Assignment 2

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Part I – Nominal Return

- **Summary Statistics of all assets(1928-2019)**

```
# nominal returns summary statistics
return_summary_table(nominal_returns)
```

	S&P	T_Bill	T_Bond	Corporate_Bond	Housing
Mean	0.116	0.034	0.051	0.072	0.042
Std	0.196	0.030	0.077	0.076	0.062
Min	-0.438	0.000	-0.111	-0.157	-0.120
Max	0.526	0.140	0.328	0.291	0.241
Kurtosis	0.087	0.901	1.721	0.629	1.304
Skewness	-0.415	1.017	1.007	0.290	0.325
Autocorrelation_1	-0.008	0.921	-0.073	0.166	0.686

- *Mean:*

The mean of S&P return is 11.6%, which is the highest among all assets. It can represent the average stock return for the past 100 years. It is high since the stock market has the highest risk among all assets.
The treasury bill has the lowest return and it indicates the risk-free return rate.
The treasury bond has an average return of 5.1%, which is between the stock return and T-bill return.
The corporate bond has an average return of 7.2%. It is higher than that of treasury bonds since companies have higher default risks than the government does.
The average housing return is 4.2%, which is the second-lowest nominal return of all assets.

- *Standard Deviation*

The S&P has the largest standard deviation, while the treasury bill has the lowest standard deviation.
If we sort standard deviations by sizes, we would observe that the sequence of assets is the same as we sort them by mean sizes. It reflects that high-yield assets have high risks.
Specifically, the standard deviation of stocks is more than 6 times of treasury bill's standard deviation, but stocks' return is less than 4 times of treasury bill's return. Therefore, choosing which asset to include in portfolios can indicate investors' risk preferences. We would talk more in Sharpe ratio analysis later.
The average standard deviation of corporate bonds is lower than that of treasury bonds, but corporate bonds have a higher mean return. Therefore, in the past 100 years, corporate bonds are more favorable than treasury bills based on just mean and average standard deviation.

- *Max & Mean*

The S&P 500 has the maximum nominal return of 52.6% which is highest among all assets as expected. Interestingly, the maximum return of Treasury bonds is 32.8%, which is larger than the maximum return of 29.1% of corporate bonds.
The housing maximum return is 24.1% in 1946. That year is widely considered as the beginning of the baby boom period when the demand for housing surged.

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The S&P 500 has the minimum nominal return of -43.8%, which is also the lowest among all minimum returns of all assets. It is in 1931, the period of the Great Depression.
The corporate bond has a lower minimum return than treasury bonds. It is reasonable since it is riskier.
The minimum of treasury bonds is 0, indicating that the risk-free return is 0 in that year.

- *Kurtosis*

All assets have kurtosis lower than 3, indicating that the return distributions have fewer returns in their tails than the normal distributions. Their distributions all follow the platykurtic distribution.
The S&P 500 has the smallest kurtosis, indicating that its return distribution has a short and broad peak. Treasury bond has the highest kurtosis and it means that the distribution has relatively more outliers.

- *Skewness:*

The S&P 500 has the smallest skewness and it is the only asset that has negative skewness. So, its return distribution is left-skewed and it means that investments in stocks in the past years would have small and frequent gains and few significant losses.
Other assets and positive skewness and their return distributions are right-skewed, and it means that investments in those assets would have small and frequent losses and few significant gains. The treasury bill and treasury bond have the two largest skewness so lots of their returns are below the average but not much, and few of their returns are above the average but very far.

- *Autocorrelation*

The S&P 500 and treasury bill has nearly 0 autocorrelations, indicating that returns of stocks and corporate bonds are unpredictable. The Treasury bill has a significant positive autocorrelation of 1, meaning that if it has a positive yield at year 0, it is very likely it would have a positive yield in year 1. Similarly, housing has a very positive autocorrelation of yields. Therefore, the return of treasury bills and housing are predictable.
Similarly, the corporate bonds a 0.166 autocorrelation.
The treasury bond has a negative autocorrelation but it is too small to be significant.

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- **Excess returns on stocks and bonds**

```
# excess nominal returns summary statistics
return_summary_table(excess_nominal_returns)
```

	S&P	T_Bond	Corporate_Bond	Housing
Mean	0.082	0.017	0.038	0.008
Std	0.199	0.074	0.079	0.065
Min	-0.461	-0.144	-0.180	-0.134
Max	0.516	0.222	0.232	0.237
Kurtosis	0.002	0.349	0.205	1.642
Skewness	-0.318	0.375	-0.079	0.785
Autocorrelation_1	0.002	-0.140	0.179	0.705

- *Mean:*

Besides the treasury bill, the mean excess returns of all assets are larger than their mean returns. They are all positive, indicating that investments in those assets outperformed treasury bills.

- *Standard deviation:*

Standard deviation of different assets all change a little bit, but those changes are small and insignificant.

- Min & Max

Both minimum and maximum excess returns of all assets are smaller than their nominal returns

- Kurtosis

Beside the housing, other assets' excess returns have smaller kurtosis, indicating their distributions have broader peaks and fewer outliers.

- Skewness

Excess returns of stocks become less left-skewed than nominal returns.
Excess returns of treasury bonds decrease significantly and become less right-skewed than nominal returns.
Interestingly, excess returns of corporate bonds are left-skewed, compared with its right-skewed distribution of nominal returns.
The housing's excess returns are much more right-skewed compared with its distribution of nominal returns.

- Autocorrelation

All assets' autocorrelations of excess returns are higher in absolute value than that of nominal returns, indicating that excess returns and relatively more predictable.

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- **Portfolios based on different asset allocations**

We use *np.linspace* function to create weights of stocks and bonds within each portfolios.

```
weights_stock = np.linspace(0, 1, 11)
weights_bond = 1 - weights_stock
weights = np.vstack((weights_stock, weights_bond))
# 2 * 11 matrix
weights
```

```
array([[0. , 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. ],
       [1. , 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0. ]])
```

Then we use a matrix to demonstrate the nominal returns of different portfolio across the past 100 years. (first 5 years are shown in the picture below)

```
# matrix multiplication
port_returns_nominal = excess_nominal_returns[['S&P', 'T_Bond']].dot(weights) # 92 * 11 matrix
port_returns_nominal.columns = [f'{i}%_Stock' for i in range(0, 110, 10)]
port_returns_nominal.head().style.format('{:.2%}')
```

Year	0%_Stock	10%_Stock	20%_Stock	30%_Stock	40%_Stock	50%_Stock	60%_Stock	70%_Stock	80%_Stock	90%_Stock	100%_Stock
1928	-2.24%	2.05%	6.35%	10.65%	14.95%	19.24%	23.54%	27.84%	32.14%	36.43%	40.73%
1929	1.04%	-0.21%	-1.46%	-2.71%	-3.96%	-5.21%	-6.46%	-7.71%	-8.96%	-10.21%	-11.46%
1930	-0.01%	-2.98%	-5.94%	-8.91%	-11.87%	-14.84%	-17.81%	-20.77%	-23.74%	-26.71%	-29.67%
1931	-4.87%	-9.00%	-13.12%	-17.25%	-21.38%	-25.51%	-29.64%	-33.76%	-37.89%	-42.02%	-46.15%
1932	7.72%	5.98%	4.23%	2.49%	0.75%	-1.00%	-2.74%	-4.48%	-6.23%	-7.97%	-9.71%

- **Summary Statistics of all portfolios**

```
# summary statistics for portfolios
return_summary_table(port_returns_nominal)
```

	0%_Stock	10%_Stock	20%_Stock	30%_Stock	40%_Stock	50%_Stock	60%_Stock	70%_Stock	80%_Stock	90%_Stock	100%_Stock
Mean	0.017	0.024	0.030	0.037	0.043	0.050	0.056	0.062	0.069	0.075	0.082
Std	0.074	0.070	0.072	0.080	0.092	0.107	0.124	0.141	0.160	0.179	0.199
Min	-0.144	-0.120	-0.131	-0.173	-0.214	-0.255	-0.296	-0.338	-0.379	-0.420	-0.461
Max	0.222	0.210	0.207	0.221	0.235	0.270	0.319	0.368	0.418	0.467	0.516
Kurtosis	0.349	0.114	-0.085	-0.139	-0.113	-0.075	-0.047	-0.028	-0.015	-0.005	0.002
Skewness	0.375	0.255	0.055	-0.124	-0.233	-0.286	-0.309	-0.317	-0.320	-0.320	-0.318
Autocorrelation_1	-0.140	-0.097	-0.044	-0.004	0.015	0.021	0.019	0.015	0.011	0.006	0.002

Based on the chart, **the minimum variance portfolio has 10% stock and 90% bonds.** This portfolio minimizes its return volatility while achieving the relatively high returns, so nobody on the market would take return lower than this portfolio's return.

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- **Sharpe Ratio**

The portfolio with 40% stocks and 60% bonds has the highest Sharpe ratio.

```
# portfolio sharpe ratios
Sharpe_Ratio(port_returns_nominal)
```

```
0%_Stock      0.235
10%_Stock     0.341
20%_Stock     0.421
30%_Stock     0.460
40%_Stock     0.470
50%_Stock     0.465
60%_Stock     0.454
70%_Stock     0.442
80%_Stock     0.430
90%_Stock     0.420
100%_Stock    0.411
```

Sharpe ratio calculated as *the mean excess return of the portfolio / standard deviation of the portfolio excess return*. The ratio adjusts the portfolio's returns from the excess risks, so it becomes the method to calculate the risk-adjusted return.
The higher the Sharpe ratio, the higher risk-adjusted-return the portfolio has. Therefore, the portfolio of 40% stock and 60% bonds has the most attractive risk-adjusted return.

Part II – Real Return

- **Summary Statistics of all assets (1928-2019)**

```
# real returns summary statistics
return_summary_table(real_returns)
```

	S&P	T_Bill	T_Bond	Corporate_Bond	Housing
Mean	0.084	0.004	0.022	0.042	0.011
Std	0.196	0.036	0.081	0.086	0.050
Min	-0.389	-0.121	-0.146	-0.149	-0.152
Max	0.582	0.127	0.251	0.378	0.147
Kurtosis	0.084	3.585	0.326	1.782	1.451
Skewness	-0.165	0.005	0.519	0.565	-0.055
Autocorrelation_1	-0.025	0.716	0.056	0.220	0.549

The real return is calculated by *((1+R_nominal)/(1+inflation)) -1.* It is inflation-adjusted return and it reflects the actual purchasing power today.

- *mean*

The mean real returns are lower than nominal returns of all assets.

- *Standard deviation*

Beside stocks and housing, the standard deviations of other assets are a little bit higher, indicating higher volatilities.

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- *Min & max*

The minimum real return of stocks is higher after adjusting the inflation. This also happens to the corporate bond.
The treasury bill's minimum return becomes negative, rather than 0 in terms of nominal returns.
The treasury bond's minimum return is lower after adjusting the inflation, This also happens to the housing.

- *Kurtosis*

Stocks have nearly the same kurtosis values compared with nominal returns
Treasury bills' kurtosis value increases dramatically so its real return distribution concentrates more toward the mean.
Treasury bond has smaller kurtosis after adjusting the inflation, meaning that its real returns have a short and broad-looking peak, as well as fewer outliers.
Both Corporate bonds and housing have higher kurtosis value of their real return than nominal returns.

- *Skewness*

Stocks become less negatively-skewed.
Treasury bills, treasury bonds, and corporate bonds all become less positively-skewed.
Interestingly, after adjusting for inflation, the real return of housing switches from right skewness to left skewness.

- *Autocorrelation*

Autocorrelation of stocks become more negative, while the autocorrelation of corporate bond become more positive. But those changes are insignificant.
Autocorrelation of treasury bills and housing decrease in absolute values, meaning that they become more unpredictable
Interestingly, the autocorrelation of treasury bonds switch from negative autocorrelation toward positive autocorrelation, though the absolute changes are very small.

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- **Excess real returns on stocks and bonds**

```
# excess real returns summary statistics
return_summary_table(excess_real_returns)
```

	S&P	T_Bond	Corporate_Bond	Housing
Mean	0.080	0.017	0.038	0.007
Std	0.197	0.072	0.078	0.063
Min	-0.507	-0.127	-0.198	-0.129
Max	0.517	0.209	0.251	0.219
Kurtosis	0.242	0.269	0.552	1.262
Skewness	-0.317	0.382	-0.015	0.625
Autocorrelation_1	0.013	-0.154	0.156	0.712

The mean, standard deviation, minimum, maximum, skewness and autocorrelation of excess real return are very similar to those of nominal returns.
The kurtosis of stocks and corporate bonds increase a lot.

- **Sharpe ratio for real return**

```
# Sharpe Ratio for real returns
sp_real = Sharpe_Ratio(excess_real_returns)

pd.DataFrame({'Sharpe_Nominal':sp_nominal, 'Sharpe_Real':sp_real})
```

	Sharpe_Nominal	Sharpe_Real
S&P	0.411	0.406
T_Bond	0.235	0.242
Corporate_Bond	0.485	0.487
Housing	0.119	0.109

For all assets, the Sharpe ratios for real returns are nearly the same as the Sharpe ratios for nominal returns.

- **Portfolios based on different asset allocations (real return)**

```
# matrix multiplication (using weights from part 1)
port_returns_real = excess_real_returns[['S&P', 'T_Bond']].dot(weights) # 92 * 11 matrix
port_returns_real.columns = [f'{i}%_Stock' for i in range(0, 110, 10)]
port_returns_real.head().style.format('{:.2%}')
```

Year	0%_Stock	10%_Stock	20%_Stock	30%_Stock	40%_Stock	50%_Stock	60%_Stock	70%_Stock	80%_Stock	90%_Stock	100%_Stock
1928	-2.27%	2.08%	6.42%	10.77%	15.12%	19.47%	23.82%	28.16%	32.51%	36.86%	41.21%
1929	1.04%	-0.21%	-1.46%	-2.71%	-3.96%	-5.21%	-6.46%	-7.71%	-8.96%	-10.21%	-11.46%
1930	-0.01%	-3.06%	-6.11%	-9.15%	-12.20%	-15.25%	-18.30%	-21.34%	-24.39%	-27.44%	-30.49%
1931	-5.35%	-9.88%	-14.41%	-18.94%	-23.48%	-28.01%	-32.54%	-37.08%	-41.61%	-46.14%	-50.67%
1932	8.61%	6.66%	4.72%	2.78%	0.83%	-1.11%	-3.05%	-5.00%	-6.94%	-8.88%	-10.83%

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- **Summary Statistics of all portfolios (real returns)**

```
# summary statistics for portfolios
return_summary_table(port_returns_real)
```

	0%_Stock	10%_Stock	20%_Stock	30%_Stock	40%_Stock	50%_Stock	60%_Stock	70%_Stock	80%_Stock	90%_Stock	100%_Stock
Mean	0.017	0.024	0.030	0.036	0.042	0.049	0.055	0.061	0.067	0.074	0.080
Std	0.072	0.068	0.070	0.078	0.090	0.105	0.122	0.140	0.158	0.177	0.197
Min	-0.127	-0.114	-0.144	-0.189	-0.235	-0.280	-0.325	-0.371	-0.416	-0.461	-0.507
Max	0.209	0.197	0.202	0.215	0.228	0.269	0.318	0.367	0.416	0.466	0.517
Kurtosis	0.269	0.056	-0.060	0.002	0.111	0.187	0.225	0.242	0.246	0.245	0.242
Skewness	0.382	0.247	0.031	-0.154	-0.258	-0.304	-0.321	-0.325	-0.324	-0.321	-0.317
Autocorrelation_1	-0.154	-0.106	-0.045	0.001	0.024	0.031	0.031	0.027	0.022	0.018	0.013

We find that the portfolio with the smallest variance is still the portfolio with 10% stock and 90% bonds

- **Sharpe Ratio**

The portfolio with 40% stocks and 60% bonds also still has the highest Sharpe ratio.

```
# summary statistics for portfolios
Sharpe_Ratio(port_returns_real)
```

```
0%_Stock      0.242
10%_Stock     0.348
20%_Stock     0.426
30%_Stock     0.462
40%_Stock     0.469
50%_Stock     0.462
60%_Stock     0.450
70%_Stock     0.437
80%_Stock     0.426
90%_Stock     0.415
100%_Stock    0.406
```

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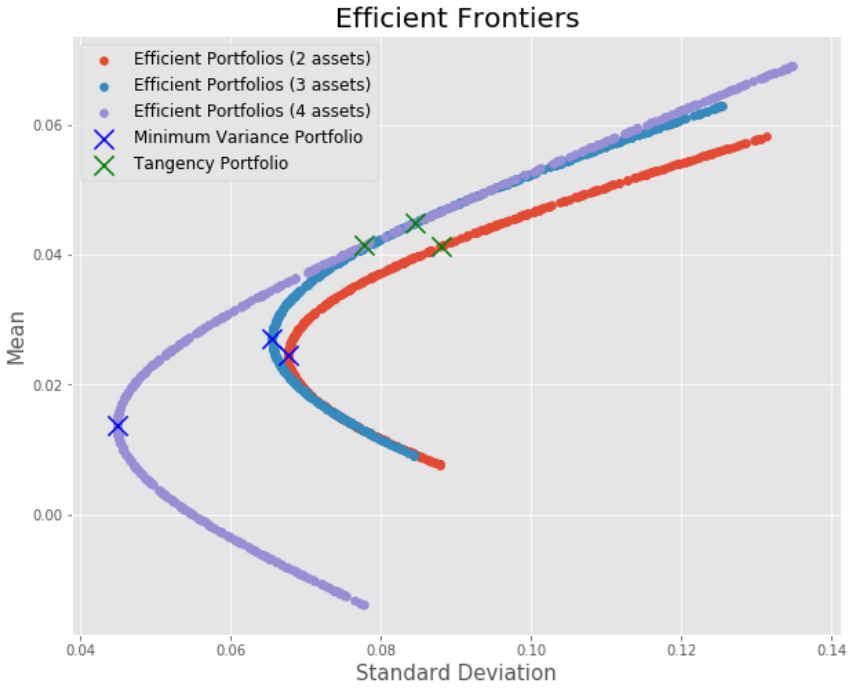
Part III – Efficient frontiers

We Compute an efficient frontier based on two fund separation theorems. Two funds are the minimum variance portfolio and the tangency portfolio.
Therefore, we created a function that returns a dictionary which stores lists of means and standard deviations from generated efficient portfolios, and also information of minimum variance portfolio and tangency portfolio.

We have created the following three assets allocation plans:
Portfolio 1: 2 asset classes - S&P, T-Bond
Portfolio 2: 3 asset classes - S&P, T-Bond, Corporate Bond
Portfolio 3: 4 asset classes - S&P, T-Bond, Corporate Bond, Housing

We draw a graph of efficient frontiers of three portfolios:



Based on the graph, we can easily see that portfolio with more types of assets tends to have a smaller standard deviation while maintaining a relatively good mean return.

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Specifically, we summarize the minimum variance portfolio and tangency portfolio of each asset allocation plan in the following two tables:

- Summary Statistics for Minimum Variance Portfolios

	two_assets	three_assets	four_assets
Mean	0.024	0.027	0.014
Std	0.068	0.066	0.045
Skewness	0.217	0.086	0.119
Kurtosis	-0.037	-0.027	-0.307

Portfolio 1: 11.35% S&P, 88.65% T-Bond
Portfolio 2: 5.35% S&P, 63.38% T-Bond, 30.82% Corporate Bond
Portfolio 3: -0.72% S&P, 34.85% T-Bond, 12.35% Corporate Bond, 53.52% Housing

Compared with individual assets, we find that each portfolio has very low standard deviations.
Moreover, although the variance of portfolio 3 is the smallest, that portfolio also has the smallest mean, proving again the tradeoff between return and risks.
All three portfolios have positive skewness and kurtosis.

- Summary Statistics for Tangency Portfolios

	two_assets	three_assets	four_assets
Mean	0.041	0.045	0.041
Std	0.088	0.085	0.078
Skewness	-0.242	-0.495	-0.570
Kurtosis	0.025	0.497	0.617

Portfolio 1: 38.38% S&P, 61.62% T-Bond
Portfolio 2: 19.73% S&P, 5.54% T-Bond, 74.74% Corporate Bond
Portfolio 3: 17.61% S&P, 5.84% T-Bond, 68.29% Corporate Bond, 8.26% Housing

Compared with individual assets, we find that each portfolio has a good mean return while keeping the low standard deviation.
All three portfolios have positive skewness and kurtosis.

By comparing the Sharpe ratios of three selected portfolios, we found portfolio 3 has the largest Sharpe ratio of 52.94%. Therefore, to maximize our risk-adjusted returns, we should choose to include stock, treasury bonds, and corporate bonds into our portfolios.