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The central limit theorem

Problem 1

A certain population has mean $\mu = 27.3$ and standard deviation $\sigma = 4.2$. For a sample of size $n = 36$,

(a) Determine the mean, variance, and standard deviation of the sample mean x.

Answer:

```
mu <- 27.3
sigma <- 4.2
n <- 36

cat("mean of x:",mu,"\n")
```

```
mean of x: 27.3
```

```
cat("variance of x:",(sigma ^ 2 / n),"\n")
```

```
variance of x: 0.49
```

```
cat("standard deviation of x:",sqrt((sigma ^ 2 / n)),"\n")
```

```
standard deviation of x: 0.7
```

(b) Determine the probability that $x \leq 26.0$.

Answer:

```r
mu <- 27.3
sigma <- 4.2
n <- 36
stndDev <- (sigma / sqrt(n))

z <- (26.0 - mu) / stndDev

pnorm(z)
```

```
[1] 0.03164542
```

(c) Determine the probability that $x \geq 25.9$.

Answer:

```r
mu <- 27.3
sigma <- 4.2
n <- 36
stndDev <- (sigma / sqrt(n))

z <- (25.9 - mu) / stndDev

1 - pnorm(z)
```

```
[1] 0.9772499
```

Problem 2

At a local grocery, apples have mean weight 0.620 pounds with standard deviation 0.165 pounds. The distribution is approximately normal.

(a) What is the probability that a randomly-selected apple weighs more than 0.650 pounds?

Answer:

```r
mu <- 0.62
sigma <- 0.165

cat("The probablity is:",1 - pnorm(0.65,mu,sigma))
```

```
The probablity is: 0.4278627
```

(b) What is the probability that 10 randomly-selected apples weigh more than 0.650 pounds, on average?

Answer:

```r
mu <- 0.62
sigma <- 0.165
n = 10
stndDev <- (sigma / sqrt(n))

z <- (0.65 - mu) / stndDev

cat("The probablity is:",1 - pnorm(z))
```

```
The probablity is: 0.2826593
```

(c) What is the probability that 50 randomly-selected apples weigh more than 0.650 pounds, on average?

Answer:

```r
mu <- 0.62
sigma <- 0.165
n = 50
stndDev <- (sigma / sqrt(n))

z <- (0.65 - mu) / stndDev

cat("The probablity is:",1 - pnorm(z))
```

```
The probablity is: 0.09928285
```

(d) What is the probability that 500 randomly-selected apples weigh more than 0.650 pounds, on average?

Answer:

```r
mu <- 0.62
sigma <- 0.165
n = 500
stndDev <- (sigma / sqrt(n))
```

```
z <- (0.65 - mu) / stndDev

pnorm(z)
```

```
[1] 0.999976
```

```
cat("The probablity is about:",1 - 0.999)
```

```
The probablity is about: 0.001
```

Problem 3

Lengths of eruptions of the Old Faithful geyser are approximately normally distributed with mean 3.49 minutes and standard deviation 1.14 minutes.

(a) Which is more likely, a single eruption longer than 3.20 minutes or 20 eruptions with mean greater than 3.20 minutes? Compute both probabilities.

Answer:

```
mu <- 3.49
sigma <- 1.14
n = 20
x = 3.20
stndDev <- (sigma / sqrt(n))

single_eruption <- 1 - pnorm(x,mu,sigma)
multi_eruption <- 1 - pnorm((x - mu) / stndDev)

print(single_eruption)
```

```
[1] 0.6004013
```

```
print(multi_eruption)
```

```
[1] 0.8723664
```

```r
if(single_eruption > multi_eruption){
  cat("The probability of a single eruption of 3.2 minutes or greater is higher, as:",single_
}else if (multi_eruption > single_eruption){
  cat("The probability of the mean of 20 eruptions being greater than 3.2 minutes is higher,
}else{
  cat("They are equally probable.")
}
```

```
The probability of the mean of 20 eruptions being greater than 3.2 minutes is higher, as: 0.8
```

(b) Which is more likely, a single eruption longer than 3.60 minutes or 20 eruptions with mean greater than 3.60 minutes? Justify your answer without computing probabilities.

Answer:

```r
mu <- 3.49
sigma <- 1.14
n = 20
x = 3.60
stndDev <- (sigma / sqrt(n))

single_eruption <- 1 - pnorm(x,mu,sigma)
multi_eruption <- 1 - pnorm((x - mu) / stndDev)

print(single_eruption)
```

```
[1] 0.4615652
```

```r
print(multi_eruption)
```

```
[1] 0.3330445
```

```r
if(single_eruption > multi_eruption){
  cat("The probability of a single eruption of 3.6 minutes or greater is higher, as:",single_
}else if (multi_eruption > single_eruption){
  cat("The probability of the mean of 20 eruptions being greater than 3.6 minutes is higher,
}else{
  cat("They are equally probable.")
}
```

```
The probability of a single eruption of 3.6 minutes or greater is higher, as: 0.4615652 > 0.3
```