HW18 – linear Algebra   Sepehr Akbari

1) $D(ax^3+bx^2+cx+d) = 3ax^2+2bx+c$

2) $M(D)\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $M(D) = \begin{bmatrix} x_{11} & \cdots & 0 \\ x_{12} & \cdots & 0 \\ x_{13} & \cdots & 0 \\ x_{14} & \cdots & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (in class)

so the las col responds to $D(\text{const}) = 0 = 0x^3+0x^2+0x+0$

doing this for all other we get;

$D(x^3) = 0x^3 + 3x^2 + 0x + 0$

$D(x^2) = 0x^3 + 0x^2 + 1x + 0$

$D(x) = 0x^3 + 0x^2 + 0x + 1$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 3a \\ 2b \\ c \end{bmatrix} = 0x^3+3ax^2+2bx+c$ ✓ :)

3) from $M(D)$:
$$\begin{cases} 0a+0b+0c+0d=0 \\ 3a+0b+0c+0d=0 \\ 0a+2b+0c+0d=0 \\ 0a+0b+c+0d=0 \end{cases} \Rightarrow \begin{array}{l} 3a=0 \Rightarrow a=0 \\ 2b=0 \Rightarrow b=0 \\ c=0 \\ d\in\mathbb{R} \end{array}$$

this means anything is zero, but d can be anything in the field.

So $\text{null}(M(D)) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ d \end{bmatrix} \middle| d\in\mathbb{R} \right\}$

4) want: $M(S)\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ c \\ 0 \end{bmatrix} \Rightarrow M(S) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

to get null space $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ c \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} b=c=0 \\ a\in\mathbb{R} \end{array}$

so $\text{null}(M(S)) = \left\{ \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \middle| a\in\mathbb{R} \right\}$

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5)

So $M(P) \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ c \\ a \end{bmatrix} \Rightarrow M(P) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

I can already say the null space is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ bcs this matrix represents the basis. But we can also show it:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ c \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} b = 0 \\ c = 0 \\ a = 0 \end{matrix} \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \text{null}(M(P)) = \{0\}.$