HW3 – MATH 221.

Q1) making sure S satisfies vector addition:

for $(a,b),(c,d) \in S$: $a+b=1$, $c+d=1$

then, $(a+c, b+d) \Rightarrow (a+c)+(b+d)=1$

however, $(a+c)+(b+d) = 2 \neq 1$

So, S is not supporting vector addition.

lets see if S satisfies scalar multi:

$x(a,b) = (xa, xb) \xrightarrow{(a+b)=1} x \cdot 1 = x$

this is only possible if $x=1$.

So, S is not supporting scalar mult either.

So, S is not a vector space over $\mathbb{R}$

Q2) Again; Vector addition:

$(a,b),(c,d) \in U \Rightarrow a,b,c,d \geq 0$

in $(a+c, b+d)$ we'll have $a+c \geq 0$, $b+d \geq 0$.

So U satisfies vector addition

Scalar mult:

$(a,b) \in U$, $x \in \mathbb{R} \Rightarrow a,b \geq 0$

if $x > 0$ then $xa, xb \geq 0$, but if $x \leq 0$, $ax, bx < 0$ which $x(a,b) \notin U$

so U does not satisfy scalar mult.

this is enough proof, but furthermore, additive inverse is not satified either:

for $(a,b) \in U$, $(a,b)+(c,d) = (0,0) \Rightarrow (c,d) = (-a,-b)$

but if $a,b \geq 0 \Rightarrow -a,-b \not\geq 0$. So U doesn't satisfy inverses.

Q3) vector addition: $(x_1, y_1) + (x_2 + y_2) = (x_1 + x_2, y_1 + y_2)$, $x_1 + x_2, y_1 + y_2 \in \mathbb{R}$ ✓

scalar mult: $c \cdot (x, y) = (cx, y)$, $cx, y \in \mathbb{R}$ ✓

additive ID: $(x, y) + (0, 0) = (x + 0, y + 0) = (x, y)$ ✓ $(0,0) \in W$

additive inv: $(x, y) + (-x, -y) = (x - x, y - y) = (0, 0)$ ✓

distributivity add: $c \cdot ((x_1, y_1) + (x_2, y_2)) = (c)(x_1, y_1) + (c)(x_2, y_2)$

$= (cx_1, y_1) + (cx_2, y_2) = (cx_1 + cx_2, y_1 + y_2)$

which is equal ($\equiv$) $c(x_1, y_1) + c(x_2, y_2)$ ✓

scalar Id: for $c = 1$, $1 \cdot (x, y) = (x, y)$ ✓

5. scalar inv: $((x, y) \cdot c) \cdot c^{-1} = (cx, y) \cdot c^{-1} = (x, y)$ so, c^{-1} is not applied to y.

so, it fails ???

distributivity mult.: WTS: $(c + d)(x * y) = c(x, y) + d(x, y)$

($\Rightarrow$) $(c + d)(x, y) = (x(c + d), y)$

($\Leftarrow$) $c(x, y) + d(x, y) = (cx + dx, y + y) \neq$ so it fails X

So, W is <u>not</u> a vector space

Q4) vector addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2) = 0 + 0 + 0$ ✓

Scalar mult: $c(x, y, z) = (cx, cy, cz) = c \cdot 0 = 0$ ✓

~~inverses~~ ids: $(x, y, z) + (0, 0, 0) = 0 + 0 + 0 = 0$ ✓, $(x, y, z) \cdot (1) = (x, y, z)$ ✓

~~ID~~ inv: $(x, y, z) + (-x, -y, -z) = (0, 0, 0)$ ✓

distributivity: ~~Proof~~ this hold bcs of $\mathbb{R}^3$ properties, on X.

So, X <u>is</u> a vector space.