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Introduction to confidence intervals

Problem 1

In a random sample of $n = 55$ repairs at a certain auto garage, the average cost was \$374.75. Construct a level 90% confidence interval for the population mean repair cost at this garage. Assume the population standard deviation is $\sigma = \$72.50$.

Answer:

```
n <- 55
x <- 374.75
stndDev <- 72.5

z <- qnorm(0.95) # 1.645
low_bound <- x - (z * (stndDev / sqrt(n)))
up_bound <- x + (z * (stndDev / sqrt(n)))

cat("The cost will be between",round(low_bound,2),"and",
    round(up_bound,2),"dollars.")
```

```
The cost will be between 358.67 and 390.83 dollars.
```

Problem 2

Repeat problem 1 with a sample of size $n = 85$. Which confidence interval is wider? Briefly explain.

Answer:

```
n <- 55
n2 <- 85
x <- 374.75
stndDev <- 72.5

z <- qnorm(0.95) # 1.645
low_bound <- x - (z * (stndDev / sqrt(n)))
```

```r
low_bound2 <- x - (z * (stndDev / sqrt(n2)))
up_bound <- x + (z * (stndDev / sqrt(n)))
up_bound2 <- x + (z * (stndDev / sqrt(n2)))

cat("The cost will be between",round(low_bound2,2),"and",
    round(up_bound2,2),"dollars.\n\n")
```

```
The cost will be between 361.82 and 387.68 dollars.
```

```r
if(round(up_bound2,2) - round(low_bound2,2) >
   round(up_bound,2) - round(low_bound,2)){
  cat("The sample size",n,"will have a narrower confidence level")
}else{
  cat("The sample size",n2,"will have a narrower confidence level")
}
```

```
The sample size 85 will have a narrower confidence level
```

As shown, the confidence interval for the sample size of 85 will be narrower than the one for 55. This is because the margin of error is inversely proportional to the square root of the sample size. With a larger sample size (n=85), the standard deviation of the sample mean becomes smaller, leading to a smaller margin of error and a narrower confidence interval.

Problem 3

In a random sample of $n = 218$ eruptions of a geyser, the mean duration was 2.42 minutes. Construct a level 95% confidence interval for the population mean. Assume the population standard deviation is $\sigma = 1.04$ minutes.

Answer:

```r
n <- 218
x <- 2.42
stndDev <- 1.04

z <- qnorm(0.975) # 1.960
low_bound <- x - (z * (stndDev / sqrt(n)))
up_bound <- x + (z * (stndDev / sqrt(n)))

cat("The eruption will be between",round(low_bound,2),"and",
    round(up_bound,2),"minutes \n\n")
```

```
The eruption will be between 2.28 and 2.56 minutes
```

Problem 4

Repeat problem 3 with a standard deviation of $\sigma = 1.92$. Which confidence interval is wider? Briefly explain.

Answer:

```
n <- 218
x <- 2.42
stndDev <- 1.04
stndDev2 <- 1.92

z <- qnorm(0.975) # 1.960
low_bound <- x - (z * (stndDev / sqrt(n)))
low_bound2 <- x - (z * (stndDev2 / sqrt(n)))
up_bound <- x + (z * (stndDev / sqrt(n)))
up_bound2 <- x + (z * (stndDev2 / sqrt(n)))

cat("The eruption will be between",round(low_bound2,2),"and",
    round(up_bound2,2),"minutes \n\n")
```

```
The eruption will be between 2.17 and 2.67 minutes
```

```
if(round(up_bound2,2) - round(low_bound2,2) >
   round(up_bound,2) - round(low_bound,2)){
  cat("The sample with standard deviation",stndDev,
      "will have a narrower confidence level")
}else{
  cat("The sample with standard deviation",stndDev2,
      "will have a narrower confidence level")
}
```

```
The sample with standard deviation 1.04 will have a narrower confidence level
```

As shown, the confidence interval for the sample's standard deviation of 1.04 will be narrower than the one for 1.92 This is because the margin of error is directly proportional to the standard deviation of the sample's standard deviation. Just opposite question 2. So as the standard deviation decreases the gap between the bounds also decrease.