HW17 - linear Algebra Sepehr Akbari

Q1) (a)

$$\begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3x+3y \\ -x-y \end{bmatrix} \Rightarrow \begin{matrix} 3x+3y=0 \\ -x-y=0 \end{matrix} \Rightarrow y=-x$$

So the includes any vector of form $\begin{bmatrix} x \\ -x \end{bmatrix}$. Since the null space is not just $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, (e.g. $\begin{bmatrix} 5 \\ -5 \end{bmatrix}$) it is not a basis!

(b)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x+z \\ y+z \\ x+y+2z \end{bmatrix} \Rightarrow \begin{matrix} x+z=0 \\ y+z=0 \\ x+y+2z=0 \end{matrix} \Rightarrow x=y=-z$$

So the null space includes vectors of form $\begin{bmatrix} -z \\ -z \\ z \end{bmatrix}$, its also not basis!

Q2)

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x-y+2z \\ 3x-3y+6z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x-y+2z=0 \Rightarrow y=x+2z \\ 3x-3y+6z=0 \end{cases} \Rightarrow \text{null space} = \begin{bmatrix} x \\ x+2z \\ z \end{bmatrix}$$

observe that; $\begin{bmatrix} x \\ x+2z \\ z \end{bmatrix} = x\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ and that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ are ~~Both~~ linearly indep and they span the spce. so they form a basis. is this what the q is asking?

Q3) In this case the dimentionality of M's range has to be less than the amount of columns (3).

if the columns of M were spanning and lin. indep, then it wou'ldn't have been nontrivial. The fact that it is means the colums are dependent which will suggest overlap in the span.

Q4) The way I think of it is that if M is nontrivial, is like our machine is malfunctioning. instead of only outputting origin when recieving and translating else, it is flattening or basically destroying some healthy vectors.

now the bigger this nontrivial null space, the more vectors the machine will ruin, and the more the dimention of the range will shrink.

Q5) Assuming M rotates around (0,0) we can just set θ to the angle of rotation.

$\cos(240) = -\cos(60) = -1/2$, $\sin(240) = -\sin(60) = -\frac{\sqrt{3}}{2}$

to calculate the resulting coordinate:

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{bmatrix} = \begin{bmatrix} -1 + 3\sqrt{3}/2 \\ -\sqrt{3} - 3/2 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}-2}{2} \\ \frac{-2\sqrt{3}-3}{2} \end{bmatrix}$$ is the new coordinate ?