Name: 　　　　No.:

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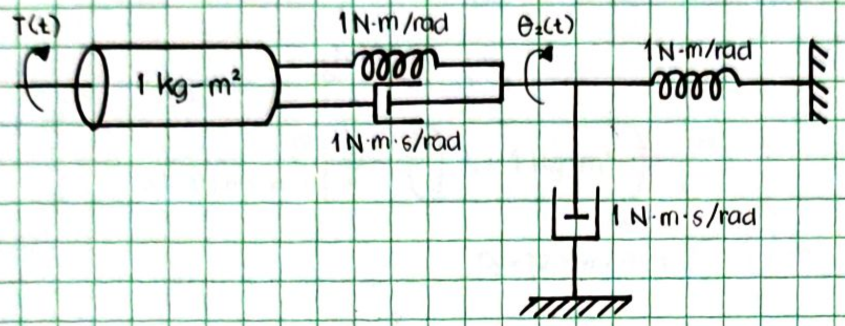
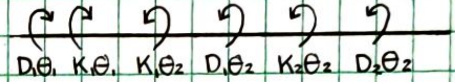
FIND THE TRANSFER FUNCTION, $G(s) = \theta_2(s)/T(s)$, FOR THE ROTATIONAL MECHANICAL SYSTEM SHOWN.



FBD 1:

$$T(t) = J_1\ddot{\theta}_1 + K_1\theta_1 + D_1\dot{\theta}_1 - K_1\theta_2 - D_1\dot{\theta}_2$$
$$\{\ T(t) = \ddot{\theta}_1 + \theta_1 + \dot{\theta}_1 - \theta_2 - \dot{\theta}_2\ \}$$
$$T(s) = s^2\theta_1(s) + \theta_1(s) + s\theta_1(s) - \theta_2(s) - s\theta_2(s)$$
$$T(s) = \theta_1(s)[s^2+s+1] - \theta_2(s)[s+1] \ \ldots\ldots \text{eq. 1}$$

FBD 2:



$$0 = K_1\theta_2 + D_1\dot{\theta}_2 + K_2\theta_2 + D_2\dot{\theta}_2 - D_1\dot{\theta}_1 - K_1\theta_1$$
$$0 = \theta_2 + \dot{\theta}_2 + \theta_2 + \dot{\theta}_2 - \dot{\theta}_1 - \theta_1$$
$$\{\ 0 = 2\theta_2 + 2\dot{\theta}_2 - \dot{\theta}_1 - \theta_1\ \}$$
$$0 = 2\theta_2(s) + 2s\theta_2(s) - s\theta_1(s) - \theta_1(s)$$
$$0 = \theta_2(s)[2s+2] - \theta_1(s)[s+1] \ \ldots\ldots \text{eq. 2}$$

$$\overset{C}{T(s)} = \theta_1(s)\overset{A}{[s^2+s+1]} - \theta_2(s)\overset{B}{[s+1]}$$
$$\underset{C}{0} = \theta_2(s)\underset{B}{[2s+2]} - \theta_1(s)\underset{A}{[s+1]}$$

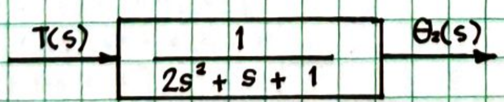
$$\theta_2(s) = \frac{\begin{bmatrix} s^2+s+1 & T(s) \\ -(s+1) & 0 \end{bmatrix}}{\begin{bmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & 2(s+1) \end{bmatrix}} = \frac{[\cancel{(s^2+s+1)(0)} - (-\cancel{(s+1)}(T(s)))]}{[(s^2+s+1)\underset{2}{\cancel{(2s+2)}} - (-\cancel{(s+1)}-(s+1))]}$$

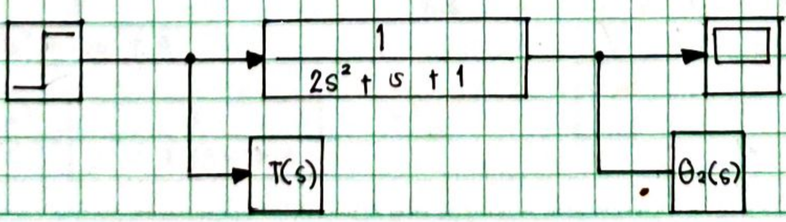
$$\theta_2(s) = \frac{T(s)}{2(s^2+s+1) - (s+1)}$$

$$\theta_2(s) = \frac{T(s)}{2s^2+2s+2-s-1}$$

$$\theta_2(s) = \frac{T(s)}{2s^2+s+1}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+s+1}$$





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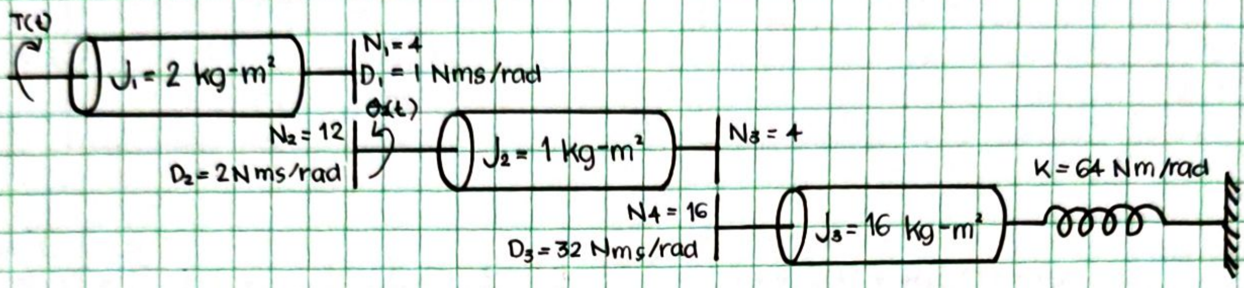
FOR THE ROTATIONAL SYSTEM SHOWN, FIND THE TRANSFER FUNCTION, $G(s) = \theta_2(s) / T(s)$.



$$R \cdot T(t) = J_e \ddot{\theta}_2(t) + D_e \dot{\theta}_2(t) + K_e \theta_2(t)$$

$$R = \frac{N_D}{N_S} = \frac{N_2}{N_1} = \frac{12}{4} = 3$$

$$J_e = \sum J \left(\frac{N_D}{N_S}\right)^2$$

$$= (2)\left(\frac{12}{4}\right)^2 + (1)\left(\frac{12}{12}\cdot\frac{4}{4}\right)^2 + (16)\left(\frac{4}{16}\right)^2$$

$$= (2)(3)^2 + (1)(1)^2 + 16\left(\frac{4}{16}\right)^2$$

$$= 18 + 1 + 1$$

$$J_e = 20$$

$$D_e = \sum D\left(\frac{N_D}{N_S}\right)^2$$

$$= (1)\left(\frac{12}{4}\right)^2 + (2)\left(\frac{12}{12}\cdot\frac{4}{4}\right)^2 + 32\left(\frac{4}{16}\right)^2$$

$$= 9 + 2 + 2$$

$$D_e = 13$$

$$K_e = K\left(\frac{N_D}{N_S}\right)^2$$

$$= 64\left(\frac{4}{16}\right)^2$$

$$K_e = 4$$

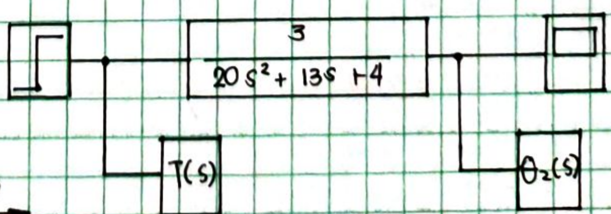
$$R \cdot T(t) = J_e \ddot{\theta}_2(t) + D_e \dot{\theta}_2(t) + K_e \theta_2(t)$$

$$\{\, 3\, T(t) = 20 \ddot{\theta}_2(t) + 13 \dot{\theta}_2(t) + 4 \theta_2(t) \,\}$$

$$3\, T(s) = 20 s^2 \theta_2(s) + 13 s\, \theta_2(s) + 4 \theta_2(s)$$

$$3\, T(s) = \theta_2(s) \left[20s^2 + 13s + 4 \right]$$

$$\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2 + 13s + 4}$$



$$T(s) \longrightarrow \boxed{\frac{3}{20s^2 + 13s + 4}} \longrightarrow \theta_2(s)$$

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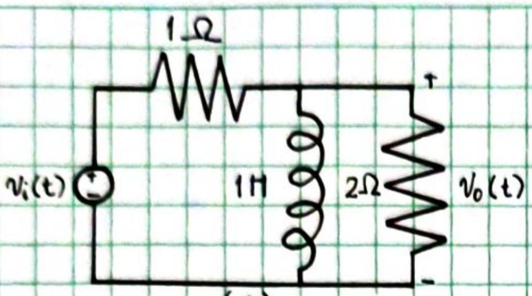
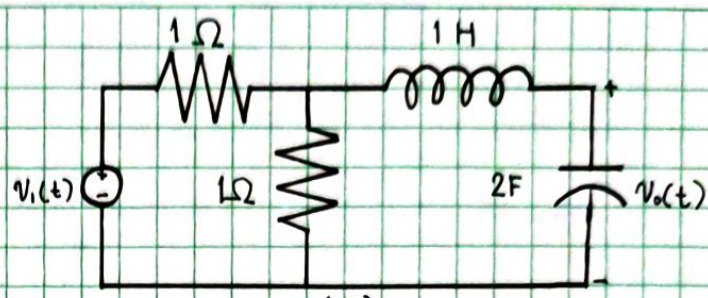


(a)



(b)

KVL @ Loop 1

$$v_i(t) = R_1 i_1(t) + L_1 \frac{di_1(t)}{d(t)} - L_1 \frac{di_2(t)}{dt}$$

$$\mathcal{L}\left\{ v_i(t) = i_1(t) + \frac{di_1(t)}{dt} - \frac{di_2(t)}{dt} \right\}$$

$$V_i(s) = I_1(s) + sI_1(s) - sI_2(s)$$

$$V_i(s) = I_1(s)[s+1] - sI_2(s) \text{..... eq.1}$$

KVL @ Loop 2

$$0 = L_1 \frac{di_2(t)}{dt} + R_2 i_2(t) - L_1 \frac{di_1(t)}{dt}$$

$$\mathcal{L}\left\{ 0 = \frac{di_2(t)}{dt} + 2i_2(t) - \frac{di_1(t)}{dt} \right\}$$

$$0 = sI_2(s) + 2I_2(s) - sI_1(s)$$

$$0 = I_2(s)[s+2] - sI_1(s) \text{..... eq.2}$$

C A B

$$V_i(s) = I_1(s)[s+1] - sI_2(s)$$

$$0 = I_2(s)[s+2] - sI_1(s)$$

C B A

$$I_2(s) = \frac{\begin{bmatrix} (s+1) & V_i(s) \\ -s & 0 \end{bmatrix}}{\begin{bmatrix} (s+1) & -s \\ -s & (s+2) \end{bmatrix}}$$

$$= \frac{\cancel{(s+1)(0)} - (V_i(s))(-s)}{(s+1)(s+2) - (-s)(-s)}$$

$$\frac{s V_i(s)}{\cancel{s^2} + 2s + s + 2 - \cancel{s^2}}$$

$$I_2(s) = \frac{s V_i(s)}{3s+2} \quad ; \quad V_i(t) = R_2 i_2(t)$$

$$V_R(s) = 2 I_2(s)$$

$$V_R(s) = 2\left[\frac{s V_i(s)}{3s+2}\right]$$

$$V_R(s) = \frac{2s V_i(s)}{3s+2}$$

$$\frac{V_R(s)}{V_i(s)} = \frac{2s}{3s+2}$$

$$V_i(s) \longrightarrow \boxed{\frac{2s}{3s+2}} \longrightarrow V_R(s)$$

KVL @ Loop 1

$$v_i(t) = R_1 i_1(t) + R_2 i_1(t) - R_2 i_2(t)$$

$$v_i(t) = i_1(t) + i_1(t) - i_2(t)$$

$$\mathcal{L}\{ v_i(t) = 2i_1(t) - i_2(t) \}$$

$$V_1(s) = 2I_1(s) - I_2(s) \text{..... eq.1}$$

KVL @ Loop 2

$$0 = R_2 i_2(t) + L_1 \frac{di_2(t)}{dt} + \frac{1}{C_1}\int i_2(t)dt - R_2 i_1(t)$$

$$\mathcal{L}\left\{ 0 = i_2(t) + \frac{di_2(t)}{dt} + \frac{1}{2}\int i_2(t)dt - i_1(t) \right\}$$

$$0 = I_2(s) + sI_2(s) + \frac{I_2(s)}{2s} - I_1(s)$$

$$0 = I_2(s)\left[\frac{2s^2 + 2s}{2s}\right] - I_1(s)$$

$$0 = I_2(s)\left[\frac{\cancel{2s}[s+1]}{\cancel{2s}}\right] - I_1(s)$$

$$0 = I_2(s)[s+1] - I_1(s)$$

$$I_2(s) = \frac{\begin{bmatrix} 2 & V_i(s) \\ -1 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & -1 \\ -1 & s+1 \end{bmatrix}}$$

$$= \frac{\cancel{(2)(0)} - (-1)(V_i(s))}{(2)(s+1) - (-1)(-1)}$$

$$= \frac{V_i(s)}{2s+2-1}$$

$$I_2(s) = \frac{V_i(s)}{2s+1} \quad ; \quad V_c(t) = \frac{1}{C_1}\int i_2(t)dt$$

$$V_c(s) = \frac{I_2(s)}{2s}$$

$$= \frac{V_i(s)}{2s+1} \cdot \frac{1}{2s}$$

$$V_c(s) = \frac{V_i(s)}{4s^2 + 2s}$$

$$\frac{V_c(s)}{V_i(s)} = \frac{1}{4s^2 + 2s}$$

$$V_i(s) \longrightarrow \boxed{\frac{1}{4s^2 + 2s}} \longrightarrow V_c(s)$$

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