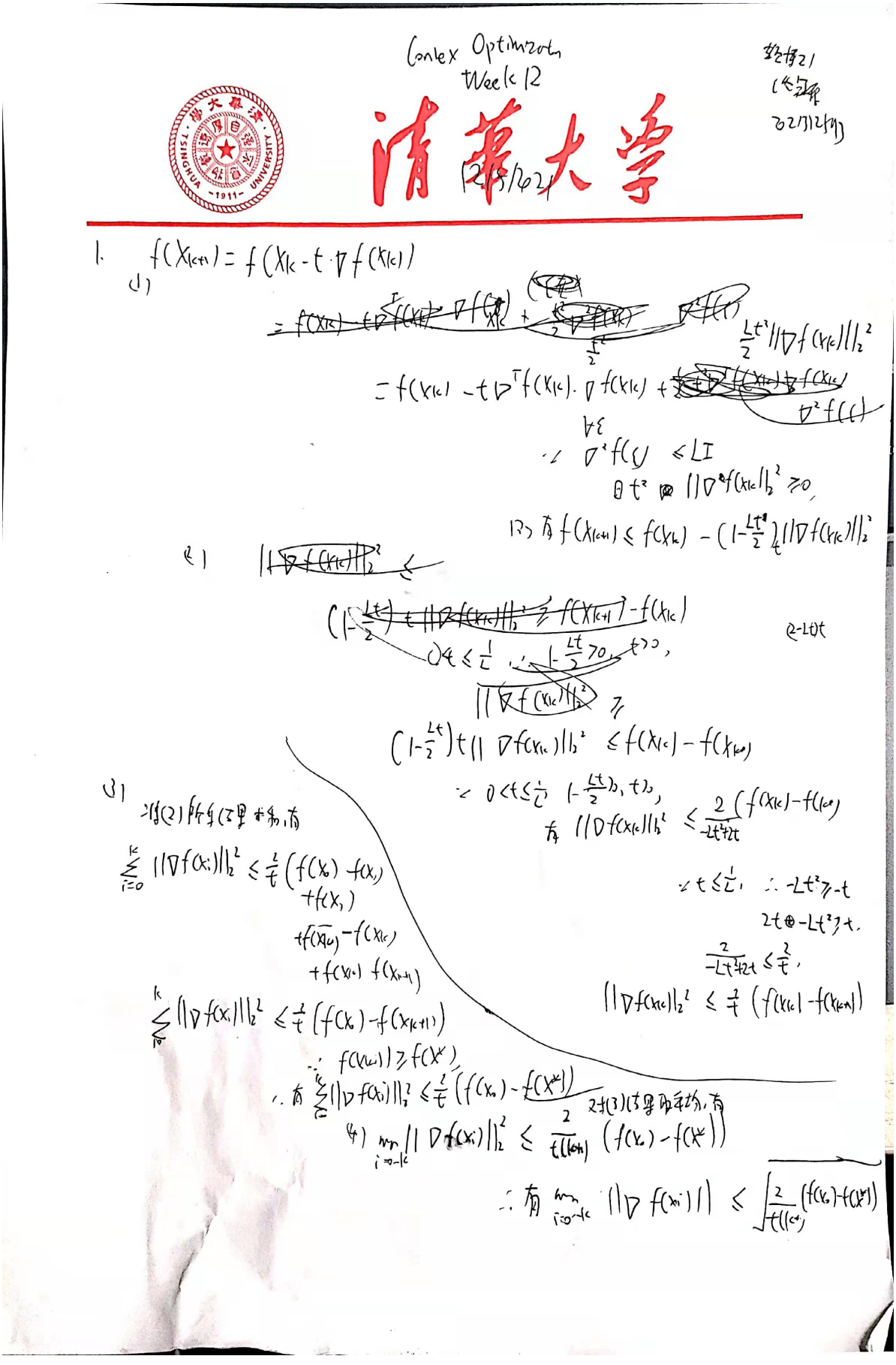
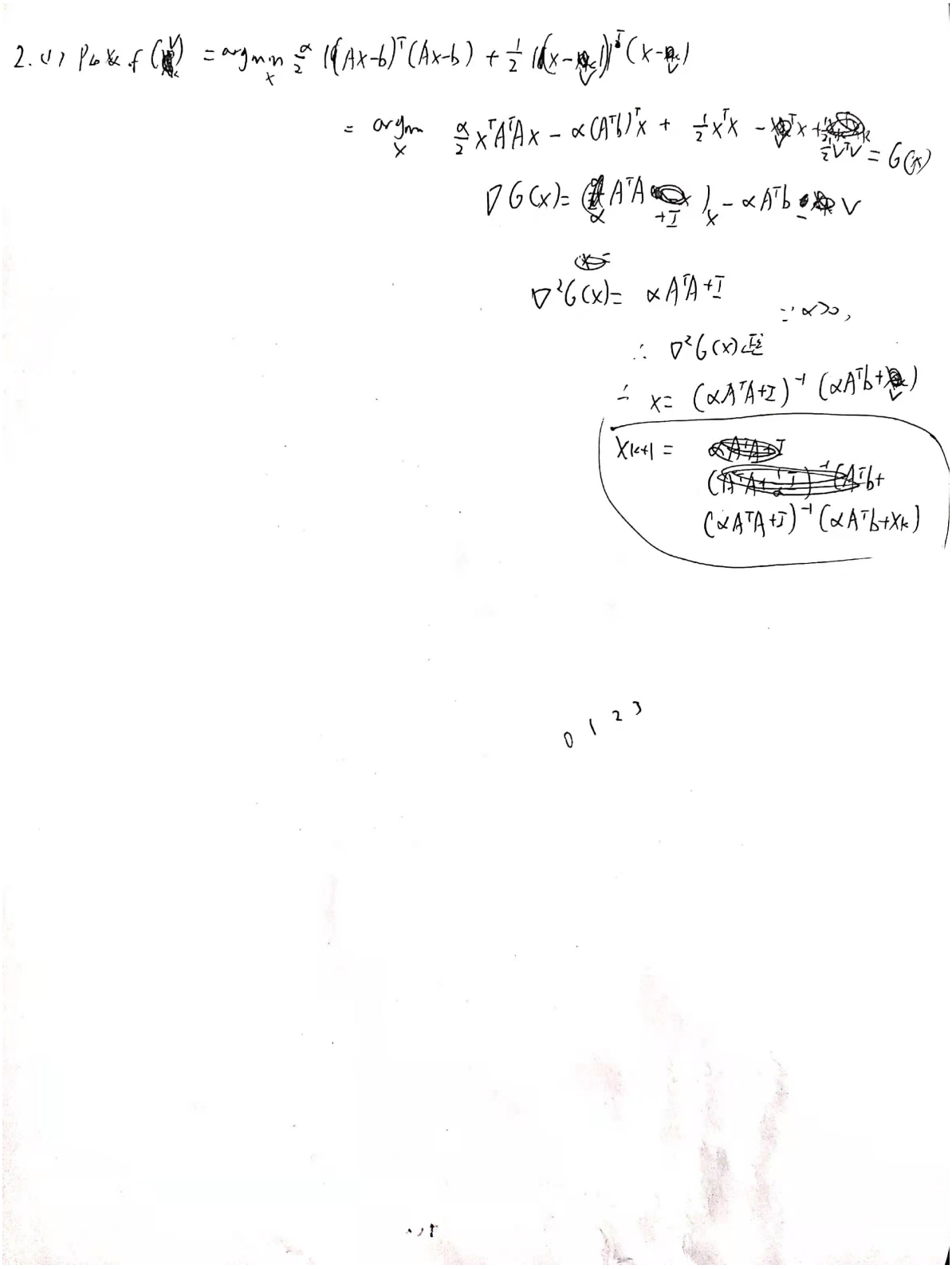
1.



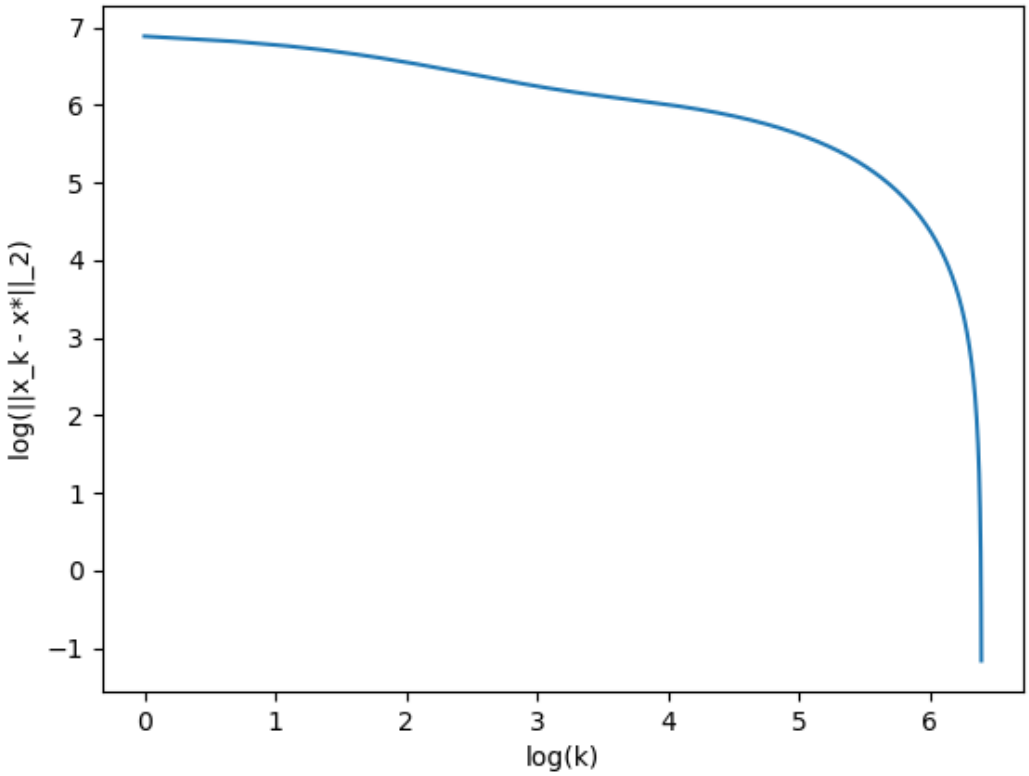
2.(1)



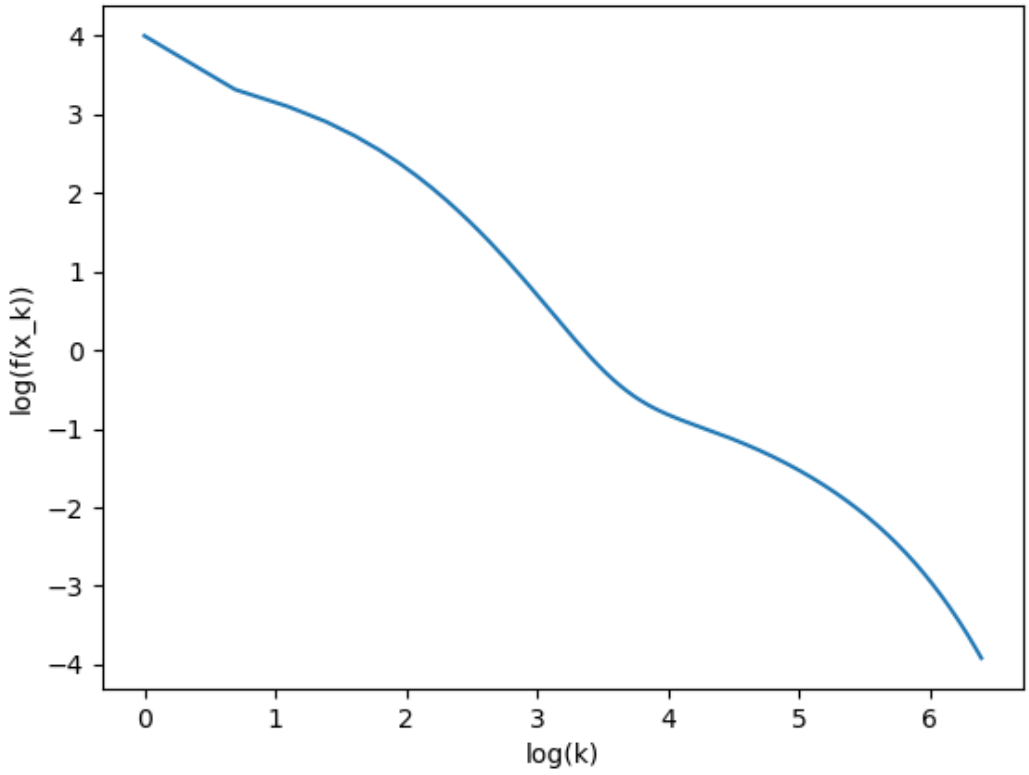
(2)

采用alpha=1000时，收敛到f(x)=0.01991554109994043，此时x结果见根目录下result/Q2/x.csv文件

log(k)和log(||x_k - x*||^2)的图像如下：



log(k)和log(f(x_k))的图像如下：

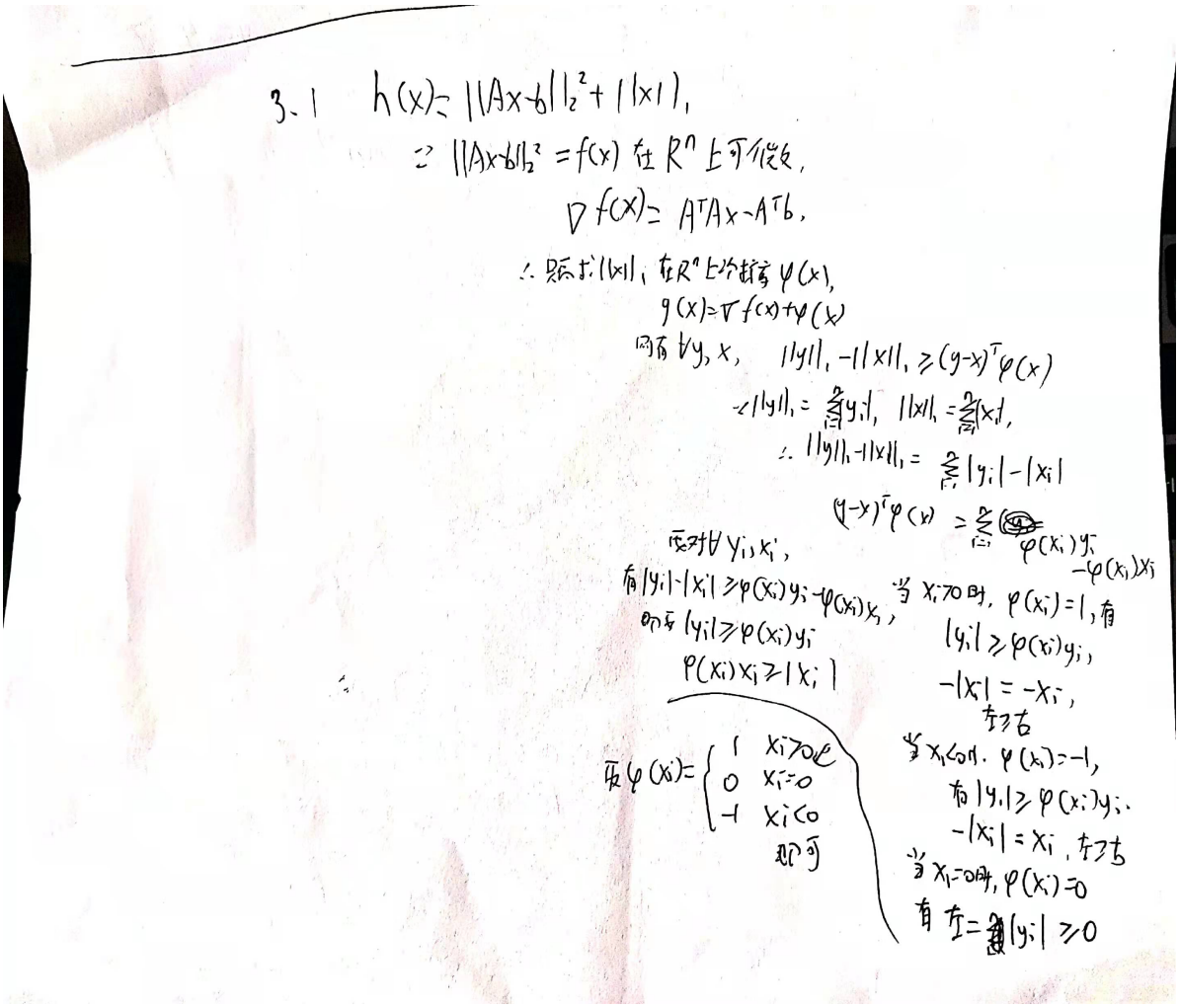


3.(1)

$min_x \frac{1}{2}||Ax-b||_2^2+||x||_1$的次梯度$g(x)$为

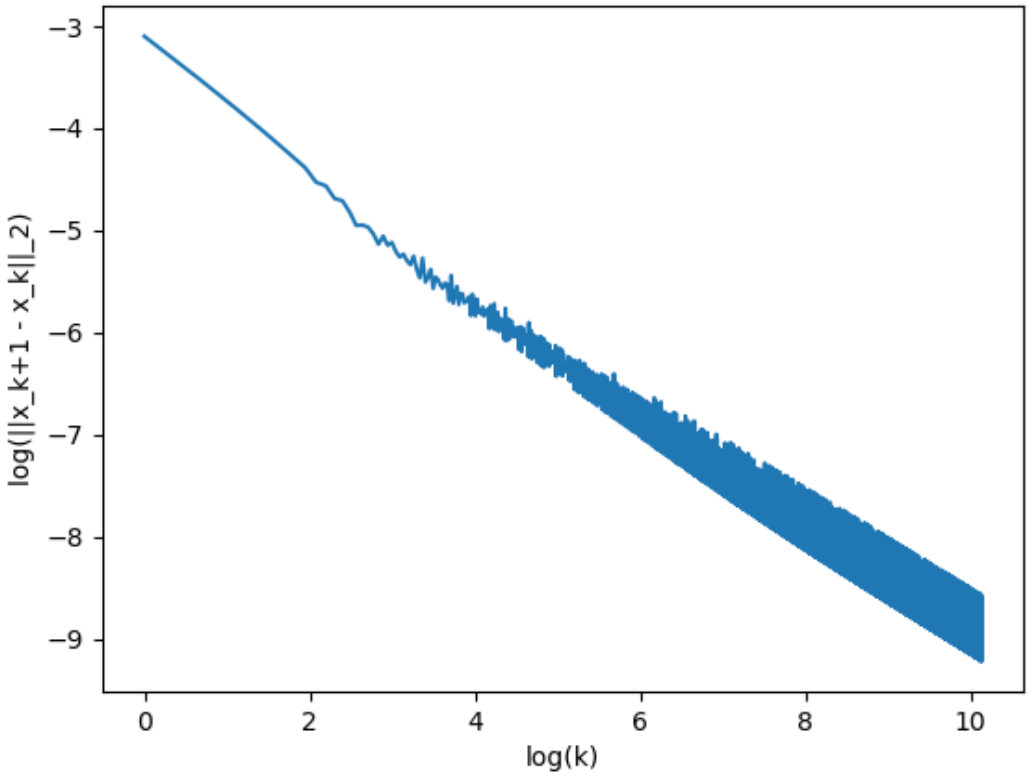
$$A^TAx-A^Tb+sign(x), \text{其中} sign(x)_i=\begin{cases}1 & x_i>0\\ 0 & x_i=0\\ -1 & x_i<0\end{cases}$$

具体推导如下：

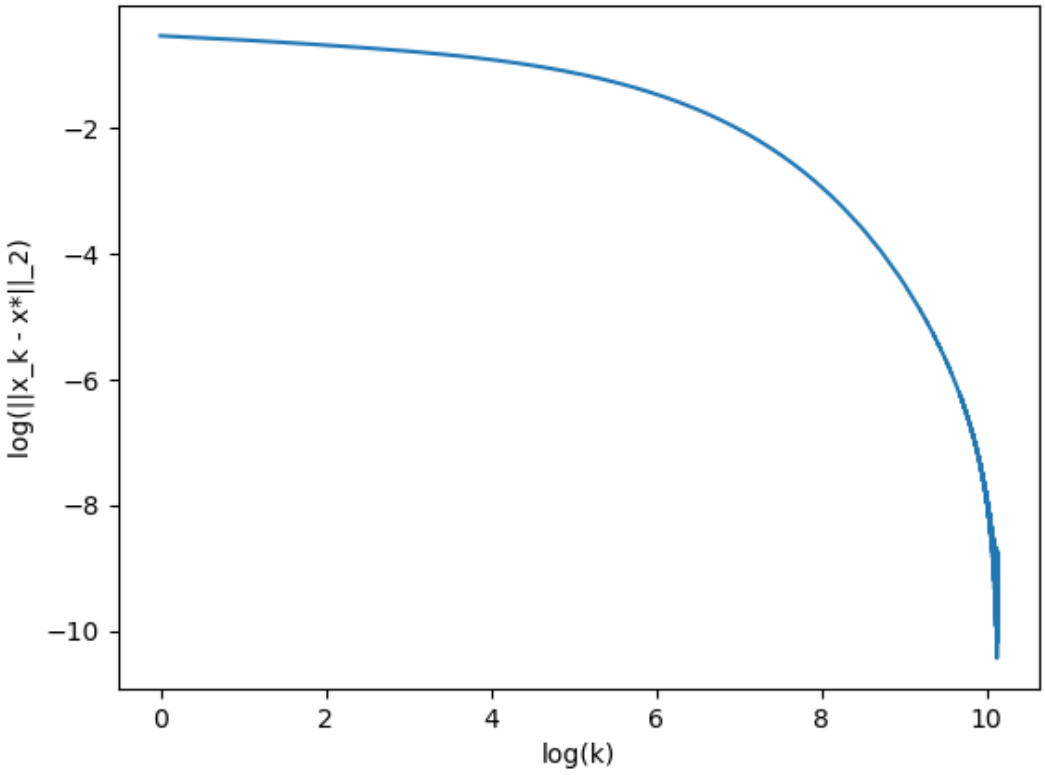


解得最优的h(x)=12.522447423125461,x存储在根目录下的result/Q3/x1.csv中

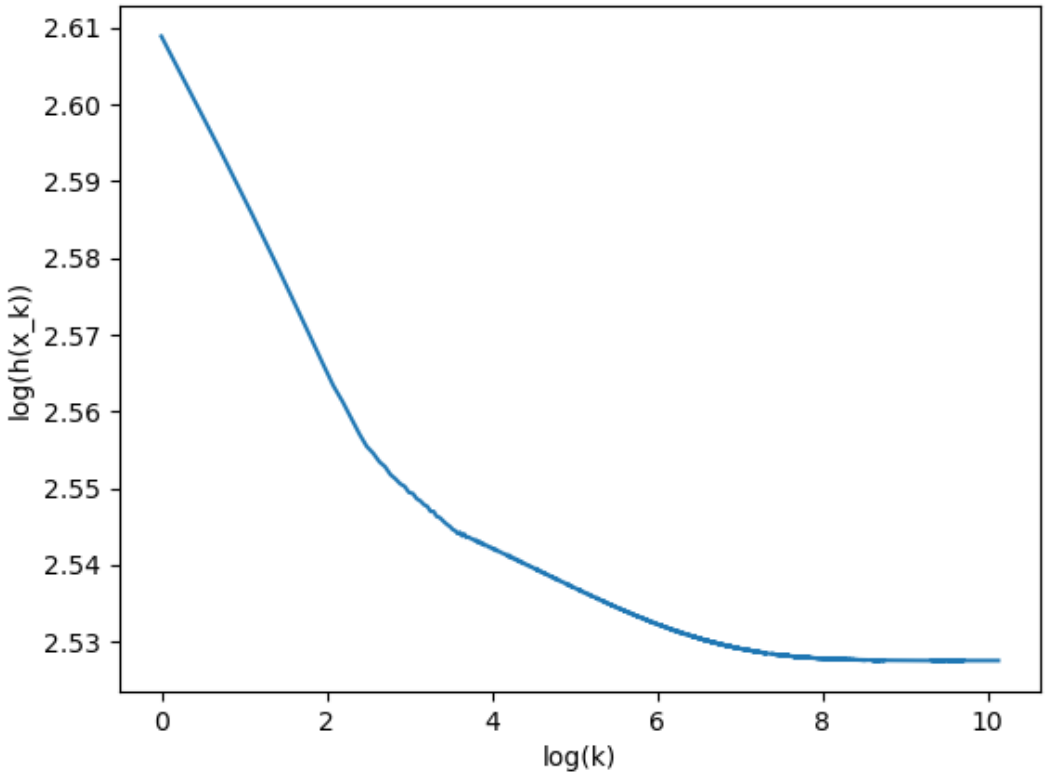
log(k)和log(||x_k+1 - x_k||_2)的图像如下：



log(k)和log(||x_k-x*||_2)的图像如下：



log(x)和log(h(x))的图像如下：



(2)

$$\text{对}f\text{做泰勒展开，有}f(y) = f(x) + \nabla f(x)^T(y-x) + \frac{1}{2}(y-x)^T\nabla^2 f(x)(y-x)$$

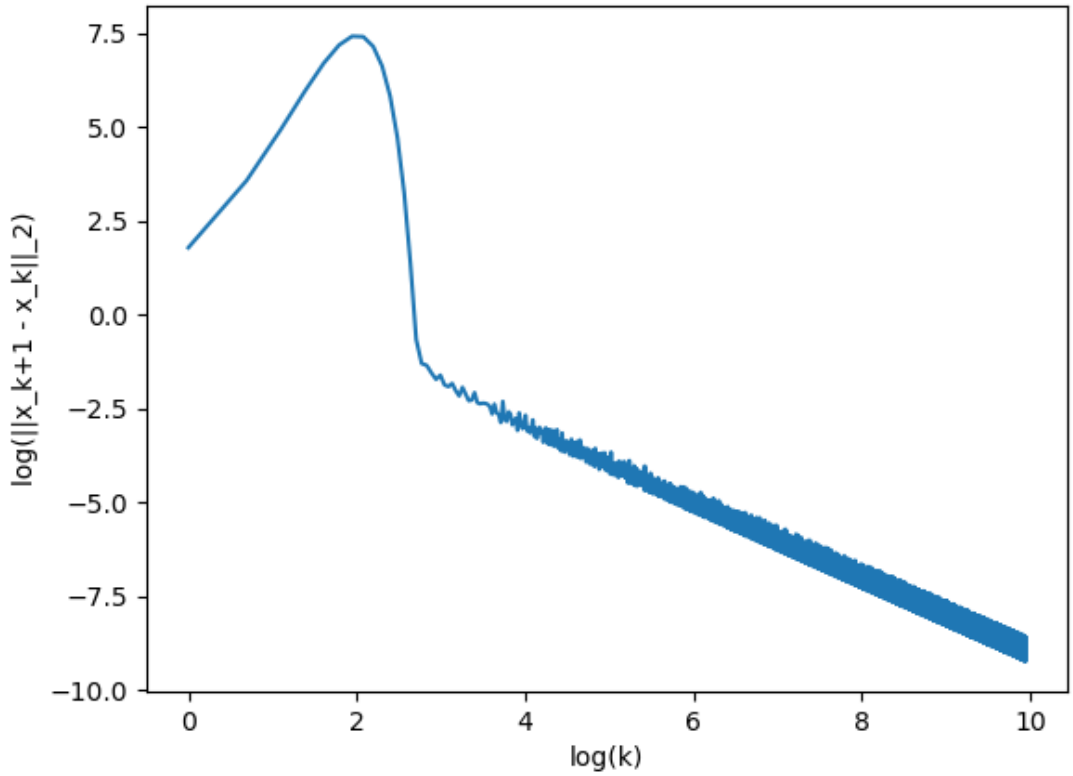
$$\geq f(x) + \nabla f(x)^T(y-x) + \frac{1}{2}m(y-x)^T(y-x), \text{其中}\nabla f(x)^2 = A^TA$$

因此有对于任意x，$y \in dom(f)$，有$(y-x)^T(A^TA - mI)(y-x) \geq 0$, 也就是$A^TA - mI$半正定

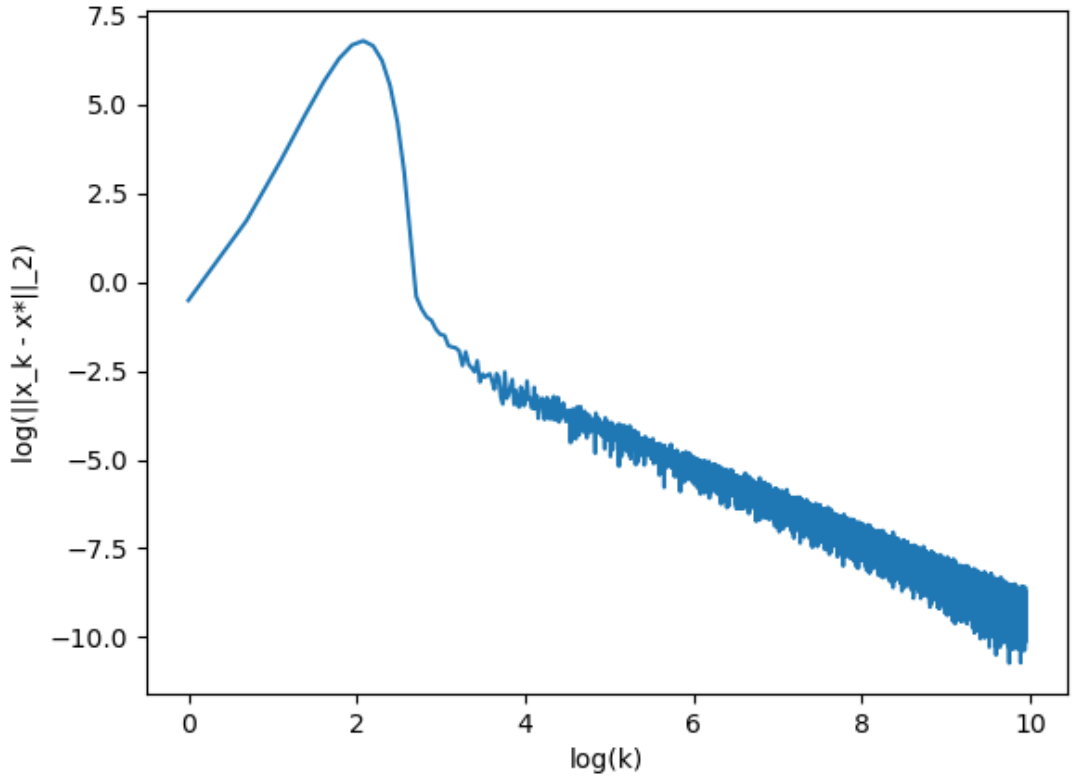
m最大值为A^TA最小特征值

解得m=0.7569455027845506， 最优的h(x)=12.52240996025351,x存储在根目录下的result/Q3/x2.csv中

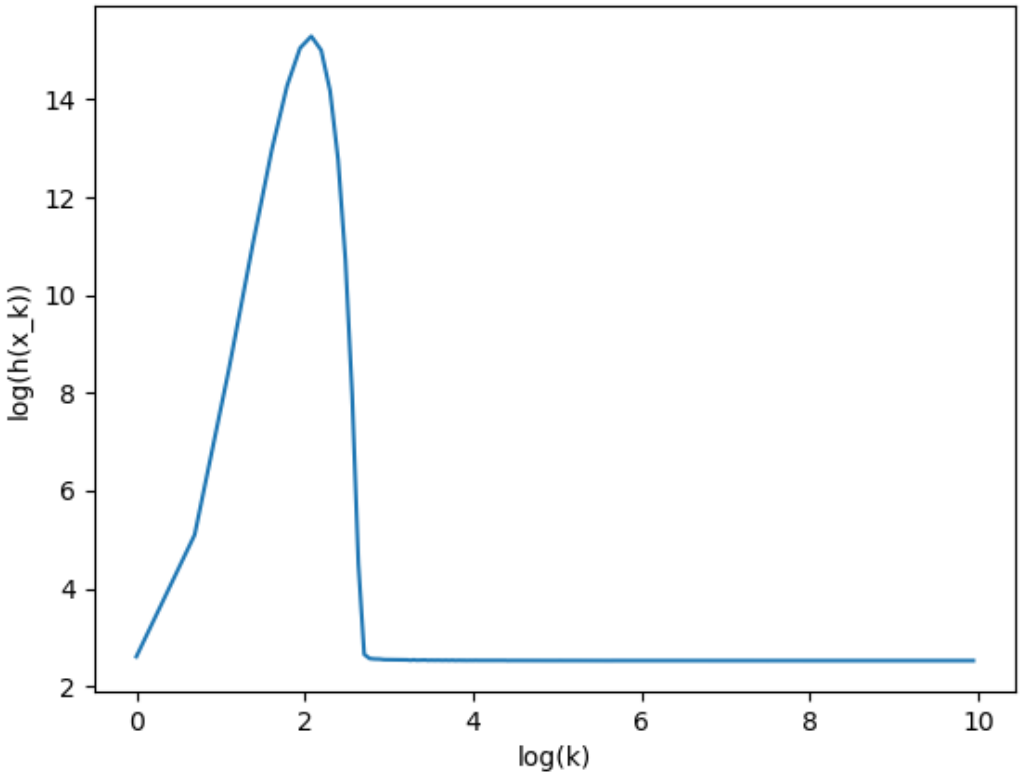
log(k)和log(||x_k+1 - x_k||_2)的图像如下：



log(k)和log(||x_k-x*||_2)的图像如下：



log(k)和log(h(x_k))的图像如下：


log(h(x_k))
log(k)
14
12
10
8
6
4
2
0
2
4
6
8
10