Bayesian Range Only SLAM with SOGs

RO-SLAM Based on Particl Filter (RBPF) for Unmanned Vehicles

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***Abstract*—This present a Range-Only SLAM (RO-SLAM) algorithm based on Rao-Blackwellized Particle Filter (RBPF) knowing only odometry and range sensor data. The algorithm is reformulated to estimate and update parameters of the particle filter sequentially with a sum of Gaussians (SOGs). The comparison of the results achieved by the proposed RO-SLAM algorithm and Monte-Carlo approximation algorithm in an experimental setup illustrates a significant improvement in accuracy, computation time, and robustness against outliers.**

Keywords—RO-SLAM; Rao-Blackwellized particle filter; Monte-Carlo aproxiamtion; SOGs;

I. INTRODUCTION

The problem of Simultaneous Localization and Mapping (SLAM) has been a topic with a very intensive research in unmanned and autonomous robotics within last decades. A very tangible example of this algorithm is marital rovers and unmanned drown which is widely used in the military applications. Almost all of the proposed approaches rely on probabilistic Bayesian filtering such as Kaman Filter (KF), EKF, and Particle Filter (PF) because of lots of uncertainty involved in the target application either by environment or by lacking precision in the measurement tools as well as sensor itself. Besides, in any practical application only noisy measurements are available and imperfect actions can be performed that's the reason why the Bayesian filter constitutes one part of any realistic scenarios. The classic SLAM (so called standard SLAM) consists three main components, i.e. Localization, Mapping and movement. The main focus of the current paper is Mapping, although is not possible to split above mentioned three parts since they are connected together by a set of recursive equations.

Many researches focused on the type of sensors employed for SLAM to make precise laser scanners, stereo cameras (both bearing and range) and monocular cameras (for bearing-only). However, only few publications have addressed the problem of building maps with sensors providing range-only (RO) data, despite of its importance in many important applications such as autonomous submarine [4] or ground vehicles in real environments [1]. From now on, this method is called RO-SLAM to distinguish standard SLAM from proposed SLAM.

A research in RO-SLAM faces with two fundamental challengings: the existence of outliers due to the sensor nature (typically sonar or radio pulses), and more importantly the high ambiguity of the measurements. The latest challenge is illustrated in Fig.1. Two problems with these sensors are: (1) the large portion of the environment where a beacon could be, given just one observation, and (2) the very likely possibility of multiple plausible hypotheses (as shown in the Fig.1).

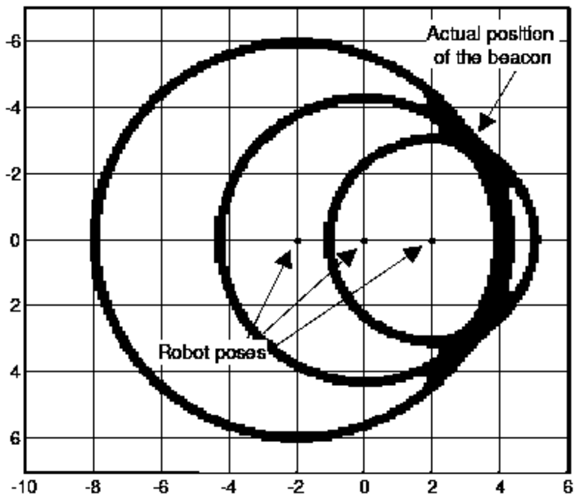

Fig. 1. High ambiguity of the measurement in RO-SLAM: One peculiarity of Range-Only SLAM is that map estimations may converge to multi-modal densities. In this example, the symmetry in the observations made by a robot over a straight path leads to two regions with a high probability of containing the sensed beacon.

The RO-SLAM algorithm proposed by Singh et al [5] reported a geometric method for adding new beacons to a map using delayed initialization, although a partially known map is required as an initial condition. A range only localization under the classic EKF were addressed in [2] and [3]. Kurth et al [3] used equation (1) and (2) as a base for Kalman formulation in their SLAM scheme. Equation (1) is called process equations, which predict state using previous simulation, pose and position of the robot. Equation (2) is the measurement equations which model the sensor and includes the sensor parameters and measurement uncertainties.

$$X_{k+1} = \begin{bmatrix} x_k + \Delta D_k \cos(\theta_k) \\ y_k + \Delta D_k \sin(\theta_k) \\ \theta_k + \Delta\theta_k \end{bmatrix} + v_k \triangleq f(X_k) + 0\times u_k + v_k \tag{1}$$

$$Z_{k+1} = \begin{bmatrix} \sqrt{(x_b - x_k)^2 + (y_b - y_k)^2} \\ \sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2} \\ \theta_k - \theta_{k-1} \end{bmatrix} + w_{k+1} = \begin{bmatrix} r_k \\ \Delta D_k \\ \Delta\theta_k \end{bmatrix} + w_{k+1} \triangleq h(X_{k+1}) + w_{k+1} \quad (2)$$

Where, X_k is the robot state at time k, which will be represented by $[x_k \quad y_k \quad \theta_k]^T$. Coordinate x_k, y_k and angle θ_k are the robot's position and orientation at time k, temporal variation ΔD_k is the odometric distance travelled, angular variation $\Delta\theta_k$ is the change in heading, r_k is the range measurement received at time k, and (x_b, y_b) is the location of the beacon from which a measurement is received.

The above process equations (1) and the sensor model (2) is non-linear, but the error distribution is still Gaussian which implies using a type of Kalman filter. Since the process and sensor model are not linear using Kalman filter formulation directly is not possible and as consequence Extended Kalman Filter (EKF) algorithm is applied. Author linearized process and sensor model equations about the current state estimate discrete time system by (3) and (4).

$$A_k = \frac{\partial f}{\partial x}\Big|_{X = \hat{X}_{k+1}^-} \quad (3)$$

$$H_{k+1} = \frac{\partial h}{\partial x}\Big|_{X = \hat{X}_{k+1}^-} \quad (4)$$

Then, the non-linear set of estate estimation equations (1) and (2) becomes a linear set as (5) and (6).

$$X_{k+1} = A_k X_k + 0 \times u_k + v_k \quad (5)$$

$$Z_{k+1} = H_{k+1} X_{k+1} + w_{k+1} \quad (6)$$

Now, the Kalman Filter formulation [6] can be applied to estimate the system state in SLAM.

Blanco et al. [7] propose an approximation of the sensor model inspired by the circular shaped distributions obtained for range sensors to resolve high ambiguity of the measurement. They also address map building but assuming a prior knowledge about the beacon locations. Thus a non-linear process and sensor model as well as non-Gaussian error distribution, which imply using particle filter or Bayesian, filter. The main difference of [4] with [7] is the usage of a least-square error minimization procedure instead of a probabilistic filter.

The current work presents three main contributions with respect to previous works: (1) a new inverse sensor model for initializing map distributions as weighted SOGs, (2) the explanation of how to update those Gaussians and their weights using a multi-hypothesis EKF, and (3) the derivation of the corresponding observation model required for the RBPF.

To this end, the paper organized as follow: in section II, the problem will be explained and a new set of recursive equations for RO-SLAM will be derived. The RO-SLAM algorithm will be reformulated for current assumption and based on RBPF prediction equations. Section III will present some experimental setup to evaluate the performance of the algorithm and provides some results. This section will end up to some discussions over result achieved. The findings of the paper will be concluded in section IV and future work will be proposed. Short-listed papers will be provided at the end of the paper for further reference and future work.

II. Problem and Solution

A. Problem statement

First of all, we limit our application two only robotics in this paper. Besides, time step t instead of index k will be used from now on. The same standard notation of the SLAM as [6] will be followed here. We denote the robot path as a sequence of poses in time $x^t = \{x_1, x_2, \cdots, x_t\}$, robot actions (odometry) and observations (range measurements) at each time step t will be represented by u_t and z_t, respectively. The RBPF approach to estimating the joint SLAM posterior $p(x^t, m | u^t, z^t)$ consists of approximating the marginal distribution of the robot path x^t using importance sampling, then computing the map as a set of conditional distributions given each path hypothesis. The motivation for choosing a RBPF approach is that it allows factoring the distribution of the map associated to each particle as shown in (7).

$$p(m|x^t, z^t, u^t) = \prod_l p(m_l | x^t, z^t) \quad (7)$$

Where, m_l denotes the different individual beacon positions in the map m. Suppose a set of particles for the previous time step $x_{t-1}^{[i]}$ are approximately distributed according to the real posterior (for the first time step all the particles are arbitrarily initialized to the origin). New particles are drawn using the robot motion model (derived from odometry readings) at each time stamp (8). Then, the important weights are updated by (9).

$$x_t^{[i]} = p\left(x_t \middle| x_{t-1}^{[i]}, u_t\right) \quad (8)$$

$$\omega_t^{[i]} = \omega_{t-1}^{[i]} p\left(z_t \middle| x_t^{[i]}, z_{t-1}\right) \quad (9)$$

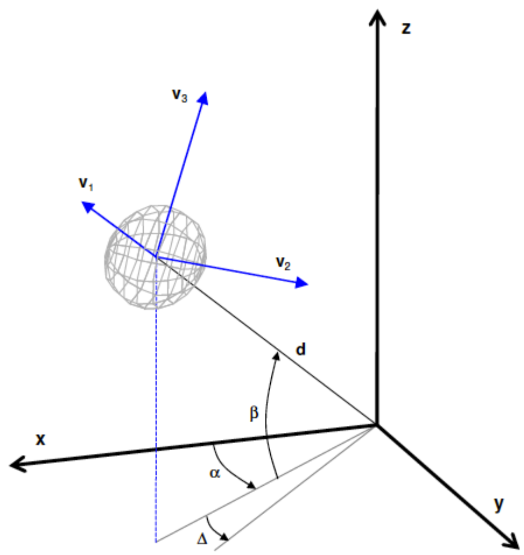

Fig. 2. The variables involved in the generation of the Gaussian modes within each SOG. Azimuth and elevation angles from the sensor position (the coordinate origin in the figure) are represented by α and β, respectively. The

covariance matrix is computed by mapping the uncertainties in the radial (v_1) and tangent (v_2, v_3) directions using the appropriate transformation matrix.

Two key questions shall be answered now to make above update at each time step feasible: 1- "*how is the probability distribution (7) estimated and updated in each iteration of RBPF?"2-How do we calculate (9)?* Following section will answer above questions.

B. Solution and answare to the questions

Once defined the prior belief as a uniform, it follows from (10) that the initial map distribution becomes simply the inverse sensor model $p(z_t|m, x^t, z^{t-1})$ (or its evaluation over half of the 3D space in the special case).

Assuming a range sensor model with additive Gaussian noise of variance σ_s^2, the probabilistic model is as (11). Equation (11) cannot be filtered iteratively in any analytical form, thus we must rely on approximations in RO-SLAM. As a result, a Sum of Gaussians (SOG) is adopted to approximate this distribution as (12). Mean and variance of each Gaussian in equation (12), $\widehat{m}_t^k$ and $\sum_t^k$, are calculated using (13) and (14).

$$p(m|x^t, z^t) \propto \underbrace{p(m|x^{t-1}, z^{t-1})}_{prior} \underbrace{p(z_t|m, x^t, z^{t-1})}_{invers\ sensor\ model} \quad (10)$$

$$p(z_t|m, x^t, z^{t-1}) \propto exp\left(-\frac{1}{2}\frac{|x_t - m|^2}{\sigma_s^2}\right) \quad (11)$$

$$p(z_t|m, x^t, z^{t-1}) \approx \sum_{k=1}^{N} v_t^k N(m, \widehat{m}_t^k, \Sigma_t^k) \quad (12)$$

$$\widehat{m}_t^{ij} = \begin{pmatrix} x_0 + r\cos(\alpha_i)\cos(\beta_j) \\ y_0 + r\sin(\alpha_i)\cos(\beta_j) \\ z_0 + r\sin(\alpha_i) \end{pmatrix} \quad (13)$$

$$\Sigma_t^{ij} = (v_1 \quad v_2 \quad v_3)\begin{pmatrix} \sigma_s^2 & 0 & 0 \\ 0 & \sigma_t^2 & 0 \\ 0 & 0 & \sigma_t^2 \end{pmatrix}\begin{pmatrix} v_1^T \\ v_2^T \\ v_3^T \end{pmatrix} \quad (14)$$

$$v_t^k = v_{t-1}^k N\left(z_t, h_t^k, {\sigma_t^k}^2\right) \quad (15)$$

$${\sigma_t^k}^2 = H\Sigma_t^k H^T + \sigma_s^2 \quad (16)$$

$$h_t^k = h(x_t^{[i]}, \widehat{m}_t^k) \quad (17)$$

In the above set of equations, $(x_0 \quad y_0 \quad z_0)^T$ stands for the absolute coordinates of the robot range sensor, which are known since we are assuming a robot pose hypothesis $x_t^{[i]}$. Angle Δ in Fig.2 denote the angular increments between consecutive Gaussians along α ($\pi \geq \alpha \geq -\pi$) or β ($\frac{\pi}{2} \geq \beta \geq -\frac{\pi}{2}$), azimuth and elevation angles from the sensor position. Variable v_t^k represents the weight of each Gaussian mode and calculated in iterative manner using (15), where h_t^k and ${\sigma_t^k}^2$ are calculated by (17) and (16) respectively. As explained mean $\widehat{m}_t^k$ is calculated by (13). To compute the covariance of the Gaussian, Σ_t^{ij}, let define three unit orthogonal vectors with origin at the center of a sphere. The new coordination system is called spherical coordinate, which better fits to the nature of RO-SLAM problem. For convenience, the first vector $v_1$will be always pointing radially, hence the others (v_2 and v_3) are tangential to the sphere, as illustrated in Fig. 2. The covariance is calculated using (14). In (14), $\sigma_t = r\Delta K$ with K being a proportionality factor. The best value for k in this calculation is K=0.4.

III. RESULTS AND DISCUSSION

The proposed RO-SLAM scheme has been implemented for a robot to move in a given environment with a certain becomes arrangement and the result will be presented.

A. RO-SLAM simulation setup

The robot will move on a rectangular path starting from one corner and return to the same corner where it had been departed. The simulation setup is shown in Fig.3, the white small points demonstrates bacons, blue line is the robot path and red region (barely visible) points at the region where RO-SLAM is searching for bacon. Fig.3 (A)-(B) shows the robot location and pose on the path, position of the bacons and finally mapping candidate. As seen in Fig.3 (a), the ambiguity presented in the mapping but later as shown in Fig.3 (B) this ambiguity reduced dramatically and in Fig.3(C) to (E) the ambiguity has been disappeared. Fig.4 demonstrates results for 3D mapping simulation. As seen, the same results as Fig.3 is achieved in 3D and the ambiguity presented in Fig.4 (A) has been completely disappeared in Fig.4(C). Therefore, it can be concluded that the above objectives which has been introduced in section I are achieved.

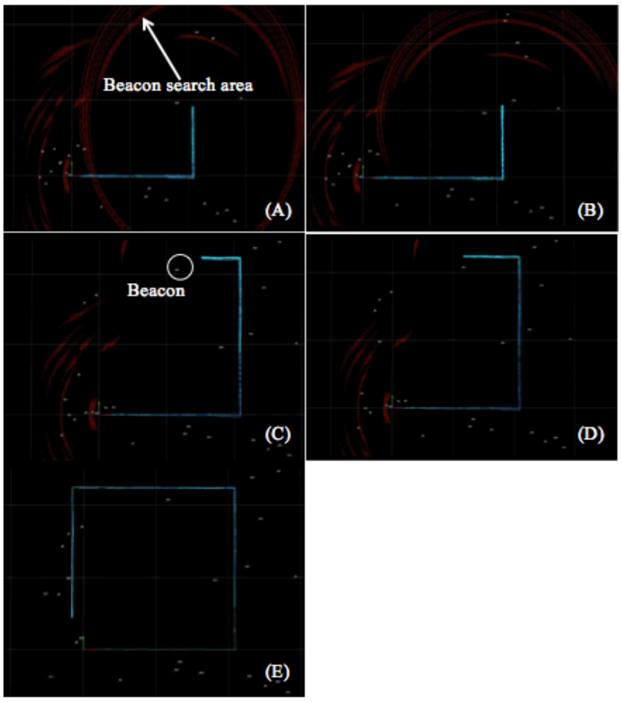

Fig. 3. Robot path, coordinate of the bacon and mapping. (A) - (E) shows robot localization when it is moving from a departure to a destination.

B. Results and discussion

As seen, the ambiguity has been removed when robot progressed in its path and the search area becomes very limited. The reason why the search area is very large at the beginning is because of the dependency of the error distribution to prior distribution, which is not available as the algorithm parameters are initiated with random values or zeros most of the time.

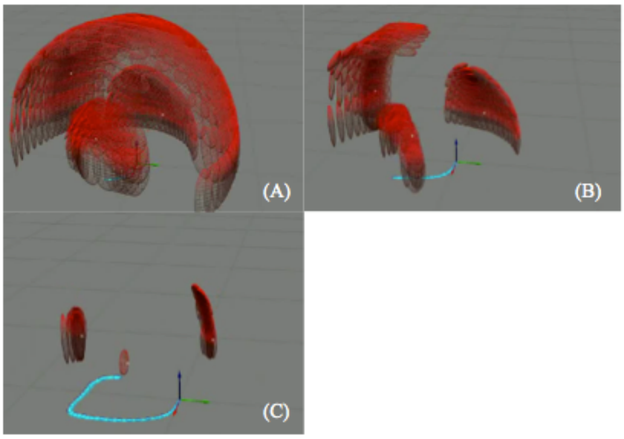

Fig. 4. A 3D map for the simulation setup presented in Fig.3. (A)- (C) shows the map for three points in robot path.

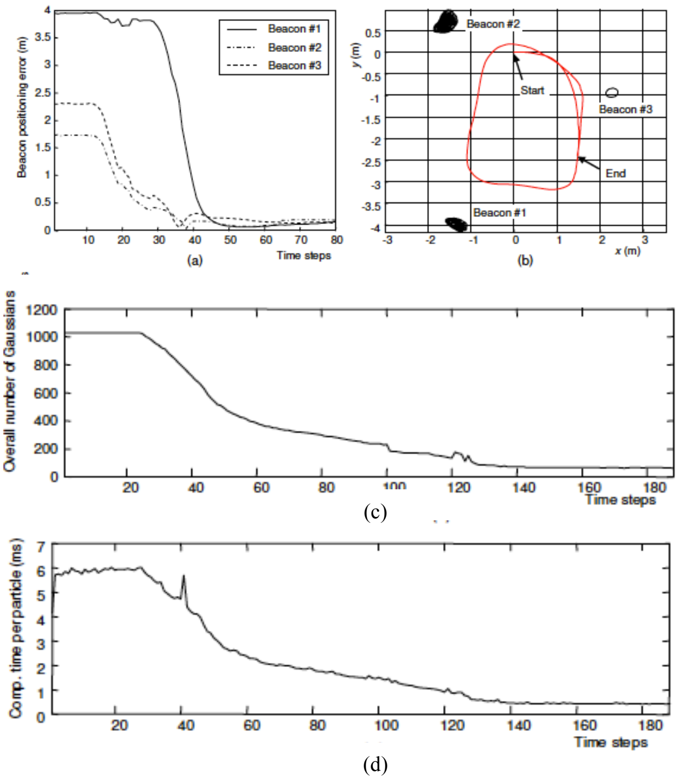

Fig. 5. Results of real robot equipped with a UWB transceiver. (a) Errors in each beacon estimate with respect to the ground truth. (b) A bird-view of the final state of the filter. (c) Overall number of Gaussians in the most likely map and (d) computation time, for each iteration of the algorithm.

C. Experimental data recording with a real sensor

Finally, the author has applied the proposed approach to a sequence of UWB range measurements [1] within an indoor environment. The experimental setup consists of a Pioneer mobile robot with a UWB transceiver onboard, while other three UWB devices act as static radio beacons. The position of the three beacons has been measured manually to provide the ground truth required to evaluate the results. In this case he has employed the a priori knowledge that beacons are above the robot to limit the map prior distribution to one half of the 3D space. The result of the measurement is shown in table I.

It is very interesting to know whether the calculation of the Gaussian series is time consuming or not? Fig.5 shows the amount of time needed to calculate Sum of Gaussian decreases when robot moves forward. In this example, the time of SOG is significantly reduces after step 25 which helps faster RBPF calculation.

TABLE I. SUMMARY OF ERRORS FOR THE 3D MAP BUILT FROM UWB DATA

Beacon coordinate		Ground truth (m)	Estimate (m)	Error (m)
	x	0	-0.059	0.059
#1	y	0	-0.278	0.278
	z	0.912	1.17	0.257
	x	-0.320	-0.392	0.072
#2	y	4.332	4.419	0.087
	z	1.374	1.214	0.159
	x	3.403	3.513	0.109
#3	y	2.802	2.740	0.062
	z	2.175	2.108	0.067

IV. CONCLUSION

A new algorithm of RO-SLAM based on SOG and RBPF was proposed and equations were derived. Formulation of SLAM was updated and implemented. Two objectives were defined to overcome in the introduction, i.e., reducing and removing ambiguity and decreasing the size of the region where possibly a new bacon is located in. the simulation results shows that these two objectives were successfully achieved. Practical setup also was established using new sensor and the outcome shows that the estimation of the map fits to the ground truth. The simulation and experimental results achieved by the proposed RO-SLAM algorithm is promising and using this algorithm in practice will give rise to robust results.

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