- Qi) (et $U = \{(x_1, x_2, x_3) | x_1 x_2 x_3 = 0, x_i \in \mathbb{R} \}$ Based on thm 1.34, U is a v.s. if it is a subspace of v.s. \mathbb{R}^3 . (et's check that:)
 - Additive identity

 additive id is $(0,0,0) \in \mathbb{R}^3$ and 0.0.0=0. Since $(0,0,0) \in \mathbb{U}$ the condition holds $\sqrt{8}$
 - 2) closed under addition

let $\alpha = (a_1, a_2, a_3) \in U$ and $b = (b_1, b_2, b_3) \in U$. this means $a_1 a_2 a_3 = b_1 b_2 b_3 = 0$

 $\Rightarrow a+b = (a_1+b_1, a_2+b_2, a_3+b_3) = 0$ however this does not always hold for example $(1,0,1), (0,1,1) \in U$ but $(1,0,1) + (0,1,1) = (1+0,0+(,1+1) = (1,1,2) \notin U$ So addition is NOT closed P

3) closed under scalar multiplication

let a = (a,, az, az) & U, and KER => ka = k(a,, az, az) = (ka,, kaz, kaz) = 0 bcs at least on a,... az is equal to zero, one ka,... kaz is also zero so, scalar melt is closed v

Since U failed to satisfy all conditions, its not a subspace of IR.

Therefore

QZ) (et W = IR. = {(a,b,c)|a,b,c61R}

We can say: $W_1 = \{(a,b,0) \mid a \in \mathbb{R}\} \subseteq \mathbb{R}$ $W_2 = \{(0,b,0) \mid b \in \mathbb{R}\} \subseteq \mathbb{W}$

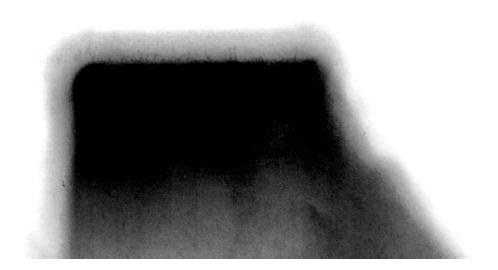
let's prove they are subspaces quick:

1 (0,0,0) is the additive id in IR3, and (0,0,0) & W., Wz. 1

(a,0,0) + (\hat{a},0,0) = (\hat{a}+\hat{a},0,0) \in W_1 (0,b,0) + (0,\hat{b},0) = (0,b+\hat{b},0) \in W_2

3 K (a,0,0) = (ka,0,0) & W1 K(0,b,0) = (0,bK,0) & W2

So lets check if $W_1 + W_2 \neq W$: $W_1 + W_2 = (a,0,0) + (o,b,0) = (a,b,0)$ bcs $(0,0,c) \in \mathbb{R}^3 = W$ but $(0,0,c) \notin W_1 + W_2$ then, $W_1 + W_2 \neq W$

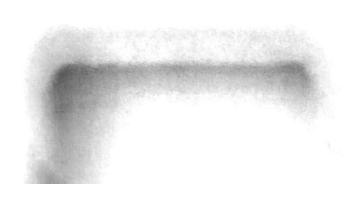


- Q3) (1) additive id in V is $(0,0,0) \in \mathbb{R}^3$ $(0,0,0) \in V_1$, bus $(0,0,0) \cdot (x,y,0) = (0,0,0)$ $(0,0,0) \notin V_2$, bus (0,y,z) = (0,0,0)
 - ② (It $(x,y,0), (\tilde{x},\tilde{y},0) \in V_1$. $(x,y,0)+(\tilde{x},\tilde{y},0) = ((x+\tilde{x}),(y+\tilde{y}),(o+o)), x+\tilde{x}, y+\tilde{y} \in \mathbb{R}_{\tilde{x}} \in \mathbb{R}_$
 - 3) (et $k \in \mathbb{R}$, $kv_1 = k(x,y,0) = (kx,ky,0), kx,ky \in \mathbb{R}, so(kx,ky,0) \neq V = \mathbb{R}^3$ $kv_2 = k(0,y,z) = (0,ky,kz), so(k0,ky,kz) \in V = \mathbb{R}^3$

* Just realized I proved $V_1, V_2 \subseteq V$, not $V_1 + U_2 \subseteq V$. 0000ps! Lets try that again: $V_1 + V_2 = (x_1y_10) + (0,y_12) = (x_1, 2y_12) = (x_2, 2y_12) = (x_1, 2y_2) = (x_2, 2y_12) = (x_1, 2y_22) = (x_2, 2y_22) = (x_1, 2y_22) = (x_2, 2y_22) = (x_1, 2y_22) = (x_2, 2y_22) = (x_2, 2y_22) = (x_1, 2y_22) = (x_2, 2y_22)$

- 1 0- A = (0,0,0), so (0,0,0) &A /
- (1) let a, à f A a+ à = (x+x, 2y+ 2y, z+2). x+x, 2y+2y, z+2 EIR, so a+ à f A
- B let α 6 A, λ + R. λα = (λχ, 2λy, λz). λχ, 2λy, λz + R, so λα + A. /

So A=V, + V2 € V. Ø



Q4) (et V = (a,b,c) and $V_1 + V_2 = (x,2y+2)$ and $V_1 + V_2 \leq V$ the $V_1 + V_2 = V$ means $\forall (a,b,c), \exists (x,2y,2)$ in this case (x,2y,z) = (a,b/2,c). $a,c \in \mathbb{R}$ done and of assuming bell $(as \ V = \mathbb{R}^3)$,

thus any real number divided by z is still real so $\underline{b} \in \mathbb{R}$.

So $a,b/2,c \in \mathbb{R}$. so, $V_1 + V_2 = V$

Q5) (If $V = (1,2,3) \in \mathbb{R}^3$. for $V_1 + V_2 = V$, $V_1 = (1,1,0)$ and $V_2 = (0,1,3)$ but in $V_1 : x + y = 1 + 1 \neq 0$ and in $V_2 : y + z = 1 + 3 \neq 0$ So, $(1,2,3) \in \mathbb{R}^3$ but $(1,2,3) \notin V_1 + U_2$. WAIT IS THAT RIGHT???

Q5-2) V_1 can be written as (x, -x, 0) V_2 can be written as (0, y, -y)So $V_1 + U_2 = \{(x, -x + y, -y) | x, y \not\in \mathbb{R}^2\}$ So for $\forall (a,b,c), \exists (x,-x + y,-y)$ So a = x, b = y - x, c = -y = y = b + x = b + a = z = -(b + a) = y = a + b + c = 0So this is a constraint, it means only (a_1b,c) s that a + b + c = 0 eye in $V_1 + V_2$ for example $(V_1 \cap V_1)$ is not in $V_2 \cap V_2 \neq V_2 \cap V_3 \cap V_4 \cap V_4 \cap V_5 \cap V_5 \cap V_5 \cap V_6 \cap V$