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HW20 - linear Algebra

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1) $\text{Area}(S) = \frac{\pi(1)^2}{2} = \frac{\pi}{2}$, $\det(A) = 4$ means area of the shape grows 4 times,

so $\text{Area}(AS) = 4 \cdot \frac{\pi}{2} = \boxed{2\pi}$

2) $\det\begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = 1\big((1)(-1)-(2)(-1)\big) - 1\big((2)(-1)-(1)(-1)\big) + (-2)\big((2)(2)-(1)(1)\big)$
$= 1 + 1 - 6 = -4$

This means the matrix increases the area 4 times and inverts the shape.

Also $\ker = \{\text{span}(0)\} = \{0\}$. Since $\det \neq 0$.

3) $T = \begin{bmatrix} 1/3 & 2/3 \\ -1/2 & 1/2 \end{bmatrix}$ which is defined ~~T: unit circle → elipse~~ T: elipse → unit circle.

$\det(T) = 1/6 - (-1/3) = 1/2$.

This means our elisepse's ~~has~~ area shrinks 50% when mapped to the (x,y)-plane. now since we now the unit circle has an area of π, this means:

$\text{Area}(\text{Elipse}) \times 1/2 = \pi \Rightarrow \text{Area}(\text{Elipse}) = \boxed{2\pi}$

4) $T^{-1} = \frac{1}{1/2}\begin{pmatrix} 1/2 & -2/3 \\ 1/2 & 1/3 \end{pmatrix} = \begin{pmatrix} 1 & -4/3 \\ 1 & 2/3 \end{pmatrix}$

5) $TT^{-1} = \begin{bmatrix} 1/3 & 2/3 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & -4/3 \\ 1 & 2/3 \end{bmatrix} = \begin{bmatrix} 1/3 + 2/3 & -4/9 + 4/9 \\ -1/2 + 1/2 & 4/6 + 2/6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\Rightarrow TT^{-1} = I$ ✓