Hypothesis Testing

Hypothesis testing is a statistical method used to determine whether there is enough evidence in a sample to infer something about a population.

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1. Key Idea

We start with two competing hypotheses:

- **Null Hypothesis (H_0)**: Assumes **no effect** or **no difference** (status quo).
- **Alternative Hypothesis (H_1 or H_a)**: The claim we want to test (suggests an effect or difference).

🔷 **Example:** Suppose we want to test if a new drug is more effective than the current one.

- H_0: The new drug is **not better** than the current one.
- H_1: The new drug **is better** than the current one.

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2. Steps in Hypothesis Testing

Step 1: State the Hypotheses

Define H_0 and H_1 clearly.

Step 2: Choose a Significance Level (α)

- The **significance level** (α) is the probability of **rejecting H_0 when it is actually true** (Type I error).
- Common values:
  - **0.05 (5%)** → 95% confidence
  - **0.01 (1%)** → 99% confidence

Step 3: Select and Compute the Test Statistic

The test statistic depends on the test type:

- **Z-test**: When the population standard deviation is known.

- **T-test**: When the population standard deviation is unknown.
- **Chi-square test**: For categorical data.
- **ANOVA/F-test**: For comparing multiple groups.

📌 **Test statistic formula (for a Z-test on the mean)**:

where:

- $\bar{X}$ = Sample mean
- μ_0 = Population mean under H_0
- σ = Standard deviation
- n = Sample size

Step 4: Compare with Critical Value or p-Value

- Find the **critical value** from the Z-table or T-table.
- Alternatively, calculate the **p-value** (probability of getting the observed result under H_0).

Step 5: Decision Rule

- **If** $p \leq \alpha$ → **Reject** H_0(evidence supports H_1).
- **If** $p > \alpha$ → **Fail to reject** H_0 (not enough evidence for H_1).

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3. Example: Hypothesis Test for a Mean

A factory claims that the **average weight of a product is 500g**. A random sample of 30 products has a mean weight of **495g** and a standard deviation of **10g**. Test if the factory's claim is correct at **5% significance level**.

Step 1: Define Hypotheses

- $H_0 : \mu = 500$ (The claim is true)
- $H_1 : \mu \neq 500$ (The claim is false)

Step 2: Choose α=0.05

Step 3: Compute Test Statistic (Z-test)

$Z = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{495 - 500}{\frac{10}{\sqrt{30}}}$

$Z = \frac{-5}{1.83} = -2.73$

Step 4: Find Critical Value or p-Value

- From the Z-table, the critical value for a **two-tailed 5% test** is **±1.96**.
- Since **Z = -2.73 is beyond -1.96**, we **reject** H_0.

Step 5: Conclusion

There is **enough evidence** to say the factory's claim is **likely false**.

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4. Common Errors in Hypothesis Testing

Error Type	Definition	Example
Type I Error (False Positive)	Rejecting H_0 when it's actually true.	Saying a drug works when it actually doesn't.
Type II Error (False Negative)	Accepting H_0 when it's false.	Saying a drug doesn't work when it actually does.
