Math 240 — Taylor Series Approximation Worksheet
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1. **Use a cubic approximation to estimate $e^{0.5}$. As best you can, what is your estimate for the value?**

   To use a cubic approximation, we can approximate the exponential function using its Taylor series expansion around 0 up to the cubic term:

   $$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}.$$

   Here, $x = 0.5$, so:

   $$e^{0.5} \approx 1 + 0.5 + \frac{(0.5)^2}{2} + \frac{(0.5)^3}{6}.$$

   Calculating each term:

   $$e^{0.5} \approx 1 + 0.5 + \frac{0.25}{2} + \frac{0.125}{6} = 1 + 0.5 + 0.125 + 0.0208333.$$

   Therefore, the cubic approximation is:

   $$e^{0.5} \approx 1.6458333.$$

   So, the estimate for $e^{0.5}$ is approximately 1.6458.

2. **Use a quadratic approximation to estimate $\cos 35°$.**

   We did these in class using degrees. But we will see in Monday's lecture why radians are better. To ensure you can study off of these notes, I provide the approach with radians.

   To use a quadratic approximation, we can approximate the cosine function using its Taylor series expansion around 0 up to the quadratic term:

   $$\cos x \approx 1 - \frac{x^2}{2}.$$

   Converting 35° to radians:

   $$x = \frac{35\pi}{180} \approx 0.6109.$$

   Using the approximation:

   $$\cos 35° \approx 1 - \frac{(0.6109)^2}{2}.$$

   Calculating:

   $$\cos 35° \approx 1 - \frac{0.3732}{2} = 1 - 0.1866 = 0.8134.$$

   So, the quadratic approximation for $\cos 35°$ is approximately 0.8134.

3. **Use a cubic approximation to estimate** $\ln(1.5)$ **using** $\ln(x+1)$ **as your function centered at** $a = 0$ **and using** $b = 0.5$**.**

To use a cubic approximation, we can expand the natural logarithm function using its Taylor series around 0 up to the cubic term:

$$\ln(1 + x) \approx x - \frac{x^2}{2} + \frac{x^3}{3}.$$

Here, $x = 0.5$, so:

$$\ln(1.5) = \ln(1 + 0.5) \approx 0.5 - \frac{(0.5)^2}{2} + \frac{(0.5)^3}{3}.$$

Calculating each term:

$$\ln(1.5) \approx 0.5 - \frac{0.25}{2} + \frac{0.125}{3} = 0.5 - 0.125 + 0.0416667.$$

Therefore, the cubic approximation is:

$$\ln(1.5) \approx 0.4166667.$$

So, the estimate for $\ln(1.5)$ is approximately 0.4167.