HW22 - linear Algebra Sepehr Akbari

1) $\det\left(\begin{pmatrix} .7 & .4 \\ .3 & .6 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = 0 = \det\begin{pmatrix} .7-\lambda & .4 \\ .3 & .6-\lambda \end{pmatrix} = (.7-\lambda)(.6-\lambda) - .12$

$= \lambda^2 - 1.3\lambda + .30 = 0 \Rightarrow \boxed{\lambda_1 = 1, \lambda_2 = 0.3}$

consider $T^n = PD^nP^{-1}$ where $D^n = \begin{pmatrix} 1^n & 0 \\ 0 & .3^n \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & .3^n \end{pmatrix}$

this means any initial state can be written as a combination of eigenvalues. overtime, the parts of ~~spa~~ the state along $\lambda < 1$ shrink (as its λ^n) and whats left is $\lambda = 1$. meaning, everything will eventully reach the steady state.

2) so $v_0 = \begin{pmatrix} .5 \\ .5 \end{pmatrix}$, so we need to calc $T^{60} v_0 = v_{60}$

$T^{60} = PD^{60}P^{-1}$

for $\lambda = 1$: $\begin{pmatrix} -0.3 & .4 \\ .3 & -.4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \Rightarrow b = \frac{3}{4}a \Rightarrow \vec{v}_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\Rightarrow P = \begin{pmatrix} 4 & 1 \\ 3 & -1 \end{pmatrix}$

for $\lambda = .3$: $\begin{pmatrix} .4 & .4 \\ .3 & .3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \Rightarrow a = -b \Rightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$P^{-1} = \frac{1}{-4-3}\begin{pmatrix} -1 & -1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 1/7 & 1/7 \\ 3/7 & -4/7 \end{pmatrix}$, $D^{60} = \begin{pmatrix} 1^{60} & 0 \\ 0 & .3^{60} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$T^{60} v_0 = \begin{pmatrix} 4 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/7 & 1/7 \\ 3/7 & -4/7 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \overset{\text{wolfram :)}}{=} \begin{pmatrix} 4/7 \\ 3/7 \end{pmatrix} \approx \begin{pmatrix} 0.571 \\ 0.429 \end{pmatrix}$

So, 57.1% mols are in cis form.

42.9% mols are in trans form.

3) here is ar diagonal matrix to start with.

$D = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ if we multiple by our standard basis we get,

$D\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$, $D\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}$ which is D, so

D is an eigenbasis. □