Lake Forest College
Math 110 Final Exam

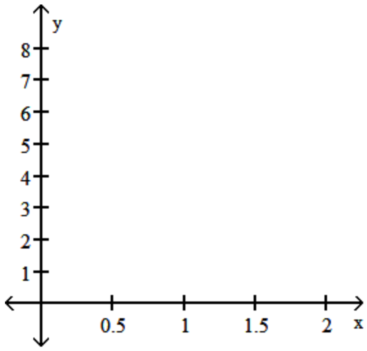
December 15, 2022

Name________________________________

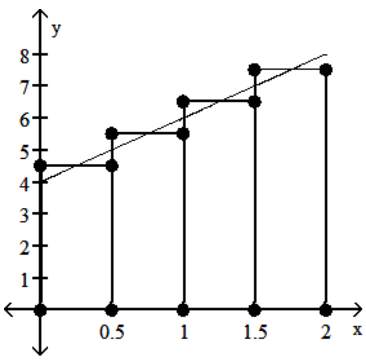
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the function f(x) over the given interval. Partition the interval into 4 subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, using the indicated point in the kth subinterval for c_k.

1) $f(x) = 2x + 4$, [0, 2], left-hand endpoint 1) ______



A)



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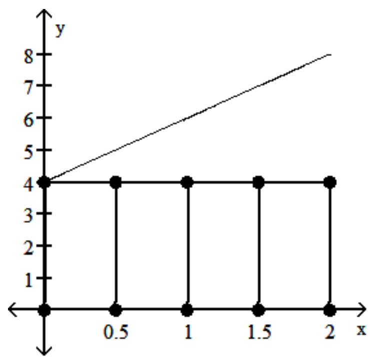

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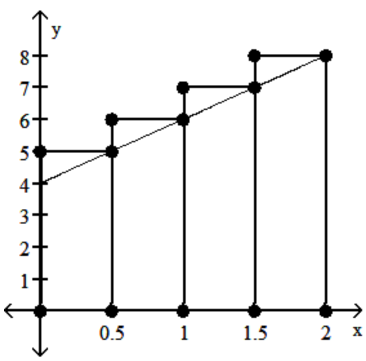

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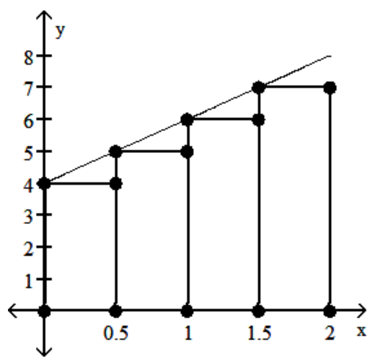


Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

2) $f(x) = \frac{1}{x}$ between x = 3 and x = 5 using a right sum with two rectangles (n=2)of equal width. 2) ________

A) $\frac{7}{20}$ B) $\frac{9}{20}$ C) $\frac{3}{4}$ D) $\frac{7}{12}$

Express the sum in sigma notation.

3) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$ 3) ________

A) $\sum_{k=0}^{4} \left(\frac{1}{4}\right)^{k+1}$ B) $\sum_{k=1}^{5} \left(\frac{1}{4}\right)^{k-1}$ C) $\sum_{k=1}^{4} \left(\frac{1}{4}\right)^{k-1}$ D) $\sum_{k=1}^{4} \left(\frac{1}{4}\right)^{k}$

Evaluate the sum.

4) $\sum_{k=1}^{13} k$ 4) ________

A) 13 B) 182 C) 91 D) $\frac{91}{2}$

5) $\sum_{k=1}^{13} k^3$ 5) ________

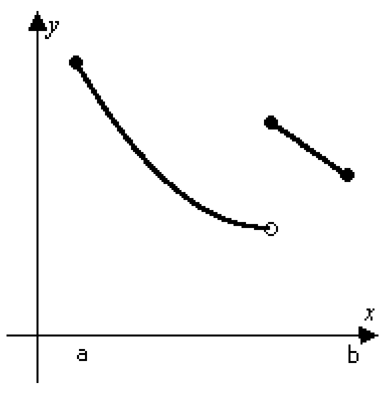
A) 819 B) 8,281 C) 2,548 D) 2,197

6) $\sum_{k=1}^{7} k^2 - 7$ 6) ________

A) 133 B) 91 C) 140 D) 42

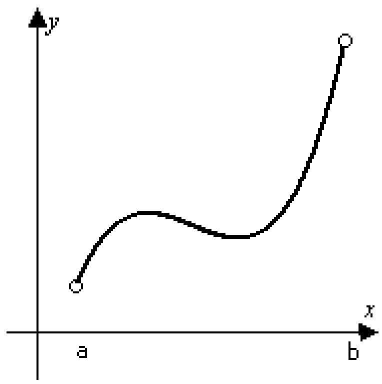
Determine from the graph whether the function has any absolute extreme values on the interval [a, b].

7) 7) ________



A) Absolute minimum and absolute maximum.
B) Absolute maximum only.
C) Absolute minimum only.
D) No absolute extrema.

8) 8) ________



A) Absolute maximum only.
B) No absolute extrema.
C) Absolute minimum and absolute maximum.
D) Absolute minimum only.

Determine all critical points for the function.

9) $f(x) = x^2 + 12x + 36$ 9) ________

A) $x = -12$ B) $x = 0$ C) $x = 6$ D) $x = -6$

Find the absolute extreme values of the function on the interval.

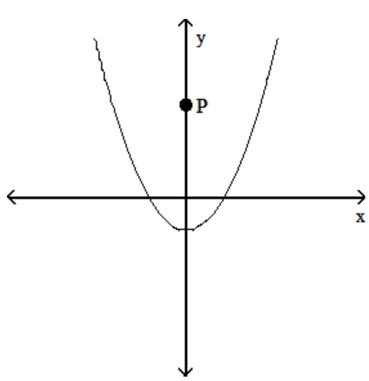
10) $f(x) = 3x - 5, \ -2 \le x \le 4$ 10) ________

A) absolute maximum is 17 at $x = -4$; absolute minimum is -11 at $x = 2$
B) absolute maximum is 7 at $x = 4$; absolute minimum is -11 at $x = -2$
C) absolute maximum is 7 at $x = -2$; absolute minimum is -1 at $x = 4$
D) absolute maximum is 17 at $x = 4$; absolute minimum is -1 at $x = -2$

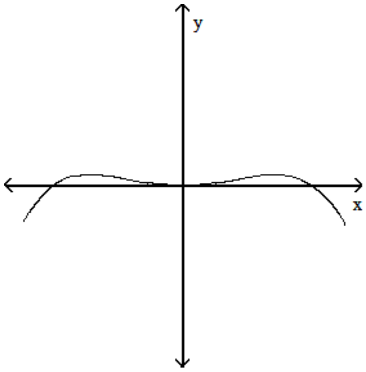
Solve the problem.

11) The graph below shows the first derivative of a function $y = f(x)$. Select a possible graph f that passes through the point P. 11) ________

f′



A)



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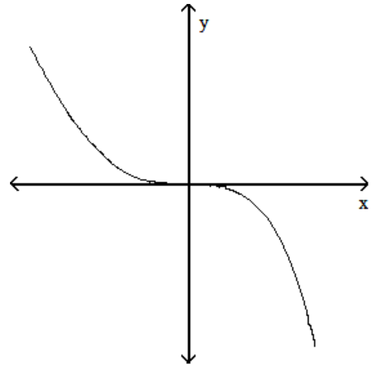

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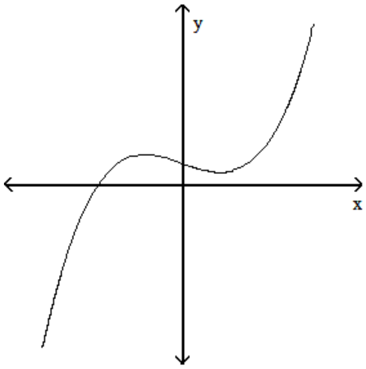
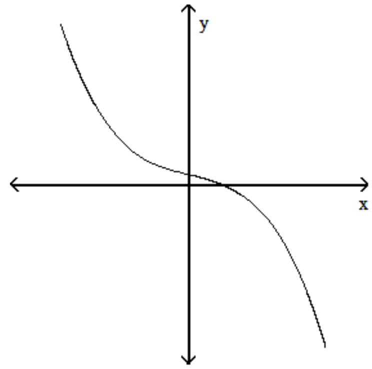


D)



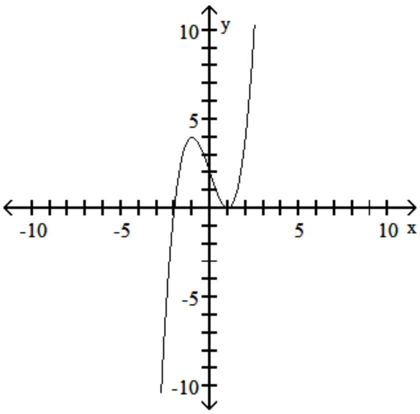
Find the largest open interval where the function is changing as requested.

12) Decreasing $f(x) = \sqrt{4 - x}$ 12) ______

A) $(4, \infty)$ B) $(-\infty, 4)$ C) $(-\infty, -4)$ D) $(-4, \infty)$

Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.

13) 13) ______



A) Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

B) Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(0, \infty)$; concave up on $(-\infty, 0)$

C) Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(-\infty, \infty)$

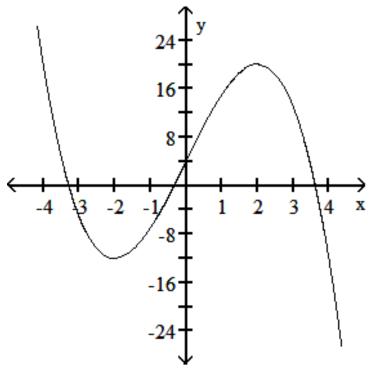
D) Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(-\infty, \infty)$

Solve the problem.

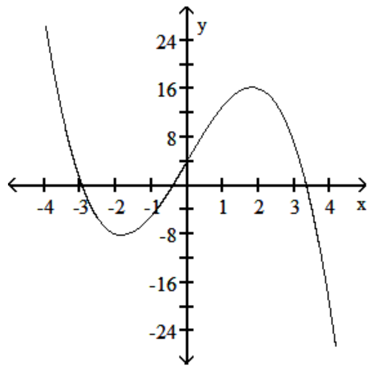
14) Using the following properties of a twice-differentiable function $y = f(x)$, select a possible graph of f. 14) ______

x	y	Derivatives
$x < 2$		$y' > 0, y'' < 0$
-2	12	$y' = 0, y'' < 0$
$-2 < x < 0$		$y' < 0, y'' < 0$
0	-4	$y' < 0, y'' = 0$
$0 < x < 2$		$y' < 0, y'' > 0$
2	-20	$y' = 0, y'' > 0$
$x > 2$		$y' > 0, y'' > 0$

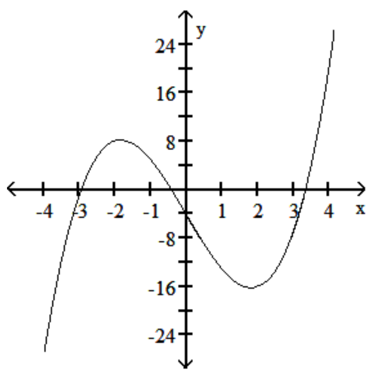
A)



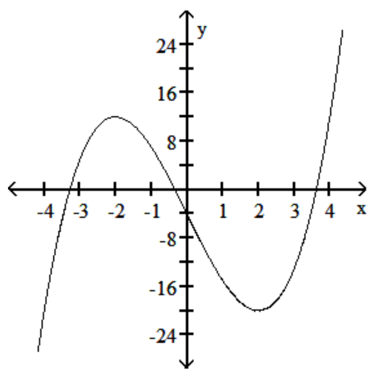
B)



C)

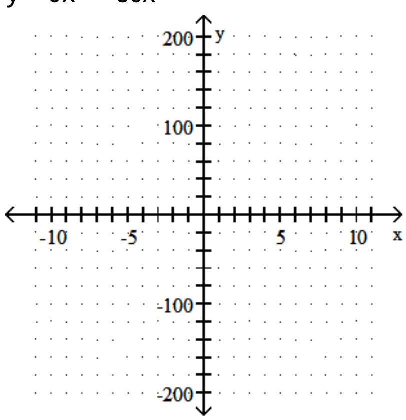


D)

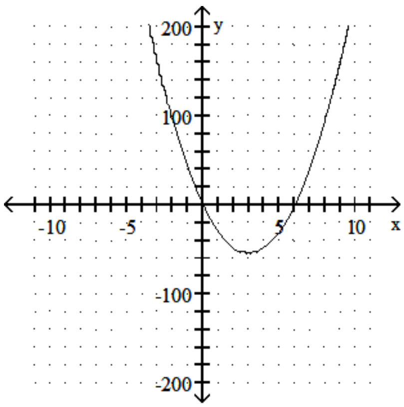


Graph the equation. Include the coordinates of any local extreme points and inflection points.

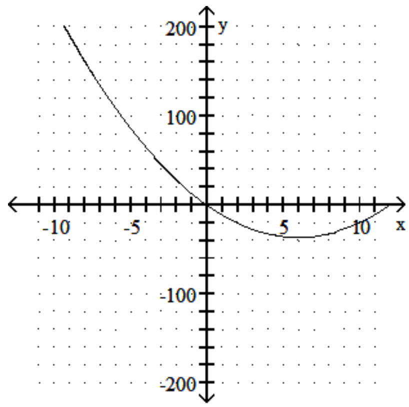
15) $y = 6x^2 + 36x$ 15) ______



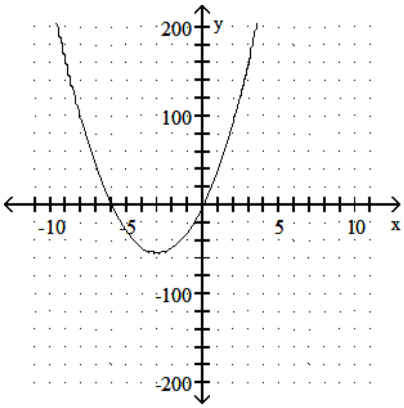
A) local minimum: (3, -54)
no inflection points



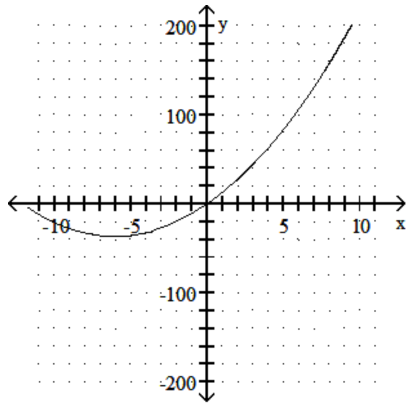
B) local minimum: (6, -36)
no inflection points



C) local minimum: (-3, -54)
no inflection points



D) local minimum: (-6, -36)
no inflection points



Solve the problem.

16) From a thin piece of cardboard 30 in. by 30 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary. 16) _______

A) 10 in. × 10 in. × 10 in.; 1,000 in^3
B) 20 in. × 20 in. × 5 in.; 2,000 in^3
C) 15 in. × 15 in. × 7.5 in.; 1,687.5 in^3
D) 20 in. × 20 in. × 10 in.; 4,000 in^3

Evaluate the limit

17) $\lim\limits_{x \to 0} \dfrac{\cos 5x - 1}{x^2}$ 17) _______

A) $-\dfrac{25}{2}$
B) 0
C) $\dfrac{5}{2}$
D) $\dfrac{25}{2}$

18) $\lim\limits_{x \to \infty} \dfrac{x^2 + 4x + 7}{x^3 + 4x^2 + 10}$ 18) _______

A) 0
B) -1
C) ∞
D) 1

19) $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$ 19) ______

A) 1/4 B) 1/2 C) Does not exist D) 0

20) $\lim\limits_{x \to 3^-} f(x)$, where $f(x) = \begin{cases} -5x + 0 & \text{for } x < 3 \\ 3x + 1 & \text{for } x \geq 3 \end{cases}$ 20) ______

A) -15 B) 2 C) 10 D) 1

21) $\lim\limits_{x \to 4^+} f(x)$, where $f(x) = \begin{cases} -2x + 1 & \text{for } x < 4 \\ 4x + 2 & \text{for } x \geq 4 \end{cases}$ 21) ______

A) 3 B) 2 C) -7 D) 18

22) $\lim\limits_{x \to 0^+} x^{-3/\ln x}$ 22) ______

A) -3 B) $\dfrac{1}{e}$ C) $\dfrac{1}{e^3}$ D) e^3

23) $\lim\limits_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$ 23) ______

A) $3x^2 + 3xh + h^2$ B) Does not exist C) $3x^2$ D) 0

24) $\lim\limits_{x \to \infty} (-4x^{18} + 15)$ 24) ______

A) 15 B) ∞ C) $-\infty$ D) 0

Find all horizontal asymptotes of the given function, if any.

25) $h(x) = \dfrac{2x - 2}{x - 7}$ 25) ______

A) y = 7 B) y = 0
C) y = 2 D) no horizontal asymptotes

Find dy/dx.

26) $2xy - y^2 = 1$ 26) ______

A) $\dfrac{y}{y - x}$ B) $\dfrac{x}{y - x}$ C) $\dfrac{x}{x - y}$ D) $\dfrac{y}{x - y}$

27) $\dfrac{x + y}{x - y} = x^2 + y^2$ 27) ______

A) $\dfrac{x(x - y)^2 - y}{x - y(x - y)^2}$ B) $\dfrac{x(x - y)^2 - y}{x + y(x - y)^2}$ C) $\dfrac{x(x - y)^2 + y}{x + y(x - y)^2}$ D) $\dfrac{x(x - y)^2 + y}{x - y(x - y)^2}$

28) $y = 4 \ln \sin^2 2x$ 28) ______

A) $\dfrac{16}{\sin 2x}$ B) $4 \tan 2x$ C) $16 \cot 2x$ D) $\dfrac{8}{\ln \sin 2x}$

29) $y = 2^x$ 29) ______

A) $2^x \ln x$ B) 2^x C) $x \ln 2$ D) $2^x \ln 2$

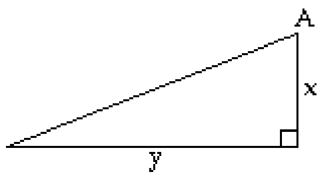
30) $x = \sec(9y)$ 30) ______

A) $\dfrac{1}{9} \cos(9y) \cot(9y)$ B) $9 \sec(9y) \tan(9y)$

C) $\dfrac{1}{9} \sec(9y) \tan(9y)$ D) $\cos(9y) \cot(9y)$

Use the inverse trig functions to express the angle in terms of the indicated unknown side.

31) 31) ______



Given that y = 9, express angle A in terms of x. Use one of the inverse trig functions $\tan^{-1}$, $\sin^{-1}$, or $\cos^{-1}$.

A) $A = \sin^{-1} \dfrac{x}{9}$ B) $A = \tan^{-1} \dfrac{9}{x}$ C) $A = \sin^{-1} \dfrac{9}{x}$ D) $A = \tan^{-1} \dfrac{x}{9}$

Find the derivative of y with respect to x.

32) $y = \tan^{-1} \dfrac{8x}{3}$ 32) ______

A) $\dfrac{9}{64x^2 + 9}$ B) $\dfrac{-24}{64x^2 + 9}$ C) $\dfrac{8}{\sqrt{9 - 64x^2}}$ D) $\dfrac{24}{64x^2 + 9}$

Answer Key
Testname: UNTITLED1

1) D
2) B
3) D
4) C
5) B
6) B
7) B
8) B
9) D
10) B
11) C
12) B
13) A
14) D
15) C
16) B
17) A
18) A
19) B
20) A
21) D
22) C
23) C
24) C
25) C
26) A
27) D
28) C
29) D
30) A
31) B
32) D