

# MTH 110 Final Exam

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1. Evaluate the following limits

(a) (5 points)

$$\lim_{t \rightarrow 1} \frac{t^4 - 1}{t^3 - 1}$$

(b) (10 points)  $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$  The answer here will be a function of  $x$ .

(c) (5 points) Use the fact that  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$  to evaluate the following

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}.$$

2. (10 points) Use the definition of the derivative to find a formula for  $f'$ , where

$$f(x) = \sqrt{9 - x}$$

State the domain of both  $f$  and  $f'$

3. (10 points) Find an equation for the line tangent to the graph of  $f(x) = \frac{\sqrt{x}}{x+1}$ , and passing through the point  $(4, f(4))$ . Put your answer in slope-intercept form

4. (5 points) Differentiate

$$f(x) = \cos^4(\sin^3(x))$$

5. (5 points) The equation

$$f''(x) + f'(x) - 2f(x) = x^2$$

is called a differential equation because it involves an unknown function  $f$  and its derivatives  $f'$  and  $f''$ . Find constants A, B, and C such that the function  $f(x) = Ax^2 + Bx + C$  satisfies the equation

6. (5 points) If  $x^2 + xy + y^3 = 1$ , find the value of  $y'$  or  $\frac{dy}{dx}$  at the point where  $x = 1$ .

7. (15 points)

Consider the function  $f(x) = x^2(x + 4)$ .

- (a) Find the x and y intercepts
- (b) Find the intervals on which  $f$  is increasing and decreasing.
- (c) Find all local maxima and minima of  $f$ .
- (d) Find the intervals on which  $f$  is concave up and concave down.
- (e) List the inflection points of  $f$ .
- (f) Draw a nice graph of  $f$

8. (10 points) A box with a square base and an open top must have volume of 32,000  $\text{cm}^3$ . Find the dimensions of the box that minimize the amount of material used to make the box.

9. Evaluate the integral

(a) (10 points)

$$\int_1^9 \frac{\sqrt{x} - 2x^2}{x} dx$$

(b) (10 points)

$$\int \frac{x + 2}{\sqrt{x^2 + 4x}} dx$$

**Good Luck**

**Total (      / 100)**