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$\frac{1}{2}w^Tw$

1. (1) 目标: $\min_{w,b} \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{n} r_i \xi_i$

约束: $\begin{cases} -y_i(w\cdot x_i + b) - \xi_i + 1 \le 0 \\ -\xi_i \le 0 \end{cases}$

$\Rightarrow L(w,b,\xi,\lambda,u) = \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{n} r_i\xi_i - \sum_{i=1}^{n}\lambda_i(y_i(w\cdot x_i+b)+\xi_i-1) - \sum_{i=1}^{n}u_i\xi_i$, $\begin{cases}\lambda_i \ge 0 \\ u_i \ge 0 \end{cases}$ $i=1\sim n$

~~(2) 对偶问题: $\max_{w,b,\lambda,u} \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{n} r_i\xi_i - \sum_{i=1}^{n}\lambda_i(y_i(w\cdot x_i+b)+\xi_i-1) - \sum_{i=1}^{n}u_i\xi_i$ s.t. $\lambda_i \ge 0$, $u_i \ge 0$ for $i \le n$~~

(2) 对偶问题: 求 $\Gamma(\lambda,u)$ =

使 $\max_{\lambda,u} \Gamma(\lambda,u) = \inf_{w,b,\xi} L(w,b,\xi,\lambda,u)$

$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{n}\lambda_i y_i x_i$ | $\frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_{i=1}^{n}\lambda_i y_i x_i$

$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^{n}\lambda_i y_i = 0$

w

$\frac{\partial L}{\partial \xi_i} = 0 \Rightarrow C r_i = \lambda_i + u_i$

代入,得:

$$\boxed{\begin{aligned} &\max_{\lambda} \sum_{i=1}^{n}\lambda_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j y_i y_j (x_i\cdot x_j) \\ &\text{s.t.} \sum_{i=1}^{n}\lambda_i y_i = 0 \\ &0 \le \lambda_i \le C r_i,\ i=1\sim n \end{aligned}}$$

3.原问题：

$$\min \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{n} r_i\varepsilon_i$$

s.t. 对 $1\le i\le n$

$$\begin{cases} -y_i(w\cdot\phi(x_i)+b) - \varepsilon_i + 1 \le 0 \\ -\varepsilon_i \le 0 \\ -r_i \le 0 \end{cases}$$

对偶问题：

$$\cancel{\max_{\alpha} \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}}$$

$$L(w,b,\alpha,\varepsilon,r,u) = \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{n} r_i\varepsilon_i + \sum_{i=1}^{n}\alpha_i\left(1-\varepsilon_i - y_i(w\cdot\phi(x_i)+b)\right) - \sum_{i=1}^{n}u_i\varepsilon_i$$

$$\begin{cases} \alpha_i\ge 0,\ \varepsilon_i\ge 0,\ r_i\ge 0 \end{cases}$$

对：

$$\frac{\partial L}{\partial w}=0 \Leftrightarrow w=\sum_{i=1}^{n}\alpha_i y_i\phi(x_i)$$

$$\frac{\partial L}{\partial b}=0 \Leftrightarrow \sum_{i=1}^{n}\alpha_i y_i=0$$

$$\frac{\partial L}{\partial \varepsilon_i}=0 \Leftrightarrow Cr_i = u_i+\alpha_i$$

代入，得：

$$\max_{\alpha} \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j k(x_i,x_j)$$

$$\text{s.t. } \sum_{i=1}^{n}\alpha_i y_i = 0$$

$$0\le\alpha_i\le Cr_i,\quad 1\le i\le n$$