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Calculating in the t-distribution

Problem 1

Compute $P(T < -1.1)$ in $t(22)$.

Answer:

```
n <- 22
t <- -1.1

pt(t,n)
```

```
[1] 0.1416118
```

Problem 2

Compute $P(-1.5 < T < 0.4)$ in $t(5)$.

Answer:

```
n <- 5
t_lower <- -1.5
t_upper <- 0.4

pt(t_upper,n) - pt(t_lower,n)
```

```
[1] 0.5502116
```

Problem 3

Find the number τ such that $P(T > \tau) = 0.05$ in $t(80)$. Note that this a right-tailed probability, not a left-tailed one.

Answer:

```r
n <- 80
x <- 1 - 0.05

cat("tau =", qt(x,n))
```

```
tau = 1.664125
```

Problem 4

Find the number τ such that 95% of the area under $t(6)$ lies between $-\tau$ and τ.

Answer:

Mathematically:

$$P(-\tau < T < \tau) = 0.95 \leftarrow t(n = 6)$$
$$= P(T < \tau) - P(-\tau < T) = 0.95$$

```r
n <- 6
x <- 0.975 # To calculate 0.95 percentile

t_lower <- qt((1 - x),n)
t_upper <- qt(x,n)

cat("lower bound:",round(t_lower,3),"\n")
```

```
lower bound: -2.447
```

```r
cat("upper bound:",round(t_upper,3))
```

```
upper bound: 2.447
```

We can test our result as well:

```r
cat("95% of t(6) is between the bounds:",
    pt(t_upper,n) - pt(t_lower,n) == 0.95,"\n")
```

```
95% of t(6) is between the bounds: TRUE
```

```r
cat("-tau = +tau:",
    abs(t_lower) == abs(t_upper))
```

```
-tau = +tau: TRUE
```

Problem 5

In a simple random sample of 10 sales clerks at convenience stores in 1989, the mean salary was $\$25,352.87$ and the standard deviation was $\$3,202.09$. Compute a level 95% confidence interval for the population mean. Carefully justify your answer.

Answer:

We can calculate this confidence interval, using the critical t-value, instead of z-value:

$$\mu = \overline{x} \pm t \times \frac{\sigma}{\sqrt{n}}$$

To calculate t, we use the R, qt( ) function, to get τ based on:

$$P(T < \tau) = \frac{1 - 0.95}{n - 1} \leftarrow t(n = 23 - 1)$$

```r
n <- 10 # sample size
m <- 25352.87 # mean
s <- 3202.09 # standard deviation
cl <- 0.95 # confidence level

t <- qt(((1 - cl) / 2),(n - 1))
mu_lower <- m - (abs(t) * ((s) / sqrt(n)))
mu_upper <- m + (abs(t) * ((s) / sqrt(n)))

cat("The mean salary of a convinient store worker in 1989",
    "\n is between"
    ,mu_lower,"and",mu_upper,"dollars. With 95% confidency rate.")
```

```
The mean salary of a convinient store worker in 1989
 is between 23062.23 and 27643.51 dollars. With 95% confidency rate.
```

This is the same approach as we had before for calculating confidence intervals. However this time, we are only interested in the sample, rather than the population. So we have to use the t_score instead of z_score. At the end, we'll get the confidence interval based on that specific sample.