HW10 - linear Algebra

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Q1) we need to check if $av_1 + bv_2 + \dots + nv_m = 0$, mean $a = b = \dots = n = 0$.

let $a, b, c \in \mathbb{R}$. and, $a(1,2,3) + b(4,5,6) + c(7,8,9) = (0,0,0)$

$= (a + 2a, 3a) + (4b, 5b, 6b) + (7c, 8c, 9c) = (0,0,0)$

$= (a + 4b + 7c, 2a + 5b + 8c, 3a + 6b + 9c) = (0,0,0)$

this means:

$a + 4b + 7c = 0 \Rightarrow \boxed{a = -4b - 7c}$

$2a + 5b + 8c = 0 \Rightarrow a = \frac{-(5b + 8c)}{2}$

$3a + 6b + 9c = 0 = 3(a + 3b + 3c)$

$\Rightarrow 3(-4b - 7c + 3b + 3c) = 0$

$= -3b + 12c = 0 \Rightarrow 12c = 3b \Rightarrow \boxed{b = 4c}, \boxed{c = \frac{1}{4}b}$

$\Rightarrow a = -4(4c) - 7c = -23c$

$= -4b - 7(\frac{b}{4}) = -\frac{16b}{4} - \frac{7b}{4} = -\frac{23}{4}$

$\overbrace{2a + 5b + 8c}^{0} - 2(\overbrace{a + 4b + 7c}^{0}) = 0$

So, $3b + 6c = 0 \Rightarrow b + 2c \Rightarrow \boxed{b = -2c}$

$\Rightarrow a + 4(-2c) + 7c = 0 = a - c = 0 \Rightarrow \boxed{a = c}$ so, $\boxed{c = a}$

hence, $a = c = -\frac{b}{2}$

So this $av_1 + bv_2 + cv_3 = 0$, can hold with

$a = c = 2, \; b = -4$

as they are not $a = b = c = 0$,

this is not linearly independent ☒

Q2) we want to see if $\forall p(x) = ax^2+bx+c$, we can express that as some $(n_1)(2) + (n_2)(x-1) + (n_3)(x^2-x) = p(x)$

so lets see. $p(x) = 2n_1 + xn_2 - n_2 + x^2 n_3 - xn_3$

$$= n_3 x^2 + n_2 x + (2n_1 - n_2)$$

lets see if this is ax^2+bx+c.

$$\begin{cases} n_3 = a \\ n_2 = b \\ 2n_1 - n_2 = c \end{cases} \Rightarrow \begin{array}{l} \boxed{n_3 = a} \\ \boxed{n_2 = b} \\ 2n_1 = c + n_2 = c+b \Rightarrow \boxed{n_1 = \frac{c+b}{2}} \end{array}$$

so for $\exists a,b,c \in \mathbb{R}$, there can be $\exists n_1, n_2, n_3 \in \mathbb{R}$ formed.

so $\text{span}(S) = P_2$ ⊠