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Sampling distributions

Problem 1

The heights of four friends, in centimeters, are 160, 165, 170, and 185.

(a) Compute the mean μ and standard deviation σ of their heights.

Answer:

```
heights <- c(160,165,170,185)

mean(heights)
```

```
[1] 170
```

```
heights <- c(160,165,170,185)

sd(heights)
```

```
[1] 10.80123
```

(b) List all possible random samples of size 2 (with replacement). Compute the sample mean $\overline{x}$ of each. Hint: order doesn't matter, so there are 10 possibilities.

Answer:

```
heights <- c(160,165,170,185)
num = 1

for (i in 1:length(heights)) {
 for (j in i:length(heights)) {
   samples <- paste0("(", heights[i], ", ", heights[j], ")")
   means <- mean(c(heights[i], heights[j]))

   cat("Sample",num,":", samples, ", Mean =",means, "cm\n")
   num = num + 1
 }
}
```

```
Sample 1 : (160, 160) , Mean = 160 cm
Sample 2 : (160, 165) , Mean = 162.5 cm
Sample 3 : (160, 170) , Mean = 165 cm
Sample 4 : (160, 185) , Mean = 172.5 cm
Sample 5 : (165, 165) , Mean = 165 cm
Sample 6 : (165, 170) , Mean = 167.5 cm
Sample 7 : (165, 185) , Mean = 175 cm
Sample 8 : (170, 170) , Mean = 170 cm
Sample 9 : (170, 185) , Mean = 177.5 cm
Sample 10 : (185, 185) , Mean = 185 cm
```

(c) Compute the mean $\mu_{\overline{x}}$ and standard deviation $\sigma_{\overline{x}}$ of these 10 values of $\overline{x}$. This is the mean and standard deviation of the sampling distribution of the sample mean.

Answer:

```r
heights <- c(160,165,170,185)
xall <- c()

for (i in 1:length(heights)) {
 for (j in i:length(heights)) {
   samples <- paste0("(", heights[i], ", ", heights[j], ")")
   means <- mean(c(heights[i], heights[j]))

   xall <- c(xall,means)
 }
}

mean(xall)
```

```
[1] 170
```

```r
heights <- c(160,165,170,185)
xall <- c()

for (i in 1:length(heights)) {
 for (j in i:length(heights)) {
   samples <- paste0("(", heights[i], ", ", heights[j], ")")
   means <- mean(c(heights[i], heights[j]))

   xall <- c(xall,means)
 }
```

```
}

sd(xall)
```

```
[1] 7.637626
```

(d) Verify that $\mu_{\overline{x}} = \mu$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ in is case.

Answer:

```
heights <- c(160,165,170,185)
xall <- c()

for (i in 1:length(heights)) {
 for (j in i:length(heights)) {
   samples <- paste0("(", heights[i], ", ", heights[j], ")")
   means <- mean(c(heights[i], heights[j]))

   xall <- c(xall,means)
 }
}

if(mean(heights) == mean(xall)){
  cat("They are equal")
}else{
  cat("They are NOT equal")
}
```

```
They are equal
```

```
heights <- c(160,165,170,185)
xall <- c()

for (i in 1:length(heights)) {
 for (j in i:length(heights)) {
   samples <- paste0("(", heights[i], ", ", heights[j], ")")
   means <- mean(c(heights[i], heights[j]))

   xall <- c(xall,means)
 }
}
```

```r
size = 2
stndDev = sd(heights)

approximity <- 0.0000000001

if (abs((stndDev / (sqrt(size))) - sd(xall)) < approximity) {
  cat("They are equal")
} else {
  cat("They are NOT equal")
}
```

```
They are equal
```