

2.64 ♦

Write code to implement the following function:

```
/* Return 1 when any even bit of x equals 1; 0 otherwise.
   Assume w=32. */
int any_even_one(unsigned x)
{
    // your code
    unsigned max = 0xffffffff;
    unsigned ans = x ^ max;
    return !ans;
}
```

Your function should follow the bit-level integer coding rules, except that you may assume that data type `int` has $w = 32$ bits.

2.73 ♦♦

Write code for a function with the following prototype:

```
/* Addition that saturates to TMin or TMax */
int saturating_add(int x, int y)
{
    int ans = x + y;
    int num_x = x & 0x7fffffff;    //maxint = 0x7fffffff
    int num_y = y & 0x7fffffff;    //maxint = 0x7fffffff

    int sign_x = x >> 31;         //31 = w - 1
    int sign_y = y >> 31;         //31 = w - 1
    int sign_ans = ans >> 31;     //31 = w - 1

    int whether_is_different_sign = (sign_x ^ sign_y);
    //if this == 1, cannot saturate

    int whether_up_saturate = (!sign_x) && (sign_ans) && (!whether_is_different_sign);
    //if this == 1, up saturate

    int whether_down_saturate = (sign_x) && (!sign_ans) && (!whether_is_different_sign);
    //if this == 1, down saturate

    int whether_change = whether_up_saturate | whether_down_saturate;

    int step = (whether_change << 31) - whether_change;    //31 = w-1
    //not saturate: step all 0
    //up saturate: step maxint
    //down saturate: step maxint

    ans -= (whether_up_saturate << 31);                    //31 = w - 1
    ans = ans | step;
    ans += whether_down_saturate;

    return ans;
}
```

Instead of overflowing the way normal two's-complement addition does, saturating addition returns *TMax* when there would be positive overflow, and *TMin* when there would be negative overflow. Saturating arithmetic is commonly used in programs that perform digital signal processing.

Your function should follow the bit-level integer coding rules.

2.81 ♦

We are running programs on a machine where values of type `int` are 32 bits. They are represented in two's complement, and they are right shifted arithmetically. Values of type `unsigned` are also 32 bits.

We generate arbitrary values `x` and `y`, and convert them to unsigned values as follows:

```
/* Create some arbitrary values */
int x = random();
int y = random();
/* Convert to unsigned */
unsigned ux = (unsigned) x;
unsigned uy = (unsigned) y;
```

For each of the following C expressions, you are to indicate whether or not the expression *always* yields 1. If it always yields 1, describe the underlying mathematical principles. Otherwise, give an example of arguments that make it yield 0.

A. $(x > y) == (-x < -y)$

$x = -2147483648$ $y = -114514$ $x > y = 0$
 $-x = -2147483648$ $-y = -114514$ $-x < -y = 1$
 Can Yield 0

B. $((x + y) < 5) + x - y == 31 * y + 33 * x$

if `x` and `y` are unsigned, the formula will always be true, since all steps equals to (a calculate `b % (2^32)`, and `%` is commutative with all other calculations. If `unsigned(x) == unsigned(y)`, `x must == y`
 so it always yields 1

C. $\sim x + \sim y == \sim(x + y)$

$x = 0, y = 0$
 $\sim x = -1$
 $\sim y = -1$
 $\sim(x + y) = -1$
 Can Yield 0

D. $(\text{int})(ux - uy) == -(y - x)$

The binary code of `ux - uy` = the binary code of `x - y`
 So the formula becomes
 $x + (-y) == -(y - x)$
 since `+` forms an abelian group, it always yields 1

E. $((x >> 1) << 1) <= x$

$x = -2147483648$
 $x >> 1 = -2147483648$
 $(x >> 1) << 1 = 0$
 Can Yield 0

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Bit-level integer coding rules

In several of the following problems, we will artificially restrict what programming constructs you can use to help you gain a better understanding of the bit-level, logic, and arithmetic operations of C. In answering these problems, your code must follow these rules:

. Assumptions

Integers are represented in two's-complement form.

Right shifts of signed data are performed arithmetically.

Data type `int` is w bits long. For some of the problems, you will be given a specific value for w , but otherwise your code should work as long as w is a multiple of 8. You can use the expression `sizeof(int)<<3` to compute w .

. Forbidden

Conditionals (`if` or `?:`), loops, switch statements, function calls, and macro invocations.

Division, modulus, and multiplication.

Relative comparison operators (`<`, `>`, `<=`, and `>=`).

Casting, either explicit or implicit.

. Allowed operations

All bit-level and logic operations.

Left and right shifts, but only with shift amounts between 0 and $w - 1$.

Addition and subtraction.

Equality (`==`) and inequality (`!=`) tests. (Some of the problems do not allow these.)

Integer constants `INT_MIN` and `INT_MAX`.