Flight Trajectory Prediction Enabled by Time-Frequency Wavelet Transform

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Skoltech course: Theoretical Methods of Deep Learning
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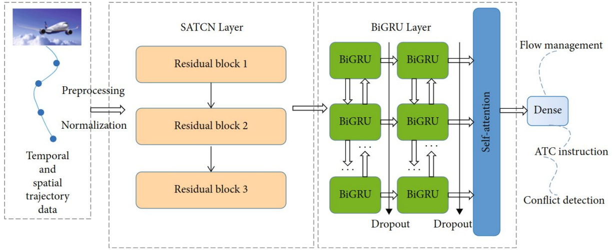
Zhang, Z., Guo, D., Zhou, S. *et al.* Flight trajectory prediction enabled by time-frequency wavelet transform. *Nat Commun* 14, 5258 (2023). https://doi.org/10.1038/s41467-023-40903-9
Paper repository: https://github.com/MusDev7/wtftp-model
Project repository: https://github.com/SerValera/wtftp-model-drones

Motivation

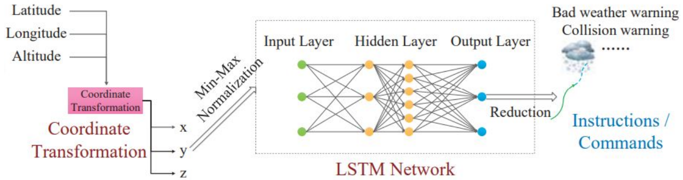
With the continual development of the global economy, the air transportation demand has significantly increased across various industries, leading to a surge in flight traffic and airspace complexity.

Research Background

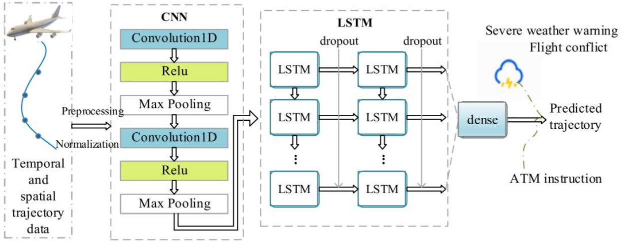
Bayesian Optimised Temporal CN [1]



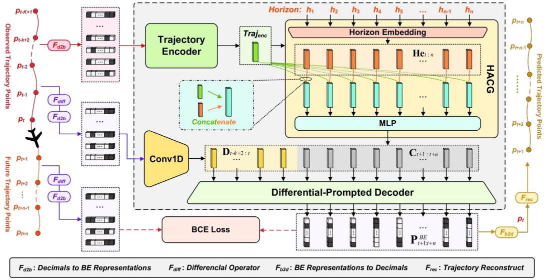
LSTM-based Flight Trajectory Prediction [2]



Hybrid CNN-LSTM Model [3]

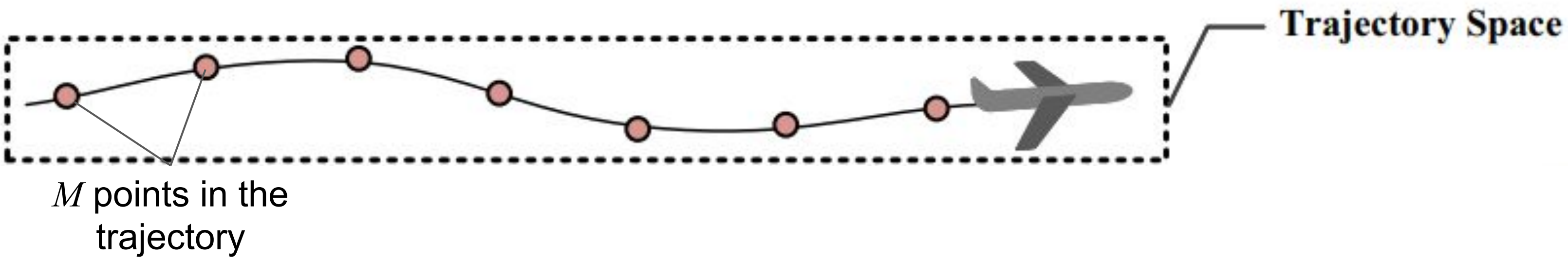


FlightBERT [4]



[1] Huang, J., Ding, W. et al. Aircraft trajectory prediction based on bayesian optimised temporal convolutional network–bidirectional gated recurrent unit hybrid neural network. Int. J. Aerosp. Eng. 2022 2086904 (2022).
[2] Shi, Z., Xu, M., Pan, Q., Yan, B. & Zhang, H. LSTM-based flight trajectory prediction. In International Joint Conference on Neural Networks 8 (IEEE, 2018).
[3] Ma, L. & Tian, S. A hybrid CNN-LSTM model for aircraft 4D trajectory prediction. IEEE Access 8, 134668–134680 (2020).
[4] Guo, D. et al. FlightBERT: binary encoding representation for flight trajectory prediction. IEEE Trans. Intell. Transp. Syst. 24, 1828–1842 (2022).

State Space of the Aircraft



The trajectory is defined as:

$$\{\mathbf{P}_i \in \mathbb{R}^d | \boldsymbol{i} = \boldsymbol{N-M}, \boldsymbol{N-M+1}, \cdots, \boldsymbol{N-1}\}$$

where:

$$\mathbf{P}_i = [Lon_i, Lat_i, Alt_i, Vx_i, Vy_i, Vz_i]^{\mathrm{T}}$$

Lon, Lat, Alt, Vx, Vy, Vz correspond to the longitude, latitude, altitude and velocities along the previous three dimensions, respectively.

Proposed WTFTP Framework

The nonlinear function $f(\cdot)$ is expected to be learned:

$$\hat{\mathbf{P}}_N = f(\mathbf{P}_{N-M:N-1})$$

where:

$$\mathbf{P}_{N-M:N-1} = [\mathbf{P}_{N-M}, \mathbf{P}_{N-M+1}, \cdots, \mathbf{P}_{N-1}]$$

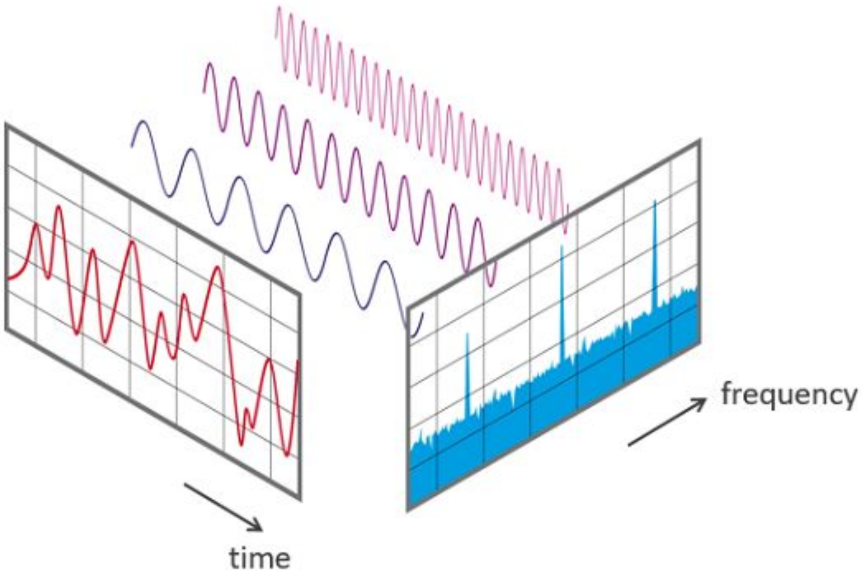
is the array of points in the trajectory.

Mathematically, the prediction process of the WTFTP can be described as:

$$[\hat{\mathbf{P}}_{N-M:N-1}, \hat{\mathbf{P}}_N] = \mathrm{WTFTP}(\mathbf{P}^{\mathrm{T}}_{N-M:N-1})$$

The function reconstructs the previous M points and predicts the next point.

Time Series and Spectral Analysis

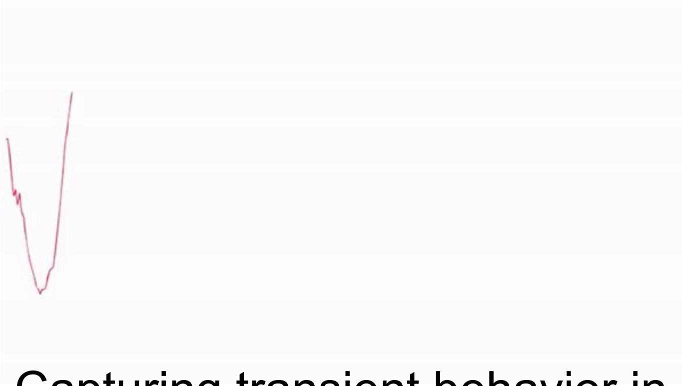


Given the time the frequency information is lost,
given the spectrum the time information is lost

The Fourier transformation formula

$$\widehat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi\xi x}\, dx.$$

Wavelet Analysis

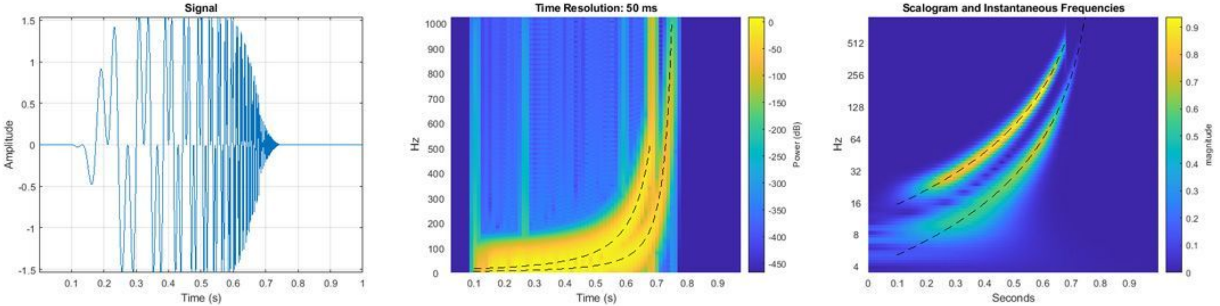


Capturing transient behavior in signals

Wavelet-transformation representation

$$C[x](a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt$$

where a is the dilation factor, b is the position factor.



Analyzing a hyperbolic chirp signal

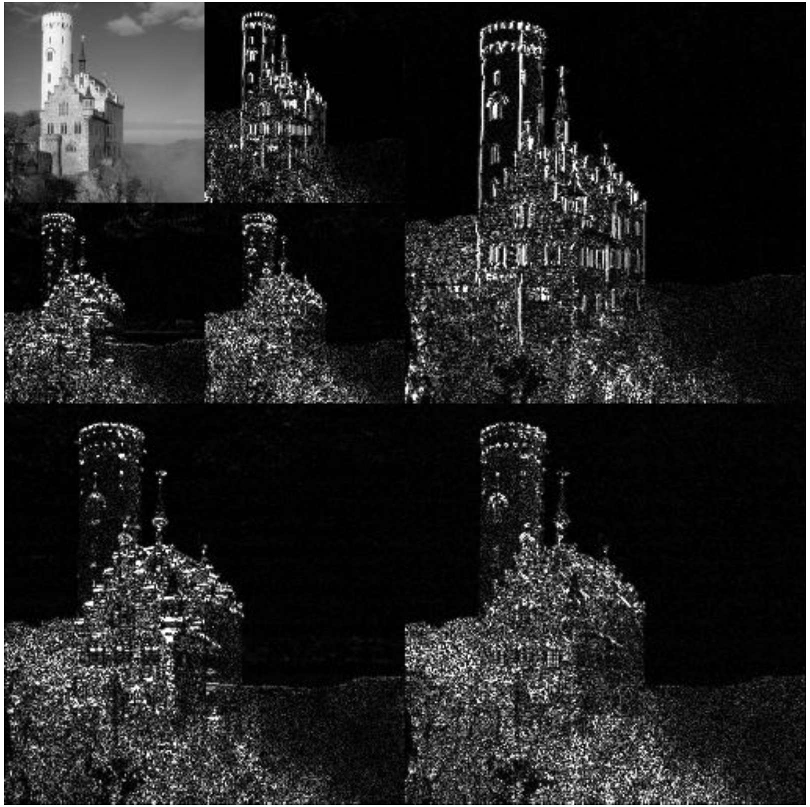
Discrete Wavelet Transform: Mallat algorithm

Mathematical representation of the Mallat algorithm for discrete wavelet transform:

$$a_{j-1,k} = \sum_{n \in \mathbb{Z}} G_{n-2k} a_{j,n}$$

$$d_{j-1,k} = \sum_{n \in \mathbb{Z}} H_{n-2k} a_{j,n}$$

where G is a low-pass filter and H is a high-pass filter.



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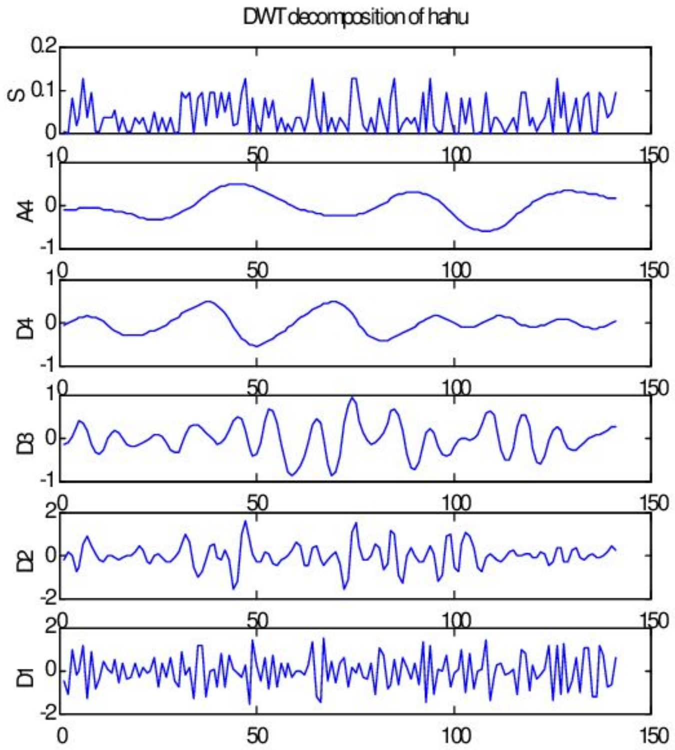
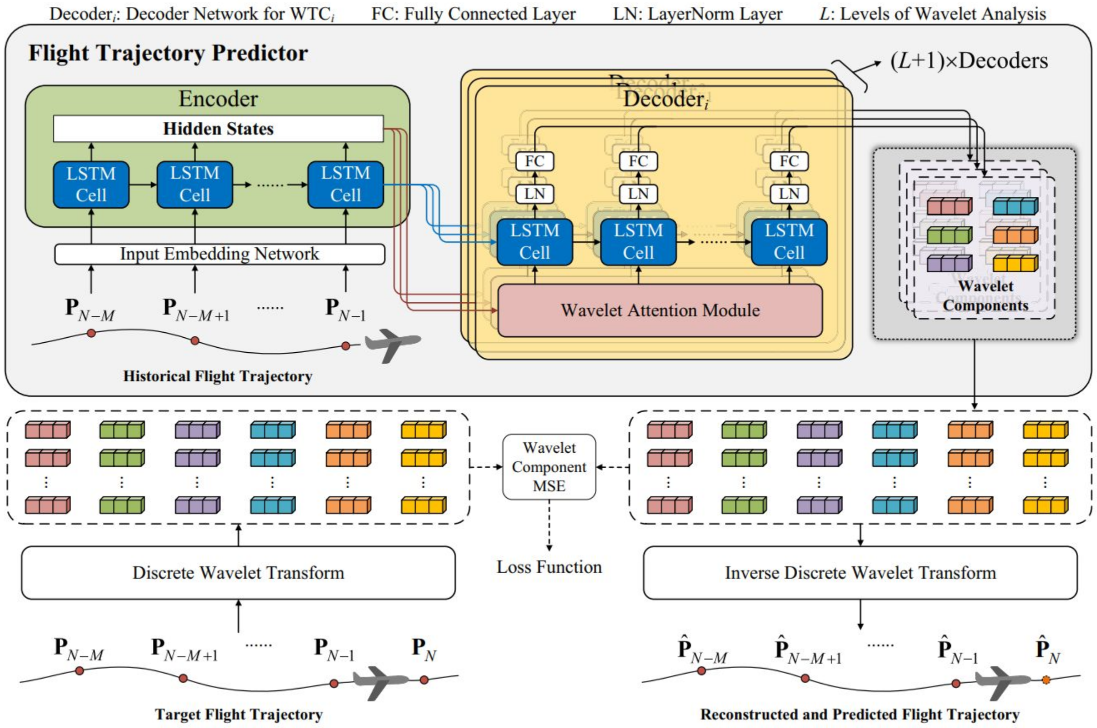


Proposed Neural Architecture



The loss function of the network

$$\mathcal{L} = \sum_{k=0}^{L} \mathcal{L}_{wavelet}^{k}$$

$$\mathcal{L}_{wavelet}^{k} = \frac{1}{h_{L-k} \cdot d} \sum_{i=1}^{h_{L-k}} \sum_{j=1}^{d} \left(c_{i,j}^{k} - \hat{c}_{i,j}^{k} \right)^2$$

where L is the level of wavelet analysis, h_{L-k} represents the length of the WTC$_k$ and d = 6 is the number of attributes in this work.

Performance Metrics

A total of four measurements are considered to evaluate the performance of prediction models, including root of mean squared error (RMSE), mean absolute error (MAE), mean relative error (MRE) and mean deviation error (MDE).

$$MAE_i = \frac{1}{N}\sum_{j=1}^{N} |\mathbf{P}_{i,j} - \hat{\mathbf{P}}_{i,j}|$$

$$MDE = \frac{1}{N}\sum_{j=1}^{N} \sqrt{\sum_{i}^{3} (pos_{i,j} - \hat{pos}_{i,j})^2}$$

$$RMSE_i = \sqrt{\frac{1}{N}\sum_{j=1}^{N} (\mathbf{P}_{i,j} - \hat{\mathbf{P}}_{i,j})^2}$$

$$MRE_i = 100\% \times \frac{1}{N}\sum_{j=1}^{N} \left|\frac{\mathbf{P}_{i,j} - \hat{\mathbf{P}}_{i,j}}{\mathbf{P}_{i,j}}\right|$$

$$\begin{cases} pos_{1,j} = (\mathrm{PR}(lon_j) + alt_j)\cos(lon_j)\cos(lat_j) \\ pos_{2,j} = (\mathrm{PR}(lon_j) + alt_j)\cos(lon_j)\sin(lat_j) \\ pos_{3,j} = \left(\frac{b^2}{a^2}\mathrm{PR}(lon_j) + alt_j\right)\sin(lon_j) \\ \mathrm{PR}(\cdot) = \dfrac{a}{\sqrt{1 - \left(1 - \frac{b^2}{a^2}\right)\sin^2(\cdot)}} \end{cases}$$

Overall Performance Evaluation

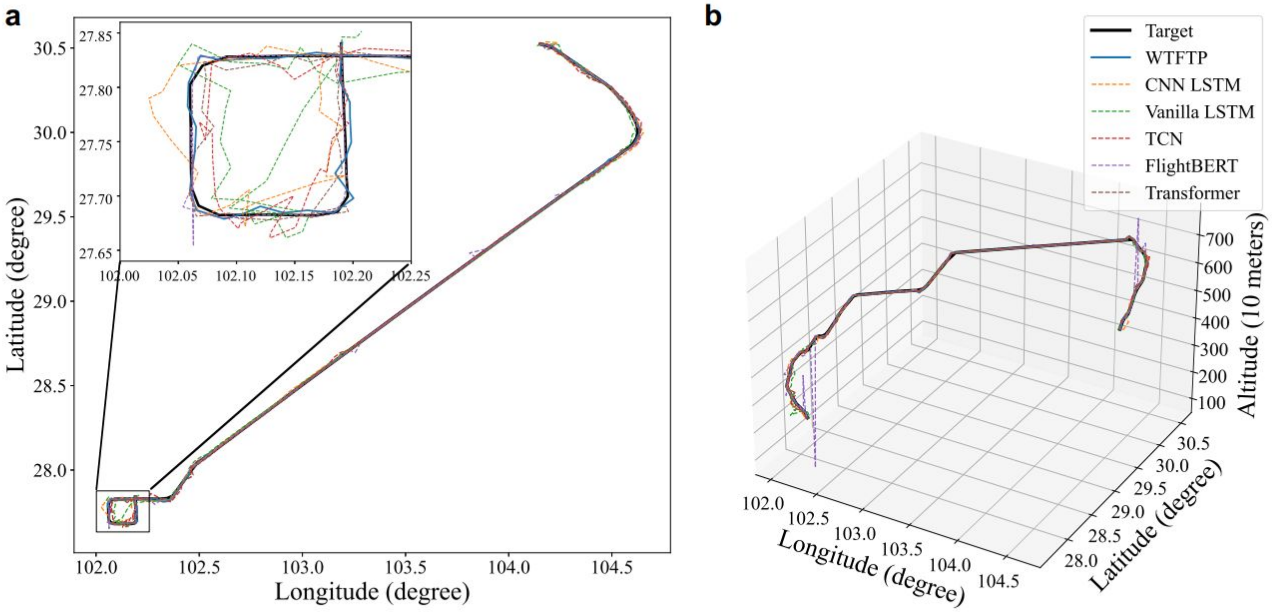
Table 1 | The overall performance evaluation

Models	MAE↓			MRE(%)↓			RMSE↓			MDE↓
	Lon	Lat	Alt	Lon	Lat	Alt	Lon	Lat	Alt	
A1	0.0056	0.0059	1.36	0.0030	0.0110	0.25	0.0163	0.0142	8.55	0.9472
A2	0.0052	0.0054	1.32	0.0049	0.0194	0.27	0.0151	0.0138	**8.13**	0.8794
A3	0.0051	0.0050	1.36	0.0048	0.0177	0.27	0.0164	0.0139	8.91	0.8299
A4	0.0049	0.0047	1.21	0.0046	0.0169	0.24	0.0148	0.0128	8.17	0.8003
A5	0.0039	0.0033	1.36	0.0029	0.0103	0.30	0.0558	0.0486	10.59	0.5910
WTFTP	**0.0025**	**0.0022**	**1.14**	**0.0023**	**0.0078**	**0.23**	**0.0148**	**0.0125**	8.91	**0.3855**

A1–A5 represent the baselines: Vanilla LSTM, TCN, CNN LSTM, Transformer and FlightBERT, respectively.

The raw flight trajectories are collected by multi-source Secondary Surveillance Radar (SSR) and Automatic Dependent Surveillance-Broadcast (ADS-B) from a real-world ATC system in China.

Overall Performance Evaluation



Visualization of the selective flight trajectory. Mean deviation error is less than 400m.

Dataset Structure & Details

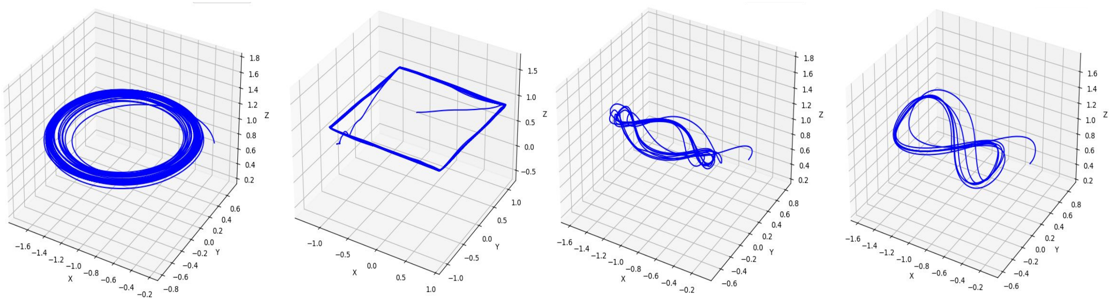
Dataset structure:
- [x, y, z, vx, vy, vz] in Vicon coordinate system [m, m/s];
- Recorded frequency 30 Hz.

Dataset Size:
- Training: 12,741 items;
- Validation: 1,657 items;
- Test: 1,391 items.

Trajectories: 4 Shapes
- Circle;
- Square;
- Spline;
- Infinity.

Hardware:
- Vicon Tracking System;
- Drone: Crazyflie 2.1

Crazyflie. Data collection process



Visualisation of collected drone's trajectories by Vicon tracking system
Flight tested area: **1.5 x 1.5 m**
Average speed **0.7 m/s**

Paper repository: https://github.com/MusDev7/wtftp-model

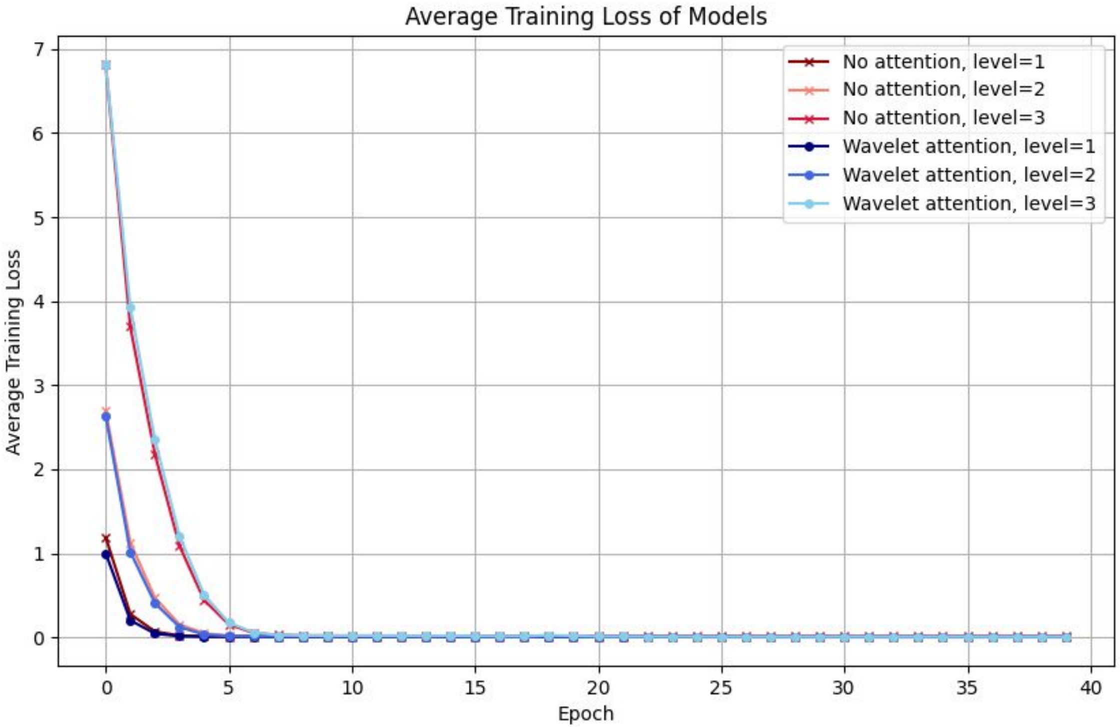
NN Training History

6 model were training with different level of wavelet analysis **(1 to 3)** and usage of the wavelet attention module in the decoder **(yes, no).**

Training Details (main parameters):

- Minibatch Length: 10
- Interval: 1
- Batch Size: 64
- Epochs: 40
- Learning Rate (lr): 0.0002

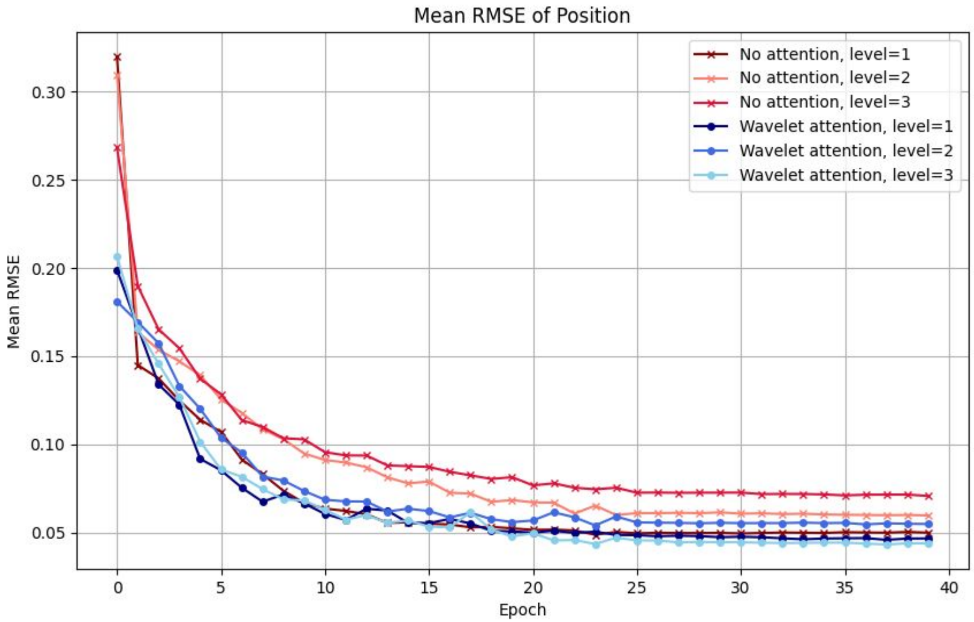
```
Epoch 40/40--------------------
2024-12-19 11:55:27,863   0/199
train_loss: 0.00177342
```

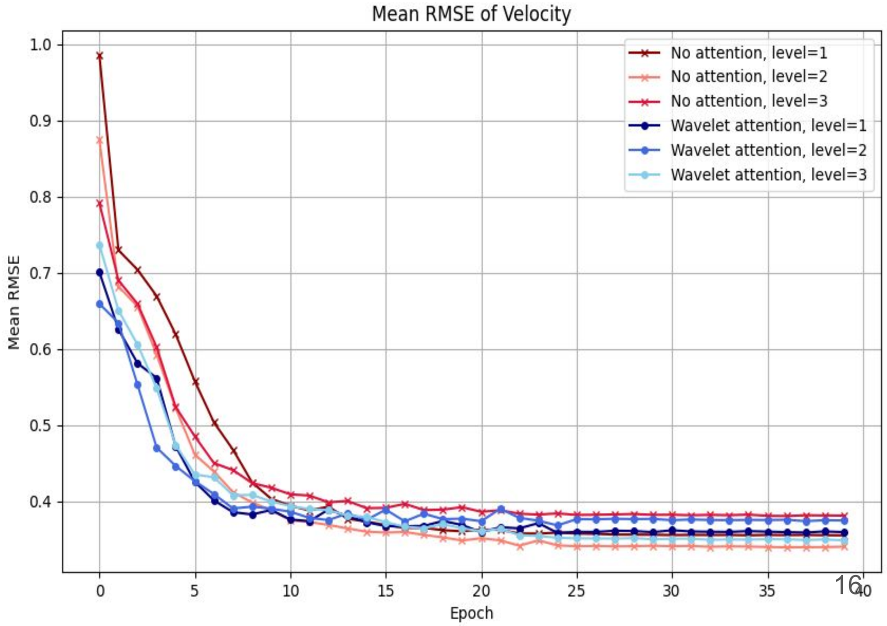


Mean RMSE of Pose and Velocity Prediction

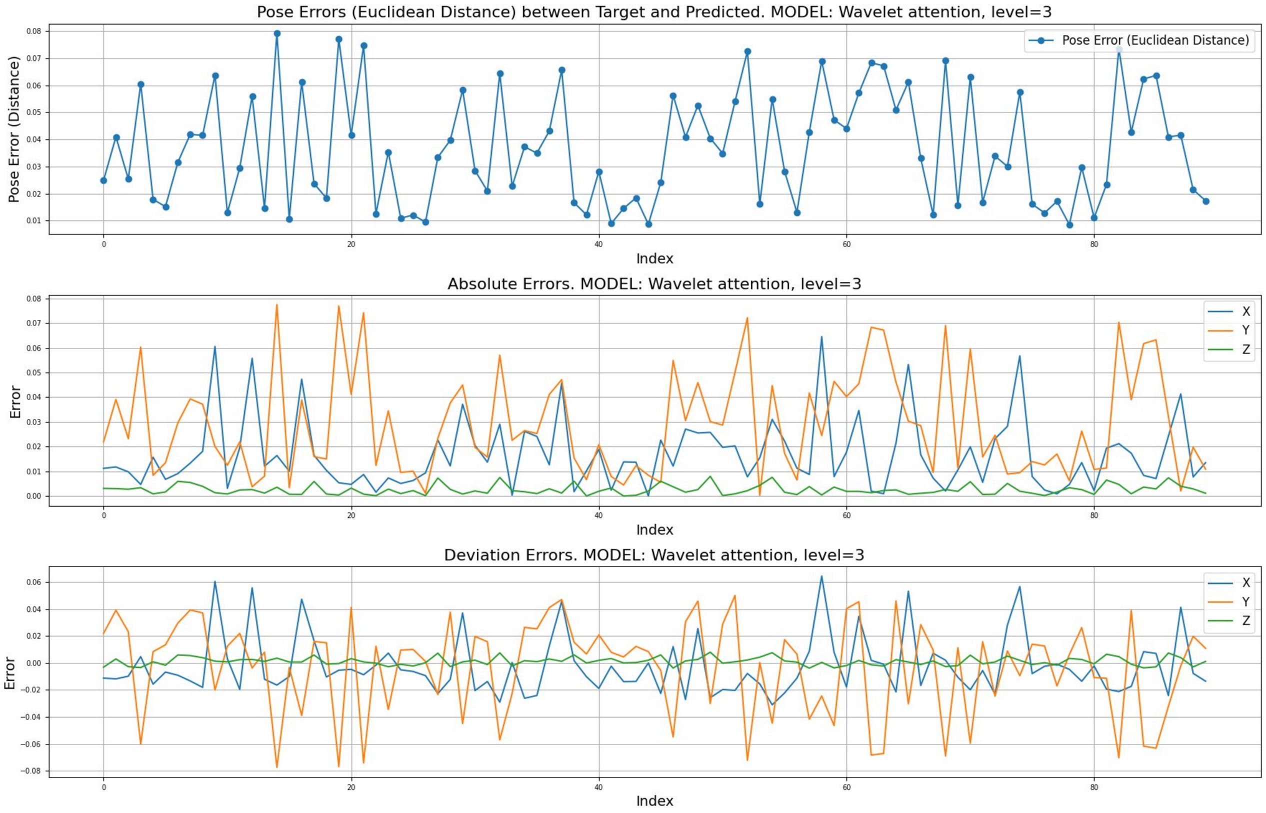
The **RMSE** for velocity and position **is lower with wavelet attention**, indicating improved accuracy. **Higher levels** of wavelet attention further **reduce the error**.

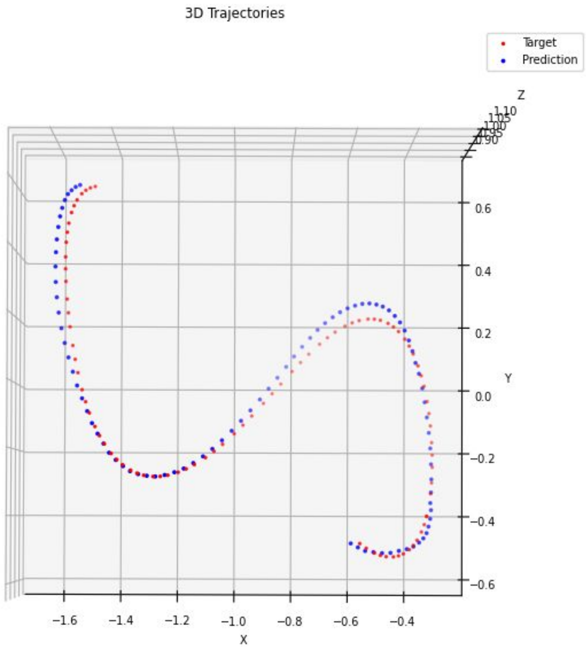
Best model: "Wavelet attention. Level=3"





Trajectory Prediction by *Best Model*

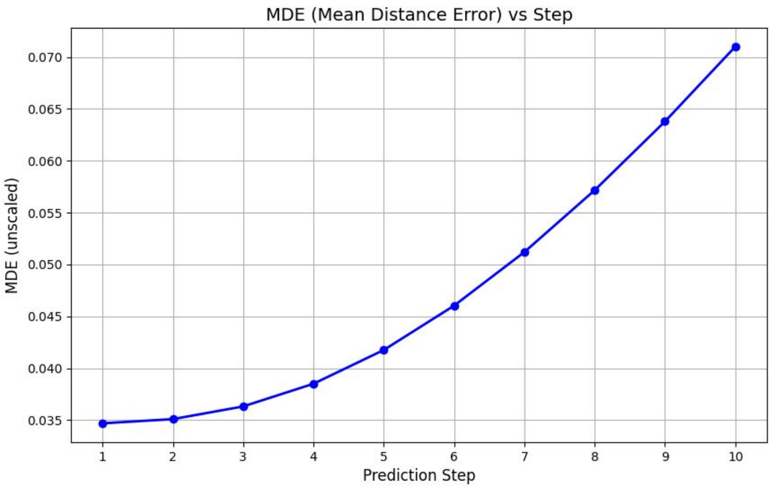




Red points represent the true trajectory, while blue points show the predicted trajectory after one future step, based on the last 10 points. (axis in m.)

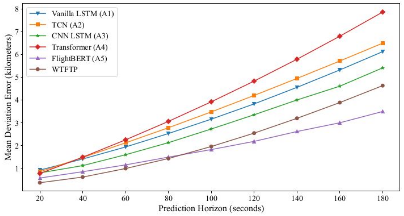
`mean_pose_error: 0.037 m`

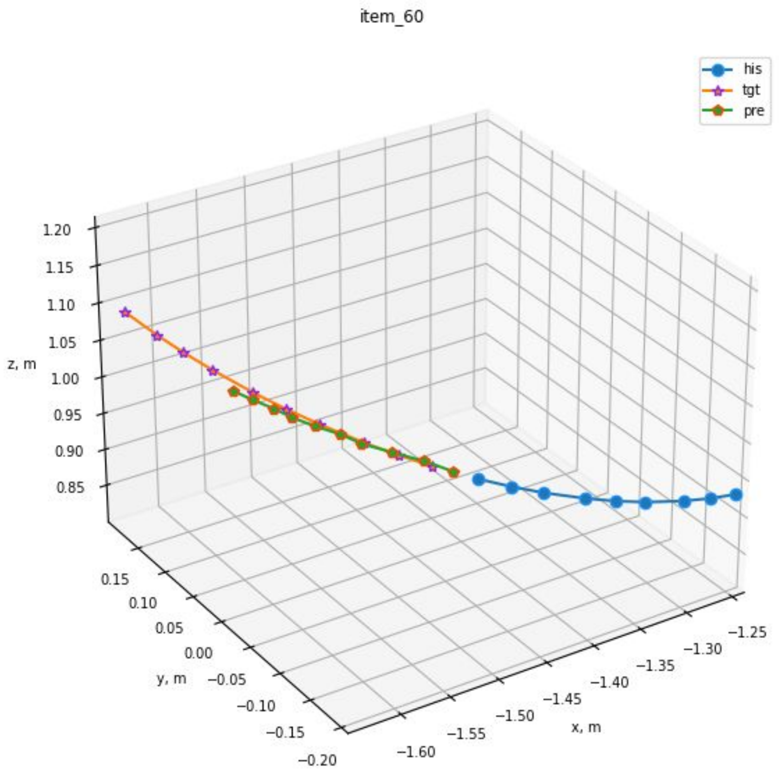
Multi-Step Prediction Analysis



Mean deviation error of the WTFTP framework of at different prediction horizons. The max horizon is 3.33 sec.

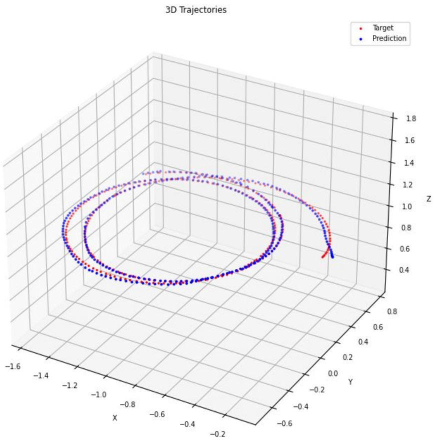
Paper results with beaslines:

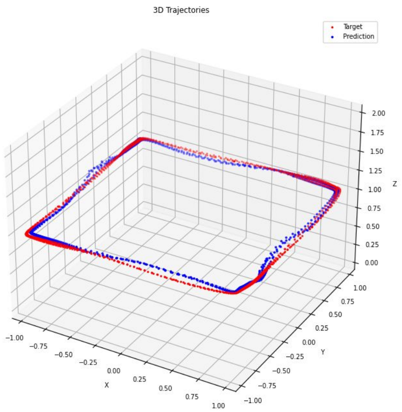


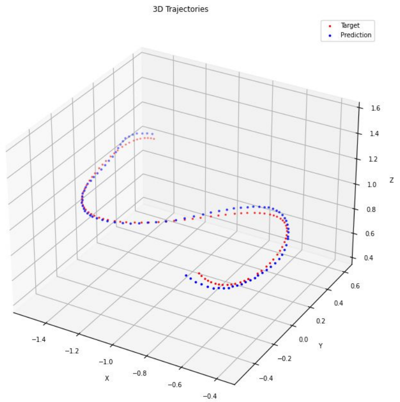


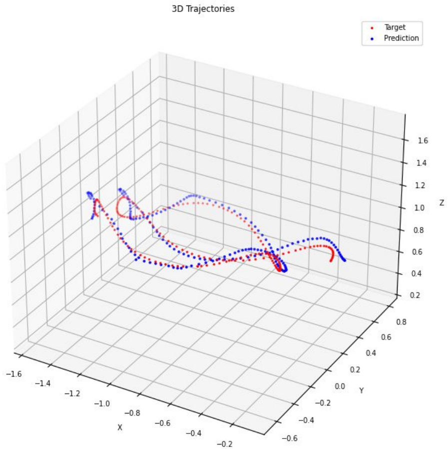
Stars represent the target poses, blue points show the trajectory history, and pentagon markers indicate the predicted poses.

Evaluation Trajectories









Mean_pose_errors: 0.016 m

Mean_pose_errors: 0.058 m

Mean_pose_errors: 0.037 m

Mean_pose_errors: 0.065 m

Red points represent the true trajectory, while **blue points show the predicted trajectory** after one future step, based on the observation of last 10 poses. (axis in m.)

Conclusion

- The wavelet transform-based approach successfully predicts flight trajectories with high accuracy.

- Time-frequency analysis is confirmed as an effective method for trajectory prediction in air traffic control.

- Our experiments with the collected dataset confirmed that the wavelet transform-based framework produces similar results, validating its effectiveness for trajectory prediction.

Configurations of the proposed framework.

Number of historical trajectory points	9
Dimension of the trajectory embeddings	64
Dimension of the enhanced trajectory embeddings	64
Dimension of the contextual embeddings	64
Layer number of the LSTM block in the encoder	4
Layer number of the LSTM block in the decoder	1
Convolution arguments for 1-level sub-band ([kernel size, stride, padding])	[2, 2, 1]
Convolution arguments for 2 level sub-band ([kernel size, stride, padding])	[3, 3, 0]
Convolution arguments for 3 level sub-band ([kernel size, stride, padding])	[5, 5, 1]
Wavelet basis	Haar
The length of the wavelet filter	2