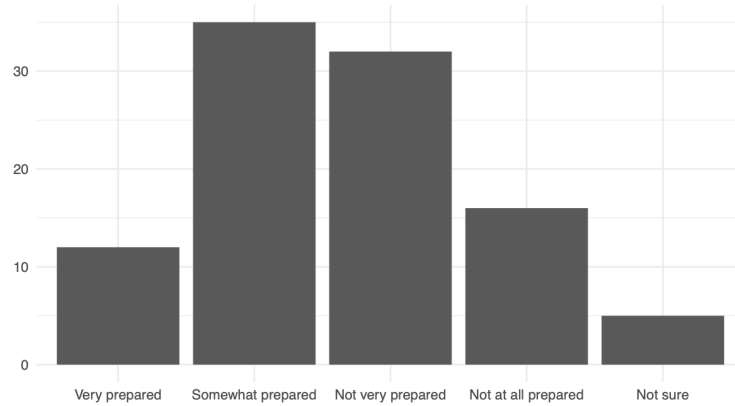
$$P(E) = \frac{|E|}{|S|} = \frac{5}{12}$$

(d) The probability of the event:

$$X = \{(H,1), (H,3), (H,5)\}$$

$$P(X) = \frac{|X|}{|S|} = \frac{3}{12} = 0.25$$

Problem 2

One hundred students are selected at random at the beginning of finals week and asked how prepared they feel. The results are summarized in the following bar chart.

(a) What is the probability that the next person surveyed says they are somewhat prepared?

(b) What is the probability that the next person surveyed does not say they are somewhat prepared?

(c) Is this an example of empirical probability, classical probability, or subjective probability?

Answer

(a) $\frac{35}{100} = 35\%$

(b) $\frac{100}{100} - \frac{35}{100} = \frac{65}{100} = 65\%$

(c) As this data is based on observed outcomes of an experiment, rather than calculations or personal judgements, it is a **empirical probability**.

Problem 3

Classify each of the following as examples of empirical, classical, or subjective probability.

(a) According to a company's records, the probability that one of their cars will need a new muffler in its first 8 years is 10%.

(b) The coach of a basketball team says they have a 50% chance of winning the championship.

(c) A passcode consists of 4 digits from 0 to 9. The probability of guessing the code correctly on the first try is $1/104 = 0.0001$.

Answer

(a) Empirical: As it is based on actual observations or experiments rather than theoretical or mathematical calculations or personal judgments or beliefs.

(b) Subjective: As it is based on personal judgments, opinions, or beliefs about the likelihood of an event occurring, rather than objective data or observations.

(c) Classical: As it assumes all outcomes in the sample space (all possible combinations of 4 digits) are equally likely, and the probability of guessing the correct passcode on the first try is calculated by dividing the number of favorable outcomes by the total number of possible outcomes.

Homework:
The Addition Rule

MATH 150

Due: Feb 16, 2024

Sepehr Akbari

Problem 1

Are the following events disjoint or not? Briefly justify your choice.

A: Randomly select a college professor who has exactly 20 years experience

B: Randomly select a college professor who is exactly 50 years old

Answer

Not Disjoint. As the probability of A and B are not exclusive, meaning there could be a scenario when both are probable. (if a 50 year old college professor has had 20 years of experience).

Problem 2

Are the following events disjoint or not? Briefly justify your choice.

A: Randomly select a student whose birthday is in May

B: Randomly select a student whose birthday is in June

Answer

Disjoint. One person cannot have birthdays in two months, therefore each probability is exclusive (we are calculating this probability based on ONE person as object).

Problem 3

A Pew Research poll asked 1,306 Americans "From what you've read and heard, is there solid evidence that the average temperature on earth has been getting warmer over the past few decades, or not?". The table below shows the distribution of responses by party and ideology, where the counts have been replaced with relative frequencies. Suppose a random person is selected from this sample.

| | | *Response* | | | |
		Earth is warming	Not warming	Don't Know Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
	Mod/Cons Democrat	0.25	0.07	0.02	0.34
	Liberal Democrat	0.18	0.01	0.01	0.20
	Total	0.60	0.34	0.06	1.00

(a) What is the probability that the person is a liberal democrat who believes that the earth is not warming?

(b) What is the probability that the person is a republican (conservative, moderate, or liberal)?

(c) What is the probability that the person is either a conservative republican OR believes the earth is warming (or both)?

Answer

(a) (Liberal Democrat) && (Not Warming) = 0.01 = 1%

(b) Total(Conservative Republican) + Total(Mod/Lib Republican) = 0.33 + 0.13 = 0.46 = 46%

(c) P(Conservative Republican) + P(Earth is Warming) - P(both) = 0.33 + 0.6 - 0.11 = 0.82 = 82%

Problem 4

A statistics class has 30 students. Of these, 8 are data science majors and 12 are sophomores. Of the sophomores, 3 are data science majors. Find the probability that a randomly-selected student is a data science major or a sophomore.

Answer

$P(DataScience) + P(Sophmore) - P(both) = \frac{8}{30} + \frac{12}{30} - \frac{3}{30} = \frac{12}{30} = 0.4 = 40\%$