



**Probability and Statistics**

**Bayes' Theorem:**  
 $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

**Joint and Marginal Distributions:**  
 $f(x,y) = f(x)f(y)$  (independent)  
 $f(x) = \int f(x,y) dy$   
 $f(y) = \int f(x,y) dx$

**Normal Distribution:**  
 $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$   
 $\mu = \frac{1}{n} \sum x_i$   
 $\sigma^2 = \frac{1}{n} \sum (x_i - \mu)^2$

**Gamma Distribution:**  
 $f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$   
 $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$

**Poisson Distribution:**  
 $P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$

**Binomial Distribution:**  
 $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

**Chi-Square Distribution:**  
 $f(x) = \frac{1}{2^{n/2} \Gamma(n/2)} x^{n/2-1} e^{-x/2}$

**F-Distribution:**  
 $f(x) = \frac{\Gamma(\frac{n_1+n_2}{2})}{\Gamma(\frac{n_1}{2}) \Gamma(\frac{n_2}{2})} \left(\frac{n_1}{n_2}\right)^{n_1/2} x^{n_1/2-1} (1+\frac{n_1}{n_2}x)^{-(n_1+n_2)/2}$

**t-Distribution:**  
 $f(x) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2}) \sqrt{n\pi}} (1+\frac{x^2}{n})^{-(n+1)/2}$

**Maximum Likelihood Estimation (MLE):**  
Find  $\theta$  that maximizes  $L(\theta) = \prod f(x_i; \theta)$   
Log-likelihood:  $\ln L(\theta) = \sum \ln f(x_i; \theta)$   
Score function:  $S(\theta) = \frac{d}{d\theta} \ln L(\theta)$   
Fisher Information:  $I(\theta) = -E[S(\theta)^2]$

**Least Squares (OLS):**  
Minimize  $\sum (y_i - \beta_0 - \beta_1 x_i)^2$   
Normal equations:  
 $\sum y_i = n\beta_0 + \beta_1 \sum x_i$   
 $\sum x_i y_i = \beta_0 \sum x_i + \beta_1 \sum x_i^2$

**Confidence Intervals:**  
Normal:  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$   
t-distribution:  $\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

**Hypothesis Testing:**  
Null hypothesis  $H_0$  vs Alternative hypothesis  $H_1$   
Test statistic  $T$   
Significance level  $\alpha$   
Power  $1 - \beta$

**Bayesian Inference:**  
Prior  $\pi(\theta)$ , Likelihood  $L(y|\theta)$ , Posterior  $\pi(\theta|y)$   
 $\pi(\theta|y) \propto L(y|\theta) \pi(\theta)$



**1. Hypothesis Testing**

**1.1 Null and Alternative Hypotheses**

- $H_0: \theta = \theta_0$  vs  $H_1: \theta \neq \theta_0$  (Two-tailed)
- $H_0: \theta \leq \theta_0$  vs  $H_1: \theta > \theta_0$  (Right-tailed)
- $H_0: \theta \geq \theta_0$  vs  $H_1: \theta < \theta_0$  (Left-tailed)

**1.2 Test Statistic**

**1.3 P-value**

**1.4 Significance Level ( $\alpha$ )**

**1.5 Critical Value**

**1.6 Decision Rule**

**1.7 Power of the Test**

**2. Confidence Intervals**

**2.1 Point Estimate**

**2.2 Standard Error**

**2.3 Margin of Error**

**2.4 Confidence Interval**

**3. Parametric Tests**

**3.1 Z-test**

**3.2 T-test**

**3.3 F-test**

**3.4 Chi-square Test**

**4. Non-parametric Tests**

**4.1 Sign Test**

**4.2 Rank Sum Test**

**4.3 Run Test**

**5. Regression Analysis**

**5.1 Simple Linear Regression**

**5.2 Multiple Regression**

**5.3 Assumptions**

**5.4 Diagnostics**

**6. Bayesian Statistics**

**6.1 Prior Distribution**

**6.2 Likelihood Function**

**6.3 Posterior Distribution**

**6.4 Decision Theory**

**7. Quality Control**

**7.1 Acceptance Sampling**

**7.2 Process Control**

**7.3 Taguchi Loss Function**

**8. Experimental Design**

**8.1 Factorial Design**

**8.2 Randomized Control Trial**

**8.3 Blocking**

**8.4 Replication**

**8.5 Randomization**

**9. Miscellaneous**

**9.1 Chebyshev's Theorem**

**9.2 Central Limit Theorem**

**9.3 Law of Large Numbers**

**9.4 Bayes' Theorem**

**9.5 Expected Value**

**9.6 Variance**

**9.7 Covariance**

**9.8 Correlation**

**9.9 Regression Coefficients**

**9.10 Residuals**

**9.11 ANOVA**

**9.12 MANOVA**

**9.13 Discriminant Analysis**

**9.14 Logistic Regression**

**9.15 Survival Analysis**

**9.16 Time Series Analysis**

**9.17 Non-linear Models**

**9.18 Robust Statistics**

**9.19 Bootstrap**

**9.20 Monte Carlo Simulation**