Señales básicas

Fernando Lozano

Universidad de los Andes

Escalón Unitario

Escalón Unitario

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

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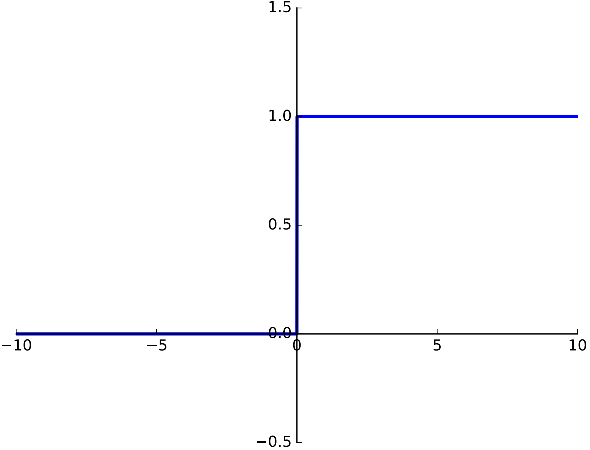
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$u(t-4)$

$$u(t-4)=\begin{cases}1 & t-4\geq 0\\ 0 & t-4<0\end{cases}$$

$$u(t-4) = \begin{cases} 1 & t-4 \geq 0 \\ 0 & t-4 < 0 \end{cases} = \begin{cases} 1 & t \geq 4 \\ 0 & t < 4 \end{cases}$$

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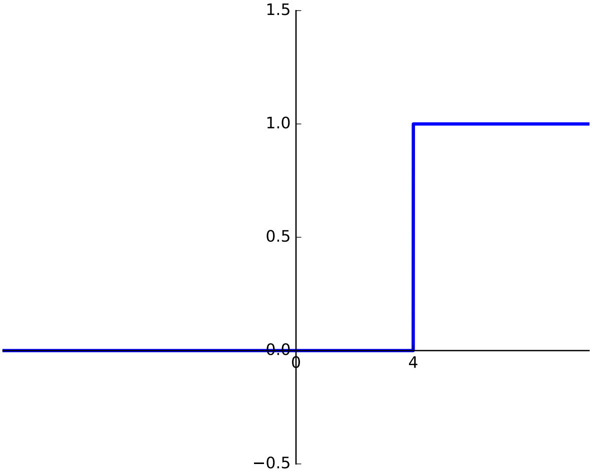
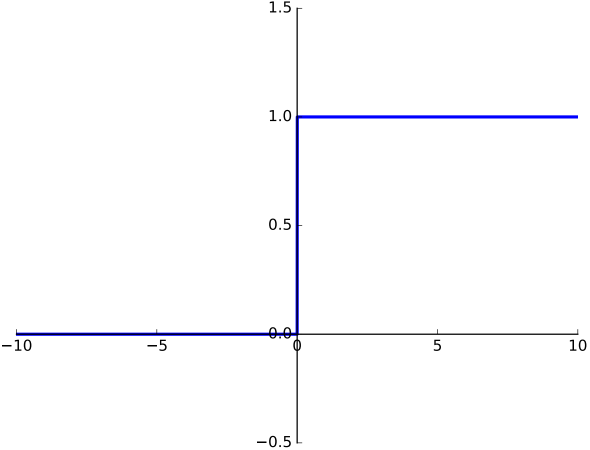
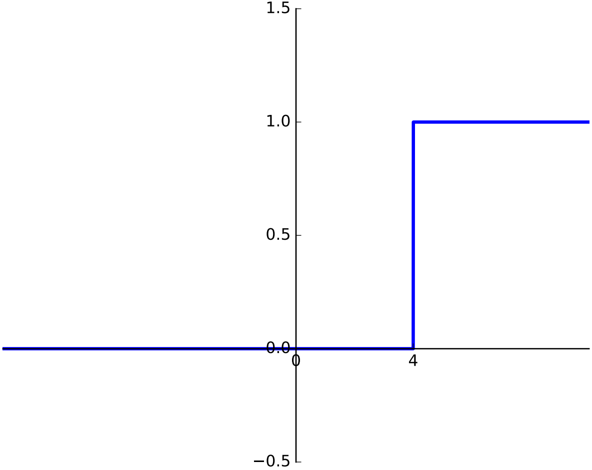


$u(t)$



$$u(t-4)$$



$$u(t+4)$$

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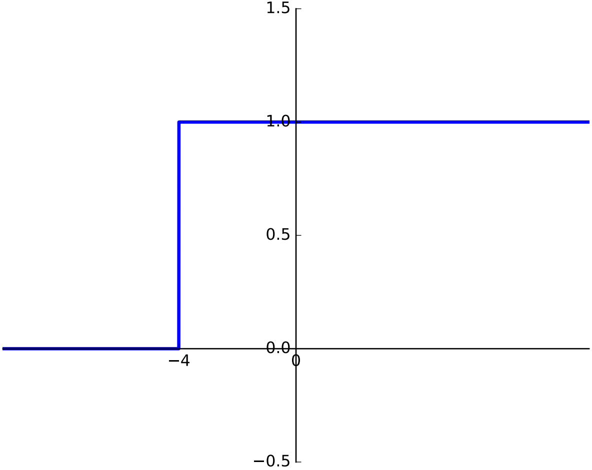

$u(-t)$

$$u(-t) = \begin{cases} 1 & -t \geq 0 \\ 0 & -t < 0 \end{cases}$$

$$u(-t) = \begin{cases} 1 & -t \geq 0 \\ 0 & -t < 0 \end{cases} = \begin{cases} 1 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

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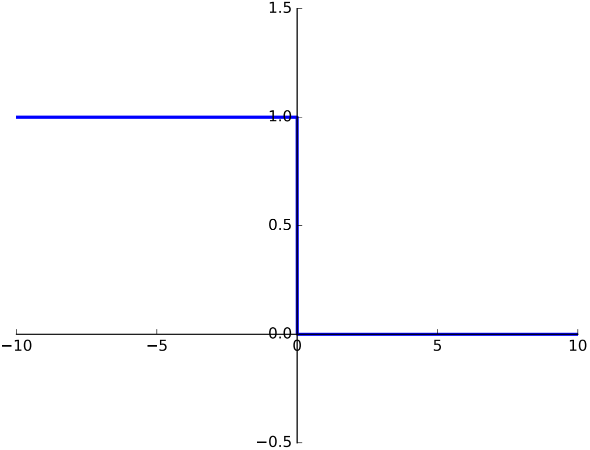
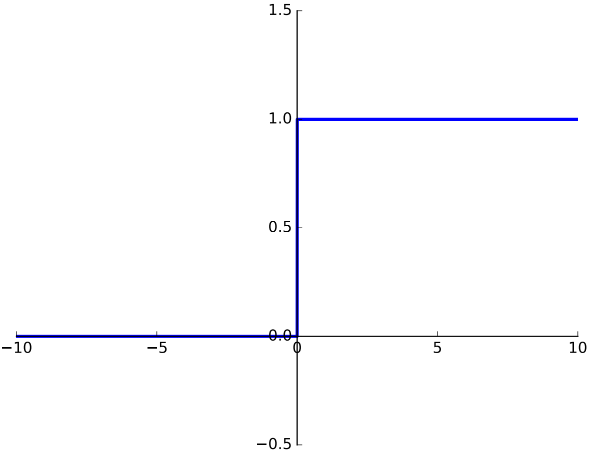
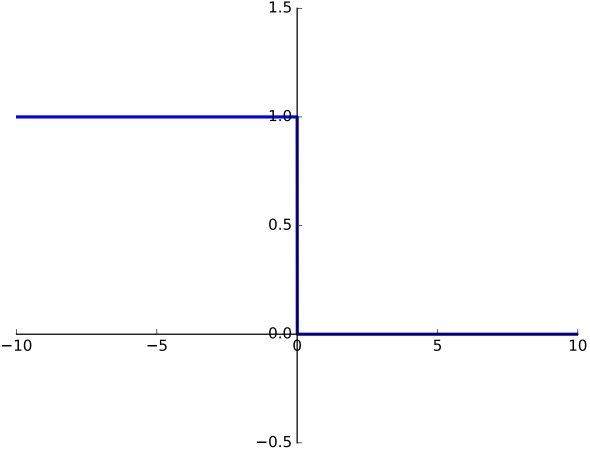


$u(t)$



$u(-t)$



$$u(-t+4)$$

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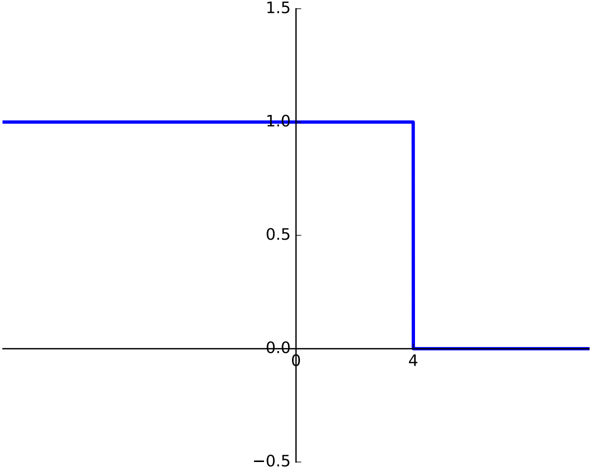
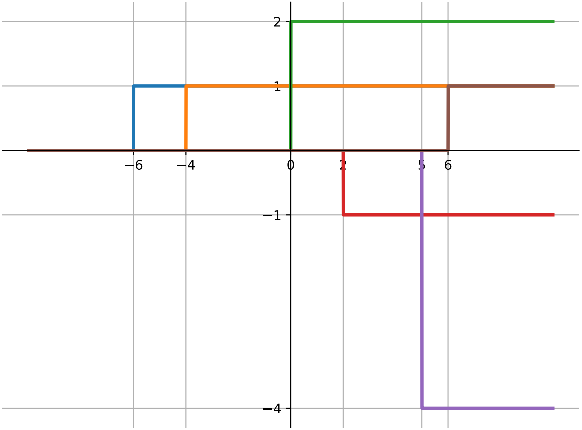


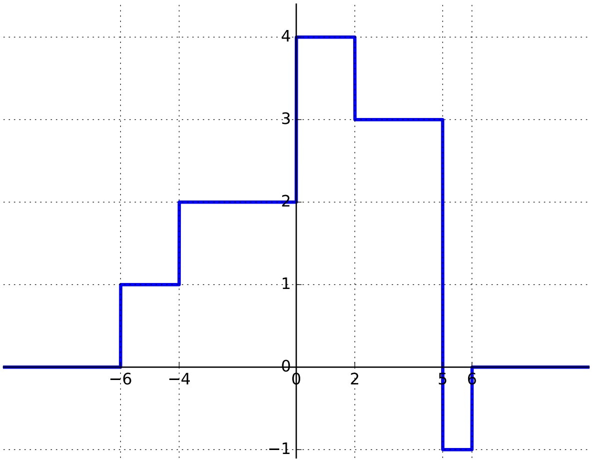
$$x(t) = u(t+6) + u(t+4) + 2u(t) - u(t-2) - 4u(t-5) + u(t-6)$$

$$x(t) = u(t+6) + u(t+4) + 2u(t) - u(t-2) - 4u(t-5) + u(t-6)$$



$$x(t) = u(t+6) + u(t+4) + 2u(t) - u(t-2) - 4u(t-5) + u(t-6)$$

$$x(t) = u(t+6) + u(t+4) + 2u(t) - u(t-2) - 4u(t-5) + u(t-6)$$



Rampa unitaria

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

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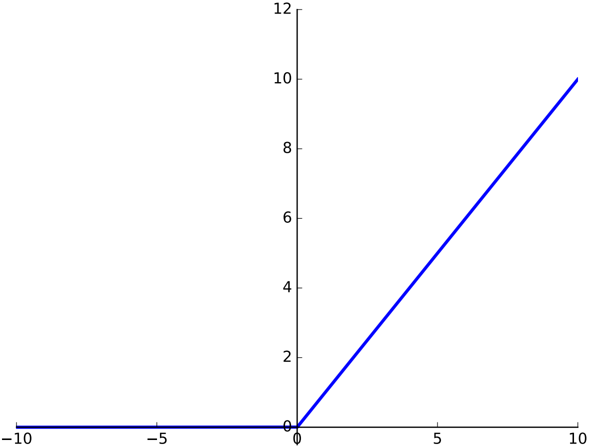

$r(t-4)$

$$r(t-4) = \begin{cases} t-4 & t-4 \geq 0 \\ 0 & t-4 < 0 \end{cases}$$

$$r(t-4) = \begin{cases} t-4 & t-4 \geq 0 \\ 0 & t-4 < 0 \end{cases} = \begin{cases} t-4 & t \geq 4 \\ 0 & t < 4 \end{cases}$$

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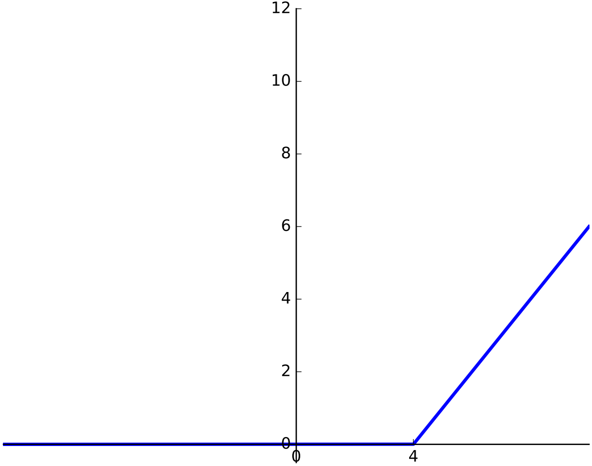
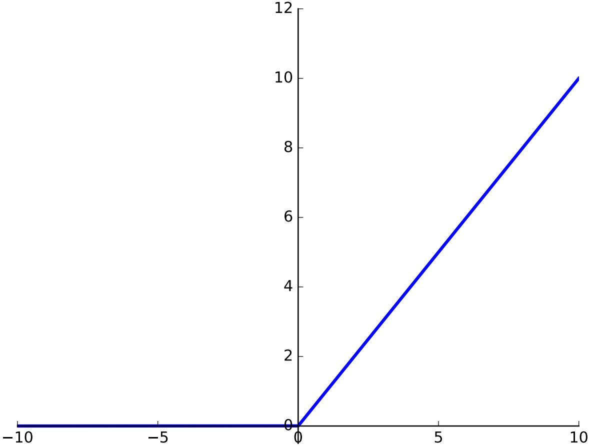
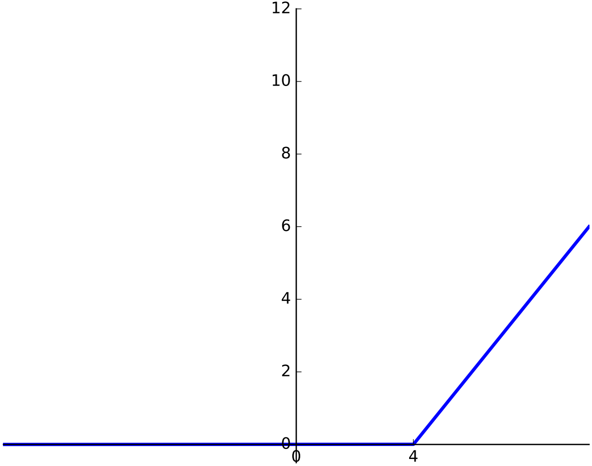


$r(t)$

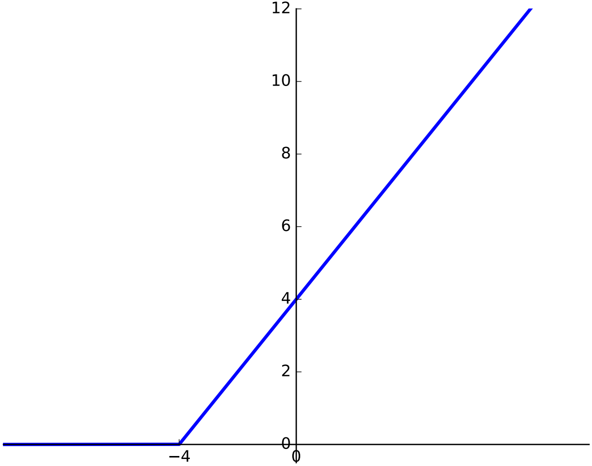


$$r(t-4)$$



$$r(t + 4)$$

$r(t+4)$



$r(-t)$

$$r(-t) = \begin{cases} -t & -t \geq 0 \\ 0 & -t < 0 \end{cases}$$

$$r(-t) = \begin{cases} -t & -t \geq 0 \\ 0 & -t < 0 \end{cases} = \begin{cases} -t & t \leq 0 \\ 0 & t > 0 \end{cases}$$

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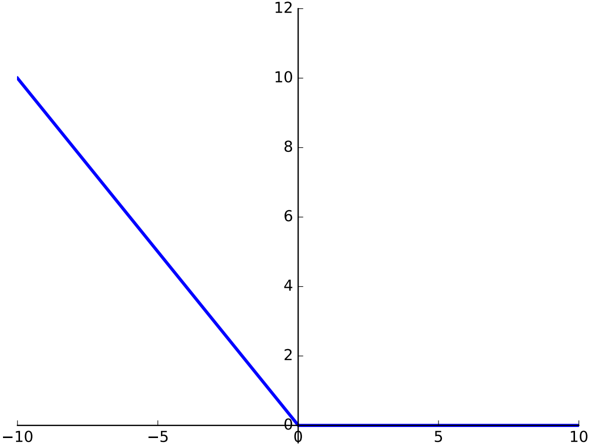
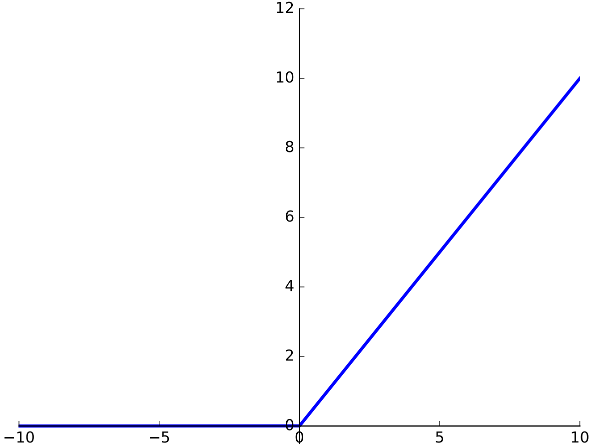
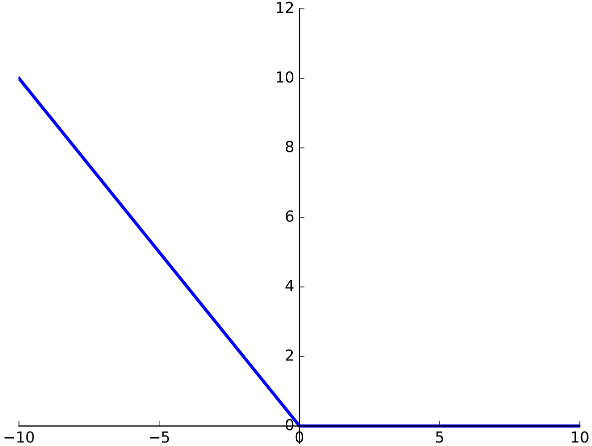


$r(t)$



$r(-t)$



Relación entre $u(t)$ y $r(t)$

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$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

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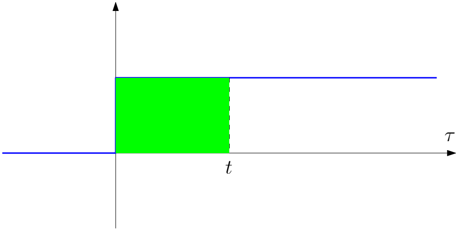
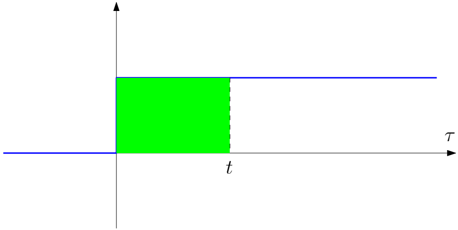


Relación entre $u(t)$ y $r(t)$

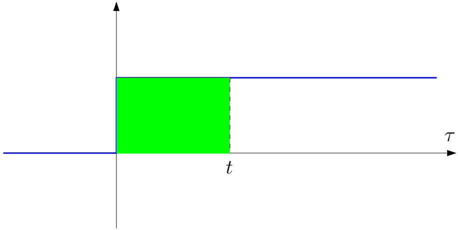
$$r(t) = \int_{-\infty}^{t} u(\tau) d\tau$$



$$u(t) = \frac{d}{dt} r(t)$$

Relación entre $u(t)$ y $r(t)$

$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

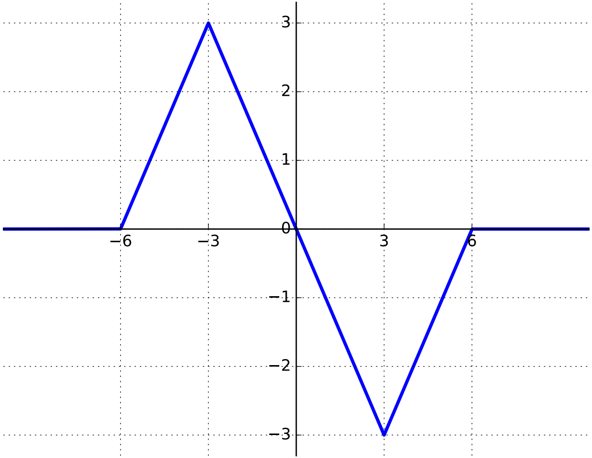


$$u(t) = \frac{d}{dt}r(t)$$

(Excepto en $t = 0$!)

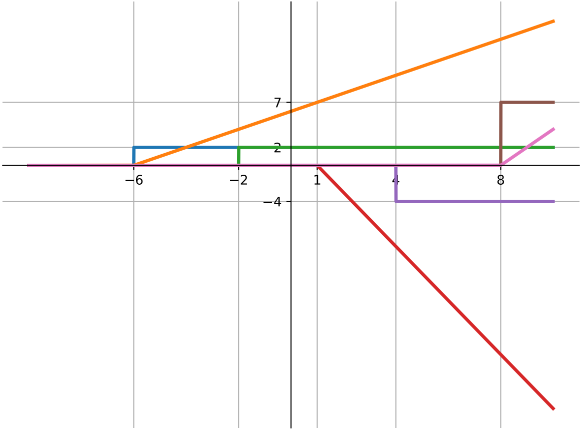
$$x(t) = r(t+6) - 2r(t+3) + 2r(t-3) - r(t-6)$$

$$x(t) = r(t+6) - 2r(t+3) + 2r(t-3) - r(t-6)$$



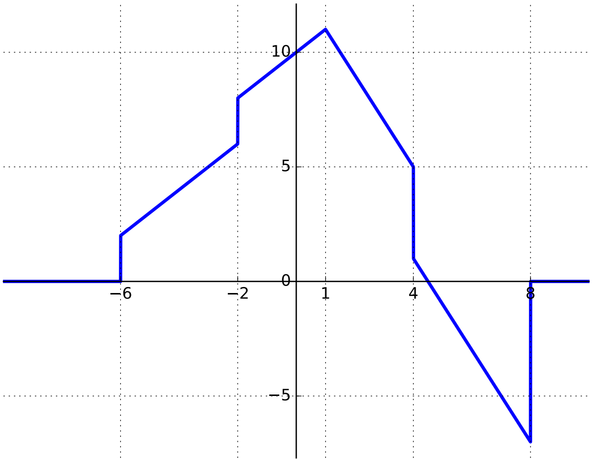
$$x(t) = 2u(t+6)+r(t+6)+2u(t+2)-3r(t-1)-4u(t-4)+2r(t-8)+7u(t-8)$$

$$x(t) = 2u(t{+}6) + r(t{+}6) + 2u(t{+}2) - 3r(t{-}1) - 4u(t{-}4) + 2r(t{-}8) + 7u(t{-}8)$$



$$x(t) = 2u(t+6)+r(t+6)+2u(t+2)-3r(t-1)-4u(t-4)+2r(t-8)+7u(t-8)$$

$$x(t) = 2u(t+6) + r(t+6) + 2u(t+2) - 3r(t-1) - 4u(t-4) + 2r(t-8) + 7u(t-8)$$



$$y(t) = x(t-2)$$

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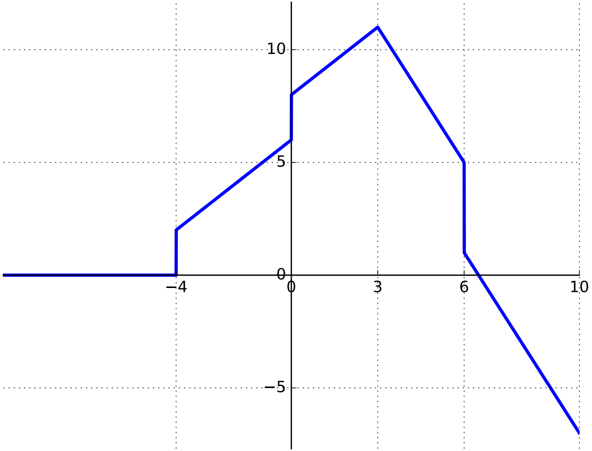
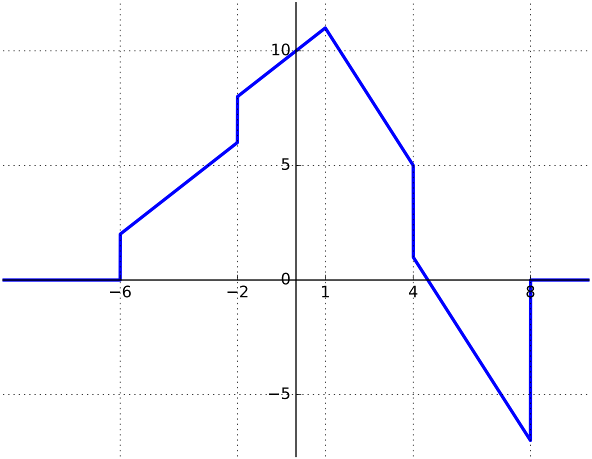
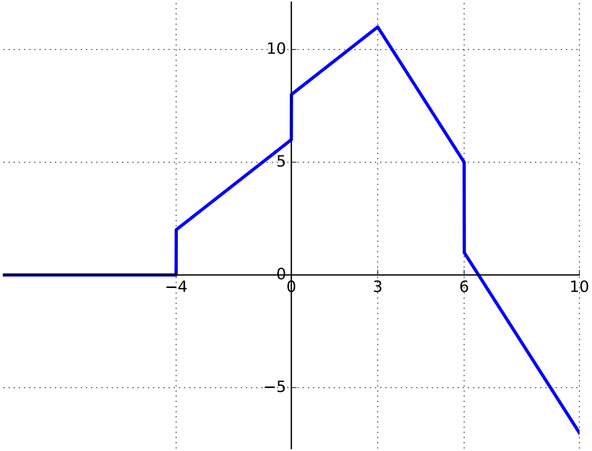


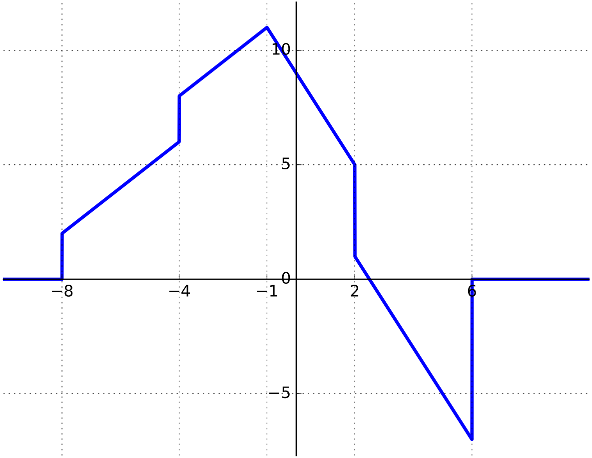
$x(t)$



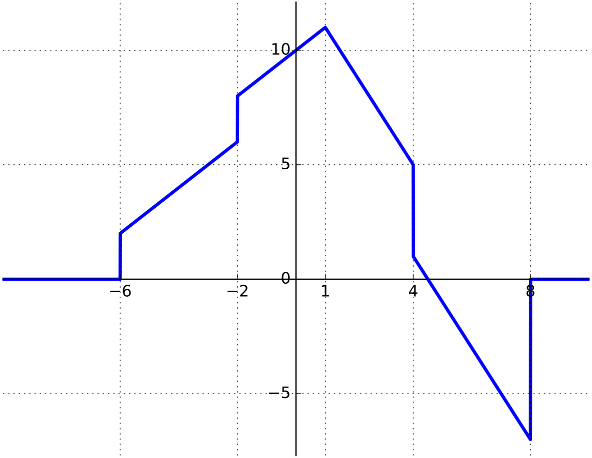
$$y(t) = x(t-2)$$



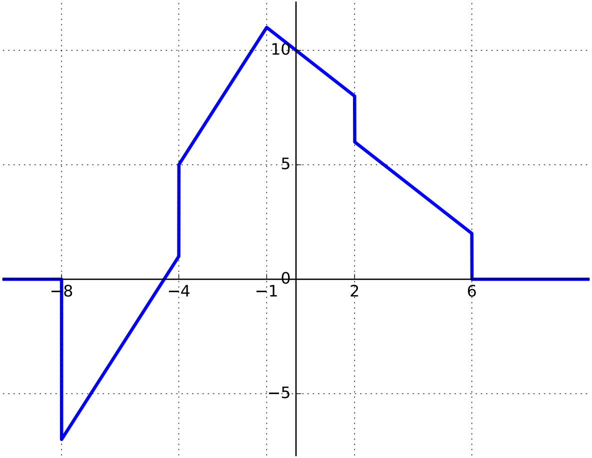
$$y(t) = x(t+2)$$



$x(t)$



$$y(t) = x(-t)$$



$$y(t) = x(2t)$$

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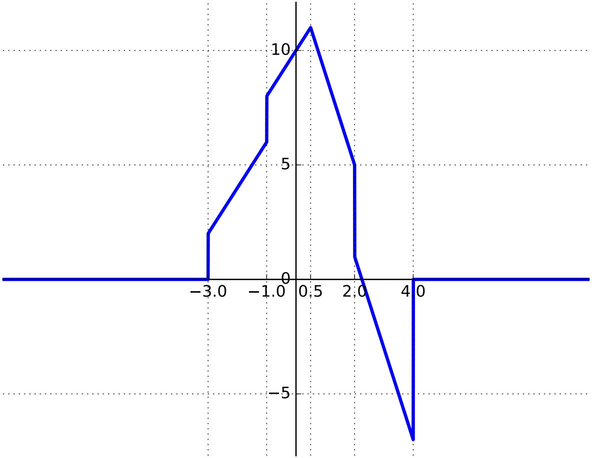
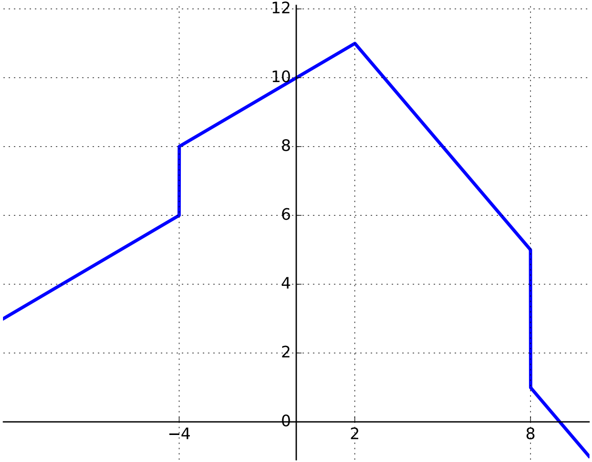
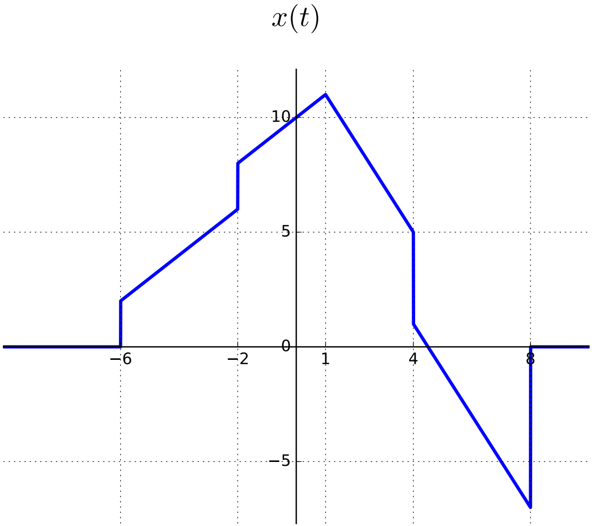


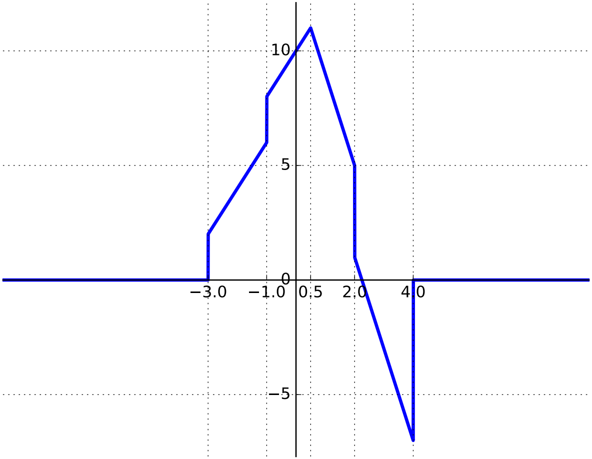
$$y(t) = x(t/2)$$

$$y(t) = x(t/2)$$

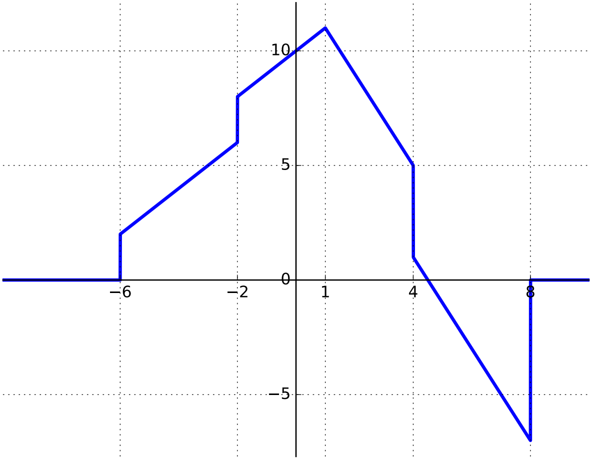



$x(t)$
10
5
0
−5
−6
−2
1
4
8

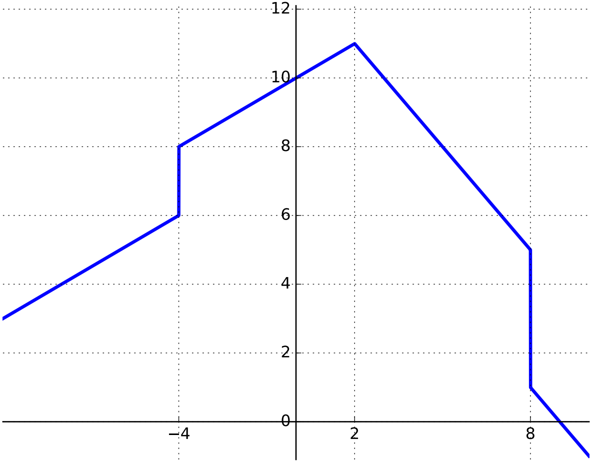
$$y(t) = x(2t)$$



$x(t)$

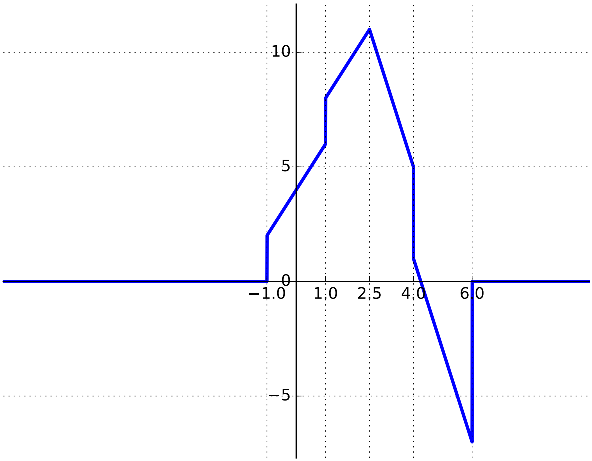


$$y(t) = x(t/2)$$



$$y(t) = x(2t - 4)$$

$$y(t) = x(2t - 4)$$



Señales Pares e Impares

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- $x(t)$ es par $\Leftrightarrow x(t) = x(-t)$

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- $x(t)$ es impar $\Leftrightarrow x(t) = -x(-t)$

Señales Pares e Impares

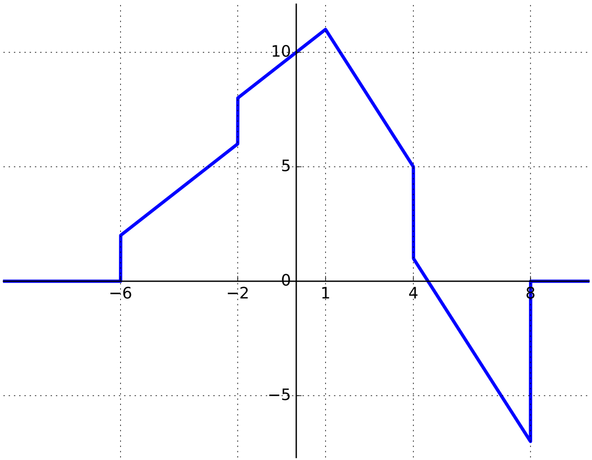
- $x(t)$ es par $\Leftrightarrow x(t) = x(-t)$
- $x(t)$ es impar $\Leftrightarrow x(t) = -x(-t)$
- Parte par e impar de una seña $x(t)$:

Señales Pares e Impares

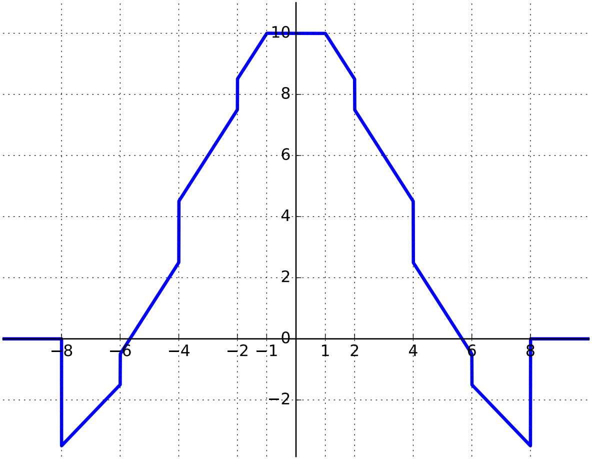
- $x(t)$ es par $\Leftrightarrow x(t) = x(-t)$
- $x(t)$ es impar $\Leftrightarrow x(t) = -x(-t)$
- Parte par e impar de una seña $x(t)$:

$$\mathcal{EV}\{x(t)\} = \frac{x(t) + x(-t)}{2}$$
$$\mathcal{ODD}\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

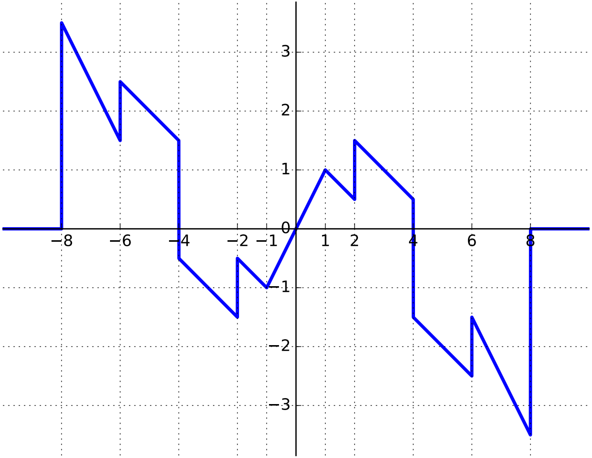
$x(t)$



$$\mathcal{EV}\left\{x(t)\right\}$$



$$\mathcal{ODD}\left\{x(t)\right\}$$



Escalón Unitario

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$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

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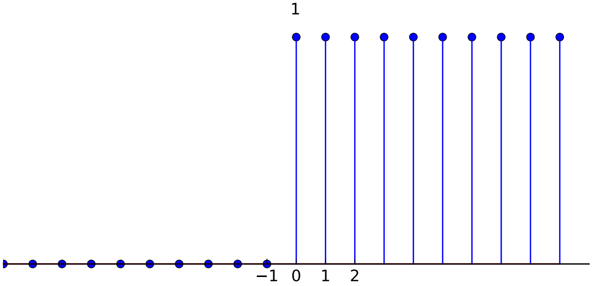
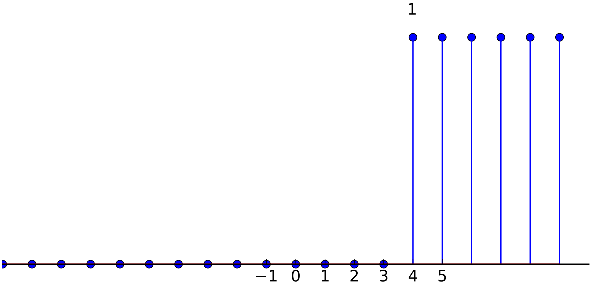


$$u[n-4] =$$

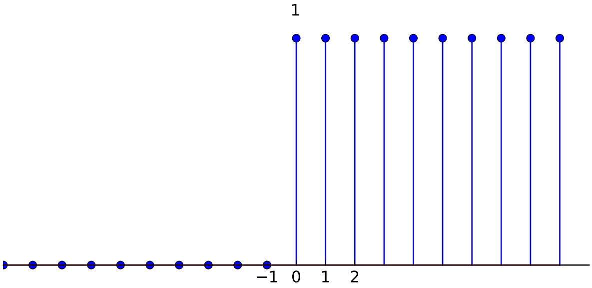
$$u[n-4] = \ = \begin{cases} 1 & n-4 \geq 0 \\ 0 & n-4 < 0 \end{cases}$$

$$u[n-4] = \;=\begin{cases}1 & n-4 \geq 0\\ 0 & n-4 < 0\end{cases} = \begin{cases}1 & n \geq 4\\ 0 & n < 4\end{cases}$$

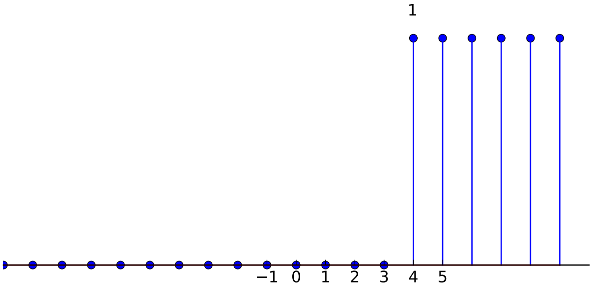
$$u[n-4] = \; = \begin{cases} 1 & n-4 \geq 0 \\ 0 & n-4 < 0 \end{cases} = \begin{cases} 1 & n \geq 4 \\ 0 & n < 4 \end{cases}$$



$u[n]$

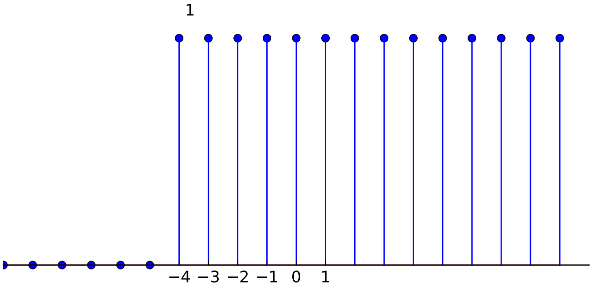


$$u[n-4]$$



$$u[n+4]$$

$$u[n+4]$$



$u[-n]$

$$u[-n] = \begin{cases} 1 & -n \geq 0 \\ 0 & -n < 0 \end{cases}$$

$$u[-n] = \begin{cases} 1 & -n \geq 0 \\ 0 & -n < 0 \end{cases} = \begin{cases} 1 & n \leq 0 \\ 0 & n > 0 \end{cases}$$

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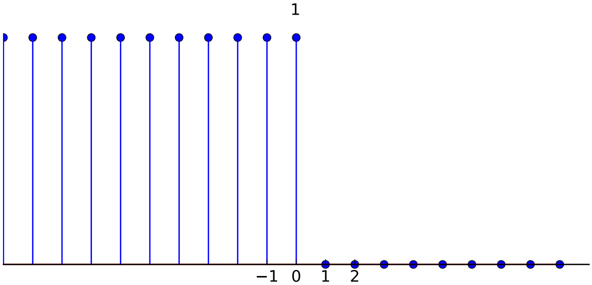
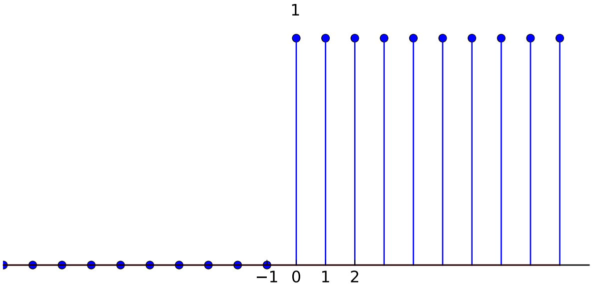
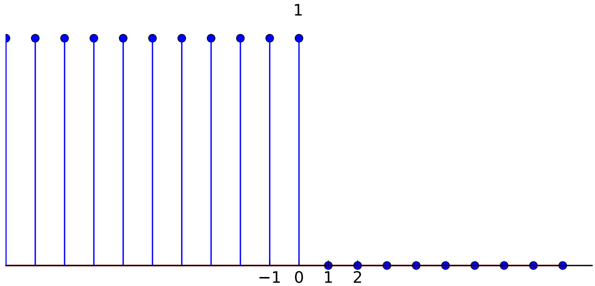


$u[n]$

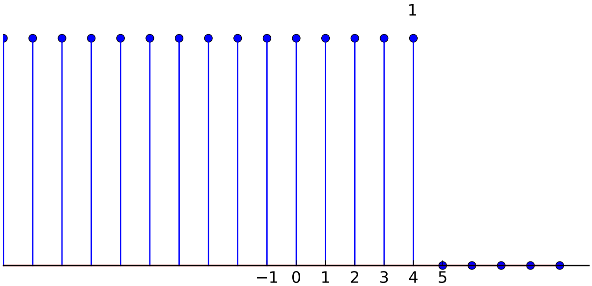


$$u[-n]$$



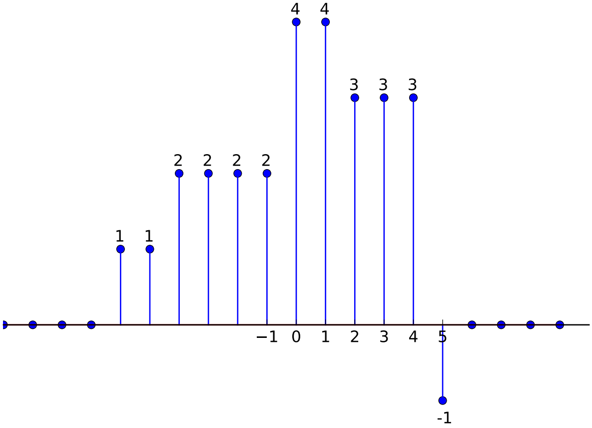
$$u[-n+4]$$

$$u[-n+4]$$



$$x[n] = u[n+6] + u[n+4] + 2u[n] - u[n-2] - 4u[n-5] + u[n-6]$$

$$x[n] = u[n+6] + u[n+4] + 2u[n] - u[n-2] - 4u[n-5] + u[n-6]$$

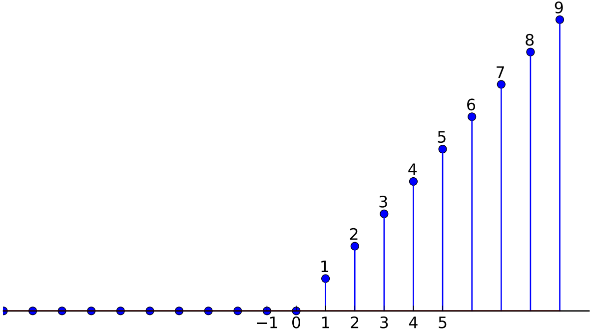


Rampa unitaria

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Rampa unitaria

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$r[n-4]$

$$r[n-4] = \begin{cases} n-4 & n-4 \geq 0 \\ 0 & n-4 < 0 \end{cases}$$

$$r[n-4] = \begin{cases} n-4 & n-4 \geq 0 \\ 0 & n-4 < 0 \end{cases} = \begin{cases} n-4 & n \geq 4 \\ 0 & n < 4 \end{cases}$$

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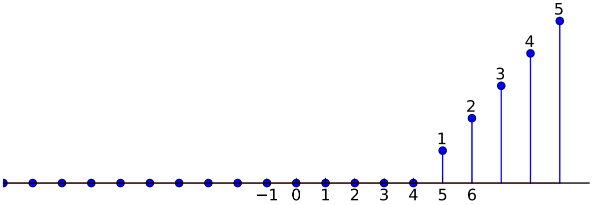
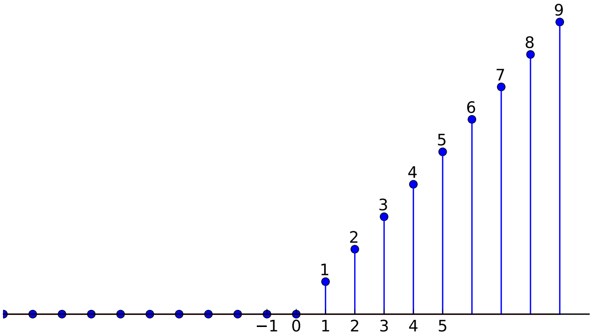
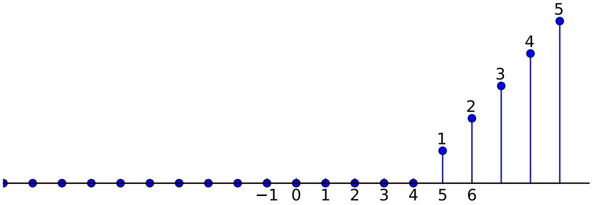


$r[n]$



$$r[n-4]$$



$$r[n+4]$$

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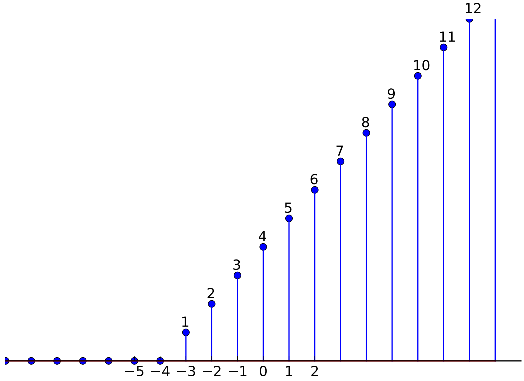

$r[-n]$

$$r[-n] = \begin{cases} -n & -n \geq 0 \\ 0 & -n < 0 \end{cases}$$

$$r[-n] = \begin{cases} -n & -n \geq 0 \\ 0 & -n < 0 \end{cases} = \begin{cases} -n & n \leq 0 \\ 0 & t > 0 \end{cases}$$

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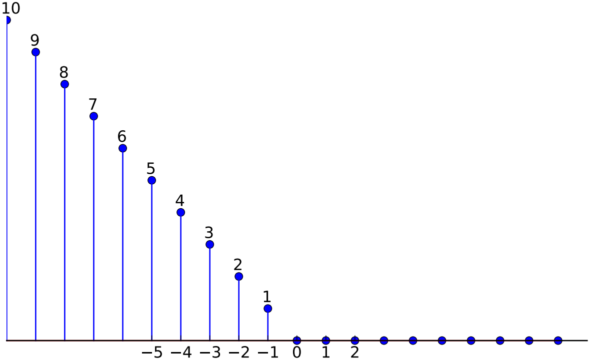
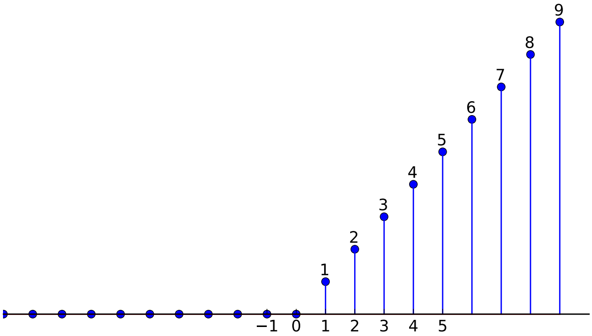
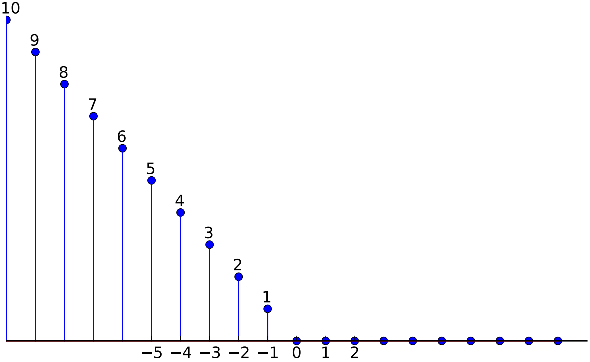


$$r[n]$$



$r[-n]$



Relación entre $u[n]$ y $r[n]$

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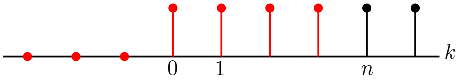

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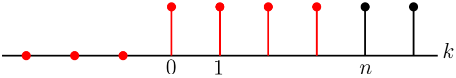
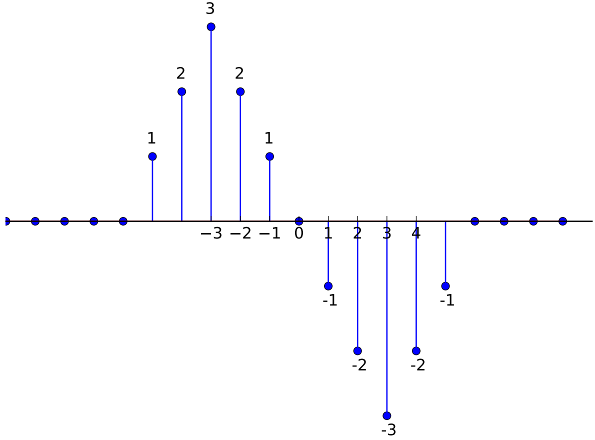


$$u[n] = r[n+1] - r[n]$$

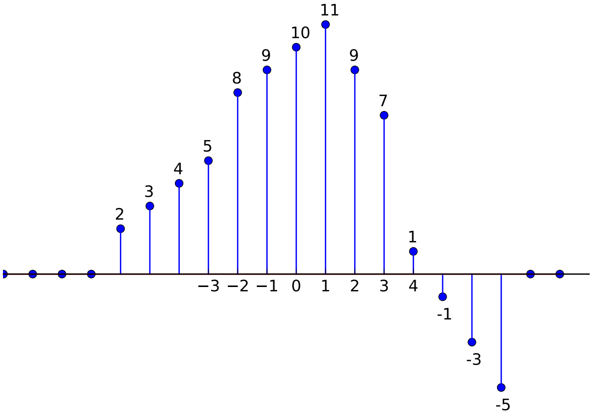
$$x[n] = r[n+6] - 2r[n+3] + 2r[n-3] - r[n-6]$$

$$x[n] = r[n+6] - 2r[n+3] + 2r[n-3] - r[n-6]$$



$$x[n] = 2u[n+6]+r[n+6]+2u[n+2]-3r[n-1]-4u[n-4]+2r[n-8]+7u[n-8]$$

$$x[n] = 2u[n+6] + r[n+6] + 2u[n+2] - 3r[n-1] - 4u[n-4] + 2r[n-8] + 7u[n-8]$$



$$y[n] = x[n-2]$$

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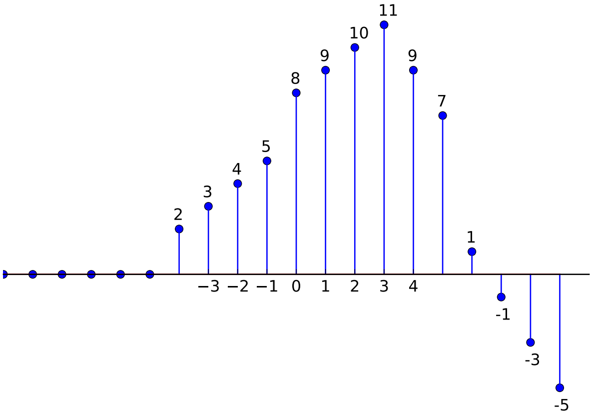
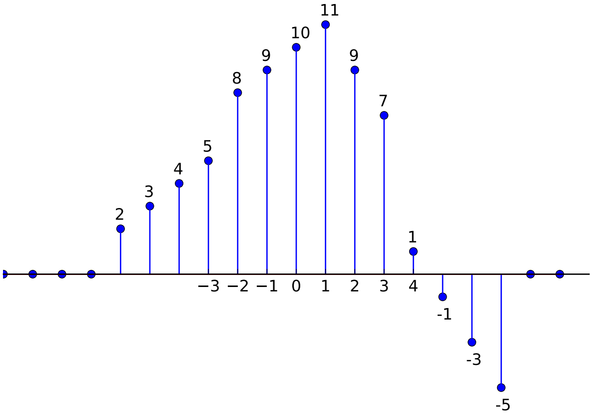


$x[n]$



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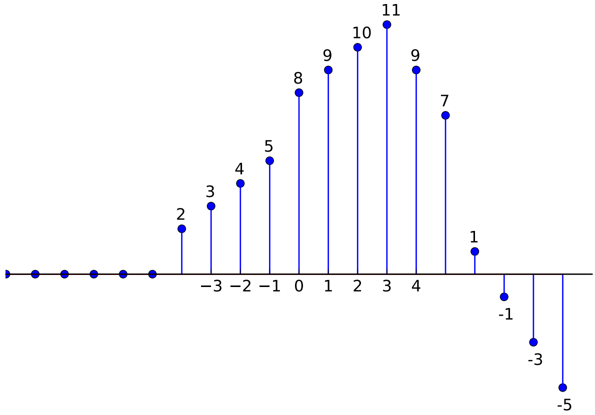
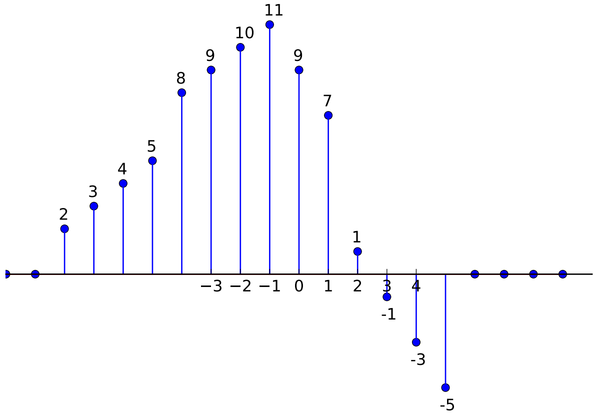
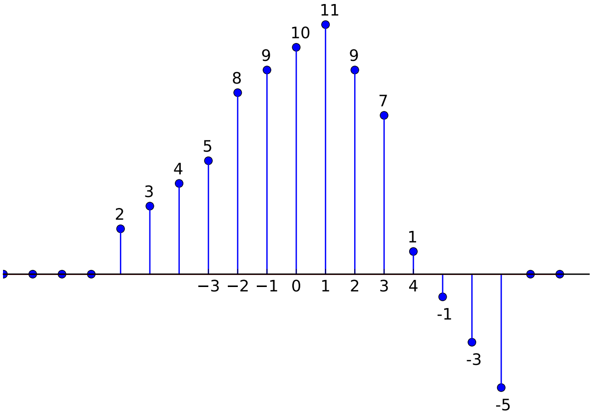


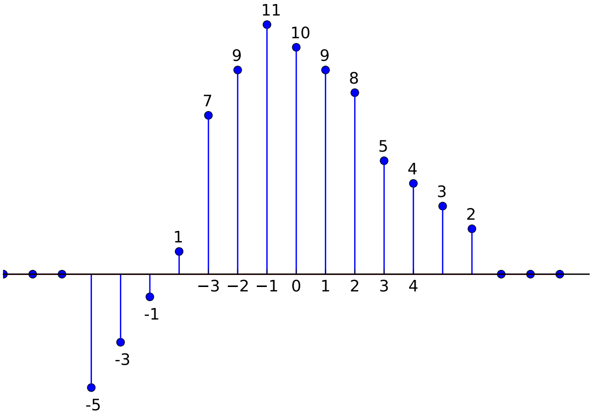
$$y[n] = x[n+2]$$



$x[n]$



$$y[n] = x[-n]$$



$$y[n] = x[2n]$$

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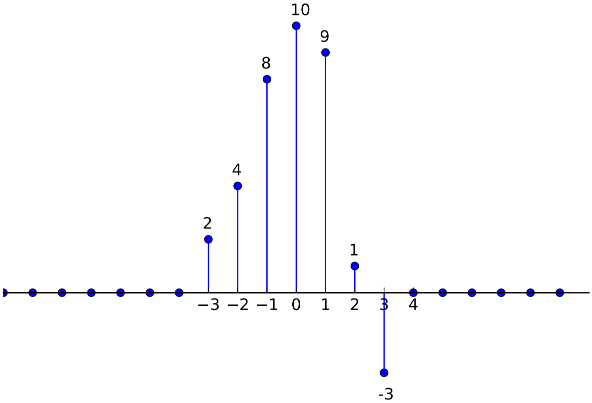

$$y[n] = x_{(2)}[n] = \begin{cases} x[n/2] & n \text{ par} \\ 0 & n \text{ impar} \end{cases}$$

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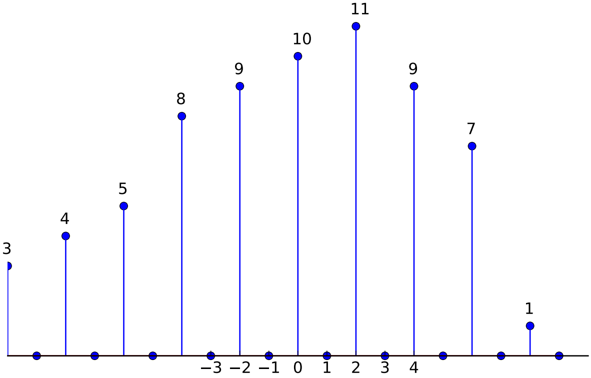
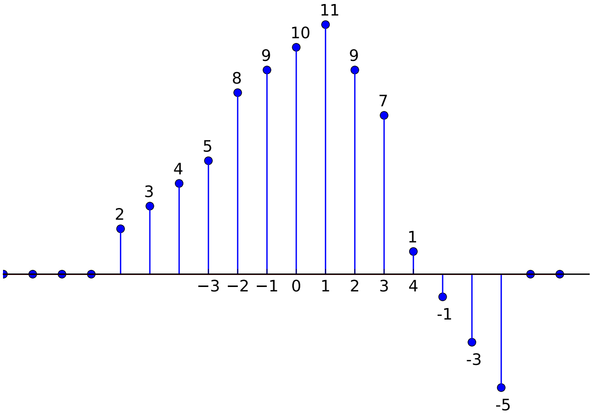


$x[n]$



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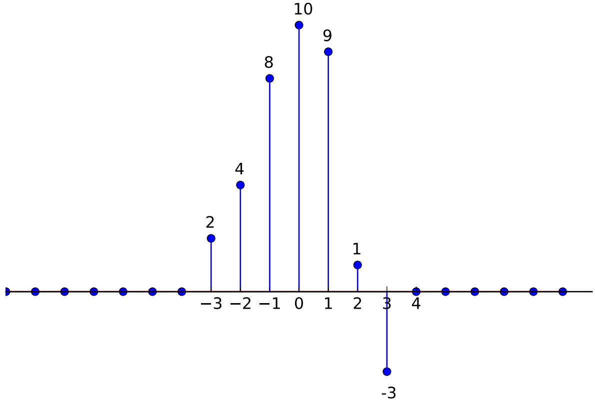
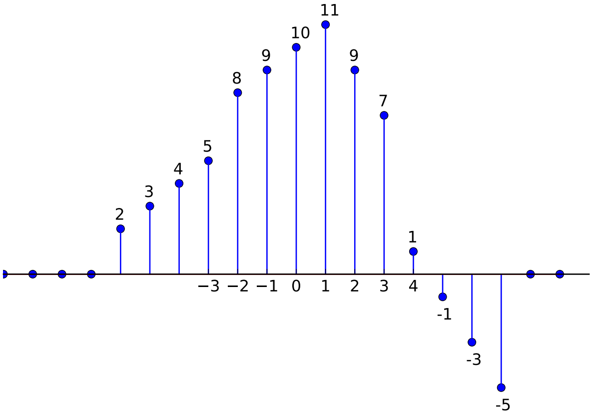
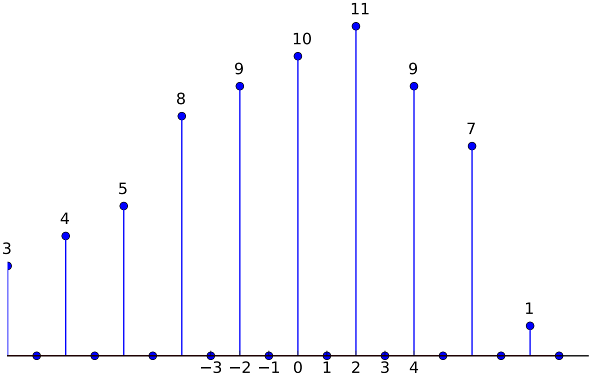


$x[n]$



$$y[n] = x_{(2)}[n]$$



$$y[n] = x[2n - 4]$$

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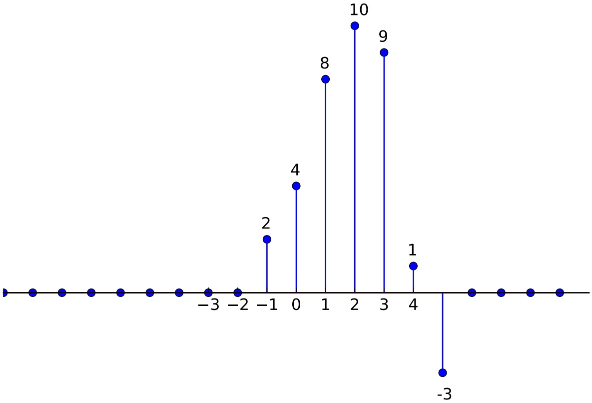

Señales Pares e Impares

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- $x[n]$ es par $\Leftrightarrow x[n] = x[-n]$

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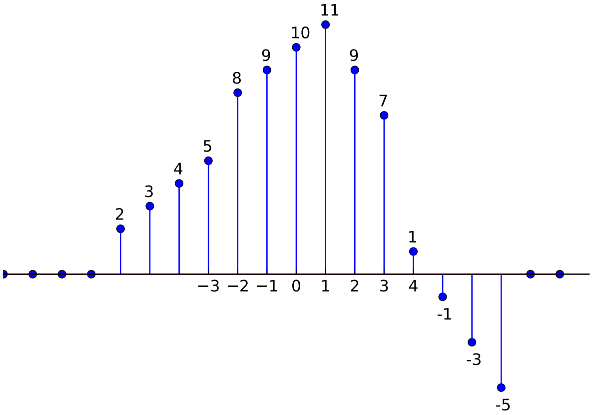
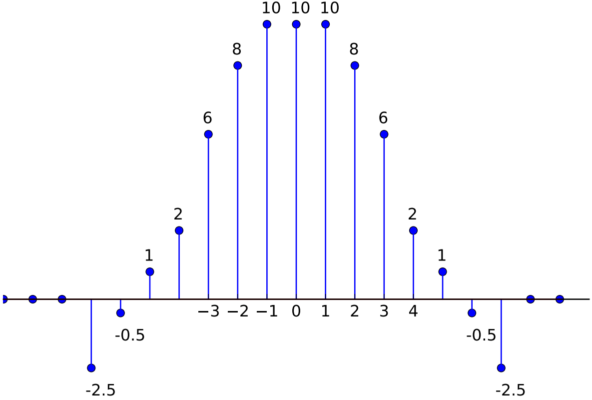
$$\mathcal{EV}\left\{x[n]\right\} = \frac{x[n] + x[-n]}{2}$$
$$\mathcal{ODD}\left\{x[n]\right\} = \frac{x[n] - x[-n]}{2}$$

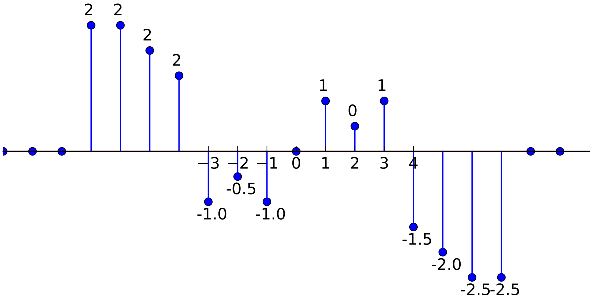
$x[n]$



$$\mathcal{EV}\{x[n]\}$$



$$\mathcal{ODD}\{x[n]\}$$



Impulso unitario (tiempo discreto)

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$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

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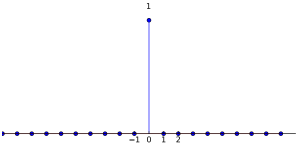
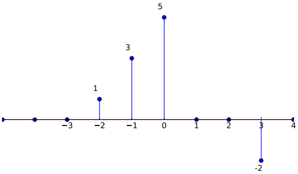
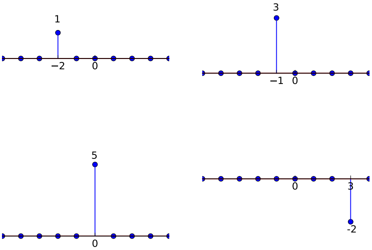
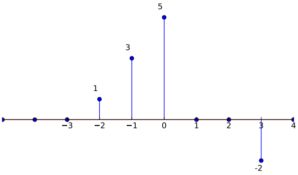
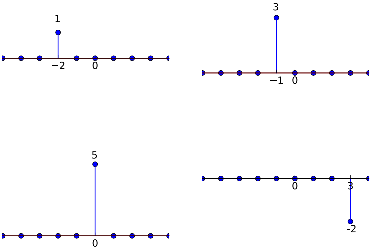



5
3
1
−3
−2
−1
0
1
2
3
4
-2


1
−2
0
3
−1
0
5
0
0
3
-2





$$x[n] = \delta[n+2] + 3\delta[n+1] + 5\delta[n] - 2\delta[n-3]$$

- En general para cualquier señal $x[n]$:

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$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

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- Por ejemplo:

$$u[n] =$$

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- Por ejemplo:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Relación entre $u[n]$ y $\delta[n]$

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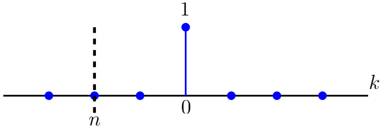

Relación entre $u[n]$ y $\delta[n]$

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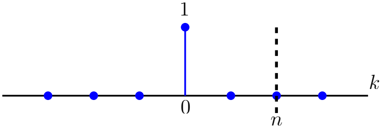

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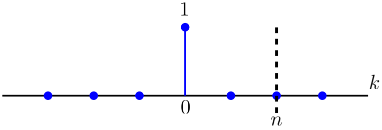

$$\delta[n] = u[n] - u[n-1]$$

Impulso unitario (tiempo contínuo)

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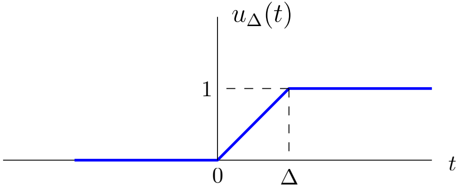

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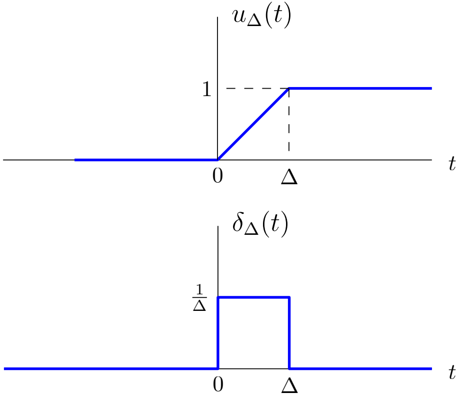
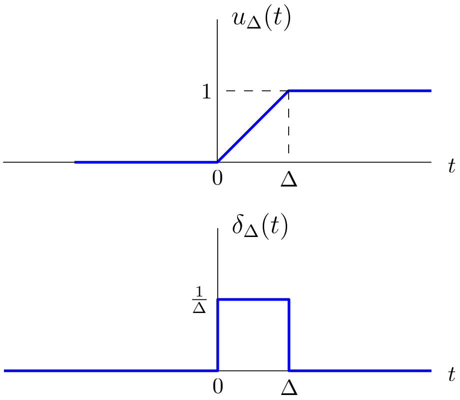
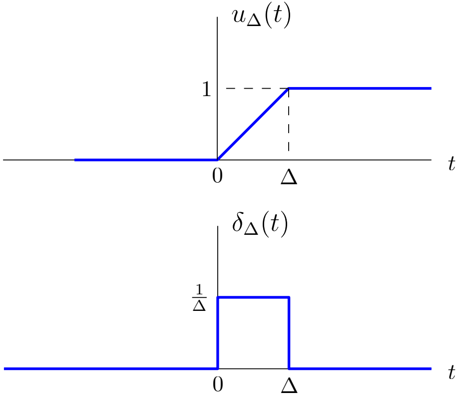


Impulso unitario (tiempo contínuo)



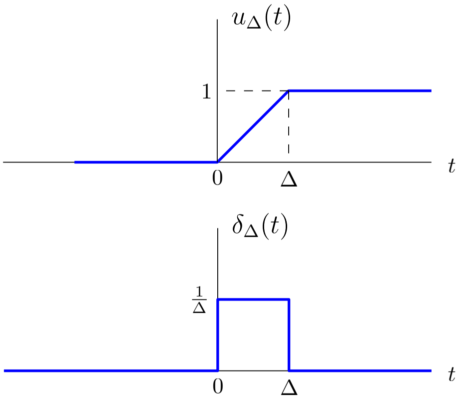
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$$

Impulso unitario (tiempo contínuo)



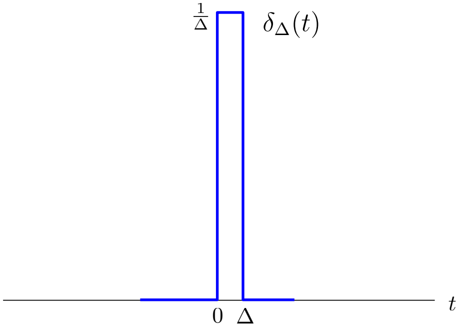
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t), \quad u_\Delta(t) =$$

Impulso unitario (tiempo contínuo)



$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t), \quad u_\Delta(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

Si se hace Δ muy pequeño:



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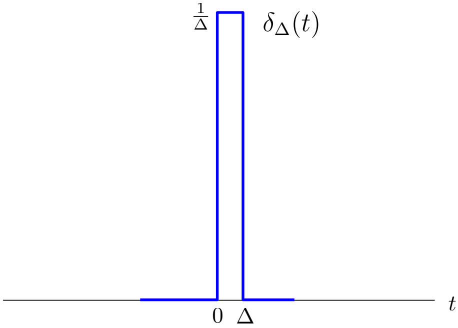

pero,

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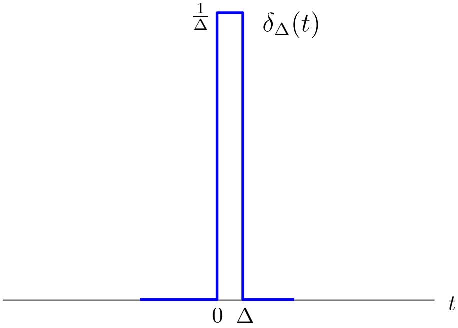

pero,

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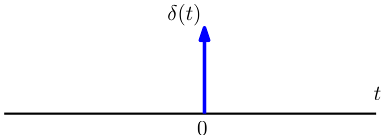

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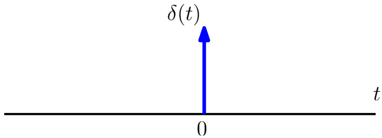

Watcha talkin bout Willis???

- Estudio formal: Teoría de distribuciones, otra definición de integral (Lebesgue, Riemann–Stieltjes).

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- Como un funcional:

$$\mathcal{F}\{x(t)\}$$

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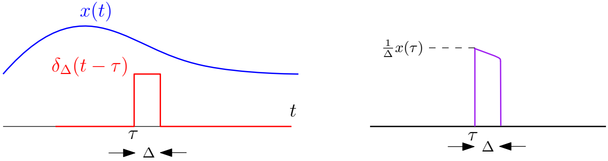

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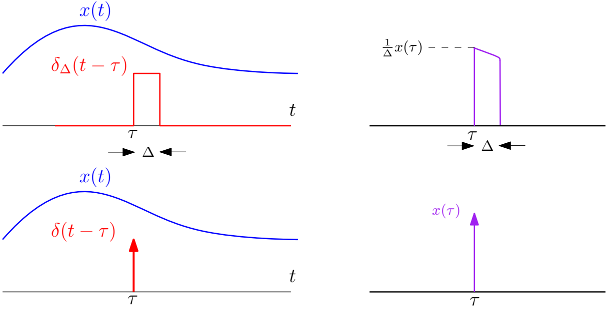

Cálculo operacional con impulsos

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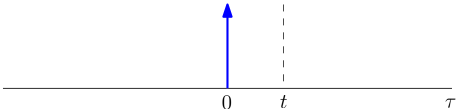

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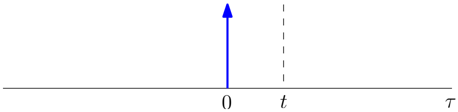

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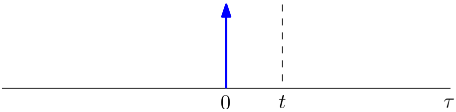
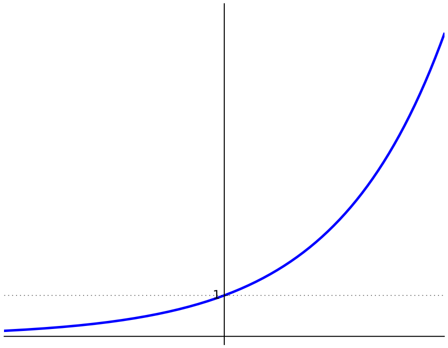


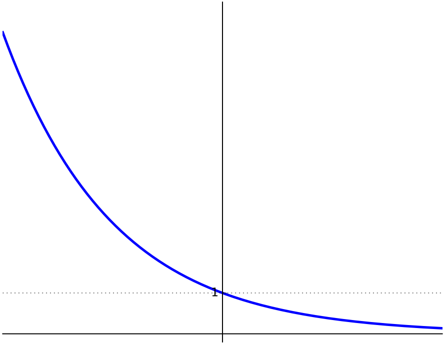
$$\frac{d}{dt}u(t) = \delta(t)$$

Señal exponencial (exponente real)

$$x(t) = e^{at}, \quad a \in \mathbb{R}^+$$



$$x(t) = e^{-at}, \quad a \in \mathbb{R}^{+}$$



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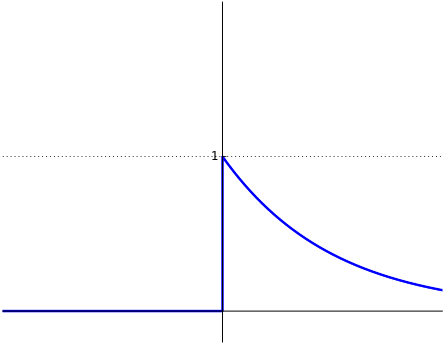

Señal exponencial (base real)

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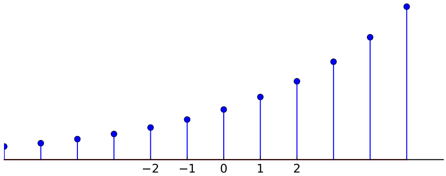

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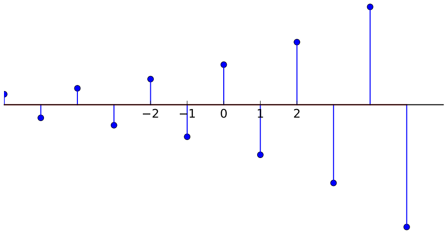

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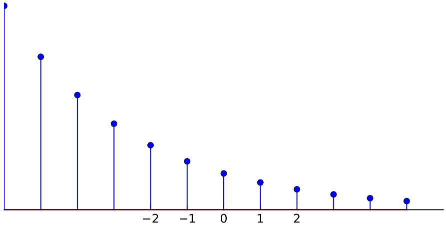

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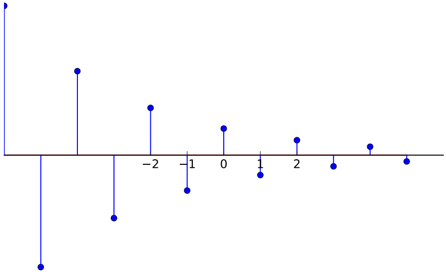

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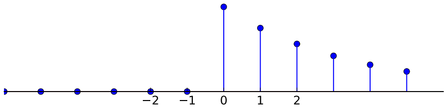

Señales periódicas (tiempo contínuo)

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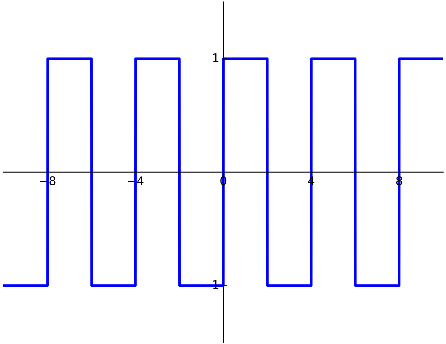

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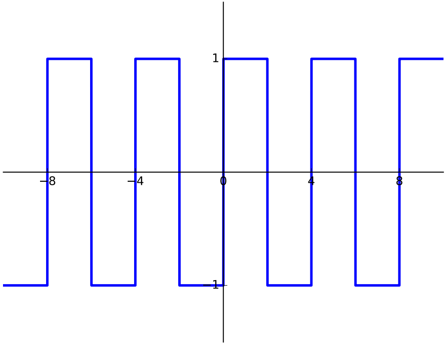

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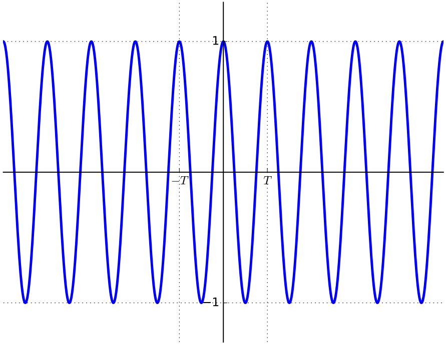

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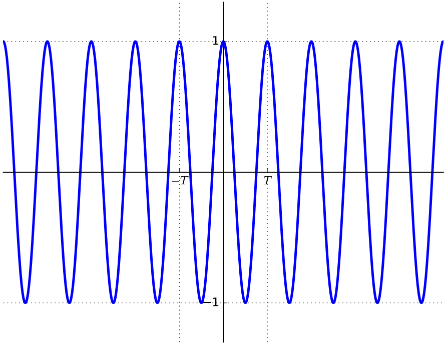

- Exponencial compleja:

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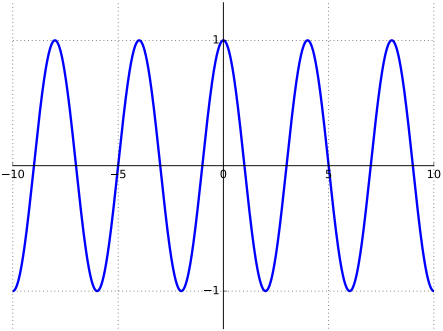

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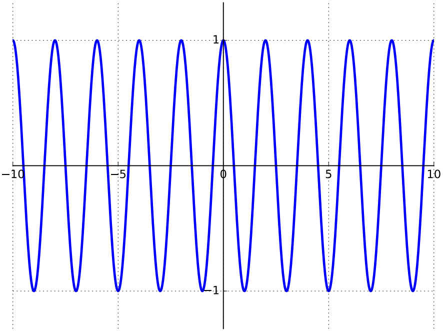

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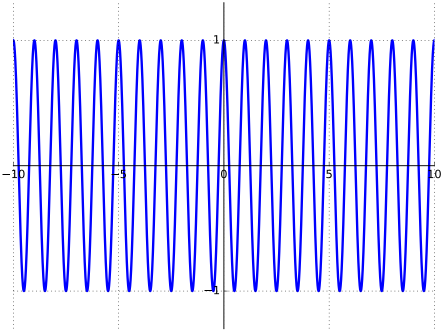

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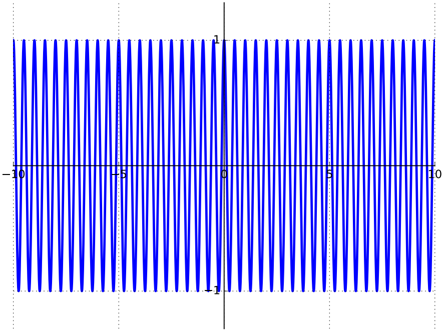

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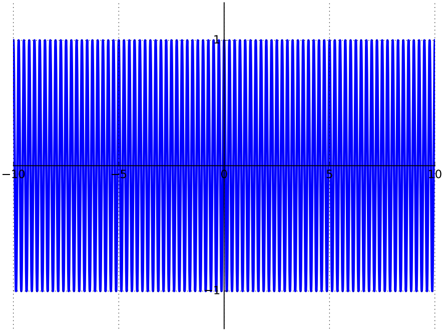

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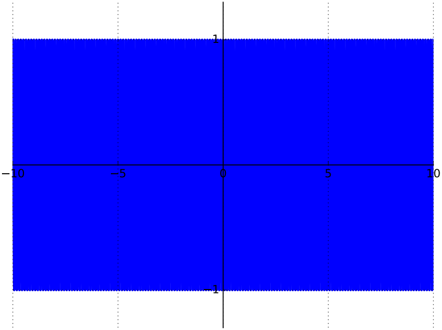

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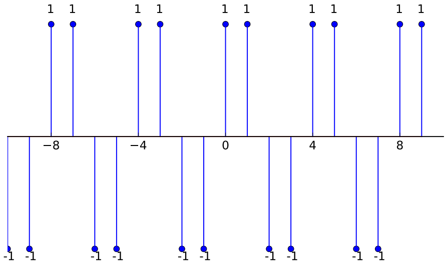

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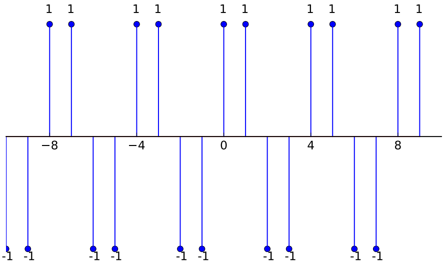

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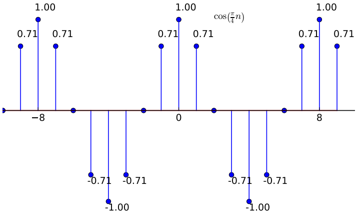

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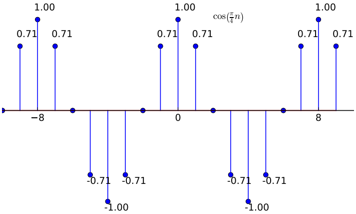

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- Es decir, para que $e^{j\omega_0 n}$ sea periódica $\frac{2\pi}{\omega_0}$ debe ser un número racional.

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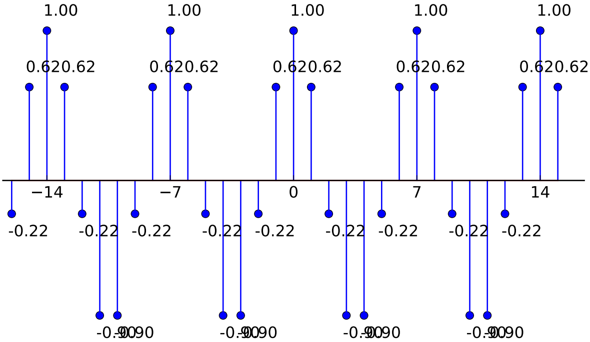

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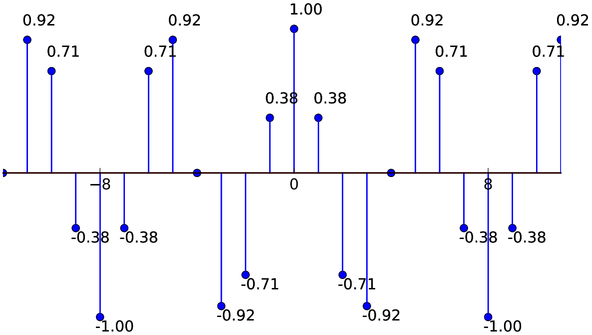

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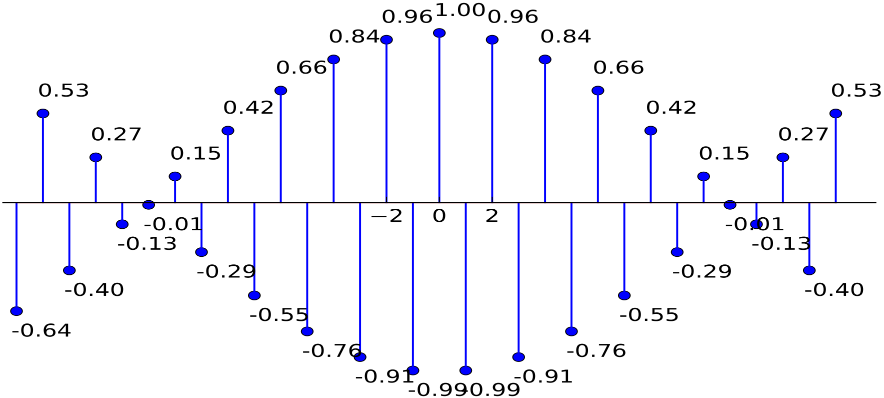

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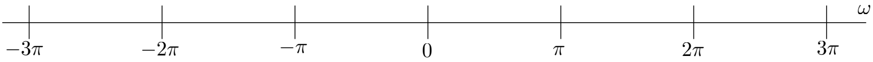

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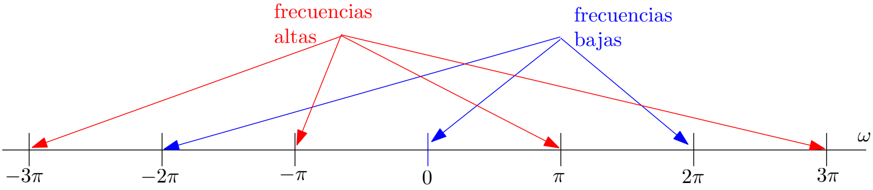

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