UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Escola Superior d'Enginyeries Industrial, Aeroespacial i Audiovisual de Terrassa

ESTRUCTURES AEROESPACIALS

3D bars structure: Glider

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Consider the hang glider depicted in Figure 1. Its structure is composed of bars of two different materials, with their properties given in Table 1. The equations of motion of the glider as a rigid body are:

$$M_{\mathrm{tot}} \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \sum_{i=1}^{n_{\mathrm{nod}}} \boldsymbol{F}_i^{\mathrm{ext}} \qquad \mathbf{I}_{\mathrm{cm}} \frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} = \sum_{i=1}^{n_{\mathrm{nod}}} \boldsymbol{r}_i \times \boldsymbol{F}_i^{\mathrm{ext}}$$

where M_{tot} is the total mass of the structure (including the pilot and upper cloth surface), $\mathbf{I}_{\mathrm{cm}}$ is the inertia tensor about the centre of mass, $\boldsymbol{V}$ is the velocity vector, $\boldsymbol{\Omega}$ is the angular (rotational) velocity vector, $\boldsymbol{r}_i$ are the position vectors of each node i with respect to the centre of mass, and $\boldsymbol{F}_i^{\mathrm{ext}}$ are the external forces acting at each node i.

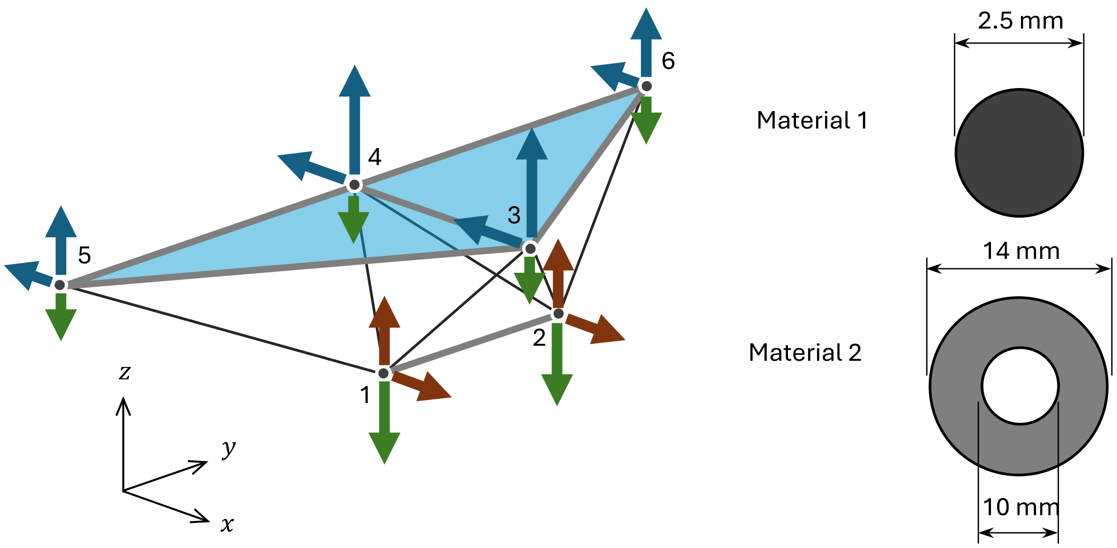

Figure 1. Sketch of the glider structure, including the loads distribution and detail of the cables (material 1) and bars (material 2) cross sections.

Table 1. Material properties and data.

Upper surface	
Surface density, ρ_S (kg/m^2)	1.55
Surface area, S (m^2)	3.75
Lift coefficient, C_L	2.6
Drag coefficient, C_D	1.45

Cables and bars		
Property	*Cables*	*Bars*
Density, ρ_i (kg/m^3)	1500	2600
Young's modulus, E_i (MPa)	150000	78000
Yield strength, σ_i^* (MPa)	150	240

In standard conditions, the glider flights at constant horizontal velocity ($\mathrm{d}V_x/\mathrm{d}t = \mathrm{d}V_y/\mathrm{d}t = 0$) and without rotating ($\Omega_x = \Omega_y = \Omega_z = 0$). Overall, the glider is subjected to:

a) **Aerodynamic loads** distributed uniformly over the upper surface, with the total lift and drag given by:

$$L = \frac{1}{2}\rho_\mathrm{a} S C_\mathrm{L} V_z^2 \quad \text{and} \quad D = \frac{1}{2}\rho_\mathrm{a} S C_\mathrm{D} V_z^2$$

where $\rho_\mathrm{a} = 1.225$ kg/m^3 is the air's density and V_z is the vertical component of the glider's descend speed (initially $V_z = 0$). The rest of the data is given in Table 1.

b) **Weight** of the overall structure, including the weight of each bar, the upper surface, and the weight of the pilot's mass $M = 85$ kg, which is distributed equally among nodes 1 and 2.

c) The **reaction force** of the pilot distributed between nodes 1 and 2 to guarantee standard flight conditions.

At time $t_\mathrm{g} = 3$ s of the descend, a gust produces a sudden increase of the lift in node 3, given by the following expression:

$$\Delta L = L_\mathrm{g} \sin\left(\pi \frac{t - t_\mathrm{g}}{\Delta t_\mathrm{g}}\right); \quad L_\mathrm{g} = \frac{1}{2}\rho_\mathrm{a} S C_\mathrm{L} V_\mathrm{g}^2$$

for $t_\mathrm{g} < t < t_\mathrm{g} + \Delta t_\mathrm{g}$, with $V_\mathrm{g} = 2.4$ m/s and $\Delta t_\mathrm{g} = 0.7$ s. At time $t_\mathrm{r} = t_\mathrm{g} + 0.5$ s, the pilot reacts to this gust applying an additional vertical force distributed between nodes 1 and 2 to return the glider to its standard flight conditions:

$$\Delta R = -L_\mathrm{g}\left(c_3(t - t_\mathrm{r})^3 - c_2(t - t_\mathrm{r})^2 + c_1(t - t_\mathrm{r})\right)$$

for $t_\mathrm{r} < t < t_\mathrm{r} + \Delta t_\mathrm{r}$, with $c_1 = 4.512$, $c_2 = 6.0996$, $c_3 = 1.9218$, and $\Delta t_\mathrm{r} = 2$ s.

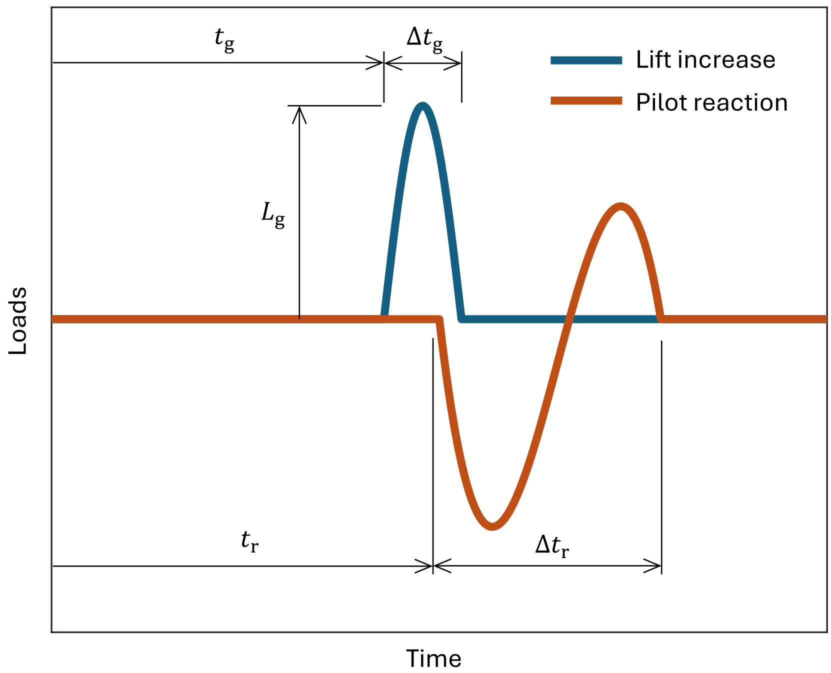

Figure 2. Gust loads evolution over time.

The purpose is to determine the critical stress (at traction and compression) of the structure at different time instants (consider timesteps of $\Delta t = 0.01$ s) from the moment the glider starts descending until it reaches its terminal velocity.

The following is requested:

Part 1 Dynamic problem:

1.1) Find the total mass, M_{tot}, and the inertia tensor with respect to its center of mass, $\mathbf{I}_{\mathrm{cm}}$.

1.3) Obtain the components of the reaction force the pilot applies in nodes 1 and 2 assuming standard flight conditions.

1.2) Postulate the equations of motion accounting for the effect of the gust and the pilot's reaction to it. Since the gust is likely to produce a net pitching moment, $\mathrm{d}\Omega_y/\mathrm{d}t \neq 0$ must be considered.

1.4) Propose a numerical integration scheme to get the linear and angular velocity, $\boldsymbol{V}$ and $\boldsymbol{\Omega}$, and the linear and angular acceleration, $\mathrm{d}\boldsymbol{V}/\mathrm{d}t$ and $\mathrm{d}\boldsymbol{\Omega}/\mathrm{d}t$, of the glider at each timestep. Then, compute the acceleration at each node according to:

$$\boldsymbol{a}_i = \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} + \frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \boldsymbol{r}_i + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}_i)$$

Hint: If the timestep Δt is small enough, derivatives can be approximated by:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} \approx \frac{\phi^n - \phi^{n-1}}{\Delta t}$$

Part 2 Quasi-static problem:

2.1) Determine a set of degrees of freedom to prescribe.

2.2) For each timestep, solve the structural problem to get the displacements, reactions and stresses of the structure.

2.3) Determine the safety factor for stresses at traction.

2.4) Determine the safety factor for buckling, assuming the critical stress given by:

$$\sigma_{\mathrm{cr}} = \frac{\pi^2 EI}{L^2 A}$$

where I is the second moment of area of the bar's cross section, and L its length.

Hint: The script file "`input_data`" contains the nodal coordinates matrix "`x`" (in meters), the nodal connectivities matrix "`Tn`" and the material connectivities matrix "`Tm`". The nodal and material indexing is as depicted in Figure 1. For postprocessing the results, use the provided "`plot3Dbars`" function. It takes as inputs (in this order): (1) the nodal coordinates matrix "`x`", (2) the nodal connectivities matrix "`Tn`" (both as provided in "`input_data`"), (3) a scale factor (to visualize the deformed structure), (4) the time vector (length N_{t}, it contains the time of each iteration step). (5) the global displacements matrix (size $n_{\mathrm{dof}} \times N_{\mathrm{t}}$, each column contains the displacements vector at each iteration step), and (6) the stress vector matrix (size $n_{\mathrm{el}} \times N_{\mathrm{t}}$, each column contains the element stresses vector at each iteration step). To just plot the results for a given time instant provide input (4) as a scalar value and inputs (5) and (6) as column vectors with the results corresponding at the time instant. You can optionally add an input (7) indicating the units in which you provided the stress.

The assignment can be done in groups of maximum two people. The MATLAB® script files and a report with the required results (including a brief discussion of the same) must be submitted to Atenea by either one of the members (not both).