On the Fairness of Time-Critical Influence Maximization in Social Networks

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ABSTRACT

Influence maximization has found applications in a wide range of real-world problems, for instance, viral marketing of products in an online social network, and information propagation of valuable information such as job vacancy advertisements and health-related information. While existing algorithmic techniques usually aim at maximizing the total number of people influenced, the population often comprises several socially salient groups, e.g., based on gender or race. As a result, these techniques could lead to disparity across different groups in receiving important information. Furthermore, in many of these applications, the spread of influence is time-critical, i.e., it is only beneficial to be influenced before a time deadline. As we show in this paper, the time-criticality of the information could further exacerbate the disparity of influence across groups. This disparity, introduced by algorithms aimed at maximizing total influence, could have far-reaching consequences, impacting people's prosperity and putting minority groups at a big disadvantage. In this work, we propose a notion of *group fairness* in *time-critical influence maximization*. We introduce surrogate objective functions to solve the influence maximization problem under fairness considerations. By exploiting the submodularity structure of our objectives, we provide computationally efficient algorithms with guarantees that are effective in enforcing fairness during the propagation process. We demonstrate the effectiveness of our approach through synthetic and real-world experiments.

1 INTRODUCTION

The problem of *Influence Maximization* has been widely studied due to its application in multiple domains such as viral marketing [32], social recommendations [39], propagation of information related to jobs, financial opportunities or public health programs [3, 38]. The idea is to identify a set of initial sources (i.e., *seed nodes*) in a social network who can influence other people (e.g., by propagating key information), and traditionally the goal has been to maximize the total number of people influenced in the process (e.g., who received the information being propagated) [27].

Real-world social networks, however, are often not homogeneous and comprise different groups of people. Due to the disparity in group sizes and the potentially high propensity towards creating within-group links (a property known as *homophily* [29]), the structure of the social network can cause a disparity in the influence maximization process. For example, selecting most of the seed nodes from the majority group might maximize the total number of influenced nodes, but it is possible that very few members of the minority group get influenced. In many application scenarios such as propagation of job or health-related information, such disparity can end up impacting people's livelihood and some groups may become impoverished in the process.

Moreover, some applications are also *time-critical* in nature [11]. For example, many job applications typically have a deadline by which one needs to apply; if information related to the application reaches someone after the deadline, it is not useful. Similarly, in viral marketing, many companies offer discount deals only for few days (hours); getting this information late doesn't serve the recipient(s). More worryingly, if one group of people gets influenced (i.e., they get the information) faster than other groups, it ends up exacerbating the inequality in information access. This is possible if the majority group is better connected than the minority group, and they are more central in the network. Thus, in time-critical application scenarios, focusing on the traditional criteria of maximizing the number of influenced nodes can have a disparate impact on different groups present in the network. This disparity in time-critical applications, in turn, can bring minority and under-represented groups at a big disadvantage with far-reaching consequences. In this paper, we attempt to mitigate such unfairness in time-critical influence maximization (TCIM), and we focus on two settings: (i) where the budget (i.e., the number of seeds) is fixed and the goal is to find a seed set which maximizes the time-critical influence, we call this as TCIM-Budget problem, and (ii) where a certain quota or fraction of the population should be influenced under the prescribed time deadline, and the goal is to find such a seed set of minimal size, we call this as TCIM-Cover problem.

1.1 Our Contributions

Our first contribution is to formally introduce the notion of fairness in time-critical influence maximization, which requires that *within a prescribed time deadline, the fraction of influenced nodes should be equal across different groups*. We highlight, via experiments and illustrative example, that the standard algorithmic techniques for solving TCIM-Budget and TCIM-Cover problems lead to unfair solutions, and the disparity across groups could get worse with tighter time deadline.

We introduce two formulations of TCIM problems under fairness considerations, namely FairTCIM-Budget and FairTCIM-Cover. However, directly optimizing FairTCIM-Budget problem and FairTCIM-Cover problem are computationally challenging, and comes without any structural properties. As our second contribution, we propose *monotone submodular* surrogates for solving both of these problems that capture the tradeoff between maximizing total influence and minimizing unfairness (i.e., disparity across groups). Although the surrogate problems are still NP-Hard, we propose a greedy approximation with provable guarantees.

We evaluate our approach over both synthetic and real-world social networks and show that our proposed surrogate objectives can successfully enforce the aforementioned fairness notion. As expected, enforcing this fairness comes at the cost of a reduction in performance i.e., lowering the total influence, in case of FairTCIM-Budget problem, and solutions with larger seed set sizes, in case of FairTCIM-Cover problem. However, as guaranteed by our theoretical results, our experiments indeed demonstrate that this cost of fairness, i.e., reduction in performance, is bounded for our approach.

2 RELATED WORK

In this section, we briefly review the related literature on influence maximization and algorithmic fairness.

Influence Maximization. Richardson et al. [32] first introduced Influence Maximization as an algorithmic problem, and proposed a heuristic approach to find a set of nodes whose initial adoption of a certain idea/product can maximize the number of further adopters. Over the years, extensive research efforts have focused on the cascading behavior, diffusion and spreading of ideas, or containment of diseases by identifying the set of influential nodes which maximizes the influence through a network [22, 25, 26, 32, 37].

Typically, identifying the most influential nodes is studied in two ways: (i) using network structural properties to find the set of most central nodes [22, 23], and (ii) formulating the problem as discrete optimization [2, 19, 22]. Kempe et al. [22] studied influence maximization under different social contagion models and showed that submodularity of the influence function can be used to obtain provable approximation guarantees. Since then, there has been a large body of work studying various extensions [5, 7, 8, 19]. However, the notion of fairness in the influence maximization problem has not been studied by this line of previous works.

Fairness in Algorithmic Decision Making. Recently a growing amount of work has focused on bias and unfairness in algorithmic decision-making systems [9, 10, 34]. The aim here is to examine and mitigate unfair decisions that may lead to discrimination. Fairness has been divided into two broad areas: individual and group-level fairness. The notion of *individual fairness*, first proposed by Dwork et al. [13], requires that similar individuals who have similar attributes to be treated similarly. Whereas, the concept of *Demographic Parity* falls into a larger category called *group fairness* [20], which requires that the outcomes of an algorithm should equally benefit different groups with different sensitive attributes (e.g., groups based on race, gender or age) [17, 21]. Although fairness along different dimensions of political science, moral philosophy, economics, and law [12, 14, 33, 35] has been extensively studied, only a few contemporary works have investigated fairness in influence maximization, as described next.

Contemporary Works. Very recently, Fish et al. [18] proposed a notion of individual fairness in information access, but did not consider the group fairness aspects. In addition, some prior works have proposed constrained optimization problems to encourage diversity in selecting the most influential nodes [1, 4, 6, 15].

In a concurrent work, Tsang et al. [36] propose a method to achieve group fairness in influence maximization. However, their work is very different from our approach in three ways: i) they propose a different problem formulation with objective that does not have submodular structural properties, ii) they only study the problem under budget constraint, and iii) they do not consider the time-critical aspect of influence in their definition of fairness for influence maximization. This could result in majority groups being influenced before the minority, and can lead to disparity in applications where the timing of being influenced/informed is critical. In our work, we introduce a submodular objective that directly addresses the time-criticality in influence maximization problem under budget constraint as well as coverage constraint.

3 BACKGROUND ON TIME-CRITICAL INFLUENCE MAXIMIZATION (TCIM)

In this section, we provide the necessary background on the problem of time-critical influence maximization (henceforth, referred to as TCIM for brevity). First, we formally introduce a well-studied influence propagation model and specify the notion of time-critical influence that we consider in this paper. Then, we discuss two discrete optimization formulations to tackle the TCIM problem.

3.1 Influence Propagation in Social Network

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of directed edges connecting these nodes. For instance, in a social network the nodes could represent people and edges could represent friendship links between people. An undirected link between two nodes can be represented by simply considering two directed edges between these nodes.

There are two classical influence propogation models that are studied in the literature [22]: (i) Independent Cascade model (IC) and (ii) Linear Threshold (LT) model. In this paper, we will consider IC model and our results can easily be extended to the LT model.

In the IC model, there is a probability of influence associated with each edge denoted as $p_{\mathcal{E}} := \{p_e \in [0, 1] : e \in \mathcal{E}\}$. Given an initial seed set $S \subseteq \mathcal{V}$, the influence propagation proceeds in discrete time steps $t = \{0, 1, 2, \ldots, \}$ as follows. At $t = 0$, the initial seed set S is "activated" (i.e., influenced). Then, at any time step $t > 0$, a node $v \in \mathcal{V}$ which was activated at time $t-1$ gets a chance to influence its neighbors (i.e., set of nodes $\{w : (v, w) \in \mathcal{E}\}$). The influence propagation process stops at time $t > 0$ if no new nodes get influenced at this time. Under the IC model, once a node is activated it stays active throughout the process and each node has only one chance to influence it's neighbors.

Note that the influence propagation under IC model is a stochastic process: the stochasticity here arises because of the random outcomes of a node v influencing its neighbor w based on the Bernoulli distribution $p_{(v,w)}$. An outcome of the influence propagation process can be denoted via a set of timestamps $\{t_v \geq 0 : v \in \mathcal{V}\}$ where t_v represents the time at which a node $v \in \mathcal{V}$ was activated. We have $t_v = 0$ iff $v \in S$ and for convenience of notation, we define $t_v = -1$ to indicate that the node v was not activated in the process.

3.2 Utility of Time-Critical Influence

As motivated in the introduction, we focus on the application settings where the spread of influence is time-critical, i.e., it is more beneficial to be influenced earlier in the process. In particular, we adopt the well-studied notion of time-critical influence as proposed by Chen et al. [11]. Their time-critical model is captured via a deadline τ: If a node is activated before the deadline, it receives a utility of 1, otherwise it receives no utility. This simple model captures the notion of timing in many important real-world applications such as viral marketing of an online product with limited availability, information propagation of job vacancy information, etc.

Given the influence propagation model and the notion of time-critical aspect via a deadline τ, we quantify the utility of time-critical influence for a given seed set S on a set of target nodes $Y \subseteq \mathcal{V}$ via the following:

$$f_\tau(S; Y, \mathcal{G}) = \mathbb{E}\Big[\sum_{v \in Y, t_v \geq 0} \mathbb{I}(t_v \leq \tau)\Big], \qquad (1)$$

where the expectation is w.r.t. the randomness of the outcomes of the IC model. The function is parametrized by deadline τ, set $Y \subseteq \mathcal{V}$ representing the set of nodes over which the utility is measured (by default, one can consider $Y = \mathcal{V}$), and the underlying graph $\mathcal{G}$ along with edge activation probabilities $p_\mathcal{E}$. Given a fixed value of these parameters, the utility function $f_\tau : 2^\mathcal{V} \rightarrow \mathbb{R}_{\geq 0}$ is a set function defined over the seed set $S \subseteq \mathcal{V}$. Note that the constraint $t_v \geq 0$ represents the node was activated and the constraint $t_v \leq \tau$ represents that the activation happened before the deadline τ.

3.3 TCIM as Discrete Optimization Problem

Next, we present two settings under which we study TCIM by casting it as a discrete optimization problem.

3.3.1 Maximization under Budget Constraint (TCIM-Budget). In the maximization problem under budget constraint, we are given a fixed budget $B > 0$ and the goal is to find an optimal set of seed nodes that maximize the expected utility. Formally, we state the problem as

$$\max_{S \subseteq \mathcal{V}} f_\tau(S; \mathcal{V}, \mathcal{G}) \quad \text{subject to } |S| \leq B \qquad \text{(P1)}$$

3.3.2 Minimization under Coverage Constraint (TCIM-Cover). In the minimization problem under coverage constraint, we are given a quota $Q \in [0, 1]$ representing the minimal fraction of nodes that must be activated or “covered" by the influence propagation in expectation. The goal is then to find an optimal set of seeds of minimal size that achieves the desired coverage constraint. We formally state the problem as

$$\min_{S \subseteq \mathcal{V}} |S| \quad \text{subject to } \frac{f_\tau(S; \mathcal{V}, \mathcal{G})}{|\mathcal{V}|} \geq Q \qquad \text{(P2)}$$

3.4 Submodularity and Approximate Solutions

Next, we present some key properties of the utility function $f_\tau(.)$ to get a better understanding of the above-mentioned optmization problems. In their seminal work, Kempe et al. [22] showed that the utility function without time-critical deadline, i.e., $f_\infty(.) : S \rightarrow \mathbb{R}_+$, is a non-negative, monotone, submodular set function w.r.t. the optimization variable $S \subseteq \mathcal{V}$. Chen et al. [11] showed that the utility function for the general time-critical setting for any τ also satisfies these properties. Submodularity is an intuitive notion of diminishing returns, stating that, for any sets $A \subseteq A' \subseteq \mathcal{V}$, and any node $a \in \mathcal{V} \setminus A'$, it holds that (omitting the parameters $\mathcal{V}$ and $\mathcal{G}$ for brevity):

$$f_\tau(A \cup \{a\}) - f_\tau(A) \geq f_\tau(A' \cup \{a\}) - f_\tau(A')$$

The fact that the utility function is submodular in turn implies that these two problems P1 and P2 are NP-Hard and hence finding the optimization solution is intractable [16, 24, 31]. However, on a positive note, one can exploit the submodularity property of the function to design efficient approximation algorithms with provable guarantees [24, 31]. In particular, we can run the following greedy heuristic: start from an empty set, iteratively add a new node to the set that provides the maximal marginal gain in terms of utility, and stop the algorithm when the desired constraint on budget or coverage is met. This greedy algorithm provides the following guarantees for these two problems:.

- for the TCIM-Budget problem P1, the greedy algorithm returns a set $\hat{S}$ that guarantees the following lower bound on the utility: $f_\tau(\hat{S}; \mathcal{V}, \mathcal{G}) \geq (1 - \frac{1}{e}) \cdot f_\tau(S^*; \mathcal{V}, \mathcal{G})$ where S^* is an optimal solution to problem P1.
- for the TCIM-Cover problem P2, the greedy algorithm returns a set $\hat{S}$ that guarantees the following upper bound on the seed set size: $|\hat{S}| \leq \ln(1+|\mathcal{V}|) \cdot |S^*|$ where S^* is an optimal solution to problem P2.

4 DISPARITY IN TCIM AND INTRODUCING NOTIONS OF FAIRNESS

In this section, we highlight the disparity in utility across population resulting from the solution to the standard TCIM problem formulations, and introduce a notion of fairness in TCIM.

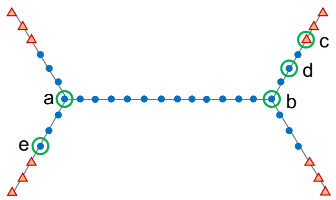
4.1 Socially Salient Groups and Their Utilities

The current approaches to TCIM consider all the nodes in $\mathcal{V}$ to be homogeneous. We capture the presence of different socially salient groups in the population by dividing individuals into k disjoint groups. Here, socially salient groups could be based on some sensitive attribute such as gender or race. We denote the set of nodes in each group $i \in \{1, 2, \ldots, k\}$ as $\mathcal{V}_i \subseteq \mathcal{V}$, and we have $\mathcal{V} = \cup_i \mathcal{V}_i$. For any given seed set S, we define the utilities for a group i as $f_\tau(S; \mathcal{V}_i, \mathcal{G})$ by setting target nodes $Y = \mathcal{V}_i$ in Eq. 1.

4.2 Disparity in Utility Across Groups

In the standard formulations for TCIM problem, i.e., TCIM-Budget problem P1 and TCIM-Cover problem P2, the utility $f_\tau(S; \mathcal{V}, \mathcal{G})$ is optimized for the whole population $\mathcal{V}$ without considering their groups. Clearly, a solution to TCIM problem can, in general, lead to high disparity in utilities of different groups.

In particular, this disparity in utility across groups arises from several factors in which two groups differ from each other. One of the factors is that the groups are of different sizes, i.e., one group is a minority. The different group sizes could, in turn, lead to selecting seed nodes from the majority group when optimizing for utility $f_\tau(S; \mathcal{V}, \mathcal{G})$ in problems P1 and P2. Another factor is related to the connectivity and centrality of nodes from different groups. The



	solution to TCIM-Budget P1				solution to FairTCIM-Budget P4			
	S	$\frac{f(S;\mathcal{V},\mathcal{G})}{\lvert\mathcal{V}\rvert}$	$\frac{f(S;\mathcal{V}_1,\mathcal{G})}{\lvert\mathcal{V}_1\rvert}$	$\frac{f(S;\mathcal{V}_2,\mathcal{G})}{\lvert\mathcal{V}_2\rvert}$	S	$\frac{f(S;\mathcal{V},\mathcal{G})}{\lvert\mathcal{V}\rvert}$	$\frac{f(S;\mathcal{V}_1,\mathcal{G})}{\lvert\mathcal{V}_1\rvert}$	$\frac{f(S;\mathcal{V}_2,\mathcal{G})}{\lvert\mathcal{V}_2\rvert}$
$\tau = \infty$	$\{a, b\}$	0.38	0.48	0.16	$\{a, c\}$	0.31	0.33	0.27
$\tau = 4$	$\{a, b\}$	0.32	0.44	0.08	$\{d, e\}$	0.25	0.26	0.22
$\tau = 2$	$\{a, b\}$	0.24	0.36	0.00	$\{a, c\}$	0.21	0.22	0.18

Figure 1: An example to illustrate the disparity across groups in the standard approaches to TCIM. (Left) Graph with $|\mathcal{V}| = 38$ nodes belonging to two groups shown in "blue dots" ($|\mathcal{V}_1| = 26$) and "red triangles" ($|\mathcal{V}_2| = 12$). (Right) We compare an optimal solution to the standard TCIM-Budget problem P1 and an optimal solution to our formulation of TCIM-Budget with fairness considerations given by FairTCIM-Budget problem P4. For different time critical deadlines τ, normalized utilities are reported for the whole population $\mathcal{V}$, for the "blue dots" group $\mathcal{V}_1$, and for the "red triangles" group $\mathcal{V}_2$. As τ reduces, the disparity between groups is further exacerbated in the solution to TCIM-Budget problem P1. Solution to FairTCIM-Budget problem P4 achieves high utility and low disparity for different time critical deadlines τ.

solution to the optimization problems P1 and P2 tend to favor nodes which are more central and have high-connectivity. Finally, given the above two factors, we note that the disparity in influence across groups can be further exacerbated for lower values of deadline τ in the time-critical influence maximization.

In Figure 1, we provide an example to illustrate the disparity across groups in the standard approaches to TCIM. In particular, to show this disparity, we consider the TCIM-Budget problem P1, and it is easy to extend this example to show disparity in TCIM-Cover problem P2. The graph that we consider in this example (see Figure 1 caption for details) has the two characteristic properties that we discussed above: (i) group V_2 is in minority with less than half of the size of group V_1, (ii) group V_1 has more central nodes compared to group V_2, and (iii) nodes in group V_1 have higher connectivity than nodes in group V_2. We consider the probability of influence in the graph to be $p_e = 0.7$ for all edges, and study the optimization problem P1 for budget $B = 2$.

For different time critical deadlines τ, we report the following normalized utilities: $\frac{f(S;\mathcal{V},\mathcal{G})}{|\mathcal{V}|}$ for the whole population $\mathcal{V}$, $\frac{f(S;\mathcal{V}_1,\mathcal{G})}{|\mathcal{V}_1|}$ for the group $\mathcal{V}_1$, and $\frac{f(S;\mathcal{V}_2,\mathcal{G})}{|\mathcal{V}_2|}$ for the group $\mathcal{V}_2$. Here, normalization captures the notion of "average" utility per node in a group, and automatically allows us to account for the differences in the group sizes. As can be seen in Figure 1, the optimal solution to the problem consistently picks set $S = \{a, b\}$ comprising of the most central and high-connectivity nodes. While these nodes maximize the total utility, they lead to a high disparity in the normalized utilities across groups. As the influence becomes more time-critical, i.e., τ is reduced, we see an increasing disparity as discussed above. For $\tau = 2$, the utility of group $\mathcal{V}_2$ reduces to 0.

4.3 Notion of Fairness

Next, in order to guide the design of fair solutions to TCIM problems, we introduce a formal notion of group fairness in TCIM. In particular, we measure the (un-)fairness or disparity of an algorithm by the maximum *disparity in normalized utilities* across all pairs of socially salient groups, given by:

$$\max_{i,j \in \{1,2,\ldots,k\}} \left| \frac{f_\tau(S;\mathcal{V}_i,\mathcal{G})}{|\mathcal{V}_i|} - \frac{f_\tau(S;\mathcal{V}_j,\mathcal{G})}{|\mathcal{V}_j|} \right| \quad (2)$$

As discussed above (see Section 4.2), normalization w.r.t. group sizes captures the notion of average utility per node in a group and hence makes the measure agnostic to the group size. In the next section, we seek to design fair algorithms for TCIM problems that have low disparity (or more fairness) as measured by Eq. 2.

5 ACHIEVING FAIRNESS IN TCIM

In this section, we seek to develop efficient algorithms for TCIM problems under fairness considerations that have low disparity measured by Eq. 2 while maintaining high performance.

5.1 Fair Maximization under Budget Constraint (FairTCIM-Budget)

5.1.1 Introducing fairness considerations in TCIM-Budget. A fair TCIM algorithm under budget constraint should seek to achieve the following two objectives: (i) maximizing total influence for the whole population $\mathcal{V}$ as was done in the standard TCIM-Budget problem P1, and (ii) enforcing fairness by ensuring that disparity across different groups as per Eq. 2 is low. Clearly, enforcing fairness would lead to a reduction in total influence, and we seek to design algorithms that can achieve a good trade-off between these two objectives. We formulate the following fair variant of TCIM-Budget problem P1 that captures this trade-off:

$$\max_{S \subseteq \mathcal{V}} \left(f_\tau(S;\mathcal{V},\mathcal{G}) - \gamma \cdot \max_{i,j} \left| \frac{f_\tau(S;\mathcal{V}_i,\mathcal{G})}{|\mathcal{V}_i|} - \frac{f_\tau(S;\mathcal{V}_j,\mathcal{G})}{|\mathcal{V}_j|} \right| \right) \quad \text{(P3)}$$
$$\text{subject to } |S| \leq B$$

where $\gamma \geq 0$ is a fixed regularization factor to trade-off the total influence and disparity of the solution.

5.1.2 Surrogate problem for FairTCIM-Budget with guarantees. We note that problem P3 is a challenging discrete optimization problem and the objective function does not have structural properties of submodularity as was the case for the standard TCIM-Budget problem P1. Instead of directly solving problem P3, we introduce a novel surrogate problem that would allow us to indirectly trade-off the two objectives of maximizing total influence and minimizing disparity across groups, as follows:

$$\max_{S \subseteq \mathcal{V}} \sum_{i=1}^{k} \lambda_i \mathcal{H}(f_\tau(S;\mathcal{V}_i,\mathcal{G})) \quad \text{subject to } |S| \leq B \quad \text{(P4)}$$

where $\mathcal{H}$ is a non-negative, monotone concave function and $\lambda_i \geq 0$ are fixed scalars.

The key idea of using the surrogate objective function in problem P4 is the following: Passing the group influence functions through a monotone concave function $\mathcal{H}$ rewards selecting seeds that would lead to higher influence on under-represented groups early in the selection process; this in turn helps in reducing disparity across groups. It is important to note that controlling the curvature of the concave function $\mathcal{H}$ provides an indirect way to trade-off the total influence and the disparity of the solution. For instance, using $\mathcal{H}(z) := \log(z)$ has higher curvature than using $\mathcal{H}(z) := \sqrt{z}$ and hence leads to lower disparity (this is demonstrated in the experimental results in Figure 2a). For our illustrative example from Section 4, we report the results for an optimal solution to FairTCIM-Budget problem P4 with $\mathcal{H}(z) := \log(z)$. As can be seen in Figure 1, the solution leads to a drastic reduction in disparity across groups for different values of deadline τ compared to an optimal solution of the standard TCIM-Budget problem P1.

While it is intuitively clear that using the concave function $\mathcal{H}(z)$ in problem P4 reduces disparity, we also need to ensure that the solution to this problem has high influence for the whole population $\mathcal{V}$ and that the solution can be computed efficiently. As proven in the theorem below, we can find an approximate solution to problem P4, with guarantees on the total influence, by running the greedy heuristic (as was introduced in Section 3.4).

Theorem 1. *Let $\hat{S}$ denote the output of the greedy algorithm for problem P4 with $\lambda_i = 1 \; \forall i \in [k]$. Let S^* be an optimal solution to problem P1. Then, the total influence of the greedy algorithm is guaranteed to have the following lower bound: $f_\tau(\hat{S}; \mathcal{V}, \mathcal{G}) \geq (1 - \frac{1}{e}) \cdot \mathcal{H}\big(f_\tau(S^*; \mathcal{V}, \mathcal{G})\big)$.*

This is equivalent to the fact that the multiplicative approximation factor of the utility of FairTCIM-Budget using greedy algorithm w.r.t. the utility of an optimal solution to TCIM-Budget scales as $\big((1 - \frac{1}{e}) \cdot \frac{\mathcal{H}(f_\tau(S^*; \mathcal{V}, \mathcal{G}))}{f_\tau(S^*; \mathcal{V}, \mathcal{G})}\big)$. Note that as the curvature of the concave function $\mathcal{H}$ increases, the approximation factor gets worse—this further highlights how the curvature of the function $\mathcal{H}$ provides a way to trade-off the total influence and disparity of the solution. We omit the detailed proofs due to lack of space, instead provide a sketch of the key ideas used in the proof.

Proof sketch. There are two key steps in proving the above theorem. The first step is showing that the objective function in problem P4 is monotone submodular function, where we use a result from [28] that composition of a non-decreasing concave and a non-decreasing submodular function is submodular. The second step is to bound the utility of the optimal solution of problem P4 w.r.t. the optimal solution of problem P1. We prove that this can be bounded by a multiplicative factor of $\frac{\mathcal{H}(f_\tau(S^*; \mathcal{V}, \mathcal{G}))}{f_\tau(S^*; \mathcal{V}, \mathcal{G})}$. Combining bounds from individual steps and by applying inequalities resulting from the concavity of $\mathcal{H}$ gives us the final result. □

5.2 Fair Minimization under Coverage Constraint (FairTCIM-Cover)

5.2.1 Introducing fairness considerations in TCIM-Cover. A fair TCIM algorithm under coverage constraint should seek to achieve the following two objectives: (i) minimizing the size of the seed set that achieves the desired coverage constraint as was done in the standard TCIM-Cover problem P2, and (ii) enforcing fairness by ensuring that disparity across different groups as per Eq. 2 is low. As was the case for FairTCIM-Budget problem above, enforcing fairness would lead to increasing the size of the required seed set, and we seek to design algorithms that can achieve a good trade-off between these two objectives. We formulate a fair variant of TCIM-Cover problem P2 that captures this trade-off as follows:

$$\min_{S \subseteq \mathcal{V}} \left(|S| + \gamma \cdot \max_{i,j} \left| \frac{f_\tau(S; \mathcal{V}_i, \mathcal{G})}{|\mathcal{V}_i|} - \frac{f_\tau(S; \mathcal{V}_j, \mathcal{G})}{|\mathcal{V}_j|} \right| \right) \qquad \text{(P5)}$$
$$\text{subject to } \frac{f_\tau(S; \mathcal{V}, \mathcal{G})}{|\mathcal{V}|} \geq Q$$

where $\gamma \geq 0$ is a fixed regularization factor to trade-off the size of seed set and disparity of the solution.

5.2.2 Surrogate problem for FairTCIM-Cover with guarantees. As in Section 5.1, we note that problem P5 is a challenging discrete optimization problem and does not have structural properties as was the case for the standard TCIM-Cover problem P2. Instead of directly solving problem P5, we introduce a novel surrogate problem that indirectly trade-offs the two objectives of minimizing the size of selected seed set and minimizing disparity, as follows:

$$\min_{S \subseteq \mathcal{V}} |S| \quad \text{subject to} \quad \sum_{i=1}^{k} \min \left\{ \frac{f_\tau(S; \mathcal{V}_i, \mathcal{G})}{|\mathcal{V}_i|}, Q \right\} \geq k \cdot Q \qquad \text{(P6)}$$

The key idea of using the surrogate objective function in problem P6 is the following: the problem has a constraint that enforces that at least Q fraction of nodes in each group are influenced by the selected seed set S; this in turn directly provides a bound on the disparity of any feasible solution to the problem as $(1 - Q)$.

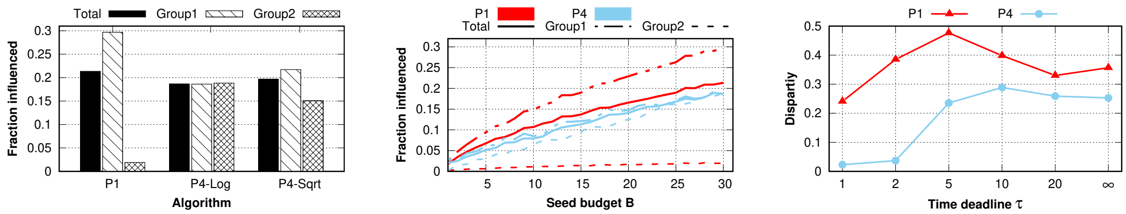
While it is intuitively clear that the solution to problem P6 reduces disparity, we also would like to bound the size of the final seed set and that the solution can be computed efficiently. As proven in the theorem below, we can find an approximate solution to problem P6, with guarantees on the final seed set size, by running the greedy heuristic (as was introduced in Section 3.4).

Theorem 2. *Let us denote the output of the greedy algorithm for problem P6 by set $\hat{S}$. For group $i \in \{1, \ldots, k\}$, let S^*_i denote an optimal solution to the coverage problem P2 for the target nodes set to $\mathcal{V}_i$, i.e., solving problem P2 with constraint given by $\frac{f_\tau(S; \mathcal{V}_i, \mathcal{G})}{|\mathcal{V}_i|} \geq Q$. Then, the size of the seed set $\hat{S}$ returned by the greedy algorithm is guaranteed to have the following upper bound: $|\hat{S}| \leq \ln(1 + |\mathcal{V}|)\Big(\sum_{i=1}^{k} |S^*_i|\Big)$.*

Proof sketch. There are two key steps in proving the above theorem. The first step is showing that the objective function in the constraint of problem P6 is monotone submodular function by using several composition properties of submodular functions [24]. The second step is to provide an upper bound on the optimal solution of problem P6 as $\Big(\sum_{i=1}^{k} |S^*_i|\Big)$. □

6 EVALUATION ON SYNTHETIC DATASET

In this section, we compare the solutions of different problems on a synthetic dataset. We show the effects of different properties of the synthetic graph and parameters of the algorithms.

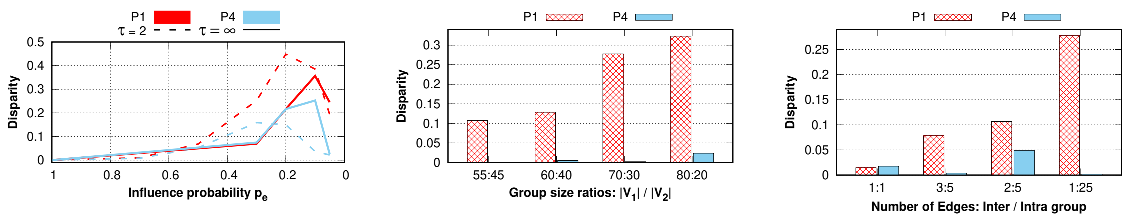


(a) Total and group influence

(b) Varying budget B

(c) Varying time deadline τ

Figure 2: [Synthetic Dataset: Budget Problem] The figures show that solving TCIM-Budget problem P1 can lead to disparity in number of influenced nodes belonging to different groups, while FairTCIM-Budget problem P4 fares better in terms of achieving parity of influence, with marginally lower total influence. See Section 6.2 for further details.



(a) Varying influence probability p_e

(b) Varying group size proportions

(c) Varying cliquishness

Figure 3: [Synthetic Dataset: Budget Problem] These figures demonstrate that lower activation probabilities, uneven group sizes, and cliquishness can lead to higher disparity of influence between different groups with TCIM-Budget problem P1. In comparison our proposed method, FairTCIM-Budget given by problem P4, leads to solutions which yield lower disparity. For further details, see Section 6.2.

6.1 Dataset and Experimental Setup

First, we discuss the synthetic dataset generation process and the setup used in our experiments.

Synthetic dataset. We consider an undirected graph with 500 nodes, where each node belongs to either group $\mathcal{V}_1$ or group $\mathcal{V}_2$. The fraction of nodes belonging to each group is determined by a parameter g (e.g., setting $g = 0.7$ results in 70% of the nodes to be randomly assigned to group $\mathcal{V}_1$). Nodes are connected based on two probabilities: (i) within-group edge probability (*Homophily*) p_{hom} and (ii) across-group edge probability (*Heterophily*) p_{het}. Placing an edge between two nodes goes as follows: given a pair of nodes (v, w), if they belong to the same group, we perform a Bernoulli trial with parameter p_{hom}; otherwise we use the parameter p_{het}. If the outcome of the trial is 1, we place an undirected edge e between these two nodes. Each edge has a probability of activation, $p_e \in [0, 1]$, with which the nodes can activate each other.

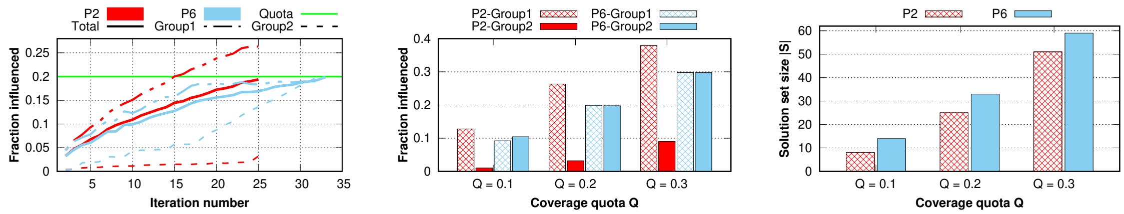
Experimental Setup. In our experiments, we used $g = 0.7$ yielding 350 nodes in $\mathcal{V}_1$ and 150 nodes in $\mathcal{V}_2$. We set $p_{hom} = 0.025$ and $p_{het} = 0.001$, which yielded 3606 total edges, out of which 2965 edges were within group $\mathcal{V}_1$, 514 within $\mathcal{V}_2$, and 127 edges connecting nodes across two groups. We used a constant activation probability on all edges given by $p_e = 0.05$. Finally, we consider the time deadline $\tau = 20$, unless explicitly stated otherwise. Evaluating utilities, as described in Eq. 1, in closed form is intractable, so we used Monte Carlo sampling to estimate these utilities. We used 200 samples for this estimation, which yielded a stable estimation of the utility function. In all the experiments, we pick a seed set by solving the corresponding problem. Then, we use this seed set to estimate the expected number of nodes influenced in the graph using TCIM. We report the following normalized utilities: $\frac{f(S;\mathcal{V},\mathcal{G})}{|\mathcal{V}|}$ for the whole population $\mathcal{V}$, $\frac{f(S;\mathcal{V}_1,\mathcal{G})}{|\mathcal{V}_1|}$ for the group $\mathcal{V}_1$, and $\frac{f(S;\mathcal{V}_2,\mathcal{G})}{|\mathcal{V}_2|}$ for the group $\mathcal{V}_2$.

6.2 TCIM under Budget Constraints

Next, we compare the solutions of TCIM-Budget problem P1 with our solution to FairTCIM-Budget problem P4, obtained through the greedy algorithm, *i.e.*, by greedily picking B seeds which maximize the objective functions. In all the figures discussed in this section, red color represents the results of TCIM-Budget problem P1, and blue color represents the results of our solution to the FairTCIM-Budget problem P4. For the experiments in this section, we used a budget of $B = 30$ seeds.

Effect of different $\mathcal{H}(z)$. Figure 2a presents the comparison of three algorithms: one solving TCIM-Budget problem P1, using the greedy heuristic; the other two solving FairTCIM-Budget problem P4, where we use two realizations of the concave monotone function, $\mathcal{H}(z)$, given by: (i) $\mathcal{H}(z) := \log(z)$ and (ii) $\mathcal{H}(z) := \sqrt{z}$. Figure 2a shows the fraction of population influenced, both overall and for every group. We can observe that solving the traditional TCIM-Budget problem leads to large disparity between the fraction of nodes influenced from each group: while 30% of nodes in group $\mathcal{V}_1$ are influenced, this fraction is only 2% for group $\mathcal{V}_2$.

On the other hand, our proposed solution to FairTCIM-Budget problem results in lower disparity between the groups, ensuring similar fraction of influenced nodes. We can further see that $\sqrt{z}$ performs worse than $\log(z)$ in removing the disparity, however incurring lower loss in total influence, which is expected as it has



(a) Greedy selection iterations **(b) Varying quota Q: Influence** **(c) Varying quota Q: Solution set size |S|**

Figure 4: [Synthetic Dataset: Cover Problem] These figures show a comparison of TCIM-Cover problem P2, in red, and FairTCIM-Cover problem P6, in blue. They show that FairTCIM-Cover achieves lower disparity of influence between different groups with slightly bigger solution set sizes. See Section 6.3 for further details.

lower curvature than $\log(z)$. One could consider higher powers of the root to increase the curvature or increase the weights λ in problem P4 for the under-represented group. The *key point* to notice is that the reduction in the total influence is only marginal as guaranteed by Theorem 1. In the subsequent figures, we only show the results of $\mathcal{H}(z) := \log(z)$ for the solution to problem P4.

Effect of seed budget. Figure 2b shows the effect of different seed budgets on the number of influenced nodes (from different groups). Dotted and dash-dotted lines correspond to groups $\mathcal{V}_2$ and $\mathcal{V}_1$ respectively, while solid lines represent the total influence. We can see that problem P4, with a small reduction in total influence, reduces a lot of disparity between the groups. We also observe that higher budget could lead to more disparity in an imbalanced graph.

Effect of deadline. Figure 2c compares disparity as we vary the value of the deadline τ, in problems P1 and P4. Disparity is computed as the absolute difference between the fraction of individuals influenced in each group, given by Eq. 2. The figure shows that the disparity depends on the value of τ and, in this case, our method is more effective for lower values of τ.

Effect of activation probabilities. Figure 3a shows the disparity in influence for different activation probabilities $p_e \in \{0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1.0\}$. The results show that lower values of τ and lower activation probabilities could result in larger disparity and our method consistently performs better.

Effect of group sizes. Figure 3b shows the effect of group sizes $g \in \{0.55, 0.6, 0.7, 0.8\}$. x-axis represents ratio of the nodes belonging to different groups and y-axis represents disparity. The figure confirms our hypothesis that *imbalance in a graph could lead to disparate influence*, as motivated in the illustrative example given in Figure 1.

Effect of graph structure. Figure 3c demonstrates the importance of the graph structure $(p_{het}, p_{hom}) \in \{(0.025, 0.025), (0.015, 0.025), (0.01, 0.025), (0.001, 0.025)\}$. x-axis shows the ratio of across and within group edge probabilities. The figure validates our hypothesis that the majority group containing more influential nodes fares better in TCIM-Budget problem, as proposed in Figure 1.

Takeaways. In this section we demonstrated that: (i) solving TCIM-Budget problem can lead to disparity of influence in different groups; (ii) the amount of disparity depends on the time limit, activation probability, relative group sizes, budget, and connectivity of the graph; and (iii) instead, solving FairTCIM-Budget results in lower disparity of influence, with marginal reduction in overall influence, as guaranteed by Theorem 1.

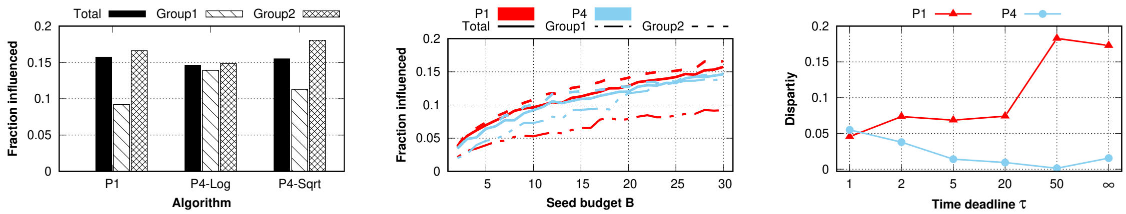
6.3 TCIM under Coverage Constraints

Next, we compare solutions of TCIM-Cover problem P2, and our solution to FairTCIM-Cover problem P6. The goal is to reach the prescribed quota Q, while trying to minimize the number of seeds. In all the figures discussed in this section, red color represents the results of TCIM-Cover problem P2, and blue color represents the results of our solution to FairTCIM-Cover problem P6.

Effect of iterations. Figure 4a shows how the fraction of population influenced changes with seed selection at each iteration. Solid lines represent total influence while dash-dotted lines and dotted lines represent groups $\mathcal{V}_1$ and $\mathcal{V}_2$, respectively. In this experiment, Q was set to 0.2 which is represented by the horizontal green line. The figure demonstrates that in FairTCIM-Cover, the fraction of influenced individuals are more than Q in both groups. On the other hand, in the TCIM-Cover, the fraction of influenced individuals in group $\mathcal{V}_1$ is larger than Q; however, the fraction of influenced individuals in group $\mathcal{V}_2$ is much smaller than Q. The figure also shows that this fairness is achieved with only marginal increase in the solution set sizes, as guaranteed in Theorem 2.

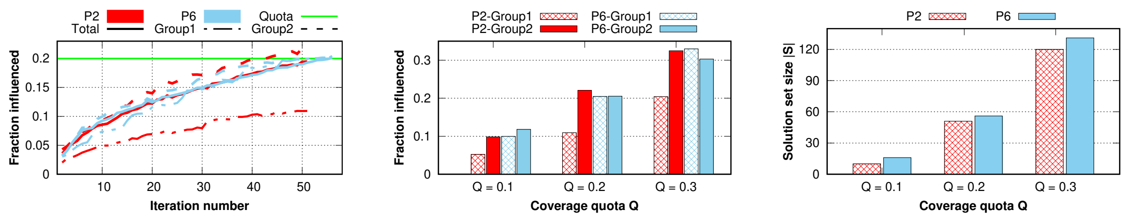
Effect of quota Q. Figure 4b shows fractions of individuals that are influenced for different quota Q: (i) for the TCIM-Cover problem, the disparity in the fraction of influenced individuals in different groups persists with increased quota; (ii) however, our proposed method results in both groups reaching the specified coverage Q. Figure 4c shows the coverage Q on the x-axis and number of seeds used on the y-axis. The figure shows that our method results in only slightly larger seed sets compared to the TCIM-Cover problem, as guaranteed by Theorem 2.

Takeaways. We compared the results of TCIM-Cover problem, P2, and our solution to FairTCIM-Cover problem, P6. The results demonstrate that: (i) both methods influence Q fraction of the total population; (ii) however, the TCIM-Cover approach can result in disparate coverage, i.e., the coverage in some groups could be much lower than the prescribed fraction Q; (iii) FairTCIM-Cover, on the other hand, ensures coverage of at least Q fraction in both groups; and (iv) additionally, FairTCIM-Cover only yields slightly larger solution set sizes, as guaranteed by Theorem 2.



(a) Total and Group Influence (b) Varying budget B (c) Varying time deadline τ

Figure 5: [Rice-Facebook Dataset: Budget Problem] These figures show similar results as in Figure 2, on real-world data. They demonstrate that our proposed methods yields seed set which propagate influence in a more fair manner, while incurring very small cost of total influence.



(a) Greedy selection iterations (b) Varying quota Q: Influence (c) Varying Q: Solution set size |S|

Figure 6: [Rice-Facebook Dataset: Cover Problem] These figures demonstrate that our proposed methods are effective in propagating influence fairly in TCIM problem with coverage constraints. The results are similar to Figure 4, but on real-world data.

7 EVALUATION ON REAL-WORLD DATASET

In this section, we first discuss the details of the experiments and then demonstrate results using a real-world dataset.

7.1 Dataset and Experimental Setup

Next, we describe the dataset we used to evaluate our proposed methods, followed by the experimental setup.

Rice-Facebook dataset. To evaluate our proposed methods, we used *Rice-Facebook* dataset collected by Mislove et al. [30], where they capture the connections between students at the Rice University. The resulting network consists of 1205 nodes and 42443 undirected edges. Each node has 3 attributes: (i) the residential college id (a number between $[1-9]$), (ii) age (a number between $[18-22]$), and (iii) a major ID (which is in the range $[1-60]$).

We grouped the nodes (students) into two groups based on their age attributes. We considered nodes with ages 18 and 19 as group $\mathcal{V}_1$ and age above 20 as group $\mathcal{V}_2$. Group $\mathcal{V}_1$ has 97 nodes and 513 within group edges. On the other hand, group $\mathcal{V}_2$ has 344 nodes and 7441 within group edges. Overall, there are 3350 across group edges going between nodes in $\mathcal{V}_1$ and $\mathcal{V}_2$.

Experimental Setup. In all the experiments in this section, we used activation probability $p_e = 0.01$. All the other parameter were the same as described in Section 6.1.

7.2 TCIM under Budget Constraint

We perform same experiments as done in Section 6.2 on the *Rice-Facebook* dataset, using the same settings as in Section 6.2.

In Figure 5a, we compare the results of TCIM-Budget problem P1 and FairTCIM-Budget problem P4 using two realizations of $\mathcal{H}(z)$, given by: (i) $\mathcal{H}(z) := \log(z)$ and (ii) $\mathcal{H}(z) := \sqrt{z}$. The figure demonstrates that: (i) At a marginal reduction of total influence, our proposed method reduces disparity in influence, as guaranteed by Theorem 1; (ii) as hypothesized in Section 5.1, a higher curvature function, $\mathcal{H}(z) := \log(z)$, leads to a bigger reduction in disparity compared to $\mathcal{H}(z) := \sqrt{z}$, also on a real-word dataset.

x-axis in Figure 5b represents the seed budget allowed. The y-axis shows the fraction of the population influenced/activated. Groups $\mathcal{V}_1$ and $\mathcal{V}_2$ are represented by dash-dotted lines and dotted lines respectively and solid lines correspond to total influence. The figure demonstrates that: (i) FairTCIM-Budget problem results in less disparity of average group influence compared to TCIM-Budget problem; (ii) the reduction in disparity is achieved at a very low cost to the total influence.

Figure 5c represents different time deadlines on the x-axis and disparity, as calculated by Eq. 2, on the y-axis. We can see that (i) our proposed method yields solutions which result in lower disparity, and (ii) the disparity is dependent on the time deadline.

Takeaways. We demonstrated that: (i) FairTCIM-Budget, our proposed method, yields less disparate solutions; (ii) this fairness is achieved at a very small reduction of the total influence, compared to TCIM-Budget problem, as guaranteed by Theorem1; and (iii) the disparity of influence is higher for bigger time deadlines, and unlike in the synthetic dataset our method is also effective in removing disparity for the higher time deadlines on this real-world data.

7.3 TCIM under Coverage Constraint

The experiments done in this section are the same as described in Section 6.3, except that we use *Rice-Facebook* dataset. Figures 6a, 6b, and 6c show similar results as Figures 4a, 4b, and 4c.

Takeaways. We compared the result of TCIM-Cover problem P2 and our solution to FairTCIM-Cover problem P6. The results show that: (i) both methods reach the same fraction of the population; (ii) only FairTCIM-Cover problem results in seed sets influencing the required quota in all the groups; and (iii) lastly, FairTCIM-Cover yields only slightly larger solution sets as guaranteed by Theorem 2.

8 CONCLUSIONS

In this paper, we considered the important problem of time-critical influence maximization (TCIM) under (i) budget constraint (TCIM-Budget) and (ii) coverage constraint (TCIM-Cover). We showed that the existing algorithmic techniques aimed at maximizing total influence in the population could lead to a huge disparity in utility across the underlying groups. This can put minority groups at a big disadvantage with far-reaching consequences. To ensure that different groups are fairly treated, we proposed a notion of fairness and formulated two novel problems to solve TCIM under fairness considerations, namely, FairTCIM-Budget and FairTCIM-Cover. By introducing surrogate objective functions with submodular structural properties, we provided computationally efficient algorithms with desirable guarantees. Experiments over synthetic and a real-world datasets demonstrated that our algorithms lead to low disparity in the time-critical influence propagation. This work opens up a variety of new research problems, including extensions to different notions of fairness, considering more complex models of time-criticality in information propagation (such as discounting with time), and developing new optimization methods for solving the fair TCIM problem formulations.

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