9.5 MAXWELL'S EQUATIONS IN FINAL FORMS

The Scottish physicist James Clerk Maxwell (1831–1879) is regarded as the founder of electromagnetic theory in its present form. Maxwell's celebrated work led to the discovery of electromagnetic waves.[4] Through his theoretical efforts when he was between 35 and 40 years old, Maxwell published the first unified theory of electricity and magnetism. The theory comprised all previously known results, both experimental and theoretical, on electricity and magnetism. It further introduced displacement current and predicted the existence of electromagnetic waves. Maxwell's equations were not fully accepted by many scientists until 1888, when they were confirmed by Heinrich Rudolf Hertz (1857–1894). The German physicist was successful in generating and detecting radio waves.

The laws of electromagnetism that Maxwell put together in the form of four equations were presented in Table 7.2 in Section 7.6 for static conditions. The more generalized forms of these equations are those for time-varying conditions shown in Table 9.1. We notice from the table that the divergence equations remain the same, while the curl equations have been modified. The integral form of Maxwell's equations depicts the underlying physical laws, whereas the differential form is used more frequently in solving problems. For a field to "qualify" as an electromagnetic field, it must satisfy all four Maxwell's equations. The importance of Maxwell's equations cannot be overemphasized because they summarize all known laws of electromagnetism. We shall often refer to them in the remainder of this text.

Since this section is meant to be a compendium of our discussion in this text, it is worthwhile to mention other equations that go hand in hand with Maxwell's equations. The Lorentz force equation

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{9.28}$$

TABLE 9.1 Generalized Forms of Maxwell's Equations

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\dfrac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$	Ampère's circuit law

*This is also referred to as Gauss's law for magnetic fields.

[4] Maxwell's work can be found in his two-volume, *Treatise on Electricity and Magnetism* (New York: Dover, 1954).