1. $D_{t+1}(i) = \dfrac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$

$D_{T+1}(i) = \dfrac{1}{m}\dfrac{\exp(-y_i f_t(x_i))}{\prod_{t=1}^{T} Z_t}$

$\hat{\varepsilon}(h) \le \frac{1}{m}\sum_{i=1}^{m} 1[y_i f_t(x_i) \le 0] = \frac{1}{m}\sum_{i=1}^{m} e^{-y_i f_t(x_i)}$

$= \sum_{i=1}^{m}\prod_{t=1}^{T} Z_t D_{T+1}(i) = \prod_{t=1}^{T} Z_t \sum_{i=1}^{m} D_{T+1}(i) = \prod_{t=1}^{T} Z_t$

欲证 $\prod_{t=1}^{T} Z_t \le e^{-2\sum_{t=1}^{T}(\frac{1}{2}-\varepsilon_t)^2} =$

即 $\sum_{t=1}^{T}\log Z_t \le \sum_{t=1}^{T} -2(\frac{1}{2}-\varepsilon_t)^2$

$d(\varepsilon_t) = \log Z_t + 2(\frac{1}{2}-\varepsilon_t)^2$

$= 4\varepsilon_t^2 + 4(\varepsilon_t + 1) + \ln 2 + \frac{1}{2}\ln\varepsilon_t + \frac{1}{2}\ln(1-\varepsilon_t)$

$\dfrac{\partial d(\varepsilon_t)}{\partial \varepsilon_t} = \left(8 - \dfrac{2}{\varepsilon_t(1-\varepsilon_t)}\right)(\varepsilon_t - \frac{1}{2})$

$d(\varepsilon_t)$ 在 $(0,\frac{1}{2})$ 单调增，$(\frac{1}{2}, 1)$ 单调减

在 $\frac{1}{2}$ 处取最大值 0

∴ $(0,1)$ 上 $d(\varepsilon_t) \le 0$.

∴ 有 $\hat{\varepsilon}(h) \le \prod_{t=1}^{T} Z_t \le e^{-2\sum_{t=1}^{T}(\frac{1}{2}-\varepsilon_t)^2}$，得证

2. $H \subseteq \{-1,+1\}^X$

for $i \leftarrow 1$ to m do

$D_1(i) \leftarrow w_i$

for $t \leftarrow 1$ to T do

$h_t \leftarrow$ base classifier in H with $\epsilon_t = \Pr_{i \sim D_t}[h_t(x) \neq y_i]$

$\alpha_t \leftarrow \frac{1}{2}\log\frac{1-\epsilon_t}{\epsilon_t}$

$Z_t \leftarrow 2[\epsilon_t(1-\epsilon_t)]^{\frac{1}{2}}$

for $i \leftarrow 1$ to m do

$D_{t+1}(i) \leftarrow \frac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$

$f_t \leftarrow \sum_{s=1}^{t} \alpha_s h_s$

return $h = \mathrm{sgn}(f_T)$

证明，令 $\bar{D}_t(i) = \frac{w_i e^{-y_i \sum_j \bar{\alpha}_{t-1,j} h_j(x_i)}}{\bar{Z}_t}$

$$D_{t+1}(i) = \frac{D_{t-1}(i)\, e^{-\alpha_{t-1} y_i h_{t-1}(x_i)}\, e^{-\alpha_t y_i h_t(x_i)}}{Z_{t-1} Z_t} = \frac{\frac{w_i}{n} e^{-y_i f_t(x_i)}}{\prod_{s=1}^{t} Z_s} = \frac{\frac{w_i}{n} e^{-y_i \sum_{s=1}^{t} \alpha_s h_s(x_i)}}{\prod_{s=1}^{t} Z_t}$$

$$F(\bar{\alpha}) = \sum_{i=1}^{n} w_i e^{-y_i f(x_i)} = \sum_{i=1}^{n} w_i e^{-y_i \sum_j \bar{\alpha}_j h_j(x_i)}$$

$$F(\bar{\alpha}_{t-1} + \eta e_k) = \sum_{i=1}^{n} w_i e^{-y_i \sum_j \bar{\alpha}_j h_j(x_i) - \eta y_i h_k(x_i)}$$

$$F'(\bar{\alpha}_{t-1}, e_k) = -\sum_{i=1}^{n} y_i h_k(x_i)\, w_i e^{-y_i \sum_j \bar{\alpha}_{t-1,j} h_j(x_i)}$$

$$= -\sum_{i=1}^{n} y_i h_k(x_i)\, \bar{D}_t(i)\, \bar{Z}_t$$

$$= -\Big[\sum_{i=1}^{n} \bar{D}_t(i)\, 1[y_i h_k(x_i)=1] - \sum_{i=1}^{n} \bar{D}_t(i)\, 1[y_i h_k(x_i)=-1]\Big] \bar{Z}_t$$

$$= -\big[(1-\bar{\epsilon}_{t,k}) - \bar{\epsilon}_{t,k}\big]\frac{\bar{Z}_t}{n}$$

$$= [2\bar{\epsilon}_{t,k} - 1]\frac{\bar{Z}_t}{n}$$

$$\frac{\partial F(\bar{\alpha}_t)}{\partial \eta} = \sum_{i=1}^{n} y_i h_k(x_i)\, \bar{D}_t(i)\, \bar{Z}_t\, e^{-\eta y_i h_k(x_i)}$$

$$= \sum_{i=1}^{n} \bar{D}_t(i)\, 1[y_i h_k(x_i)=1] e^{-\eta} - \sum_{i=1}^{n} \bar{D}_t(i)\, 1[y_i h_k(x_i)=-1] e^{\eta}$$

$$\Leftrightarrow \big[(1-\bar{\epsilon}_{t,k}) e^{-\eta} - \bar{\epsilon}_{t,k} e^{\eta}\big] =$$

$$\eta = \frac{1}{2}\log\frac{1-\bar{\epsilon}_{t,k}}{\bar{\epsilon}_{t,k}}$$