HW6 - linear Algebra

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Q1) let $U = \{ (x_1, x_2, x_3) \mid x_1 x_2 x_3 = 0, x_i \in \mathbb{R} \}$

Based on thm 1.34, U is a v.s. if it is a subspace of v.s. $\mathbb{R}^3$.
let's check that :)

① Additive identity

additive id is $(0,0,0) \in \mathbb{R}^3$. and $0 \cdot 0 \cdot 0 = 0$. since $(0,0,0) \in U$
the condition holds ✓

② closed under addition

let $a = (a_1, a_2, a_3) \in U$ and $b = (b_1, b_2, b_3) \in U$.

this means $a_1 a_2 a_3 = b_1 b_2 b_3 = 0$

$\Rightarrow a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3) = 0$ however this does not always hold. for example $(1,0,1), (0,1,1) \in U$

but $(1,0,1) + (0,1,1) = (1+0, 0+1, 1+1) = (1,1,2) \notin U$

so addition is NOT closed

③ closed under scalar multiplication

let $a = (a_1, a_2, a_3) \in U$, and $k \in \mathbb{R}$

$\Rightarrow ka = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3) = 0$

bcs at least one $a_1 \dots a_3$ is equal to zero, one $ka_1 \dots ka_3$ is also zero so,

scalar mult is closed ✓

Since U failed to satisfy all conditions, its not a subspace of $\mathbb{R}^3$.

Therefore [illegible] vector spa[illegible]

Q2) let $W = \mathbb{R}^3 := \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$

we can say: $W_1 = \{(a,0,0) \mid a \in \mathbb{R}\} \subseteq \cancel{\mathbb{R}}\, W$

$W_2 = \{(0,b,0) \mid b \in \mathbb{R}\} \subseteq W$

let's prove they are subspaces quick:

① $(0,0,0)$ is the additive id in $\mathbb{R}^3$, and $(0,0,0) \in W_1, W_2$. ✓

②
$(a,0,0) + (\tilde{a},0,0) = (a+\tilde{a},0,0) \in W_1$ ✓
$(0,b,0) + (0,\tilde{b},0) = (0,b+\tilde{b},0) \in W_2$

③ $k(a,0,0) = (ka,0,0) \in W_1$ ✓
$k(0,b,0) = (0,bk,0) \in W_2$

So lets check if $W_1 + W_2 \neq W$:

$W_1 + W_2 = (a,0,0) + (0,b,0) = (a,b,0)$

bcs $(0,0,c) \in \mathbb{R}^3 = W$ but $(0,0,c) \notin W_1 + W_2$

then, $W_1 + W_2 \neq W$ $\boxtimes$

Q3) ① additive id in V is $(0,0,0) \in \mathbb{R}^3$

$(0,0,0) \in V_1$, bcs $\cancel{(0,0,0)}^{0} \cdot (x,y,0) = (0,0,0)$ ✓

$(0,0,0) \in V_2$, bcs $0 \cdot (0,y,z) = (0,0,0)$

② let $(x,y,0), (\tilde{x},\tilde{y},0) \in V_1$.

$(x,y,0)+(\tilde{x},\tilde{y},0) = ((x+\tilde{x}),(y+\tilde{y}),(0+0))$, $x+\tilde{x}, y+\tilde{y} \in \mathbb{R} \Rightarrow \in \mathbb{R}^3 = V$

let $(0,y,z), (0,\tilde{y},\tilde{z}) \in V_2$.

$(0,y,z)+(0,\tilde{y},\tilde{z}) = (0+0, y+\tilde{y}, z+\tilde{z})$, $y+\tilde{y}, z+\tilde{z} \in \mathbb{R}$ so $(0,y+\tilde{y},z+\tilde{z}) \in \mathbb{R}^3 = V$

③ let $k \in \mathbb{R}$,

$kv_1 = k(x,y,0) = (kx,ky,0)$, $kx, ky \in \mathbb{R}$, so $(kx,ky,0) \in V = \mathbb{R}^3$

$kv_2 = k(0,y,z) = (0,ky,kz)$, so $(0,ky,kz) \in V = \mathbb{R}^3$.

* Just realized I proved $V_1, V_2 \subseteq V$, not $V_1 + V_2 \subseteq V$. Oooops!
lets try that again: $V_1 + V_2 = (x,y,0) + (0,y,z) = \underline{(x, 2y, z) = A}$

① $0 \cdot A = (0,0,0)$, so $(0,0,0) \in A$ ✓

② let $a, \tilde{a} \in A$

$a + \tilde{a} = (x+\tilde{x}, 2y+2\tilde{y}, z+\tilde{z})$. $x+\tilde{x}, 2y+2\tilde{y}, z+\tilde{z} \in \mathbb{R}$, so $a+\tilde{a} \in A$ ✓

③ let $a \in A$, $\lambda \in \mathbb{R}$.

$\lambda a = (\lambda x, 2\lambda y, \lambda z)$. $\lambda x, 2\lambda y, \lambda z \in \mathbb{R}$, so $\lambda a \in A$. ✓

So $A = V_1 + V_2 \subseteq V$. ⊠

Q4) let $V = (a,b,c)$ and $V_1 + V_2 = (x, 2y, z)$ and $V_1 + V_2 \subseteq V$

$V_1 + V_2 = V$ means $\forall (a,b,c), \exists (x, 2y, z)$

in this case $(x, 2y, z) = (a, b/2, c)$.

$a, c \in \mathbb{R}$ done and assuming $b \in \mathbb{R}$ (as $V = \mathbb{R}^3$),

then any real number divided by 2 is still real so $\frac{b}{2} \in \mathbb{R}$.

so $a, b/2, c \in \mathbb{R}$. so, $V_1 + V_2 = V$ ⊠

Q5) let $V = (1,2,3) \in \mathbb{R}^3$.

for $U_1 + V_2 = V$, $U_1 = (1,1,0)$ and $V_2 = (0,1,3)$

but in U_1: $x + y = 1+1 \neq 0$

and in U_2: $y + z = 1+3 \neq 0$

So, $(1,2,3) \in \mathbb{R}^3$ but $(1,2,3) \notin V_1 + U_2$. WAIT IS THAT RIGHT???

Q5-2) V_1 can be written as $(x, -x, 0)$

V_2 can be written as $(0, y, -y)$

So $V_1 + U_2 = \{(x, -x+y, -y) \mid x, y \in \mathbb{R}\}$

so for $\forall (a,b,c), \exists (x, -x+y, -y)$

so $a = x$, $b = y - x$, $c = -y$ $\Rightarrow y = b + x = b + a$

$\Rightarrow c = -(b+a) \Rightarrow a+b+c = 0$

so this is a constraint, it means only (a,b,c)s that

$a+b+c=0$ are in $V_1 + V_2$. for example $(1,1,1)$ is not in!

So, $U_1 + V_2 \neq V$. ⊠

This looks better still not sure tho.