HW 23 - linear Algebra

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1) $\bar{x}_{math} = \frac{85 + \dots + 70}{5} = 82.6$

$\Rightarrow \frac{(85-82.6)^2 + \dots + (70-82.6)^2}{5-1} \approx 75.8 \sim var(math)$

$\bar{x}_{science} = \frac{90 + \dots + 75}{5} = 85.8$

$\Rightarrow \frac{(90-85.8)^2 + \dots + (75-85.8)^2}{5-1} \approx 55.2 \sim var(science)$

2)

$x_i - \bar{x}$	$y - \bar{y}$	$(x-\bar{x})(y-\bar{y})$
2.4	4.2	10.8
-4.6	-3.8	17.48
9.4	2.2	20.68
5.4	8.2	44.28
-12.6	-10.8	136.08

$\Rightarrow Cov(m, s) = \frac{10.8 + \dots + 136.08}{5-1} = 57.15$

3)

	M	S
M	var(M)	cov(M,S)
S	cov(S,m)	var(S)

$= \begin{bmatrix} 75.8 & 57.15 \\ 57.15 & 55.2 \end{bmatrix}$

4) $(75.8 - \lambda)(55.2 - \lambda) = (57.15)^2 \xrightarrow{wa} \begin{array}{l} \lambda_1 = 123.645 \\ \lambda_2 = 7.355 \end{array}$

the matrix is already dim=2. It would make sence to reduce to dim=1, since one eigenvalue (PC) is much bigger (explaining more) than the other. so we need to reduce it to the eigenvector of λ_1.

5) $\begin{bmatrix} 75.8-123.645 & 57.15 \\ 57.15 & 55.2-123.645 \end{bmatrix} \Rightarrow \begin{bmatrix} -47.845 & 57.15 \\ 57.15 & -68.445 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow b = \dfrac{47.845}{57.15} a \Rightarrow \vec{v}_1 \approx \begin{bmatrix} 1 \\ 0.837 \end{bmatrix}$ of $\lambda_1 = 123.6$

6) Something is wrong...

with this math scores are remained untouched!

So it'll be $\text{Score}_m + \text{Score}_s (0.837)$:

A → 160.33

B → 146.63

C → 165.66

D → 166.64

E → 132.78

I think this is incorrect (although AI disagrees haha), cause ~~..~~ λ_1 is not covering 100% of the variance, so having Math scores unchanged does not make sense