A Flexible Mathematical Framework for Model Comparison: U-TIM (version 1.0)

João Lucas Meira Costa (ideas)
ChatGPT, DeepSeek & Gemini (equations, code & documentation)

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Abstract

The Universal Theory Incoherence Measure (U-TIM) is a mathematical framework for comparing and evaluating models across diverse domains by quantifying the inconsistencies between their predictions. This document introduces the U-TIM framework, details its mathematical formulation and Python implementation, and discusses its potential applications, particularly in physics for evaluating Theories of Everything (TOEs). The flexible design of UTIM allows for adaptation to various domains, including physics, biology, economics, and more, with the goal of providing a universal tool for model assessment and comparison.

1 Introduction

The Universal Theory Incoherence Measure (U-TIM) is a flexible **mathematical** framework designed to identify and quantify inconsistencies between different models in various domains. It is not tied to any specific physical interpretation or domain-specific assumptions. Instead, it provides a general mathematical structure that can be adapted and applied to a wide range of models, from physical theories to economic models, focusing on the **incoherence** between their predictions. This document explains the mathematical foundation of U-TIM, its implementation, and how it can be applied to physics, specifically in the context of evaluating Theory of Everything (TOE) candidates. Detailed instructions for applying U-TIM to specific domains will be provided in separate documentation.

2 Mathematical Formulation

U-TIM is based on the idea of comparing the outputs of different models across a shared input space. It is defined as a **mathematical equation** designed for flexibility and adaptability. The key components are the following.

- $\mathcal{M} = \{M_1, M_2, ..., M_n\}$: A set of n models. These could be different theories, simulations, or any other mathematical representation of a system.
- M_r: A designated reference model. This serves as a baseline for comparison. In physics, this could be the Standard Model, General Relativity, or a combination thereof.
- X: The input space. This represents the set of all possible input parameters that the models can accept. In physics, these could be fundamental constants, initial conditions, or other relevant parameters.
- Y: The output space. This represents the set of all possible outputs that the models can produce. In physics, these could be predicted observables, experimental results, or other measurable quantities.
- $f_i(x)$: The output of the model M_i for a given input $x \in X$. Thus $f_i : X \to Y$.
- $d(y_1, y_2)$: A distance metric defined on the output space Y. This measures how "different" the two outputs $y_1, y_2 \in Y$ are. The choice of distance metric is crucial and depends on the specific application.
- $w(x)$: A weight function is defined on the input space X. This assigns different "importance" to different regions of the input space.

2.1 U-TIM Score

The U-TIM score for a model M_i relative to the reference model M_r is defined as:

$$\text{U-TIM}(M_i) = \int_X w(x) \cdot d(f_i(x), f_r(x))\, dx \tag{1}$$

This integral represents the weighted average of the distances between the outputs of model M_i and the reference model M_r across all possible inputs $x \in X$. A lower U-TIM score indicates that the model M_i is "closer" to the reference model M_r in terms of its predictions.

2.2 Pairwise Coherence

We can also define a pairwise coherence measure between any two models M_i and M_j:

$$C(M_i, M_j) = \int_X w(x) \cdot d(f_i(x), f_j(x))\, dx \tag{2}$$

This measures the similarity or difference between the predictions of models M_i and M_j directly.

3 Implementation (Python)

The provided Python code implements a discretized version of U-TIM, where the integral is approximated by a sum over the sampled input values.

Here is the Python code for the UniversalTIM class:

```python
import numpy as np
from scipy.spatial import distance

class UniversalTIM:
    def __init__(self, models, reference_fn,
                 X_samples,
                 distance_metric='euclidean',
                 weight_fn=lambda x: 1):
        """
        Universal TIM implementation for cross-domain models

        Parameters:
        - models: dict {name: (x) -> y} (callable models)
        - reference_fn: (x) -> y (reference model)
        - X_samples: array (n_samples, n_dims) parameter samples
        - distance_metric: str/callable (default: Euclidean)
        - weight_fn: (x) -> scalar weight (default: uniform)
        """
        self.models = models
        self.ref_fn = reference_fn
        self.X = X_samples
        self.dist_metric = distance_metric
        self.weight_fn = np.vectorize(weight_fn, signature='(n)->()')

    def _compute_distances(self, fn1, fn2):
        """Compute distances between two functions across X samples"""
        Y1 = np.array([fn1(x) for x in self.X])
        Y2 = np.array([fn2(x) for x in self.X])
        return distance.cdist(Y1, Y2, self.dist_metric).diagonal()

    def calculate_utim(self):
        """Calculate U-TIM scores for all models"""
        scores = {}
        w = self.weight_fn(self.X)

        for name, model in self.models.items():
            dists = self._compute_distances(model, self.ref_fn)
            scores[name] = np.sum(w * dists)

        return scores
```

```
def pairwise_coherence_matrix(self):
    """Create full coherence matrix between all models"""
    model_names = list(self.models.keys())
    n_models = len(model_names)
    matrix = np.zeros((n_models, n_models))
    w = self.weight_fn(self.X)

    for i in range(n_models):
        for j in range(i, n_models):
            fn1 = self.models[model_names[i]]
            fn2 = self.models[model_names[j]]
            dists = self._compute_distances(fn1, fn2)
            matrix[i,j] = matrix[j,i] = np.sum(w * dists)

    return matrix, model_names
```

The code uses ‘scipy.spatial.distance.cdist‘ to calculate distances between model outputs and ‘numpy‘ for numerical operations. The weight function is applied element-wise to the distances.

4 Applying U-TIM to Physics (TOE Evaluation)

U-TIM can be a valuable tool in theoretical physics, particularly in the search for a Theory of Everything (TOE). It provides a *mathematical* framework for comparison, but its application requires careful consideration of the specific physical context. Detailed instructions for applying U-TIM to physics and other domains will be provided in separate documents. However, we can outline the general process in the context of TOE evaluation:

1. **Define the TOE Candidates:** Identify a set of candidate TOEs $(M_1, M_2, ..., M_n)$ that you want to compare. These could be different theoretical frameworks attempting to unify all fundamental forces.

2. **Choose a Reference Model:** Select a reference model (M_r) that represents our current understanding of physics. This could be the Standard Model combined with General Relativity.

3. **Define the Input Space (X):** Identify the relevant input parameters for the TOE candidates. These could be fundamental constants (e.g., gravitational constant, speed of light), free parameters within the theories, or initial conditions for cosmological models. The input space should be common to all the models being compared. This is likely the most challenging step and requires careful consideration of the physical meaning of the parameters.

4. **Define the Output Space (Y):** Determine the relevant outputs that the TOE candidates predict. These could be predicted particle masses, scattering cross-sections, cosmological parameters, or other observable quantities. Again, the output space should be defined in a way that allows for comparison between

the different models. The physical meaning and measurability of these outputs are crucial.

5. **Choose a Distance Metric (d_Y):** Select a distance metric that is appropriate for comparing the outputs in Y. If the outputs are numerical, you could use Euclidean distance, or a weighted version of it. If the outputs are more complex (e.g., probability distributions), you might need a more specialized metric (e.g., Kullback-Leibler divergence). Consider the physical significance of the differences you're measuring.

6. **Define the Weight Function (w(x)):** Assign weights to different regions of the input space X. You might want to emphasize regions that are more experimentally accessible or regions that are considered more theoretically important. The physical justification for the chosen weights is essential.

7. **Sample or Integrate:** If the input space X is continuous, you will need to either sample it (as in the Python code) or use numerical integration techniques to approximate the integral in the U-TIM formula. The sampling approach requires careful consideration of the sampling density to ensure accuracy. For high-dimensional spaces, Monte Carlo integration is often preferred for its efficiency.

8. **Calculate U-TIM Scores:** Use the U-TIM formula (or the Python implementation) to calculate the U-TIM score for each TOE candidate relative to the reference model.

9. **Interpret the Results:** Analyze the U-TIM scores. A lower U-TIM score suggests that a TOE candidate is more consistent with the reference model (and thus indirectly with the current experimental data). However, it is crucial to remember that U-TIM is just one tool among many. It does not prove that a theory is correct, but it can help to rank and compare different candidates. Consider the limitations of the chosen input space, output space, distance metric, and weight function. The physical interpretation of the U-TIM values is paramount.

5 Conclusion

U-TIM is a flexible **mathematical** framework for comparing different models by identifying and quantifying the inconsistencies between them. Although the mathematical formulation is relatively straightforward, practical application requires careful consideration of the specific domain. Detailed instructions for applying U-TIM to specific areas, including physics, will be provided in separate documents, one for each specific area. In physics, U-TIM can be a valuable tool for evaluating TOE candidates and guiding further theoretical development, but it should be used in conjunction with other theoretical and experimental methods. The ongoing development of U-TIM, including addressing the challenges related to truly universal application and computational efficiency, is an area of active research.

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