EE341 - Work Product Folder

UAB ECE Course-Work Compilation

Elijah T. Rose

April 27, 2020

Contents

Assignments

Course-Specific Documents

These documents are only applicable to this course and do not have inherent educational value. They are included at the request of the instructor.

Syllabus

EE 341-AV Electromagnetics
Spring 2020
Jan. 13-Apr. 24 (last day of class)

When: M/W 11-12:15PM
Instructor: Murat M. Tanik, PhD
Wallace R. Bunn Endowed Chair Professor
email: mtanik@uab.edu
Where: 267 BEC
Office: 261C BEC, *Phone:* 934-8442

Text: Matthew N. O. Sadiku
Elements of Electromagnetics, 5th Edition
Oxford University Press, 2010
Supplementary materials: as distributed in class.

Grading: 20% First exam **(Monday, Feb. 17)**
20% Homework, quizzes & participation – Weekly
20% Second exam **(Due March 9)**
Spring Break (March 16-22)
40% Final Project presentations due **(April 20&22)**
Progress and problem solving **– Weekly (Wednesday)**
Last day of class for us **(April 22)**

Preq: EE 316, PH 221. Grade C or better in EE 316 and PH 221

Coverage: We will follow the textbook including supplementary materials very closely. You need buy the 5th edition of the textbook and bring to class. Contents of Section 1 will be the first exam. Contents of Section 2 will be the second exam.

Section 1: Vector Algebra, Coordinate systems, and Vector Calculus
- Cartesian, Cylindrical and Spherical coordinates
- Del operator, Divergence, Curl

Section 2: General Electromagnetic Principles: Electrostatics and Magnetostatics
- Coulomb's law, Gauss's Law, Maxwell's equation
- Poisson's and Laplace equations
- Midterm Exam

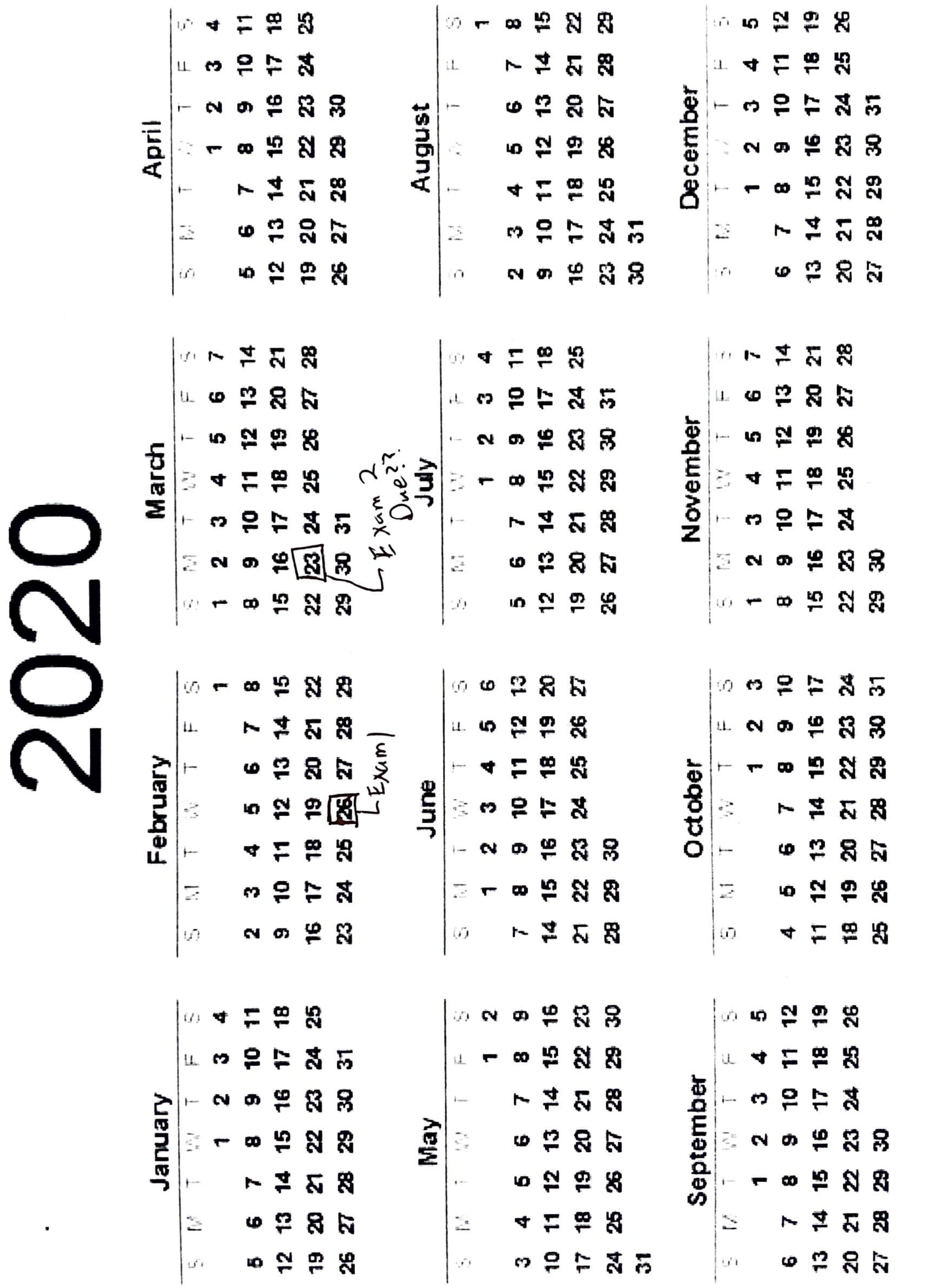
Section 3: Waves and Applications
- Faraday's law, Maxwell's equations in final forms, Time-Varying potentials, Wave Propagation, Transmission lines.
- Final Exam

This is a fast pace course therefore, students is expected to study the assigned text material BEFORE the class session that discusses that material. Students will be expected to complete homework problems and be able to present those problems to the class at the next class meeting after the problems are assigned. I will follow the following rules traditionally established in this course:

Class Attendance: Each student is expected to attend each class meeting and is, therefore, responsible for everything covered in each meeting of the class as well as for all out-of-class assignments. Students will bring all handouts and the textbook to every class. Please note that each student is expected to be in his/her seat at the beginning of class. The Instructor has been known to lock the door to the classroom for courses having students that continually arrive late for class.

Examinations and the Project: No makeup examination will be given for scheduled examinations unless the student has made arrangements with the Instructor **prior to** the examination. Consent for a

Calendar


2020
January
February
March
April
May
June
July
August
September
October
November
December
Exam
Exam 2 Due??

Group Project

Educational Documents

Vector Derivatives

Maxwell's Equations

Page 4 from Textbook

Maxwell's Equations Final

Page 414 from Textbook

Maxwell's Equations Final - Page 414

Folder
1. Table of Contents

EE 341-AV Electromagnetics
Spring 2020
GROUP PROJECT
Due Date
April 20-22, 2020

You are given a book chapter on polarization written by R. Feynman. Two links on polarization And Jones calculus are:
http://en.wikipedia.org/wiki/polarization
http://en.wikipedia.org/wiki/Jones_calculus

Industry source:
https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/polarization/

You produced as part of the test: MATLAB implementation of Polarization states given in the Feynman paper. A discussion on all these states indicating your understanding of light polarization included.

a) **25%. As part of your project folder include improved version of the take home test.**
b) **30%. MATLAB implementation of multiplication of each polarization state with a jones matrix representing an optical element. Explain your results.**
c) **30%. Develop high quality presentation slides and present in class.**
d) **15%. Consider this ethics case below. Debate among yourselves. Write your judgment as a team.**

Ethics case study: Jocelyn Tan
Catherine is a new hire at a startup that produces LCD displays for large venues, such as shopping malls. Part of her job requires her to troubleshoot malfunctioning displays.
One day, a shopping mall reported that two display units out of twelve had stopped working from their installation three months prior. The customer also noted serial and revision numbers on the two units were different from the rest of the units. At the job site, Catherine inspected the displays and realized her company had sold units that were from a bad batch (i.e. group of displays that did not have over 50% yield during manufacturing). Catherine wanted to tell the site why the units failed, but recognized that if she disclosed this information, the site would be eligible to receive replacement displays at no additional cost. On the other hand, if she blamed the failing units on a weaker cause, such as improper installation, her company would be able to charge the site for replacement units. Catherine knew her manager would want her to choose the option that would minimize the company's losses; however, she wanted to be honest with the site as they were one of the company's best customers. What should she do?
Jocelyn Tan was a 2014-2015 Hackworth Fellow in Engineering Ethics at the Markkula Center for Applied Ethics at Santa Clara University.

You will learn a great deal on this project. You are allowed to search information from anywhere. You can discuss the problems with anyone. However, you will write your project alone as a team. This will be your project work product, including your MATLAB programs.

Good luck and Good learning

Figure 1: Group Project

VECTOR DERIVATIVES

Cartesian Coordinates (x, y, z)

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Figure 2: Vector Derivatives

†1.2 A PREVIEW OF THE BOOK

The subject of electromagnetic phenomena in this book can be summarized in Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.4}$$

where ∇ = the vector differential operator
$\mathbf{D}$ = the electric flux density
$\mathbf{B}$ = the magnetic flux density
$\mathbf{E}$ = the electric field intensity
$\mathbf{H}$ = the magnetic field intensity
ρ_v = the volume charge density
$\mathbf{J}$ = the current density

Maxwell based these equations on previously known results, both experimental and theoretical. A quick look at these equations shows that we shall be dealing with vector quantities. It is consequently logical that we spend some time in Part 1 examining the mathematical tools required for this course. The derivation of eqs. (1.1) to (1.4) for time-invariant conditions and the physical significance of the quantities **D**, **B**, **E**, **H**, **J**, and ρ_v will be our aim in Parts 2 and 3. In Part 4, we shall reexamine the equations for time-varying situations and apply them in our study of practical EM devices.

1.3 SCALARS AND VECTORS

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended. We must learn its rules and techniques before we can confidently apply it. Since most students taking this course have little exposure to vector analysis, considerable attention is given to it in this and the next two chapters.[3] This chapter introduces the basic concepts of vector algebra in Cartesian coordinates only. The next chapter builds on this and extends to other coordinate systems.

A quantity can be either a scalar or a vector.

† Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

[3] The reader who feels no need for review of vector algebra can skip to the next chapter.

Figure 3: Maxwell's Equations - Page 4

Scientific Method

TheView Opener

For several decades, there has been an extensive and organized campaign intended to generate distrust in science, funded by regulated industries and libertarian think tanks whose interests and ideologies are threatened by the findings of modern science. In response, scientists have tended to stress the success of science. After all, scientists have been right about most things, from the structure of the universe (the earth does revolve around the sun) to the relativity of time and space (relativistic corrections are needed to make global positioning systems work).

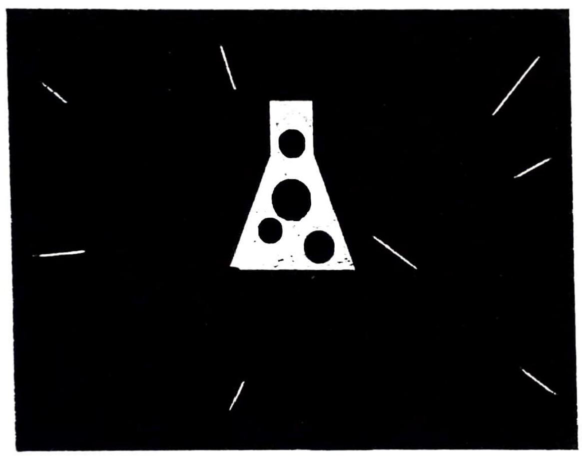
Citing successes isn't wrong, but for many people it's not persuasive. An alternative answer to the question "Why trust science?" is that scientists use the so-called scientific method. If you've got a high school science textbook lying around, you'll probably find that answer in it. But what is typically asserted to be the scientific method—develop a hypothesis, then design an experiment to test it—isn't what scientists actually do. Science is dynamic: new methods get invented; old ones get abandoned; and at any particular juncture, scientists can be found doing many different things. That's good, because the scientific method doesn't work. False theories can yield true results, so even if an experiment works, it doesn't prove that the theory it was designed to test is true.

If there is no identifiable scientific method, then what is the warrant for trust in science? How can we justify using scientific knowledge in making difficult decisions?

The answer is the methods by which those claims are evaluated. The common element in modern science, regardless of the specific field or the particular methods being used, is the critical scrutiny of claims. It's this process—of tough, sustained scrutiny—that works to ensure that faulty claims are rejected.

A scientific claim is never accepted as true until it has gone through a lengthy process of examination by fellow scientists. This process begins when scientists discuss their data and preliminary conclusions. Then the claim is shopped around at conferences and workshops. This may result in the collection of additional data or revision of the preliminary interpretation. Then the scientist writes up the results and sends the preliminary write-up to colleagues.

Until this point, scientific feedback is typically fairly friendly. But the next step is different: once the paper is ready, it is submitted to a scientific journal, where things get a whole lot tougher. Editors deliberately send scientific papers to people who are not friends or colleagues of the authors, and the job of the reviewer is to find errors or other inadequacies. We call this process



"peer review" because the reviewers are scientific peers—experts in the same field—but they act in the role of a superior who has both the right and the obligation to find fault. It is only after the reviewers and the editor are satisfied that any problems have been fixed that the paper is accepted for publication and enters the body of "science."

A KEY ASPECT of scientific judgment is that it is done collectively. It's a cliché that two heads are better than one: in modern science, no claim gets accepted until it has been vetted by dozens, if not hundreds, of heads. In areas that have been contested, like climate science and vaccine safety, it's thousands. This is why we are generally justified in not worrying too much if a single scientist, even a very famous one, dissents from the consensus. The odds that the lone dissenter is right, and everyone else is wrong, are probably in most cases close to zero. This is why diversity in science—the more people looking at a claim from different angles—is important.

In a way, science is like a trial, in which both sides get to ask tough questions in the hope that the truth becomes clear, and it is the jury that makes that call. But there are important differences: one, the jurors are not common citizens but experts who have the specialized training required to evaluate technical claims; two, the judges are all the other members of the expert community; three, double jeopardy is allowed, because there is always the possibility of reopening the case on the basis of new evidence.

Does this process ever go wrong? Of course. Scientists are human. But if we look carefully at historical cases where science went awry, typically there was no consensus.

Some people argue that we should not trust science because scientists are "always changing their minds." While examples of truly settled science being overturned are far fewer than is sometimes claimed, they do exist. But the beauty of this scientific process is that it explains what might otherwise appear paradoxical: that science produces both novelty and stability. New observations, ideas, interpretations and attempts to reconcile competing claims introduce novelty; transformative interrogation leads to collective decisions and the stability of scientific knowledge. Scientists do change their minds in the face of new evidence, but this is a strength of science, not a weakness.

Oreskes, a professor of the history of science at Harvard, is the author of Why Trust Science?

Critical Thinking

Full Source: https://images.pearsonassessments.com/images/tmrs/CriticalThinkingReviewFINAL.pdf

IEEE Code of Ethics

Full Source: https://www.ieee.org/about/corporate/governance/p7-8.html

> We, the members of the IEEE, in recognition of the importance of our technologies in affecting the quality of life throughout the world, and in accepting a personal obligation to our profession, its members, and the communities we serve, do hereby commit ourselves to the highest ethical and professional conduct and agree:

1. to hold paramount the safety, health, and welfare of the public, to strive to comply with ethical design and sustainable development practices, and to disclose promptly factors that might endanger the public or the environment;
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist;
3. to be honest and realistic in stating claims or estimates based on available data;
4. to reject bribery in all its forms;
5. to improve the understanding by individuals and society of the capabilities and societal implications of conventional and emerging technologies, including intelligent systems;
6. to maintain and improve our technical competence and to undertake technological tasks for others only if qualified by training or experience, or after full disclosure of pertinent limitations;
7. to seek, accept, and offer honest criticism of technical work, to acknowledge and correct errors, and to credit properly the contributions of others;
8. to treat fairly all persons and to not engage in acts of discrimination based on race, religion, gender, disability, age, national origin, sexual orientation, gender identity, or gender expression;
9. to avoid injuring others, their property, reputation, or employment by false or malicious action;
10. to assist colleagues and co-workers in their professional development and to support them in following this code of ethics.

Going Polarized

https://thinklucid.com/polarization-white-paper/

CRITICAL THINKING 6

Definitions of critical thinking emerging from the philosophical tradition include

- "the propensity and skill to engage in an activity with reflective skepticism" (McPeck, 1981, p. 8);
- "reflective and reasonable thinking that is focused on deciding what to believe or do" (Ennis, 1985, p. 45);
- "skillful, responsible thinking that facilitates good judgment because it 1) relies upon criteria, 2) is self-correcting, and 3) is sensitive to context" (Lipman, 1988, p. 39);
- "purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or conceptual considerations upon which that judgment is based" (Facione, 1990, p. 3);
- "disciplined, self-directed thinking that exemplifies the perfections of thinking appropriate to a particular mode or domain of thought" (Paul, 1992, p. 9);
- thinking that is goal-directed and purposive, "thinking aimed at forming a judgment," where the thinking itself meets standards of adequacy and accuracy (Bailin et al., 1999b, p. 287); and
- "judging in a reflective way what to do or what to believe" (Facione, 2000, p. 61).

Figure 4: Critical Thinking Quotes

WHITE PAPER

Cover image: False color representation for angles of polarized light from transparent objects (plastics and cell phone cover).

GOING POLARIZED

POLARIZATION ADDS A NEW PERSPECTIVE TO THE IMAGING INDUSTRY

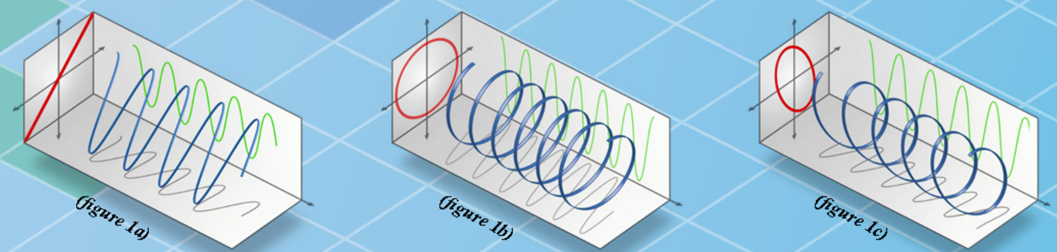
In many machine vision applications the use of polarization cameras can provide information that cannot be obtained with standard monochrome, color, multi-spectral or hyperspectral cameras. Applications that benefit from the use of polarization cameras are those in which reflected and transmitted scenes must be separated, the shape of transparent objects must be analyzed, and where removing specularities and haze is important.

To appreciate how such applications can benefit from the use of polarization, the nature of light and how it interacts with such materials must be understood. Light is an electromagnetic wave that is composed of an electric field and a perpendicular magnetic field. The direction of the electric field is used to define the polarization direction of the light. **It is the interaction of light's electric field with materials that can be leveraged in vision applications with polarization.**

Most light sources encountered every day are not polarized. Unpolarized light from the sun or from an incandescent light bulb, for example, will have electric fields that are oscillating in random directions. To polarize this light, a polarizer is used to absorb components of the random directions, passing components of the light that are aligned in only one oscillation direction. Such polarization is referred to as linear polarization since the electric field points in a single direction *(figure 2a)*. If the polarizer is polarized in the vertical direction the polarizer will block the horizontal component of all other polarizations.

Light reflecting from highly directional sources such as the sun or incandescent lamps can cause glare. Fortunately, this glare can be identified by the polarization signature it carries. When unpolarized light reflects from the surface of an object it becomes partially linear polarized. The amount of polarization is dependent on the angle of reflection and the surface characteristics. Polarizers can then be used to remove the glare.

LUCID VISION LABS



Polarized light can be represented in a coordinate system by two components. In *figure 1a*, 45° polarized light of some specific wavelength is shown broken down into X and Y components that are smaller in magnitude shown on the wall and floor of the diagram. If the polarized light were vertical, the Y component would be the same magnitude as the original polarized light, and the X component would be zero, having no energy in that component. If we take the two components of the 45° polarized light and phase shift them 90° (such as with a wave plate specifically oriented with axis in X and Y) the sum of those components will be circularly polarized since the electric field vector at any point along the Z direction turns in a circular direction *(figure 1b)*. In general, the two perpendicular light waves can have unequal amplitudes and phases in between 0 and ±90°, resulting in an electric field vector that rotates on an ellipse creating a more generalized case of elliptically polarized light *(figure 1c)*.

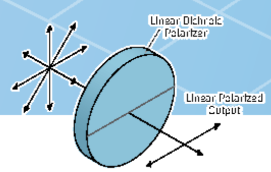


figure 2a

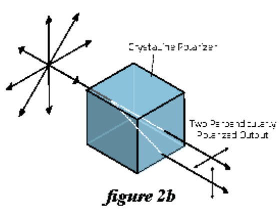


figure 2b

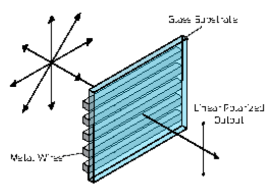


figure 2c

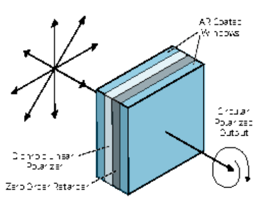


figure 2d

Polarizing Light

A number of different methods can be used to polarize light. These include the use of dichroic, crystalline, and wire grid polarizers. Linear dichroic polarizers are constructed by laminating a stretched and dyed polymer film between two coated glass windows ***(figure 2a)***. Passing unpolarized light though such a polarizer will result in linearly polarized light in the orientation of the polarizing axis of the filter.

Crystalline polarizers rely on the effects of birefringence in crystalline materials ***(figure 2b)***. Such polarizers exploit the property of double refraction of certain anisotropic crystals such as calcite. In such crystals, the refractive index differs along more than one of the crystal axes from the asymmetric molecular spacing having different electron fields in the axes interacting with light's electric field. Incident light with components in both crystal axis will split into two beams with perpendicular components moving at different speeds. These two beams that are doubly refracted can be separated resulting in two linearly polarized beams.

Fabricated lithographically, wire grid polarizers feature an array of narrow sub-wavelength parallel metal wires placed on glass or a suitable substrate ***(figure 2c)***. When unpolarized light is incident on these wire grid polarizers, the incoming light with an electric field vector components parallel to the wires is reflected or absorbed, with the remaining electric field vector components perpendicular to the wires is transmitted through. Polarization components can act differently at different wavelengths. Visible light polarizers may not function well at all UV or IR wavelengths while wave plates are usually designed for a specific wavelength.

Circular dichroic polarizers are a combination of both a quarter wave retarder and a linear dichroic polarizer ***(figure 2d)***. This quarter wave retarder ($\pi/2$) is used to transform circularly polarized light to linearly polarized light which is then blocked or passed by the linear polarizer. In the reverse direction, unpolarized light is linearly polarized by the dichroic polarizer. This linear polarization is then transformed to circular polarization by the phase shift of the retarder.

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Each polarizer type has different performance characteristics. By examining these characteristics, the correct polarizer can be chosen *(table 1)*. While all these different types of polarizers can be used in applications from biomedical, machine vision, and microscopy, linear dichroic polarizers are the least expensive and most commonly used. In applications where glare may need to be reduced, the use of dichroic polarizers may be most practical because of their low cost whereas applications that require broad spectrum ultraviolet (UV) to infrared (IR) polarization with high extinction ratios may benefit from crystalline polarizers.

	CRYSTALLINE	DICHROIC GLASS	DICHROIC POLYMER	DIELECTRIC BEAMSPLITTING	WIRE GRID
Clear Aperture (mm)	5 - 10	1 - 30.4	10 - 50+	10 - 20	1 - 200
Acceptance Angle	± 5°	± 20°	± 10°	± 2°	45°
Part Cost	\$500 - \$5000	\$500 - \$5000	\$1 - \$1000	\$100 - \$5000	\$100 - \$5000
Contrast Ratio	10^5	10^6	10^7	10^3	10^7
Damage Resistance (per cm²)	25 - 30 W	1 W	1 W	500 W	50 KW
Mechanical Thickness (mm)	22.5 - 38.1	0.5 - 6.9	3.3 - 12.7	12.7 - 25.4	0.7 - 1.5
Transmission*	85 - 95%	39 - 98%	45 - 90%	95%	70 - 85%
Wavelength Range (nm)	320 - 2300	325 - 2000	325 - 1800	440 - 1600	400 - 2000+

Table 1: Each polarizer type, whether it be linear dichroic, circular dichroic, crystalline or wire grid has different performance characteristics. * This is the transmission for light that is aligned with the polarization axis of the polarizer.

All Aboard a Single-Chip

Wire grid polarization is the principle behind Sony's IMX250MZR and IMX250MYR CMOS sensors, a 2464 x 2056 global shutter imager that uses four directional polarizing filters at 0°, 90°, 45°, and 135° on every four 3.45µm pixels *(figure 3)*. Light passing through these four filters can then be linearly interpolated to provide a single intensity pixel value with its associated angle of linear polarization (AoLP) and degree of linear polarization (DoLP). The resulting image will then be one quarter of the original 2464 x 2056 pixels or 1232 x 1028 pixels, or can be interpolated to provide the full resolution similar to traditional bayer pattern display.

Fabricated on-chip as opposed to on-glass, the polarizer array is positioned below the micro lens to reduce crosstalk from polarized angles being incorrectly detected by the wrong pixel. Since the imager employs linear polarizers, it can be used to calculate the DoLP of light. For the sensor to measure circular polarization, it would be necessary to add a quarter-wave (π/2) wave plate to the imaging path. Although at present no commercially available circular micro-polarizers have been fabricated on-chip, researchers have demonstrated how such a concept could be achieved.

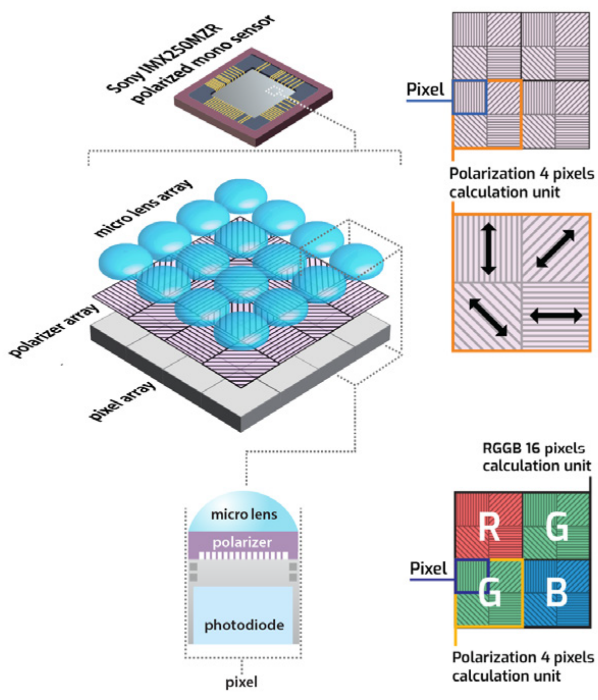
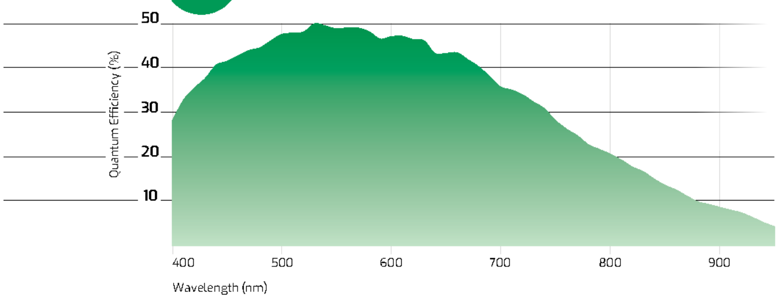


figure 3
Sony's IMX250MZR (mono) sensor layout and IMX250MYR (color) RGGB bayer pattern.

Polarization Performance

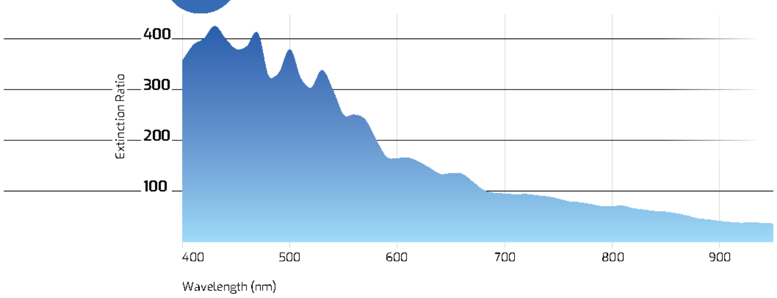
Having polarization filters on the image sensor changes the quantum efficiency (QE) of the sensor. In the case of the Sony IMX250MZR CMOS polarized sensor, measured QE across the 400-950nm spectrum results in a decrease in QE when compared with the Sony IMX250LLR. For example, at 470 nm, 525 nm and 640 nm, the QE of the IMX250 is 53%, 66% and 58% respectively while for the IMX250MZR with polarized light matching the angle of the pixel this is approximately 43%, 48% and 43% respectively ***(figure 4)***.

figure 4 Average Quantum Efficiency (QE) Using Polarized Light Matching Angle of Pixel



A key performance metric of a polarizer or polarization sensor is the ability to block the light polarized perpendicular to the polarizer's transmission axis as much as possible. The polarization extinction ratio (ER) is the ratio of the maximum signal obtained when a high quality reference polarizer is aligned to a polarization axis of the polarization sensor to the minimum signal when the reference polarizer is "crossed" or rotated by 90°. ERs can vary considerably depending on the polarizer type, quality and wavelength. The Sony IMX250MZR ER has been measured in ***figure 5***.

figure 5 Minimum Extinction Ratio (ER) Among 4 Angles at Each Wavelength.



Stokes parameters – a set of values (known as a Stokes vector) describes the polarization state of electromagnetic radiation. These were originally defined by Sir George Stokes in 1852. The four-dimensional Stokes vector is composed of the total intensity of light (I), the intensity difference between polarized components of the electromagnetic wave parallel and perpendicular to the reference plane (Q = p_0 - p_90), the intensity difference between polarized components in planes 45° and 135° to the reference plane (U = p_45 - p_135) and the difference between left and right circularly polarized radiation (V).

Since circular polarization cannot be measured directly using a simple implementation of the Sony IMX250MZR CMOS polarized sensor, only the I, Q and U components are available. From these, we can calculate the degree of linear polarization (DoLP) and the angle of linear polarization (AoLP) using the following equations:

$$\text{DoLP} = \frac{\sqrt{Q^2 + U^2}}{I}$$

$$\text{AoLP} = \frac{1}{2} \arctan \frac{U}{Q}$$

Results of the intensity of the 2 x 2 individual directional filters can then be displayed along with the AoLP, DoLP or a combination of both AoLP and DoLP on a graphical user interface.

Counting the Ways

Many methods of performing polarization analysis exist. One of the simplest of these is to use an inexpensive linear polarizer mounted in front of a standard camera lens. By rotating this polarizer, different images can be captured. This simple method has recently been used by NVIDIA to recover the 3D shape of objects from multiple images at different polarization orientations. Using a Hoya linear polarizer mounted in front of a Canon EOS 7D camera lens, multi-view stereo methods were used to first recover the camera positions and the initial 3D shape of well-textured regions. After the phase angle maps for each view from the corresponding polarized images were then computed, NVIDIA resolved the ambiguities to estimate azimuth angles to recover depth for featureless regions and fused together depth maps to recover the objects 3D shape ***(figure 6)***.

While polarization cameras can be manufactured by mechanically rotating polarizers and/or waveplates between capturing individual image frames, more sophisticated methods have also been developed. One such method, developed by Bossa Nova Technologies, uses a polarization filter based on ferroelectric liquid crystals (FLC) to acquire the four polarization frames used to calculate the four-dimensional Stokes vector of each pixel of the image ***(figure 7)***. Thus, the camera can determine the degree of linear polarization, the degree of circular polarization, the angle of polarization and the ellipticity angle.

In other implementations, multiple imagers and prisms are used to accomplish the same task. Since at least three exposures with different analyzer or half-wave plate positions are required to estimate the degree and orientation of polarization of the image, Fluxdata uses a 3-way prism beam splitter coupled to three polarizers, each at a different orientation in the design of its FD-1665P camera ***(figure 8)***. Oriented at 0, 45, and 90°, polarized images are then captured by three independent CCD sensors and processed to yield polarization information.

figure 6:
Polarimetric multi-view stereo combines per-pixel photometric information from polarization with epipolar constraints from multiple views for 3D reconstruction.

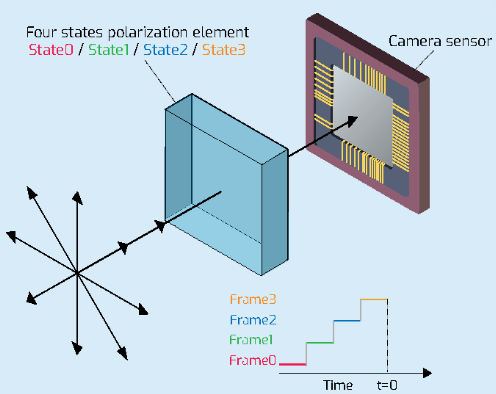


figure 7
Ferroelectric liquid crystal based polarization filter

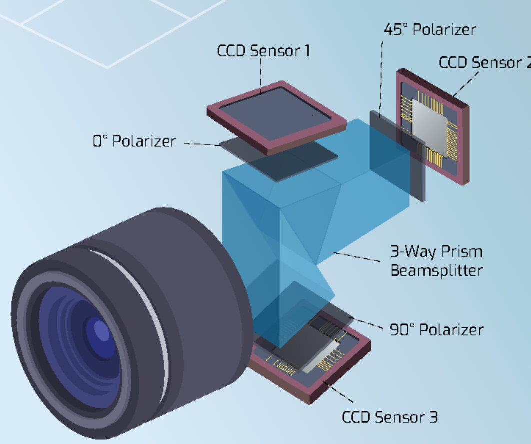


figure 8
3-Way prism beamsplitter with 3 polarizers and 3 sensors

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Reducing Stress

In many cases it is not necessary to compute the circular polarization of the image or the ellipticity angle. In industrial and machine vision applications, such as measuring stress-induced birefringence in glass, for example, computing the AoLP and DoLP is sufficient. In such applications, **using a solid state micropolarizer camera based on the Sony IMX250MZR polarization sensor, permits single shot data acquisition at frame rates while being extremely compact compared with rotating polarizer-based or prism-based implementations.** Because of this, the new Sony sensors are an ideal fit for LUCID's micro-compact Phoenix camera.

Many applications for such cameras exist including product analysis, defect detection, 3D shape reconstruction and haze removal. One of the unique properties of carbon fibers, for example, is that they polarize incident unpolarized light parallel to the direction of the fiber. To capture the orientation and the position of the carbon fiber bundle to determine which way the fiber is oriented, an unpolarized red light from a commercial ring light illuminating the surface is reflected as polarized light ***figure 9***,. Using a Phoenix polarization camera from LUCID, the camera can then be used to capture image data at 0°, 90°, 45°, and 135° angles. Polka Imaging software from Fraunhofer IIS is then used to perform the required computation for calculating the angle of linear polarization, which directly indicates the direction of the fibers for each pixel. In ***figure 10***, the bottom image shows the measured intensity while the top image shows the false color polarization information, with color depicting the measured angles of the fibers.

Such cameras can also be used for analyzing internal stress of plastics and glass that are not visible with conventional imaging techniques. Furthermore, using polarization can also highlight any scratches or other surface defects. Transparent objects, especially those fabricated from plastic, for example, alter the characteristics of light such that when placed in a beam of polarized light and imaged using a polarizing camera, internal non-uniformities turn the plane of polarization depending on the stress or surface characteristics of the material. While the monochrome image of the cellphone case in ***figure 11*** shows no such surface defects or internal stress characteristics, these are easily visualized using a polarization camera.

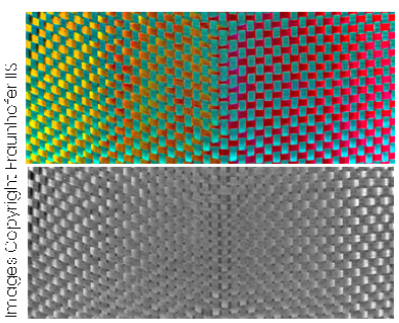

figure 10
Top: False color representing polarized angles.
Bottom: Polarization intensity.

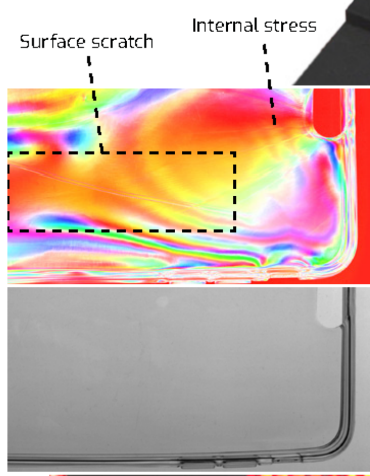

figure 11
Top: Polarization image showing hidden scratch and stress.
Bottom: Mono image.

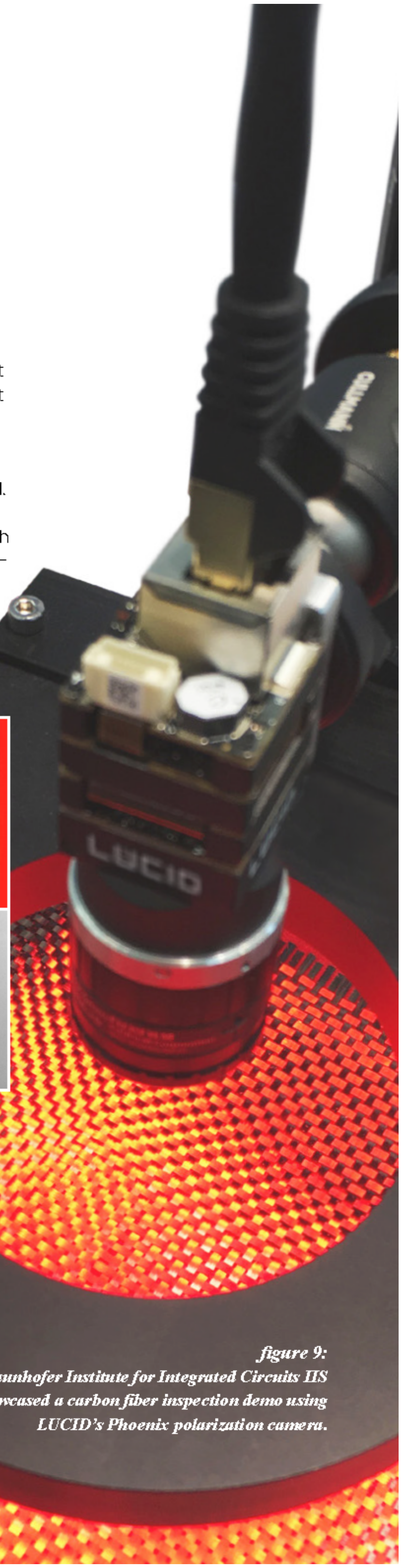

figure 9:
Fraunhofer Institute for Integrated Circuits IIS showcased a carbon fiber inspection demo using LUCID's Phoenix polarization camera.

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While processing polarized images from camera with on-chip polarizers is useful in machine vision applications, it can also be used in other applications such as aiding in de-hazing images. Based on the fact that usually natural illuminating light scattered by the atmosphere is partially polarized, polarizing techniques can be used in conjunction with optical methods to enhance hazy images as Yoav Schechner points out in his paper "Polarization-based vision through haze". Such de-hazing can be accomplished with as few as two images taken through a polarizer at different orientations ***(figure 12).***

figure 12
Left: Hazy image. Right: Enhanced image using polarizing and optical techniques.

Future Perspectives

Many applications of polarization imagery exist ranging from remote sensing, microscopy to 3D image reconstruction. Specialized laser-based applications may still demand the use of relatively costly crystalline polarizers while low-cost photography will still employ low-cost linear and circularly polarized filters.

For industrial imaging applications, the expensive mechanical or electro-optical systems required to perform such analysis has been relegated to the past thanks to the introduction of low-cost cameras based on polarizing imagers such as the Sony IMX250MZR. Not only will this eliminate the need to employ dual polarizers in such systems, it will dramatically reduce the cost of such systems. In the future, the continued trend towards applying different on-chip filters to CMOS imagers will open up new and exciting perspectives to the imaging industry.

The Phoenix camera with the Sony IMX250MZR polarization sensor.

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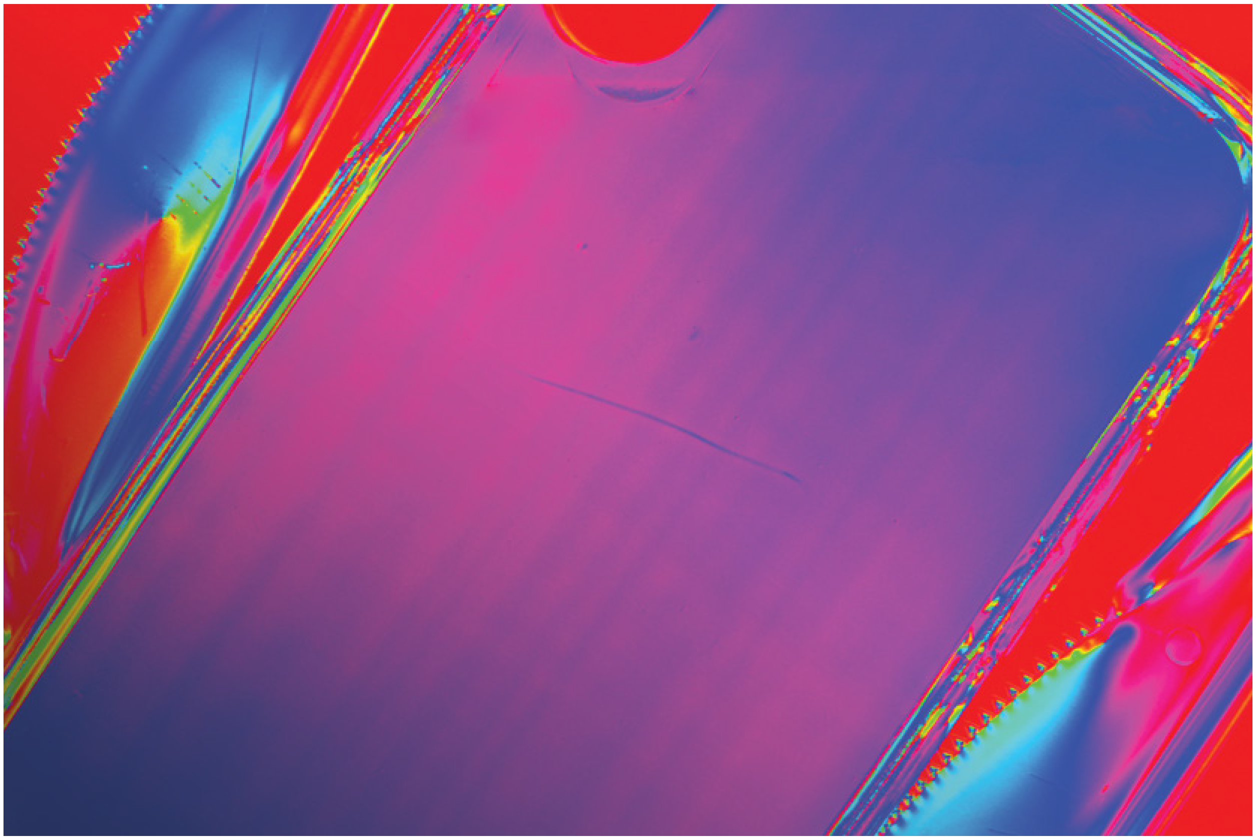

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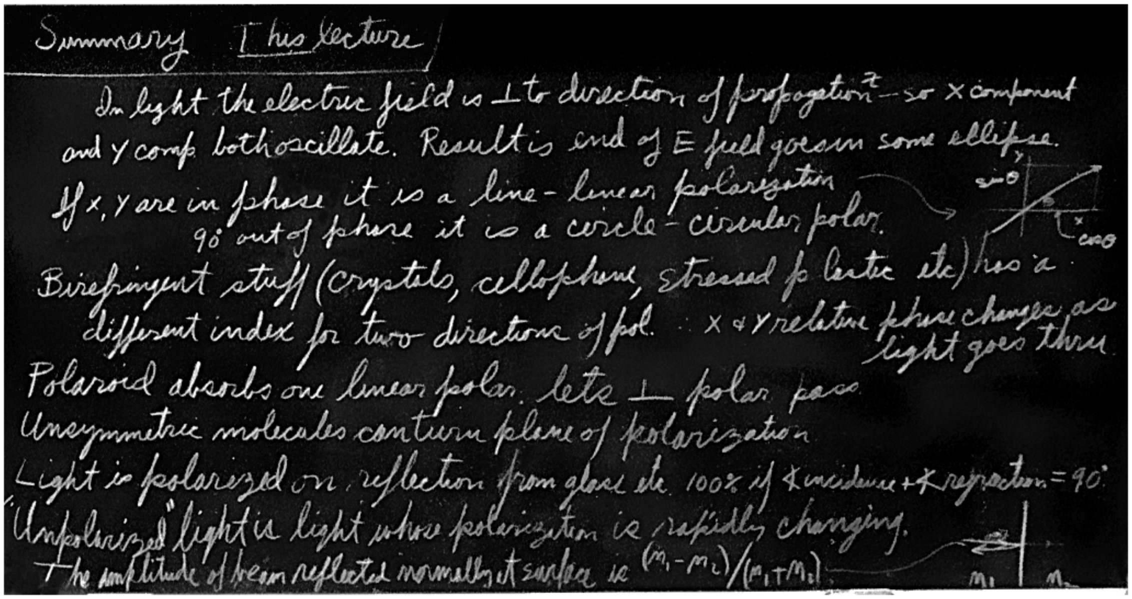
Polarization

https://www.feynmanlectures.caltech.edu/I_33.html



33–1 The electric vector of light

In this chapter we shall consider those phenomena which depend on the fact that the electric field that describes the light is a vector. In previous chapters we have not been concerned with the direction of oscillation of the electric field, except to note that the electric vector lies in a plane perpendicular to the direction of propagation. The particular direction in this plane has not concerned us. We now consider those phenomena whose central feature is the particular direction of oscillation of the electric field.

In ideally monochromatic light, the electric field must oscillate at a definite frequency, but since the x-component and the y-component can oscillate independently at a definite frequency, we must first consider the resultant effect produced by superposing two independent oscillations at right angles to each other. What kind of electric field is made up of an x-component and a y-component which oscillate at the same frequency? If one adds to an x-vibration a certain amount of y-vibration at the same phase, the result is a vibration in a new direction in the xy-plane. Figure 33–1 illustrates the superposition of different amplitudes for the x-vibration and the y-vibration. But the resultants shown in Fig. 33–1 are not the only possibilities; in all of these cases we have assumed that the x-vibration and the y-vibration are *in phase*, but it does not have to be that way. It could be that the x-vibration and the y-vibration are out of phase.

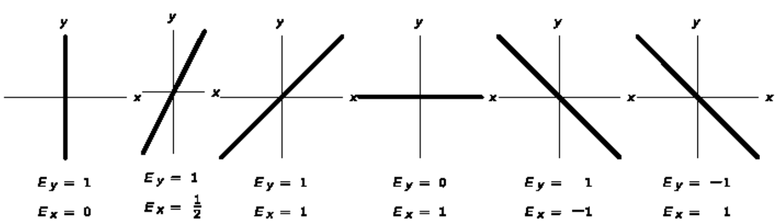


Fig. 33–1. Superposition of x-vibrations and y-vibrations in phase.

When the x-vibration and the y-vibration are not in phase, the electric field vector moves around in an ellipse, and we can illustrate this in a familiar way. If we hang a ball from a support by a long string, so that it can swing freely in a horizontal plane, it will execute sinusoidal oscillations. If we imagine horizontal x- and y-coordinates with their origin at the rest position of the ball, the ball can swing in either the x- or y-direction with the same pendulum frequency. By selecting the proper initial displacement and initial velocity, we can set the ball in oscillation along either the x-axis or the y-axis, or along any straight line in the xy-plane. These motions of the ball are analogous to the oscillations of the electric field vector illustrated in Fig. 33–1. In each instance, since the x-vibrations and the y-vibrations reach their maxima and minima at the same time, the x- and y-oscillations are in phase. But we know that the most general motion of the ball is motion in an ellipse, which corresponds to oscillations in which the x- and y-directions are *not* in the same phase. The superposition of x- and y-vibrations which are not in phase is illustrated in Fig. 33–2 for a variety of angles between the phase of the x-vibration and that of the y-vibration. The general result is that the electric vector moves around an ellipse. The motion in a straight line is a particular case corresponding to a phase difference of zero (or an integral multiple of π); motion in a circle corresponds to equal amplitudes with a phase difference of $90°$ (or any odd integral multiple of $\pi/2$).

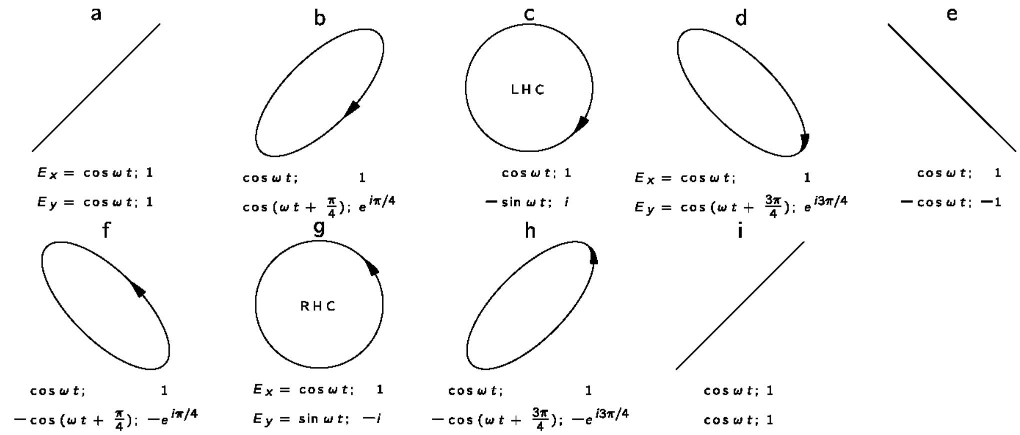


Fig. 33–2.Superposition of x-vibrations and y-vibrations with equal amplitudes but various relative phases. The components E_x and E_y are expressed in both real and complex notations.

In Fig. 33–2 we have labeled the electric field vectors in the x- and y-directions with complex numbers, which are a convenient representation in which to express the phase difference. Do not confuse the real and imaginary components of the complex electric vector in this notation with the x- and y-coordinates of the field. The x- and y-coordinates plotted in Fig. 33–1 and Fig. 33–2 are actual electric fields that we can measure. The real and imaginary components of a complex electric field vector are only a mathematical convenience and have no physical significance.

Now for some terminology. Light is *linearly polarized* (sometimes called plane polarized) when the electric field oscillates on a straight line; Fig. 33–1 illustrates linear polarization. When the end of the electric field vector travels in an ellipse, the light is *elliptically polarized.* When the end of the electric field vector travels around a circle, we have *circular polarization.* If the end of the electric vector, when we look at it as the light comes straight toward us, goes around in a counterclockwise direction, we call it right-hand circular polarization. Figure 33–2(g) illustrates right-hand circular polarization, and Fig. 33–2(c) shows left-hand circular polarization. In both cases the light is coming out of the paper. Our convention for labeling left-hand and right-hand circular polarization is consistent with that which is used today for all the other particles in physics which exhibit polarization (e.g., electrons). However, in some books on optics the opposite conventions are used, so one must be careful.

We have considered linearly, circularly, and elliptically polarized light, which covers everything except for the case of *unpolarized* light. Now how can the light be unpolarized when we know that it must vibrate in one or another of these ellipses? If the light is not absolutely monochromatic, or if the x- and y-phases are not kept perfectly together, so that the electric vector first vibrates in one direction, then in another, the polarization is constantly changing. Remember that one atom emits during 10^{-8} sec, and if one atom emits a certain polarization, and then another atom emits light with a different polarization, the polarizations will change every 10^{-8} sec. If the polarization changes more rapidly than we can detect it, then we call the light unpolarized, because all the effects of the polarization average out. None of the interference effects of polarization would show up with unpolarized light. But as we see from the definition, light is unpolarized only if we are unable to find out whether the light is polarized or not.

33–2 Polarization of scattered light

The first example of the polarization effect that we have already discussed is the scattering of light. Consider a beam of light, for example from the sun, shining on the air. The electric field will produce oscillations of charges in the air, and motion of these charges will radiate light with its maximum intensity in a plane normal to the direction of vibration of the charges. The beam from the sun is unpolarized, so the direction of polarization changes constantly, and the direction of vibration of the charges in the air changes constantly. If we consider light scattered at $90°$, the vibration of the charged particles radiates to the observer only when the vibration is perpendicular to the observer's line of sight, and then light will be polarized along the direction of vibration. So scattering is an example of one means of producing polarization.

33–3 Birefringence

Another interesting effect of polarization is the fact that there are substances for which the index of refraction is different for light linearly polarized in one direction and linearly polarized in another. Suppose that we had some material which consisted of long, nonspherical molecules, longer than they are wide, and suppose that these molecules were arranged in the substance with their long axes parallel. Then what happens when the oscillating electric field passes through this substance? Suppose that because of the structure of the molecule, the electrons in the substance respond more easily to oscillations in the direction parallel to the axes of the molecules than they would respond if the electric field tries to push them at right angles to the molecular axis. In this way we expect a different response for polarization in one direction than for polarization at right angles to that direction. Let us call the direction of the axes of the molecules the *optic axis.* When the polarization is in the direction of the optic axis the index of refraction is different than it would be if the direction of polarization were at right angles to it. Such a substance is called *birefringent.* It has two refrangibilities, i.e., two indexes of refraction, depending on the direction of the polarization inside the substance. What kind of a substance can be birefringent? In a birefringent substance there must be a certain amount of lining up, for one reason or another, of unsymmetrical molecules. Certainly a cubic crystal, which has the symmetry of a cube, cannot be birefringent. But long needlelike crystals undoubtedly contain molecules that are asymmetric, and one observes this effect very easily.

Let us see what effects we would expect if we were to shine polarized light through a plate of a birefringent substance. If the polarization is parallel to the optic axis, the light will go through with one velocity; if the polarization is perpendicular to the axis, the light is transmitted with a different velocity. An interesting situation arises when, say, light is linearly polarized at $45°$ to the optic axis. Now the $45°$ polarization, we have already noticed, can be represented as a superposition of the x- and the y-polarizations of equal amplitude and in phase, as shown in Fig. 33–2(a). Since the x- and y-polarizations travel with different velocities, their phases change at a different rate as the light passes through the substance. So, although at the start the x- and y-vibrations are in phase, inside the material the phase difference between x- and y-vibrations is proportional to the depth in the substance. As the light proceeds through the material the polarization changes as shown in the series of diagrams in Fig. 33–2. If the thickness of the plate is just right to introduce a $90°$ phase shift between the x- and y-polarizations, as in Fig. 33–2(c), the light will come out circularly polarized. Such a thickness is called a quarter-wave plate, because it introduces a quarter-cycle phase difference between the x- and the y-polarizations. If linearly polarized light is sent through two quarter-wave plates, it will come out plane-polarized again, but at right angles to the original direction, as we can see from Fig. 33–2(e).

One can easily illustrate this phenomenon with a piece of cellophane. Cellophane is made of long, fibrous molecules, and is not isotropic, since the fibers lie preferentially in a certain direction. To demonstrate birefringence we need a beam of linearly polarized light, and we can obtain this conveniently by passing unpolarized light through a sheet of polaroid. Polaroid, which we will discuss later in more detail, has the useful property that it transmits light that is linearly polarized parallel to the axis of the polaroid with very little absorption, but light polarized in a direction perpendicular to the axis of the polaroid is strongly absorbed. When we pass unpolarized light through a sheet of polaroid, only that part of the unpolarized beam which is vibrating parallel to the axis of the polaroid gets through, so that the transmitted beam is linearly polarized. This same property of polaroid is also useful in detecting the direction of polarization of a linearly polarized beam, or in determining whether a beam is linearly polarized or not. One simply passes the beam of light through the polaroid sheet and rotates the polaroid in the plane normal to the beam. If the beam is linearly polarized, it will not be transmitted through the sheet when the axis of the polaroid is normal to the direction of polarization. The transmitted beam is only slightly attenuated when the axis of the polaroid sheet is rotated through $90°$. If the transmitted intensity is independent of the orientation of the polaroid, the beam is not linearly polarized.

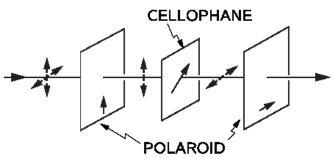


Fig. 33–3.An experimental demonstration of the birefringence of cellophane. The electric vectors in the light are indicated by the dotted lines. The pass axes of the polaroid sheets and optic axes of the cellophane are indicated by arrows. The incident beam is unpolarized.

To demonstrate the birefringence of cellophane, we use two sheets of polaroid, as shown in Fig. 33–3. The first gives us a linearly polarized beam which we pass through the cellophane and then through the second polaroid sheet, which serves to detect any effect the cellophane may have had on the polarized light passing through it. If we first set the axes of the two polaroid sheets perpendicular to each other and remove the cellophane, no light will be transmitted through the second polaroid. If we now introduce the cellophane between the two polaroid sheets, and rotate the sheet about the beam axis, we observe that in general the cellophane makes it possible for some light to pass through the second polaroid. However, there are two orientations of the cellophane sheet, at right angles to each other, which permit no light to pass through the second polaroid. These orientations in which linearly polarized light is transmitted through the cellophane with no effect on the direction of polarization must be the directions parallel and perpendicular to the optic axis of the cellophane sheet.

We suppose that the light passes through the cellophane with two different velocities in these two different orientations, but it is transmitted without changing the direction of polarization. When the cellophane is turned halfway between these two orientations, as shown in Fig. 33–3, we see that the light transmitted through the second polaroid is bright.

It just happens that ordinary cellophane used in commercial packaging is very close to a half-wave thickness for most of the colors in white light. Such a sheet will turn the axis of linearly polarized light through $90°$ if the incident linearly polarized beam makes an angle of $45°$ with the optic axis, so that the beam emerging from the cellophane is then vibrating in the right direction to pass through the second polaroid sheet.

If we use white light in our demonstration, the cellophane sheet will be of the proper half-wave thickness only for a particular component of the white light, and the transmitted beam will have the color of this component. The color transmitted depends on the thickness of the cellophane sheet, and we can vary the effective thickness of the cellophane by tilting it so that the light passes through the cellophane at an angle, consequently through a longer path in the cellophane. As the sheet is tilted the transmitted color changes. With cellophane of different thicknesses one can construct filters that will transmit different colors. These filters have the interesting property that they transmit one color when the two polaroid sheets have their axes perpendicular, and the complementary color when the axes of the two polaroid sheets are parallel.

Another interesting application of aligned molecules is quite practical. Certain plastics are composed of very long and complicated molecules all twisted together. When the plastic is solidified very carefully, the molecules are all twisted in a mass, so that there are as many aligned in one direction as another, and so the plastic is not particularly birefringent. Usually there are strains and stresses introduced when the material is solidified, so the material is not perfectly homogeneous. However, if we apply tension to a piece of this plastic material, it is as if we were pulling a whole tangle of strings, and there will be more strings preferentially aligned parallel to the tension than in any other direction. So when a stress is applied to certain plastics, they become birefringent, and one can see the effects of the birefringence by passing polarized light through the plastic. If we examine the transmitted light through a polaroid sheet, patterns of light and dark fringes will be observed (in color, if white light is used). The patterns move as stress is applied to the sample, and by counting the fringes and seeing where most of them are, one can determine what the stress is. Engineers use this phenomenon as a means of finding the stresses in odd-shaped pieces that are difficult to calculate.

Another interesting example of a way of obtaining birefringence is by means of a liquid substance. Consider a liquid composed of long asymmetric molecules which carry a plus or minus average charge near the ends of the molecule, so that the molecule is an electric dipole. In the collisions in the liquid the molecules will ordinarily be randomly oriented, with as many molecules pointed in one direction as in another. If we apply an electric field the molecules will tend to line up, and the moment they line up the liquid becomes birefringent. With two polaroid sheets and a transparent cell containing such a polar liquid, we can devise an arrangement with the property that light is transmitted only when the electric field is applied. So we have an electrical switch for light, which is called a *Kerr cell*. This effect, that an electric field can produce birefringence in certain liquids, is called the Kerr effect.

33–4Polarizers

So far we have considered substances in which the refractive index is different for light polarized in different directions. Of very practical value are those crystals and other substances in which not only the index, but also the coefficient of absorption, is different for light polarized in different directions. By the same arguments which supported the idea of birefringence, it is understandable that absorption can vary with the direction in which the charges are forced to vibrate in an anisotropic substance. Tourmaline is an old, famous example and polaroid is another. Polaroid consists of a thin layer of small crystals of herapathite (a salt of iodine and quinine), all aligned with their axes parallel. These crystals absorb light when the oscillations are in one direction, and they do not absorb appreciably when the oscillations are in the other direction.

Suppose that we send light into a polaroid sheet polarized linearly at an angle θ to the passing direction. What intensity will come through? This incident light can be resolved into a component perpendicular to the pass direction which is proportional to $\sin\theta$, and a component along the pass direction which is proportional to $\cos\theta$. The amplitude which comes out of the polaroid is only the cosine θ part; the $\sin\theta$ component is absorbed. The amplitude which passes through the polaroid is smaller than the amplitude which entered, by a factor $\cos\theta$. The energy which passes through the polaroid, i.e., the intensity of the light, is proportional to the square of $\cos\theta$. $\text{Cos}^2\,\theta$, then, is the intensity transmitted when the light enters polarized at an angle θ to the pass direction. The absorbed intensity, of course, is $\sin^2\theta$.

An interesting paradox is presented by the following situation. We know that it is not possible to send a beam of light through two polaroid sheets with their axes crossed at right angles. But if we place a third polaroid sheet *between* the first two, with its pass axis at $45°$ to the crossed axes, some light is transmitted. We know that polaroid absorbs light, it does not create anything. Nevertheless, the addition of a third polaroid at $45°$ allows more light to get through. The analysis of this phenomenon is left as an exercise for the student.

One of the most interesting examples of polarization is not in complicated crystals or difficult substances, but in one of the simplest and most familiar of situations—the reflection of light from a surface. Believe it or not, when light is reflected from a glass surface it may be polarized, and the physical explanation of this is very simple. It was discovered empirically by Brewster that light reflected from a surface is completely polarized if the reflected beam and the beam refracted into the material form a right angle. The situation is illustrated in Fig. 33–4. If the incident beam is polarized in the plane of incidence, there will be no reflection at all. Only if the incident beam is polarized normal to the plane of incidence will it be reflected. The reason is very easy to understand. In the reflecting material the light is polarized transversely, and we know that it is the motion of the charges in the material which generates the emergent beam, which we call the reflected beam. The source of this so-called reflected light is not simply that the incident beam is reflected; our deeper understanding of this phenomenon tells us that the incident beam drives an oscillation of the charges in the material, which in turn generates the reflected beam. From Fig. 33–4 it is clear that only oscillations normal to the paper can radiate in the direction of reflection, and consequently the reflected beam will be polarized normal to the plane of incidence. If the incident beam is polarized in the plane of incidence, there will be no reflected light.

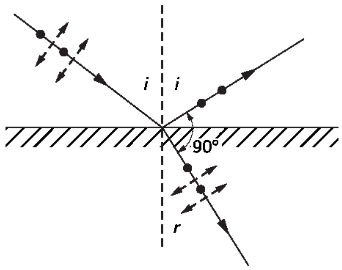


Fig. 33–4.Reflection of linearly polarized light at Brewster's angle. The polarization direction is indicated by dashed arrows; round dots indicate polarization normal to the paper.

This phenomenon is readily demonstrated by reflecting a linearly polarized beam from a flat piece of glass. If the glass is turned to present different angles of incidence to the polarized beam, sharp attenuation of the reflected intensity is observed when the angle of incidence passes through Brewster's angle. This attenuation is observed only if the plane of polarization lies in the plane of incidence. If the plane of polarization is normal to the plane of incidence, the usual reflected intensity is observed at all angles.

33–5Optical activity

Another most remarkable effect of polarization is observed in materials composed of molecules which do not have reflection symmetry: molecules shaped something like a corkscrew, or like a gloved hand, or any shape which, if viewed through a mirror, would be reversed in the same way that a left-hand glove reflects as a right-hand glove. Suppose all of the molecules in the substance are the same, i.e., none is a mirror image of any other. Such a substance may show an interesting effect called optical activity, whereby as linearly polarized light passes through the substance, the direction of polarization rotates about the beam axis.

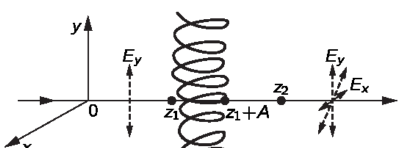


Fig. 33–5.A molecule with a shape that is not symmetric when reflected in a mirror. A beam of light, linearly polarized in the y-direction, falls on the molecule.

To understand the phenomenon of optical activity requires some calculation, but we can see qualitatively how the effect might come about, without actually carrying out the calculations. Consider an asymmetric molecule in the shape of a spiral, as shown in Fig. 33–5. Molecules need not actually be shaped like a corkscrew in order to exhibit optical activity, but this is a simple shape which we shall take as a typical example of those that do not have reflection symmetry. When a light beam linearly polarized along the y-direction falls on this molecule, the electric field will drive charges up and down the helix, thereby generating a current in the y-direction and radiating an electric field E_y polarized in the y-direction. However, if the electrons are constrained to move along the spiral, they must also move in the x-direction as they are driven up and down. When a current is flowing up the spiral, it is also flowing into the paper at $z=z_1$ and out of the paper at $z=z_1+A$, if A is the diameter of our molecular spiral. One might suppose that the current in the x-direction would produce no net radiation, since the currents are in opposite directions on opposite sides of the spiral. However, if we consider the x-components of the electric field arriving at $z=z_2$, we see that the field radiated by the current at $z=z_1+A$ and the field radiated from $z=z_1$ arrive at z_2 separated in time by the amount A/c, and thus separated in phase by $\pi+\omega A/c$. Since the phase difference is not exactly π, the two fields do not cancel exactly, and we are left with a small x -component in the electric field generated by the motion of the electrons in the molecule, whereas the driving electric field had only a y-component. This small x-component, added to the large y-component, produces a resultant field that is tilted slightly with respect to the y-axis, the original direction of polarization. As the light moves through the material, the direction of polarization rotates about the beam axis. By drawing a few examples and considering the currents that will be set in motion by an incident electric field, one can convince himself that the existence of optical activity and the sign of the rotation are independent of the orientation of the molecules.

Corn syrup is a common substance which possesses optical activity. The phenomenon is easily demonstrated with a polaroid sheet to produce a linearly polarized beam, a transmission cell containing corn syrup, and a second polaroid sheet to detect the rotation of the direction of polarization as the light passes through the corn syrup.

33–6The intensity of reflected light

Let us now consider quantitatively the reflection coefficient as a function of angle. Figure 33–6(a) shows a beam of light striking a glass surface, where it is partly reflected and partly refracted into the glass. Let us suppose that the incident beam, of unit amplitude, is linearly polarized normal to the plane of the paper. We will call the amplitude of the reflected wave b, and the amplitude of the refracted wave a. The refracted and reflected waves will, of course, be linearly polarized, and the electric field vectors of the incident, reflected, and refracted waves are all parallel to each other. Figure 33–6(b) shows the same situation, but now we suppose that the incident wave, of unit amplitude, is polarized in the plane of the paper. Now let us call the amplitude of the reflected and refracted wave B and A, respectively.

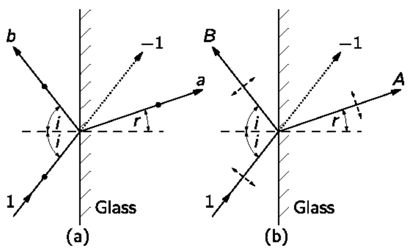


Fig. 33–6.An incident wave of unit amplitude is reflected and refracted at a glass surface. In (a) the incident wave is linearly polarized normal to the plane of the paper. In (b) the incident wave is linearly polarized in the direction shown by the dashed arrows.

We wish to calculate how strong the reflection is in the two situations illustrated in Fig. 33–6(a) and 33–6(b). We already know that when the angle between the reflected beam and refracted beam is a right angle, there will be no reflected wave in Fig. 33–6(b), but let us see if we cannot get a quantitative answer—an exact formula for B and b as a function of the angle of incidence, i.

The principle that we must understand is as follows. The currents that are generated in the glass produce two waves. First, they produce the reflected wave. Moreover, we know that if there were no currents generated in the glass, the incident wave would continue straight into the glass. Remember that all the sources in the world make the net field. The source of the incident light beam produces a field of unit amplitude, which would move into the glass along the dotted line in the figure. This field is not observed, and therefore the currents generated in the glass must produce a field of amplitude -1, which moves along the dotted line. Using this fact, we will calculate the amplitude of the refracted waves, a and A.

In Fig. 33–6(a) we see that the field of amplitude b is radiated by the motion of charges inside the glass which are responding to a field a inside the glass, and that therefore b is proportional to a. We might suppose that since our two figures are exactly the same, except for the direction of polarization, the ratio B/A would be the same as the ratio b/a. This is not quite true, however, because in Fig. 33–6(b) the polarization directions are not all parallel to each other, as they are in Fig. 33–6(a). It is only the component of the electric field in the glass which is perpendicular to B, $A\cos(i+r)$, which is effective in producing B. The correct expression for the proportionality is then

$$\frac{b}{a}=\frac{B}{A\cos(i+r)}. \tag{33.1}$$

Now we use a trick. We know that in both (a) and (b) of Fig. 33–6 the electric field in the glass must produce oscillations of the charges, which generate a field of amplitude -1, polarized parallel to the incident beam, and moving in the direction of the dotted line. But we see from part (b) of the figure that only the component of the electric field in the glass that is normal to the dashed line has the right polarization to produce this field, whereas in Fig. 33–6(a) the full amplitude a is effective, since the polarization of wave a is parallel to the polarization of the wave of amplitude -1. Therefore we can write

$$\frac{A\cos(i-r)}{a}=\frac{-1}{-1}, \tag{33.2}$$

since the two amplitudes on the left side of Eq. (33.2) each produce the wave of amplitude -1.

Dividing Eq. (33.1) by Eq. (33.2), we obtain

$$\frac{B}{b}=\frac{\cos(i+r)}{\cos(i-r)}, \tag{33.3}$$

a result which we can check against what we already know. If we set $(i+r)=90°$, Eq. (33.3) gives $B=0$, as Brewster says it should be, so our results so far are at least not obviously wrong.

We have assumed unit amplitudes for the incident waves, so that $|B|^2/1^2$ is the reflection coefficient for waves polarized in the plane of incidence, and $|b|^2/1^2$ is the reflection coefficient for waves polarized normal to the plane of incidence. The ratio of these two reflection coefficients is determined by Eq. (33.3).

Now we perform a miracle, and compute not just the ratio, but each coefficient $|B|^2$ and $|b|^2$ individually! We know from the conservation of energy that the energy in the refracted wave must be equal to the incident energy minus the energy in the reflected wave, $1-|B|^2$ in one case, $1-|b|^2$ in the other. Furthermore, the energy which passes into the glass in Fig. 33–6(b) is to the energy which passes into the glass in Fig. 33–6(a) as the ratio of the squares of the refracted amplitudes, $|A|^2/|a|^2$. One might ask whether we really know how to compute the energy inside the glass, because, after all, there are energies of motion of the atoms in addition to the energy in the electric field. But it is obvious that all of the various contributions to the total energy will be proportional to the square of the amplitude of the electric field. Therefore we can write

$$\frac{1-|B|^2}{1-|b|^2}=\frac{|A|^2}{|a|^2}. \tag{33.4}$$

We now substitute Eq. (33.2) to eliminate A/a from the expression above, and express B in terms of b by means of Eq. (33.3):

$$\frac{1-|b|^2\dfrac{\cos^2(i+r)}{\cos^2(i-r)}}{1-|b|^2}=\frac{1}{\cos^2(i-r)}. \tag{33.5}$$

This equation contains only one unknown amplitude, b. Solving for $|b|^2$, we obtain

$$|b|^2=\frac{\sin^2(i-r)}{\sin^2(i+r)} \tag{33.6}$$

and, with the aid of (33.3),

$$|B|^2=\frac{\tan^2(i-r)}{\tan^2(i+r)}. \tag{33.7}$$

So we have found the reflection coefficient $|b|^2$ for an incident wave polarized perpendicular to the plane of incidence, and also the reflection coefficient $|B|^2$ for an incident wave polarized in the plane of incidence!

It is possible to go on with arguments of this nature and deduce that b is real. To prove this, one must consider a case where light is coming from both sides of the glass surface at the same time, a situation not easy to arrange experimentally, but fun to analyze theoretically. If we analyze this general case, we can prove that b must be real, and therefore, in fact, that $b=\pm\sin(i-r)/\sin(i+r)$. It is even possible to determine the sign by considering the case of a very, very thin layer in which there is reflection from the front and from the back surfaces, and calculating how much light is reflected. We know how much light should be reflected by a thin layer, because we know how much current is generated, and we have even worked out the fields produced by such currents.

One can show by these arguments that

$$b=-\frac{\sin(i-r)}{\sin(i+r)},\quad B=-\frac{\tan(i-r)}{\tan(i+r)}. \tag{33.8}$$

These expressions for the reflection coefficients as a function of the angles of incidence and refraction are called Fresnel's reflection formulas.

If we consider the limit as the angles i and r go to zero, we find, for the case of normal incidence, that $B^2\approx b^2\approx(i-r)^2/(i+r)^2$ for both polarizations, since the sines are practically equal to the angles, as are also the tangents. But we know that $\sin i/\sin r=n$, and when the angles are small, $i/r\approx n$. It is thus easy to show that the coefficient of reflection for normal incidence is

$$B^2=b^2=\frac{(n-1)^2}{(n+1)^2}.$$

It is interesting to find out how much light is reflected at normal incidence from the surface of water, for example. For water, n is $4/3$, so that the reflection coefficient is $(1/7)^2\approx 2\%$. At normal incidence, only two percent of the light is reflected from the surface of water.

33–7Anomalous refraction

The last polarization effect we shall consider was actually one of the first to be discovered: anomalous refraction. Sailors visiting Iceland brought back to Europe crystals of Iceland spar ($CaCO_3$) which had the amusing property of making anything seen through the crystal appear doubled, i.e., as two images. This came to the attention of Huygens, and played an important role in the discovery of polarization. As is often the case, the phenomena which are discovered first are the hardest, ultimately, to explain. It is only after we understand a physical concept thoroughly that we can carefully select those phenomena which most clearly and simply demonstrate the concept.

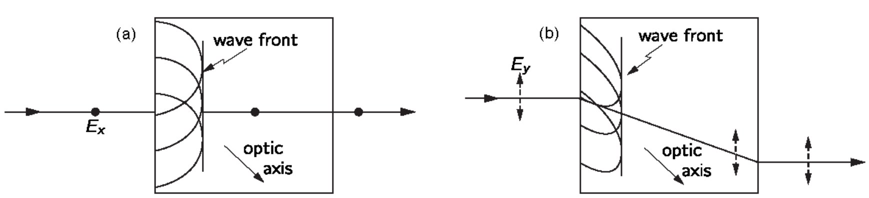


Fig. 33–7.Part (a) of the diagram shows the path of the ordinary ray through a doubly refracting crystal. The extraordinary ray is shown in part (b). The optic axis lies in the plane of the paper.

Anomalous refraction is a particular case of the same birefringence that we considered earlier. Anomalous refraction comes about when the optic axis, the long axis of our asymmetric molecules, is *not* parallel to the surface of the crystal. In Fig. 33–7 are drawn two pieces of birefringent material, with the optic axis as shown. In part (a) of the figure, the incident beam falling on the material is linearly polarized in a direction perpendicular to the optic axis of the material. When this beam strikes the surface of the material, each point on the surface acts as a source of a wave which travels into the crystal with velocity $v_\perp$, the velocity of light in the crystal when the plane of polarization is normal to the optic axis. The wavefront is just the envelope or locus of all these little spherical waves, and this wavefront moves straight through the crystal and out the other side. This is just the ordinary behavior we would expect, and this ray is called the *ordinary ray*.

In part (b) of the figure the linearly polarized light falling on the crystal has its direction of polarization turned through 90°, so that the optic axis lies in the plane of polarization. When we now consider the little waves originating at any point on the surface of the crystal, we see that they do not spread out as spherical waves. Light travelling along the optic axis travels with velocity $v_\perp$ because the polarization is perpendicular to the optic axis, whereas the light travelling perpendicular to the optic axis travels with velocity $v_\parallel$ because the polarization is parallel to the optic axis. In a birefringent material $v_\parallel \neq v_\perp$, and in the figure $v_\parallel < v_\perp$. A more complete analysis will show that the waves spread out on the surface of an ellipsoid, with the optic axis as major axis of the ellipsoid. The envelope of all these elliptical waves is the wavefront which proceeds through the crystal in the direction shown. Again, at the back surface the beam will be deflected just as it was at the front surface, so that the light emerges parallel to the incident beam, but displaced from it. Clearly, this beam does not follow Snell's law, but goes in an extraordinary direction. It is therefore called the *extraordinary ray*.

When an unpolarized beam strikes an anomalously refracting crystal, it is separated into an ordinary ray, which travels straight through in the normal manner, and an extraordinary ray which is displaced as it passes through the crystal. These two emergent rays are linearly polarized at right angles to each other. That this is true can be readily demonstrated with a sheet of polaroid to analyze the polarization of the emergent rays. We can also demonstrate that our interpretation of this phenomenon is correct by sending linearly polarized light into the crystal. By properly orienting the direction of polarization of the incident beam, we can make this light go straight through without splitting, or we can make it go through without splitting but with a displacement.

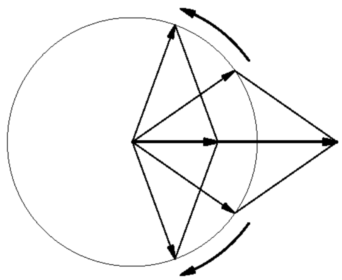


Fig. 33–8.Two oppositely rotating vectors of equal amplitude add to produce a vector in a fixed direction, but with an oscillating amplitude.

We have represented all the various polarization cases in Figs. 33–1 and 33–2 as superpositions of two special polarization cases, namely x and y in various amounts and phases. Other pairs could equally well have been used. Polarization along any two perpendicular axes x', y' inclined to x and y would serve as well [for example, any polarization can be made up of superpositions of cases (a) and (e) of Fig. 33–2]. It is interesting, however, that this idea can be extended to other cases also. For example, any *linear* polarization can be made up by superposing suitable amounts at suitable phases of right and left *circular* polarizations [cases (c) and (g) of Fig. 33–2], since two equal vectors rotating in opposite directions add to give a single vector oscillating in a straight line (Fig. 33–8). If the phase of one is shifted relative to the other, the line is inclined. Thus all the pictures of Fig. 33–1 could be labeled "the superposition of equal amounts of right and left circularly polarized light at various relative phases." As the left slips behind the right in phase, the direction of the linear polarization changes. Therefore optically active materials are, in a sense, birefringent. Their properties can be described by saying that they have different indexes for right- and left-hand circularly polarized light. Superposition of right and left circularly polarized light of different intensities produces elliptically polarized light.

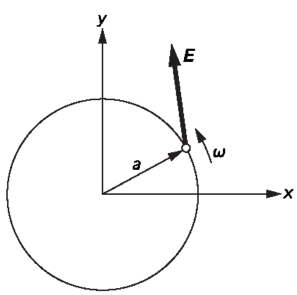


Fig. 33–9.A charge moving in a circle in response to circularly polarized light.

Circularly polarized light has another interesting property—it carries *angular momentum* (about the direction of propagation). To illustrate this, suppose that such light falls on an atom represented by a harmonic oscillator that can be displaced equally well in any direction in the plane xy. Then the x-displacement of the electron will respond to the E_x component of the field, while the y-component responds, equally, to the equal E_y component of the field but 90° behind in phase. That is, the responding electron goes around in a circle, with angular velocity ω, in response to the rotating electric field of the light (Fig. 33–9). Depending on the damping characteristics of the response of the oscillator, the direction of the displacement a of the electron, and the direction of the force q_eE on it need not be the same but they rotate around together. The E may have a component at right angles to a, so work is done on the system and a torque τ is exerted. The work done per second is $\tau\omega$. Over a period of time T the energy absorbed is $\tau\omega T$, while τT is the angular momentum delivered to the matter absorbing the energy. We see therefore that *a beam of right circularly polarized light containing a total energy $\mathcal{E}$ carries an angular momentum* *(with vector directed along the direction of propagation)* $\mathcal{E}/\omega$. For when this beam is absorbed that angular momentum is delivered to the absorber. Left-hand circular light carries angular momentum of the opposite sign, $-\mathcal{E}/\omega$.

Jones Matrices

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Jones calculus

In optics, polarized light can be described using the **Jones calculus**, discover
Polarized light is represented by a **Jones vector**, and linear optical elements
matrices. When light crosses an optical element the resulting polarization of th
by taking the product of the Jones matrix of the optical element and the Jo
light. Note that Jones calculus is only applicable to light that is already fully
randomly polarized, partially polarized, or incoherent must be treated using Mu

Contents

The Jones vector

The Jones vector describes the polarization of light in free space or another hor
attenuating medium, where the light can be properly described as transvers
monochromatic plane wave of light is travelling in the positive z-direction, with
wavevector $\mathbf{k} = (0,0,k)$, where the wavenumber $k = \omega/c$. Then the electric and
are orthogonal to $\mathbf{k}$ at each point; they both lie in the plane "transverse" to
Furthermore, $\mathbf{H}$ is determined from $\mathbf{E}$ by 90-degree rotation and a fixed mu
wave impedance of the medium. So the polarization of the light can be deterr
complex amplitude of $\mathbf{E}$ is written

$$\begin{pmatrix} E_x(t) \\ E_y(t) \\ 0 \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i(kz-\omega t+\phi_x)} \\ E_{0y} e^{i(kz-\omega t+\phi_y)} \\ 0 \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \\ 0 \end{pmatrix} e^{i(kz-\omega t)}.$$

Note that the physical $\mathbf{E}$ field is the real part of this vector; the complex mult
information. Here i is the imaginary unit with $i^2 = -1$.

https://en.wikipedia.org/wiki/Jones_calculus

The Jones vector is then

$$\begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix}.$$

Thus, the Jones vector represents the amplitude and phase of the electric field in the x and y directions.

- $|R\rangle$ and $|L\rangle$

The polarization of any point not equal to $|R\rangle$ or $|L\rangle$ and not on the circle that passes through $|H\rangle, |D\rangle, |V\rangle, |A\rangle$ is known as elliptical polarization.

Jones matrices

The Jones matrices are operators that act on the Jones vectors defined above. These matrices are implemented by various optical elements such as lenses, beam splitters, mirrors, etc. Each matrix represents projection onto a one-dimensional complex subspace of the Jones vectors. The following table gives examples of Jones matrices for polarizers:

Optical element	Jones matrix
Linear polarizer with axis of transmission horizontal[1]	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Linear polarizer with axis of transmission vertical[1]	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Linear polarizer with axis of transmission at ±45° with the horizontal[1]	$\frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$
Right circular polarizer[1]	$\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$
Left circular polarizer[1]	$\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

Phase retarders

Phase retarders introduce a phase shift between the vertical and horizontal component of the field and thus change the polarization of the beam. Phase retarders are usually made out of birefringent uniaxial crystals such as calcite, MgF_2 or quartz. Uniaxial crystals have one crystal axis that is different from the other two crystal axes (i.e., $n_i \neq n_j = n_k$). This unique axis is called the extraordinary axis and is also referred to as the optic axis. An optic axis can be the fast or the slow axis for the crystal depending on the crystal at hand. Light travels with a higher phase velocity along an axis that has the smallest refractive index and this axis is called the fast axis. Similarly, an axis which has the largest refractive index is called a slow axis since the phase velocity of light is the lowest along this axis. "Negative" uniaxial crystals (e.g., calcite $CaCO_3$, sapphire Al_2O_3) have $n_e < n_o$ so for these crystals, the extraordinary axis (optic axis) is the fast axis, whereas for "positive" uniaxial crystals (e.g., quartz SiO_2, magnesium fluoride MgF_2, rutile TiO_2), $n_e > n_o$ and thus the extraordinary axis (optic axis) is the slow axis.

Any phase retarder with fast axis equal to the x- or y-axis has zero off-diagonal terms and thus can be conveniently expressed as

$$\begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix}$$

where ϕ_x and ϕ_y are the phase offsets of the electric fields in x and y directions respectively. In the phase convention $\phi = kz - \omega t$, define the relative phase between the two waves as $\epsilon = \phi_y - \phi_x$. Then a positive ϵ (i.e. $\phi_y > \phi_x$) means that E_y doesn't attain the same value as E_x until a later time, i.e. E_x leads E_y. Similarly, if $\epsilon < 0$, then E_y leads E_x.

For example, if the fast axis of a quarter wave plate is horizontal, then the phase velocity along the horizontal direction is ahead of the vertical direction i.e., E_x leads E_y. Thus, $\phi_x < \phi_y$ which for a quarter wave plate yields $\phi_y = \phi_x + \pi/2$.

In the opposite convention $\phi = \omega t - kz$, define the relative phase as $\epsilon = \phi_x - \phi_y$. Then $\epsilon > 0$ means that E_y doesn't attain the same value as E_x until a later time, i.e. E_x leads E_y.

Phase retarders	Corresponding Jones matrix
Quarter-wave plate with fast axis vertical[2][note 1]	$e^{\frac{i\pi}{4}}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$
Quarter-wave plate with fast axis horizontal[2]	$e^{-\frac{i\pi}{4}}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
Quarter-wave plate with fast axis at angle θ w.r.t the horizontal axis	$e^{-\frac{i\pi}{4}}\begin{pmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$
Half-wave plate with fast axis at angle θ w.r.t the horizontal axis[3]	$e^{-\frac{i\pi}{2}}\begin{pmatrix} \cos^2\theta - \sin^2\theta & 2\cos\theta\sin\theta \\ 2\cos\theta\sin\theta & \sin^2\theta - \cos^2\theta \end{pmatrix}$
Arbitrary birefringent material (as phase retarder)[4]	$e^{-\frac{i\eta}{2}}\begin{pmatrix} \cos^2\theta + e^{i\eta}\sin^2\theta & \left(1-e^{i\eta}\right)e^{-i\phi}\cos\theta\sin\theta \\ \left(1-e^{i\eta}\right)e^{i\phi}\cos\theta\sin\theta & \sin^2\theta + e^{i\eta}\cos^2\theta \end{pmatrix}$

The special expressions for the phase retarders can be obtained by taking suitable parameter values in the general expression for a birefringent material. In the general expression:

- The relative phase retardation induced between the fast axis and the slow axis is given by $\eta = \phi_y - \phi_x$
- θ is the orientation of the fast axis with respect to the x-axis.
- ϕ is the circularity.

Note that for linear retarders, ϕ = 0 and for circular retarders, $\phi = \pm\pi/2$, $\theta = \pi/4$. In general for elliptical retarders, ϕ takes on values between $-\pi/2$ and $\pi/2$.

Axially rotated elements

Assume an optical element has its optic axis perpendicular to the surface vector for the plane of incidence and is rotated about this surface vector by angle $\theta/2$ (i.e., the principal plane, through which the optic axis passes, makes angle $\theta/2$ with respect to the plane of polarization of the electric field of the incident TE wave). Recall that a half-wave plate rotates polarization as *twice* the angle between incident polarization and optic axis (principal plane). Therefore, the Jones matrix for the rotated polarization state, M(θ), is

$$M(\theta) = R(-\theta)\, M\, R(\theta),$$

where $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$

This agrees with the expression for a half-wave plate in the table above. These rotations are identical to beam unitary splitter transformation in optical physics given by

$$R(\theta) = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

where the primed and unprimed coefficients represent beams incident from opposite sides of the beam splitter. The reflected and transmitted components acquire a phase θ_r and θ_t, respectively. The requirements for a valid representation of the element are [5]

$$\theta_{\mathrm{t}} - \theta_{\mathrm{r}} + \theta_{\mathrm{t'}} - \theta_{\mathrm{r'}} = \pm\pi$$

and $r^* t' + t^* r' = 0.$

Both of these representations are unitary matrices fitting these requirements; and as such, are both valid.

Arbitrarily rotated elements

This would involve a three-dimensional rotation matrix. See Russell A. Chipman[6] and Garam Yun for work done on this.[7][8][9]

Polarization axis from Jones vector

The angle of polarization ellipse of the Jones vector $|\psi\rangle$ can be calculated as below,

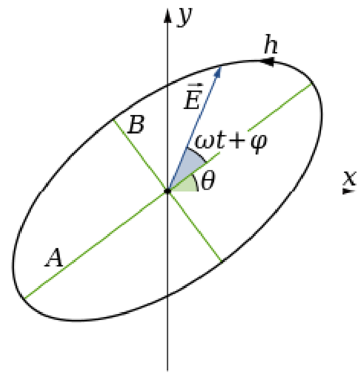
$$\tan 2\theta = \frac{\langle\psi|\,\mathrm{Ref}(45\,\mathrm{deg})|\psi\rangle}{\langle\psi|\,\mathrm{Ref}(0\,\mathrm{deg})|\psi\rangle} = \frac{2E_{0x}E_{0y}\cos(\phi_x - \phi_y)}{E_{0x}^2 - E_{0y}^2}$$

where θ is the angle of either a major or a minor axis and **Ref** is a reflection matrix.

See also

- Polarization
- Scattering parameters

- Stokes parameters
- Mueller calculus
- Photon polarization



Notes

1. The prefactor $e^{i\pi/4}$ appears only if one defines the phase delays in a symmetric fashion; that is, $\phi_x = -\phi_y = \pi/4$. This is done in Hecht[2] but not in Fowles.[1] In the latter reference the Jones matrices for a quarter-wave plate have no prefactor.

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External links

- *Jones Calculus written by E. Collett on Optipedia* (http://spie.org/x32380.xml)

Linear vs Direct Proportionality

https://www.mytutor.co.uk/answers/17552/A-Level/Physics/What-is-the-difference-between-linearly-

MyTutor

Answers > Physics > A Level > Article

What is the difference between linearly, directl proportional relationships?

A relationship is a way of describing how one variable can affect another. If a rela change in one variable will cause a change in another variable by a fixed amount is Force against the Extension of a spring (up until the limit of proportionality!). A relationship is a special type of linear relationship. When one variable is equal to have a value of 0. On a graph, there would be a straight line through going the "o proportional relationship is one where an increase in one variable will lead to a c quickly this occurs may vary. An example of this would be Gravitational field stre centre of a planet. As the distance increases, the strength of gravity decreases.

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directly-and-inversely-proportional-relationships/

Bertrand Russell's Essential Rules of Critical Thinking

https://www.teachthought.com/critical-thinking/bertrand-russells-10-rules-of-critical-thinking/ http://www.criticalthinking.org
and-university-students/799 <– Not the link, but included on the page

1. Do not feel absolutely certain of anything.
2. Do not think it worth while to proceed by concealing evidence, for the evidence is sure to come to light.
3. Never try to discourage thinking for you are sure to succeed.
4. When you meet with opposition, even if it should be from your husband or your children, endeavor to overcome it by argument and not by authority, for a victory dependent upon authority is unreal and illusory.
5. Have no respect for the authority of others, for there are always contrary authorities to be found.
6. Do not use power to suppress opinions you think pernicious, for if you do the opinions will suppress you.
7. Do not fear to be eccentric in opinion, for every opinion now accepted was once eccentric.
8. Find more pleasure in intelligent dissent than in passive agreement, for, if you value intelligence as you should, the former implies a deeper agreement than the latter.
9. Be scrupulously truthful, even if the truth is inconvenient, for it is more inconvenient when you try to conceal it.
10. Do not feel envious of the happiness of those who live in a fool's paradise, for only a fool will think that it is happiness.

- Not Included
  - Scan of Pg. 314 and 315 of the Book
  - Group Project Assignment

Reflections

Critical Thinking

> The objective analysis and evaluation of an issue in order to form a judgment.

There's not much more to it. Simply, it is knowing how to use information to connect and form relationships in order to deduce further information. It does *not* relate to being able to store or know a lot of things, but rather knowing how to obtain more information and apply it, an important distinction. While it often behooves someone to be both knowledgeable and a critical thinker, i.e. how to apply said knowledge, these two do not necessarily follow. As such, it is more important to learn critical thinking and the methods in which one solves problems over memorizing rote cases due to the ever-changing knowledge-scape.

Relevant Video (funny, but has some curses/inappropriate humor): https://youtu.be/qMrnVkDH2Ak

Deductive Reasoning: Certain Logic with Uncertain Results

Deductive reasoning stands as a very useful daily tool and even when not applied explicitly is used in daily presumptions (not assumptions) and decision-making processes. Deductive reasoning uses general, "known" premises presumed true to create a logical conclusion. A simple example: "Greens and salads are healthy, so the salad from this store is healthy." That can be further extended with "Greasy meats are not healthy, ergo the hamburger from this store is not healthy." Which when combined can allow more effective decision-making rather than blind-guessing (note that the decision-making part and choosing the salad over the hamburger is NOT deductive reasoning here, the underlying conclusions are).

Deduction is important in everyday-life as well as scientific and mathematical thinking. However, it is not infallible, and must be used cautiously, as it is only as strong as its underlying premises. For instance, green food and salads are NOT inherently always 100% healthy – a moldy green meat covered in poisoned tomatoes would be a counter-argument. It is more dangerous when applied to larger biases, such as those that we make of genders and races or the like as it is dificult to separate facts mixed with exceptions and stereotypes/opinions. A healthy *skepticism* is needed in applying deduction, allowing you to presume a general conclusion but also allow for exceptions and counters to the premises of your conclusion.

The problem is not deductive reasoning itself, as logically if the premises hold true then the conclusions hold true, and deductive reasoning is about as certain as you can obtain knowledge in logical thinking (assuming, of course, our concepts of logical processes hold true and we do not delve into deeper and more abstract epistemological issue). It's how we generate premises, via inductive reasoning. You may think salads are healthy as eating them has made you feel good, kept you alive, is said to be good from various sources, etc. Each source of knowledge that you used to generate your premise that "salads are healthy" could be wrong, or the issue could be more nuanced than the premise suggests (i.e. "Vitamin X typically found in X food typically found in salads, when prepared correctly as it typically is in most human establishments, shows to generally help humans in growth/survival").

We almost *must* make assumptions at some level in order to function with any semblance of what we currently call living, and even our axioms of "a is a" and underlying faith in logical/mathematical/scientific thinking are at their core blind, dogmatic, and "self-serving" beliefs. You'd likely be correct in assuming the sun will rise tomorrow based on your past experience and sources supporting the premise that "the sun rises each morning" and applying it using deduction such that "tomorrow morning is a morning, the sun rises each morning, thus the sun will rise tomorrow morning", even if the actuality is more nuanced ("the sun does not rise, but rather appears to rise based on our changing position relative to the sun") or entirely false ("a fast massive object snuffed the sun out a few minutes ago or stopped earth's orbit thus the premise that the sun will rise each morning is false based on the current definitions of 'morning' and such"). Yet we also arguably should hold a healthy skepticism for our premises – treat your conclusions and premises alike as a probability, not a certainty, as deductive reasoning only implies certainty if the premise is certain.

Critical vs. Deductive Reasoning

Apparently there was some confusion on critical thinking and how it relates to deductive reasoning. As such, I will summarize the two and expound on their differences.

Critical Thinking Critical thinking largely relates to the ability to think "outside the box", as they say. It is the *application* of knowledge and relationships in order to form new relationships. It does *not* apply to the retention of knowledge, nor even necessarily logical thought processes. Critical thinking is a creative endeavor to grasp for new ideas and meaning than is readily apparent.

Deductive Reasoning Deductive reasoning is the seeking of individual truths and knowledge from general premises. It is a very strong manner of obtaining conclusions in logic, though it is as weak as each of its premises. A common example of deductive reasoning: "Hamburgers are unhealthy. McDouble is a hamburger. McDoubles are thus unhealthy."

Deductive reasoning stands in contrast with and relies on *inductive reasoning*, the seeking of a general truth from specific instances. We think hamburgers are unhealthy or that animals must drink water or the sun will rise each day due to a mixture of inductive and deductive reasoning. First, as children, we saw that everyone drunk water and the sun rose each day, thus we made an inference that these were general truths. Then, mostly subconsciously, we applied these "truths" to predictions in the future: animals for the most part will still need to drink water and tomorrow, as it is a day and all days before have had suns, will follow that trend.

Deduction is thusly fallible, however, in that those trends may not be true: A company could form a Health-burger, animals could evolve to accept H_2 O_2 instead of water, and the sun one day may fade. The only other major premises we have are axioms – "inherent" truths that we establish as the groundwork for our beliefs and faith in some principle. Some, for instance, establish the existence of God as an inherent truth from which other truths may be deduced; more commonly and socially acceptable, many also hold various mathematic and logical principles as "truth", such as "A is equal to A" or such. These allow us to use deductive thinking outside of what we see, observe, and induce.

Critical Thinking in contrast to Deductive Reasoning They are hardly alike.

When speaking about deductive reasoning, it is largely in contrast with inductive reasoning. This is because these are moreover lines of thought and step-by-step processes at obtaining new knowledge. They do not necessarily require critical thinking, even. Critical thinking refers to the ability to look at several relationships and pieces of knowledge and presume or glean new insights (and potentially wrong ones at that) from them. It is possible that perhaps "under the hood" in the brain it is a result of several subconscious, simultaneous lines of reasoning. These two concepts, however, are not fundamentally linked: deductive reasoning is a way to gain relatively sure knowledge based on premises, critical thinking is a manner of examining and extrapolating various relationships into a new potential idea.

Reflection on IEEE Code of Ethics

IEEE's Code of Ethics acts as a semi-abstract list of guidelines for members to follow in their actions representing IEEE. They seek to define how to act and how to not act with respect to data, individuals, and conflicts. It is also notable that they use precisely the words "ethical and professional conduct" – they do not attempt to assign some moral value of right/wrongness to their code, only that it is the expected behaviorisms of its member, an important distinction. While IEEE's Code of Ethics is largely in-line with modern-day trends/beliefs and as such does not provide too much in the way of moral/ethical conflicts, it is important yet still for an organization to remove moral assignments, as shown here.

Teaching and Discussion Paves the Path to Understanding and Acceptance

5. to improve the understanding by individuals and society of the capabilities and societal implications of conventional and emerging technologies, including intelligent systems;

Professor Myers and myself have spent many hours debating the pros and cons of upcoming trends in technology, such as the prevalence of cameras, use of Big Data to target advertisements, GMOs, etc. While our opinions may drastically differ, we do agree unilaterally that there should be more education on these topics in school systems and to the public. It appears to us that people in general are not learning enough about the upcoming changes of organizations and the government in how they view, observe, and interact with people and how this may affect our future, and without the education on these matters, it is dificult for people to make informed, free-thinking, and thought-out decisions outside of what some single person or article told them to think.

As such, we support this guideline full-heartedly, and it is on our roadmap in our newly begun organization S.E.E.D. to put educate people on new technologies and how it may affect their lives as well as spark dialogue on the matter within the campus.

Know your Limits

6. to maintain and improve our technical competence and to undertake technological tasks for others only if qualified by training or experience, or after full disclosure of pertinent limitations;

I've had a saying that goes like…

> The only thing more dangerous than someone who knows nothing is someone who knows everything.

It's probably on the internet somewhere, and mostly just a rewording of the innumerable anecdotes of the dangers of pretending to know what one does not. However, I use it as a personal guideline – never let my eagerness to do or overconfidence deceive others and, more importantly, myself. I know some disagree, and I have been told by many that instead you should "fake it till you make it" or "always appear to know what you're doing", however in my limited experiences I have found admitting short-comings and always acting below your skill-level ensures that you can deliver on promises and expectations more reliably and thus be a proficient member of your team or organization.

Now mind you, I am not saying never expand your skillset or only work comfortably in skills you already know – you would never grow. However, there are ways to do that safely and with humility. Work in the "yellow zone", as they say, on the edge of your skillset, and do not be hesitant to mention when something is out of one's league, such as a complex simultaneous equation, high-paying but experienced engineering job, or obtaining a girlfriend. Approach problem energetically, but also with caution, and be aware of where to go for help.

In short, this ethical statement resonates strongly with my personal code, as I have witnessed teams crumble due to the pretending of skills people do not have in order to "gain respect" or such.

Ethical Case Study

Reflection on Critical Thinking Quotes

Chapter Work

This section seeks to overview and demonstrate example problems within the text. Note that these summaries are *non-comprehensive* – viewers will *not* be able to read this from no background and understand the topics by the end. That is the job of lectures and textbooks, whereas instead this section is dedicated to a cursory overview of the most important and practical components of the course and showing their use in problem-solving.

Chapter-01: Vector Algebra

Vector Algebra ### Notes-01 * **Electromagnetics (EM)** is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

Maxwell Equations

$$\nabla \cdot \bm{D} = \rho_v$$
$$\nabla \cdot \bm{B} = 0$$
$$\nabla \times \bm{E} = -\frac{\partial \bm{B}}{\partial t}$$
$$\nabla \times \bm{H} = \bm{J} + \frac{\partial \bm{D}}{\partial t}$$

Symbol	Name	Units	Description
∇	Del - Vector Differential Operator		
$\bm{D}$	Electric Flux Density		
$\bm{B}$	Magnetic Flux Density		
$\bm{E}$	Electric Field Intensity		
$\bm{H}$	Magnetic Field Intensity		
ρ_v	Volume Charge Density		
$\bm{J}$	Current Density		

Scalars versus Vectors

- **Scalars** are quantities with only magnitude.
  - E.G. 1, $47 Joules$, $-28kW$, etc.
- **Vectors** have both a quantity and magnitude.
  - NOTE: this is a very rudimentary and not entirely true definition; vectors are much more complex elements that extend beyond this. However, this is perhaps a more fitting definition of *Euclidean Vectors*, and will suffice for this course.
  - Technically, vectors should be bolded as $\bm{A}$, however even the book cheats often and uses a regular A as it becomes tedious to write `$\bm{A}$` in every case.
- Scalars are multiplied in the typical manner: $2 \times 7 = 14$
- Vectors can be multiplied in two primary ways (we are assuming a 3-dimensional space, though with true vectors these definitions extend past):
  - **Dot Product**: represented as $\bm{A} \cdot \bm{B} = C$, produces a scalar output. As such, also called a *scalar product*.
  $$\bm{A} \cdot \bm{B} = A_x B_x + A_y B_y + A_z B_z$$
  - **Cross Product**: represented as $\bm{A} \times \bm{B} = \bm{C}$, produces a vector output orthogonal in direction to both $\bm{A}$ and $\bm{B}$. As such, also called a *vector product*.

$$\bm{A} \cdot \bm{B} = \det \begin{vmatrix} \bm{a}_x & \bm{a}_y & \bm{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\bm{a_x} + (A_z B_x - A_x B_z)\bm{a_y} + (A_x B_y - A_y B_x)\bm{a_z}$$

- Magnitude of vectors -
- Addition/Subtraction of Vectors
- Unit Vectors
- Determinant: det $\left|\boldsymbol{A} \times \boldsymbol{B}\right|$
- Representation
  - Bracket Form:
  - Unit Vector Form:
  - "Physics" Form:
- Law of Sines
- Law of Cosines
- Component along Vector
- Projection along Vector

Problems-01

Problem-1.1 Find the unit vector along the line joining point $(2, 4, 4)$ to point $(-3, 2, 2)$.

A classic displacement problem, it can be solved with a simple formula: $\frac{A-B}{|A-B|}$

1. Find the difference between A and B:

   $A - B = \langle 2, 4, 4\rangle - \langle -3, 2, 2\rangle = \langle (2) - (-3), (4) - (2), (4) - (2)\rangle = \langle 5, 2, 2\rangle$

   This indicates that the displacement vector between A and B is $\langle 5, 2, 2\rangle$.

2. Find the magnitude of $\langle 5, 2, 2\rangle$.

   $|\langle 5, 2, 2\rangle| = \sqrt{5^2 + 2^2 + 2^2} = \sqrt{33}$

3. Divide by the vector by the magnitude: $\frac{\langle 5,2,2\rangle}{\sqrt{33}} = \langle \frac{5}{\sqrt{33}}, \frac{2}{\sqrt{33}}, \frac{2}{\sqrt{33}}\rangle$

This vector represents the displacement vector as a unit vector, I.E. its magnitude is 1 and can be easily multiplied to obtain any vector of magnitude x in its direction by the simple product of $x\hat{C}$ ($\hat{C}$ being said Unit Vector).

Problem-1.3 Given vectors $\boldsymbol{A} = 2\boldsymbol{a}_x + 5\boldsymbol{a}_z$ and $\boldsymbol{B} = \boldsymbol{a}_x - 3\boldsymbol{a}_y + 4\boldsymbol{a}_z$, find det $\left|\boldsymbol{A} \times \boldsymbol{B}\right| + \boldsymbol{A} \cdot \boldsymbol{B}$.

The Order of Operation rules apply, thus solve the two multiplications separately. A relatively straightforward problem, just solve it in pieces:

$A \times B = \langle 2, 0, 5\rangle \times \langle 1, -3, 4\rangle = \langle [(0\cdot 4) - (-3\cdot 5)], -[(2\cdot 4) - (1\cdot 5)], [(2\cdot -3) - (1\cdot 0)] = \langle 0+15, -8+5, -6-0\rangle = \langle 15, -3, -6\rangle$

$|A \times B| = |\langle 15, -3, -6\rangle| = \sqrt{15^2 + (-3)^2 + (-6)^2} = \sqrt{270} = \sqrt{2 * 5 * 3 * 9} = 3\sqrt{30}$

$A \cdot B = \langle 2, 0, 5\rangle \cdot \langle 1, -3, 4\rangle = (2 \cdot 1) + (0 \cdot -3) + (5 \cdot 4) = (2) + (0) + (20) = 22$

det $\left|\boldsymbol{A} \times \boldsymbol{B}\right| + \boldsymbol{A} \cdot \boldsymbol{B} = 3\sqrt{30} + 22 \approx 38.43$

Problem-1.11 If $\boldsymbol{A} = 4\boldsymbol{a}_x - 6\boldsymbol{a}_y + \boldsymbol{a}_z$ and $\boldsymbol{B} = 2\boldsymbol{a}_x + 5\boldsymbol{a}_z$, find: ##### (a) $A \cdot B + 2|B|^2$ Separate the part and solve independently:

$A \cdot B = \langle 4, -6, 1\rangle \cdot \langle 2, 0, 5\rangle = (4 \cdot 2) + (-6 \cdot 0) + (1 \cdot 5) = 13$

$2|B|^2 = 2\sqrt{2^2 + 5^2}^2 = 2 \cdot 29 = 58$

Then simply add the scalars:

$A \cdot B + 2|B|^2 = 13 + 58 = 71$

(b) A unit vector perpendicular to both A and B. Recall that Cross Products produce a vector that is orthogonal (perpendicular) to both input vectors. Thus, a simple way to find a vector perpendicular to A and B is to take their cross product, then divide by their magnitude to obtain a unit vector in that direction.

First find the Cross Product, representing the vector orthogonal to both A and B…

$A \times B = C = \langle 4, -6, 1\rangle \times \langle 2, 0, 5\rangle = \langle (-6 \cdot 5) - (0 \cdot 1), -(4 \cdot 5) + (2 \cdot 1), (4 \cdot 0) - (-6 \cdot 2)\rangle = \langle -30, -18, -12\rangle$

Next, find the magnitude of that vector…

$|C| = |\langle -30, -18, -12\rangle| = \sqrt{30^2 + 18^2 + 12^2} = \sqrt{1368} = 6\sqrt{38} \approx 36.99$

Finally, divide C by its magnitude to obtain a unit vector with its direction:

$\frac{C}{|C|} = \frac{\langle -30,-18,-12\rangle}{6\sqrt{38}} = \langle \frac{-5}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{-2}{\sqrt{38}}\rangle$

This unit vector is orthogonal to both A and B. Huzzah.

Problem-1.23 Let $A = -3a_x + a_y + 2a_z$, $B = 2a_x - 5a_y + a_z$, $C = a_y + 4a_z$. Determine: ##### (a) The minimum angle between A and B. A simple way to find an angle is to use the Dot Product formula:

$$\boldsymbol{A} \cdot \boldsymbol{B} = AB \cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{AB}$$

$$\theta_{AB} = \arccos \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{AB}$$

Applying this to our problem…

$$\theta_{AB} = \arccos \frac{\langle -3, 1, 2\rangle \cdot \langle 2, -5, 1\rangle}{\sqrt{14} \cdot \sqrt{30}} = arccos \frac{-6-5+2}{2\sqrt{105}} \approx \arccos -\frac{9}{20.49} \approx 63.95°$$

Note that it specifies *minimal* angle, as technically another angle between A and B is 296.05° (if one was to travel in the opposite direction).

(b) The component of A along C. The component of A along C is a scalar representing how much of A is "in-line" with C. As given in the notes, the formula to find this…

$$comp_C A = \frac{A \cdot C}{|C|}$$

$$\frac{\langle \cdot C}{|C|}$$

(c) $D = A + 2B - 3C$**.**

(d) $(A \times B \cdot C)$**.**

Chapter-02: Coordinate Systems and Transformations

Notes

Problems

Chapter-03: Vector Calculus

Notes

Problems

Chapter-04: Electrostatic Fields

Notes

Problems

Chapter-05: Conductors

Notes

Problems

Chapter-09: Maxwell's Equations

Notes

Problems

Project Work

The project mostly extends off Exam-02 and was presented via a powerpoint presentation. Said presentation is printed to PDF and included here, however note that it was done via js-. Additionally, the presentation style was largely focused on how presentations are typically done: low amounts of text, with images and graphics to help aide the presenter rather than deliver the information itself. As such, it does not demonstrate the full knowledge of the situation. Some speaker notes are added to help convey more information, though these too are non-comprehensive.

The video of the presentation, minus some sections, has been uploaded here ().

Slideshow

Done in Reveal JS (Markdown flavored, see Slideshow.md)


Overview
1. Polarization of Light
2. ~~Jones Matrices~~
3. Ethics Case
EE341 - Polarization of Light

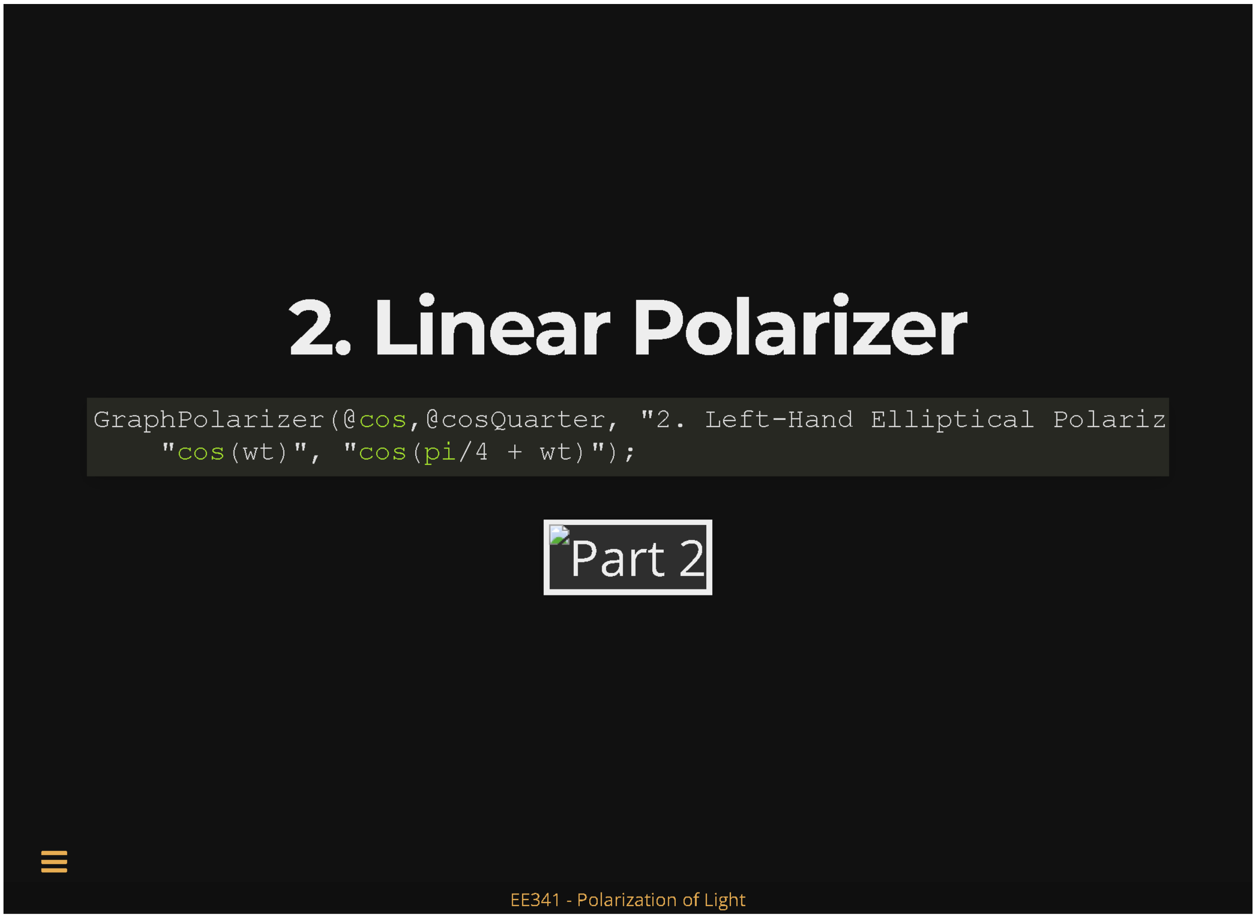

I. Polarization of Light
Light propagates in strange ways, not just along its movement...
EE341 - Polarization of Light

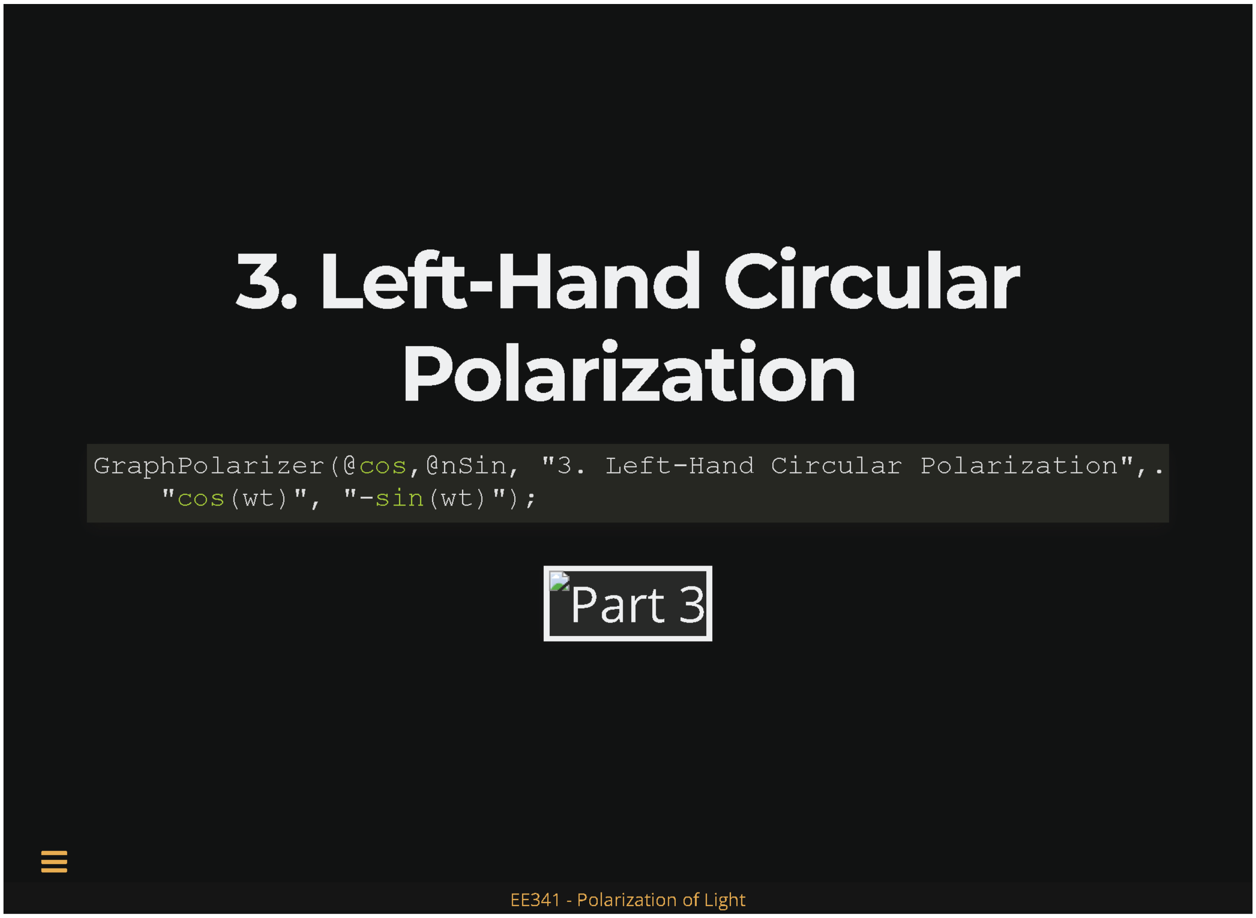
Graphing Code

```
function GraphPolarizer(xFun, yFun, varargin)
    t = 0:0.01:50;
    x=xFun(t);
    y=yFun(t);
    plot(x,y)


    %% Arrow Annotation
    % Dynamically generates directional arrows around the curve.
    for i = 0:0
        %Pt1 = randi(length(t)-1);
        Pt1 = round(length(t)/8);
        if (Pt1 == 0)
            Pt = Pt + 1;
        end
        Pt2 = Pt1 + 1;
```

EE341 - Polarization of Light


1. Linear Polarizer
GraphPolarizer(@cos,@cos,"1. Linear Polarizer at 45 Degrees");
Part 1
EE341 - Polarization of Light


2. Linear Polarizer
GraphPolarizer(@cos,@cosQuarter, "2. Left-Hand Elliptical Polariz
"cos(wt)", "cos(pi/4 + wt)");
Part 2
EE341 - Polarization of Light


3. Left-Hand Circular Polarization
GraphPolarizer(@cos,@nSin, "3. Left-Hand Circular Polarization",.
"cos(wt)", "-sin(wt)");
Part 3
EE341 - Polarization of Light


4. Left-Hand Elliptical Polarization at -45
GraphPolarizer(@cos,@cosTripleQuarter, "4. Left-Hand Elliptical P
"cos(wt)", "cos(wt + 3pi/4)");
Part 4
EE341 - Polarization of Light

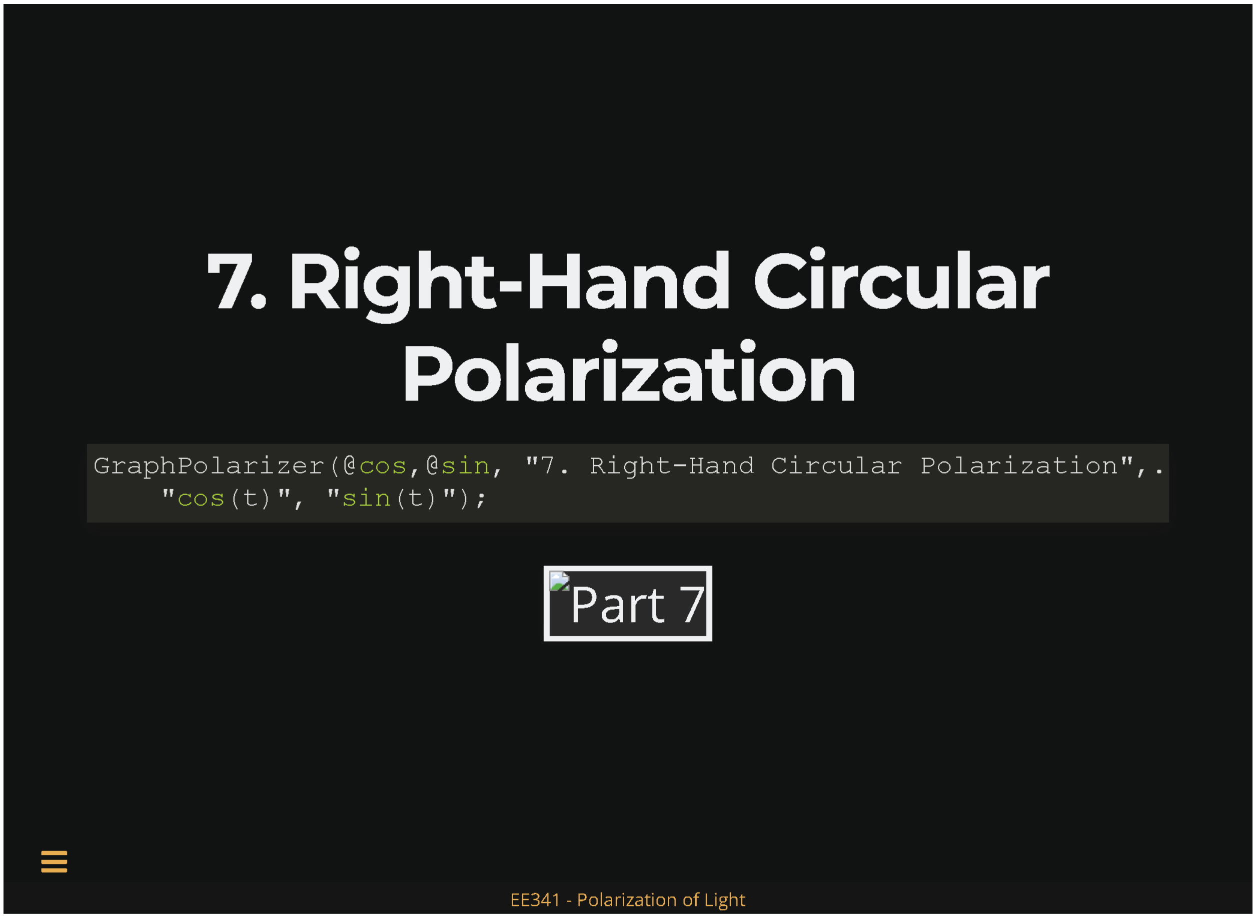
5. Linear Polarization at -45

```
GraphPolarizer(@cos,@nCos, "5. Linear Polarization at -45",...
    "cos(t)", "-cos(t)");
```

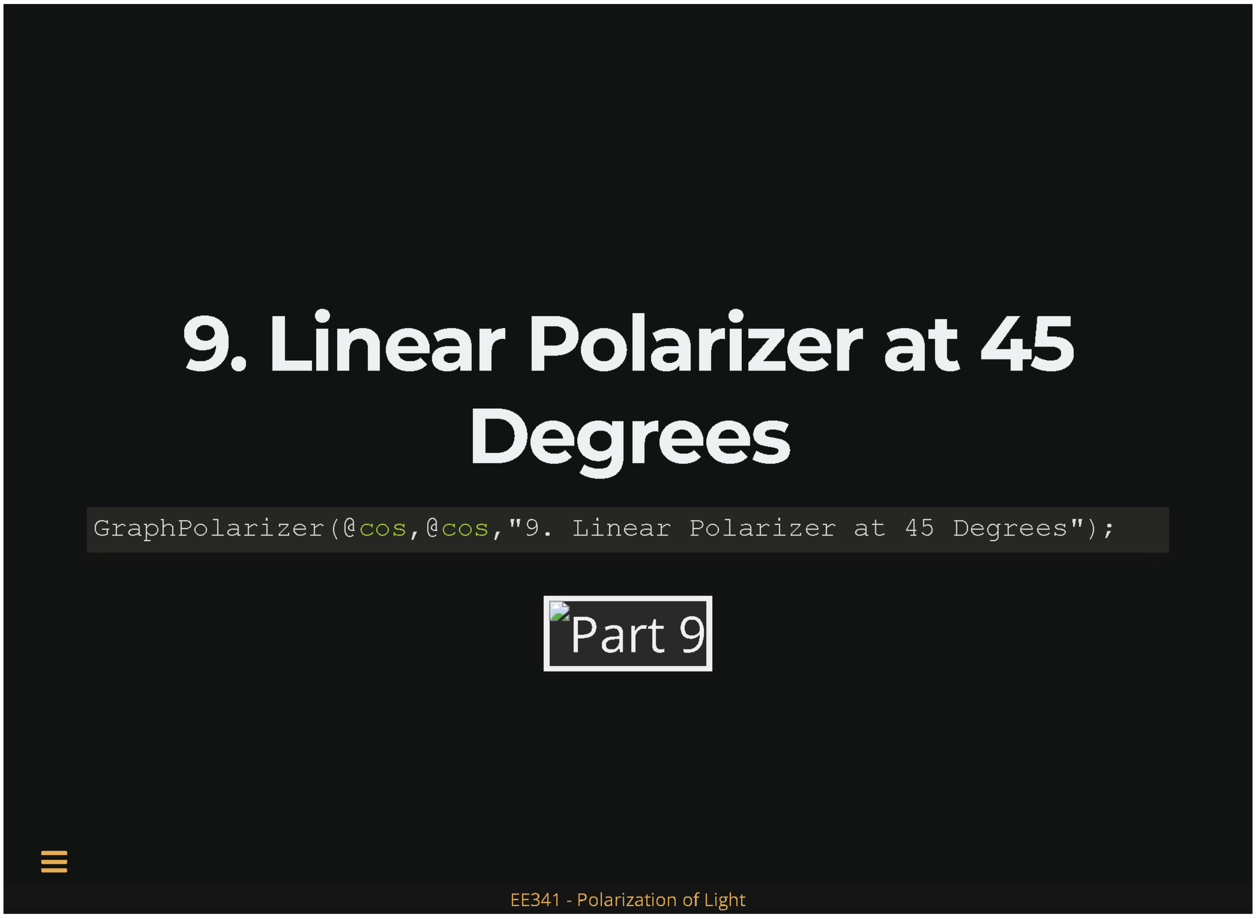
Part 5

EE341 - Polarization of Light


6. Right-Hand Elliptical Polarization at -45
GraphPolarizer(@cos,@nCosQuarter, "6. Right-Hand Elliptical Polar
"cos(t)", "-cos(t + pi/4)");
Part 6
EE341 - Polarization of Light

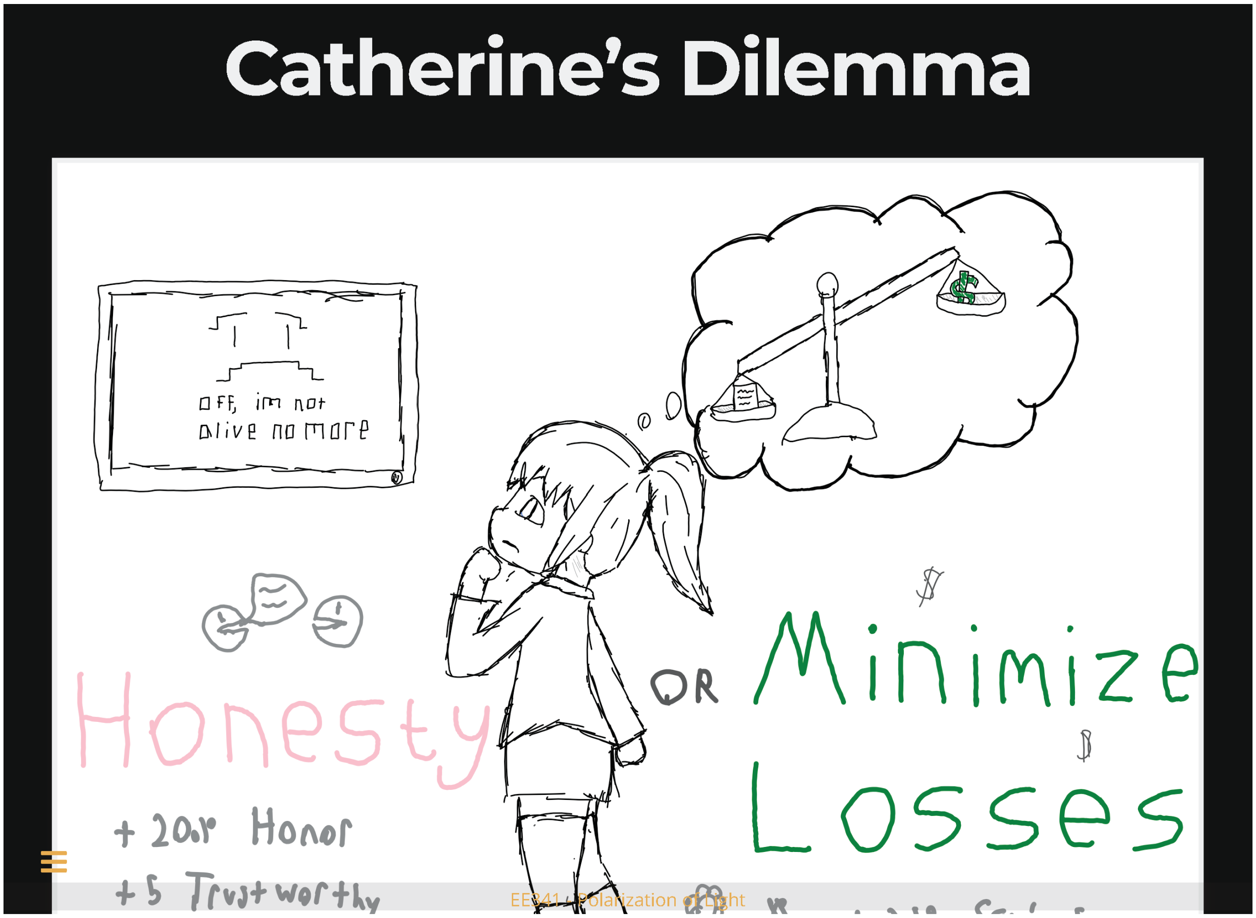

7. Right-Hand Circular Polarization
GraphPolarizer(@cos,@sin, "7. Right-Hand Circular Polarization",.
"cos(t)", "sin(t)");
Part 7
EE341 - Polarization of Light


8. Right-Hand Elliptical Polarization at 45
GraphPolarizer(@cos,@nCosTripleQuarter, "8. Right-Hand Elliptical
"cos(wt)", "-cos(wt + 3pi/4)");
Part 8
EE341 - Polarization of Light


9. Linear Polarizer at 45 Degrees
GraphPolarizer(@cos,@cos,"9. Linear Polarizer at 45 Degrees");
Part 9
EE341 - Polarization of Light

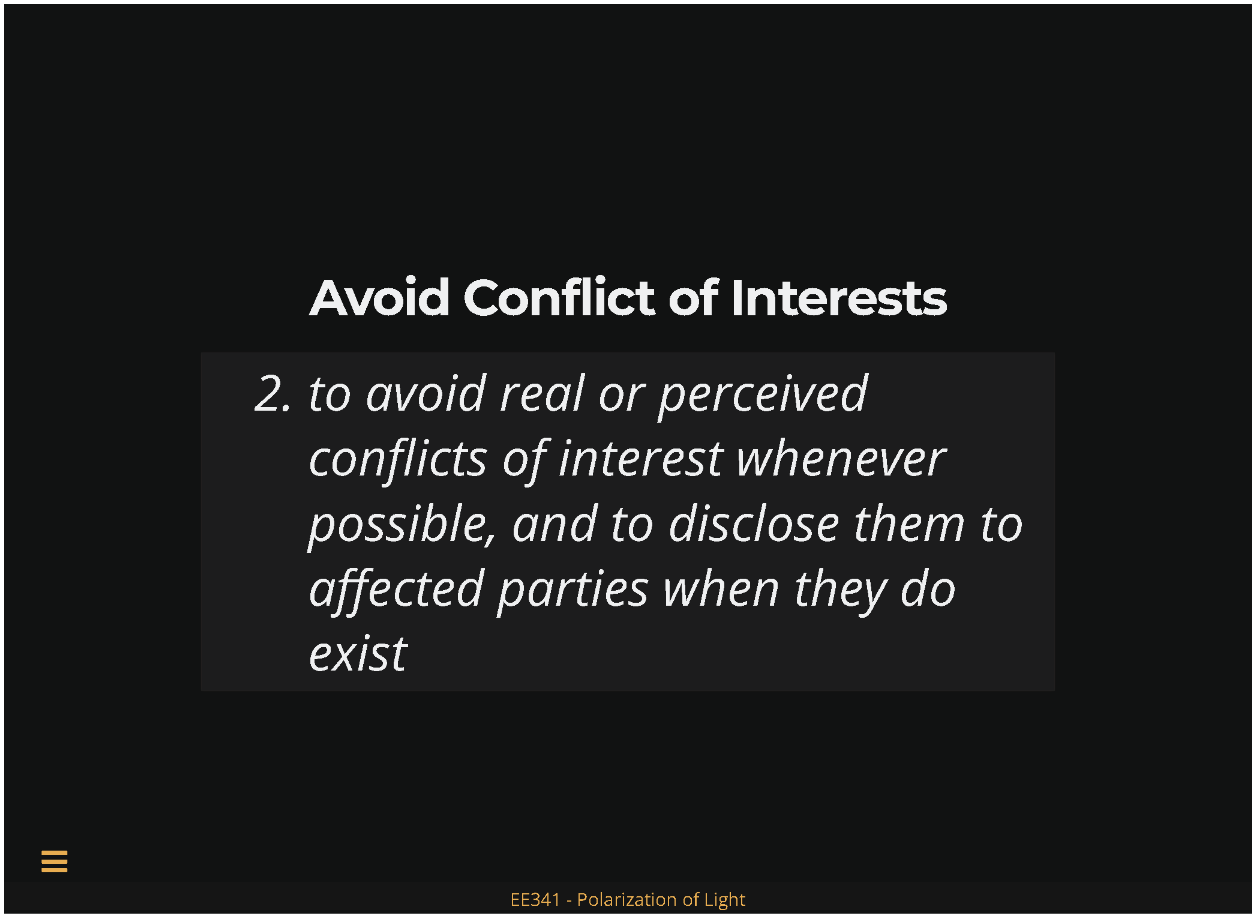

II. Ethics Case Study
"And now for something completely different!" - And Now for Something Completely Different
EE341 - Polarization of Light

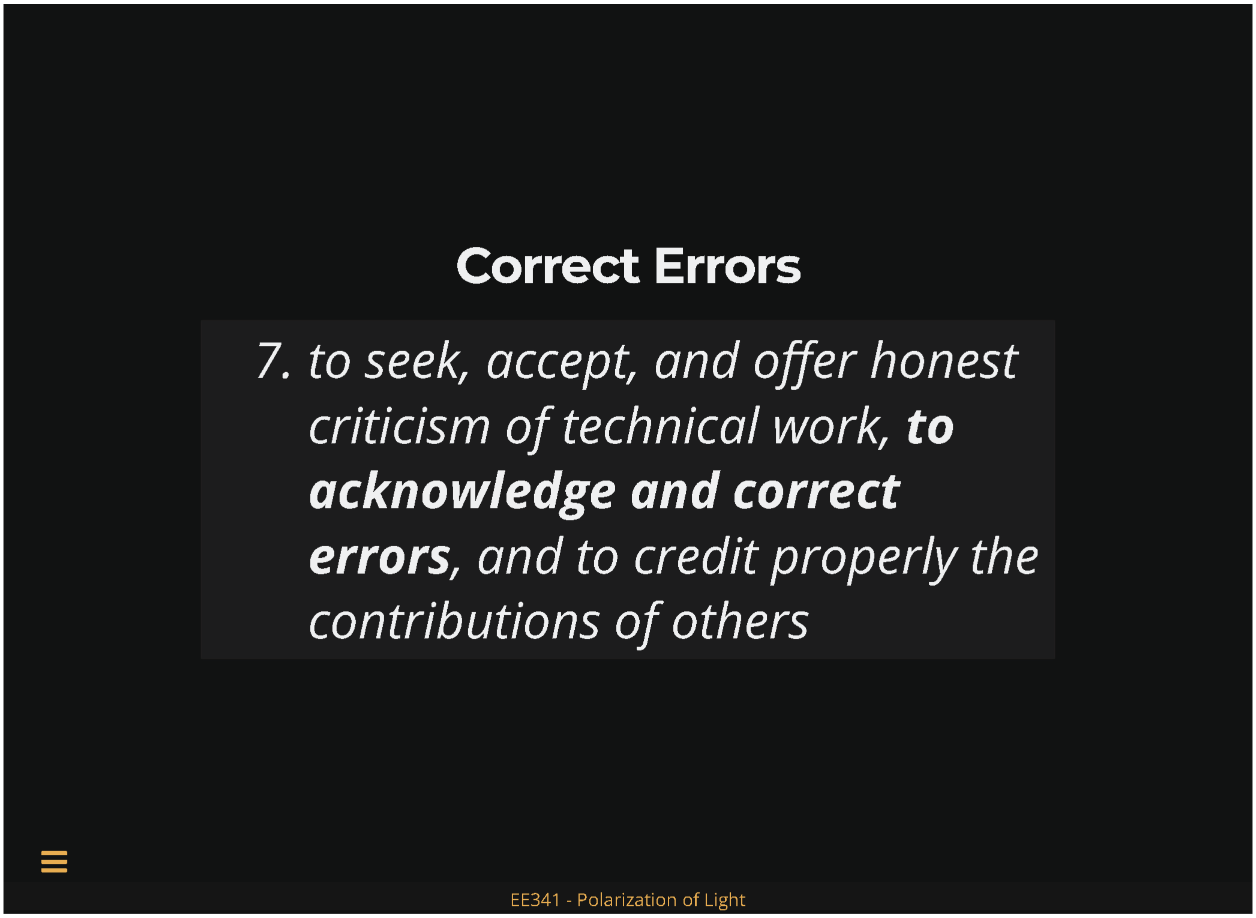

Catherine's Dilemma
Be Honest
Tell the customer that the mistake was on her own company and the units can be replaced for free.
Lie
Report the units as incorrectly installed or such to warrant a full re-purchase.
EE341 - Polarization of Light


Catherine's Dilemma
off, im not
alive no more
Honesty
OR
Minimize
Losses
+ 20or Honor
+ 5 Trustworthy
EE341 - Polarization of Light

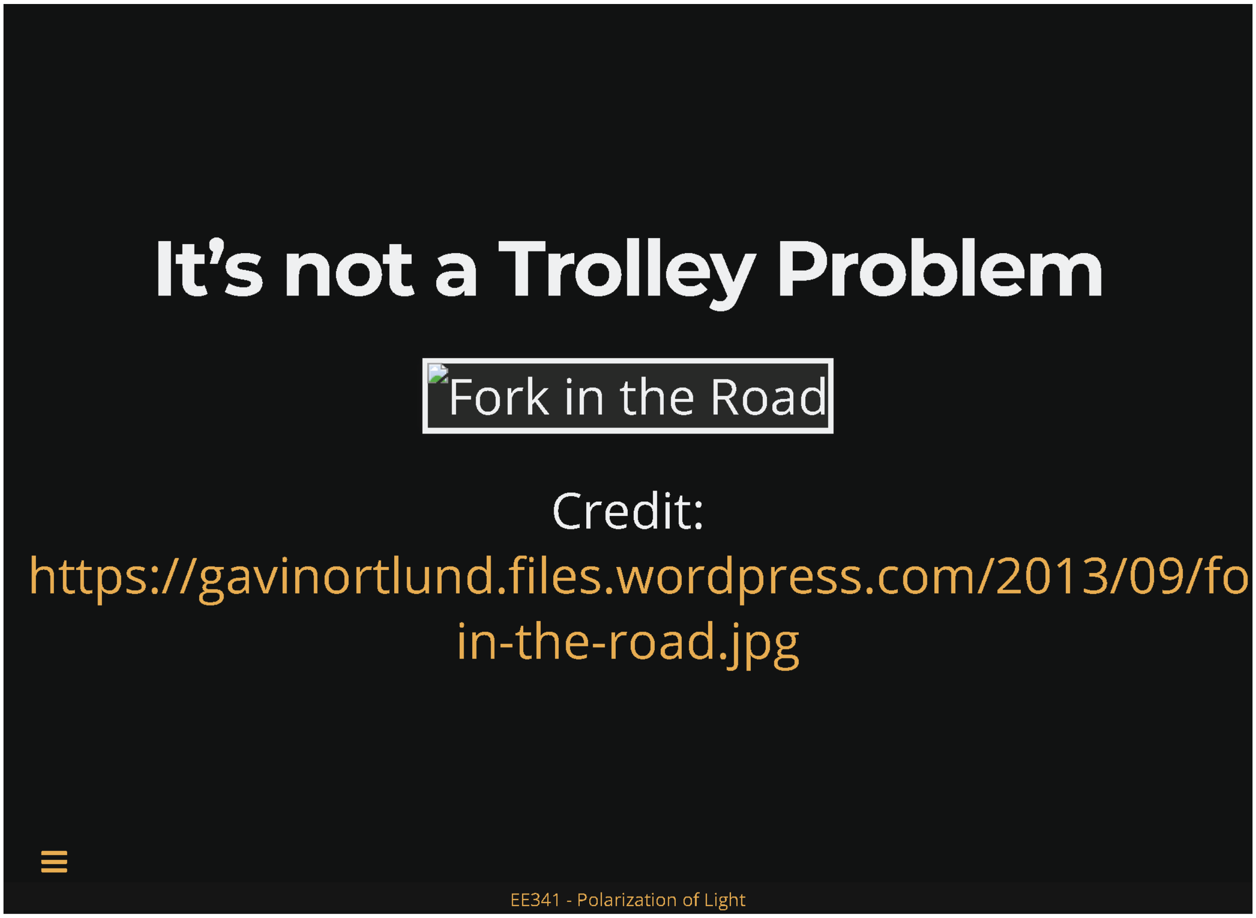

-40 Mistake
+ $7000
- Have to live with yourself
EE341 - Polarization of Light

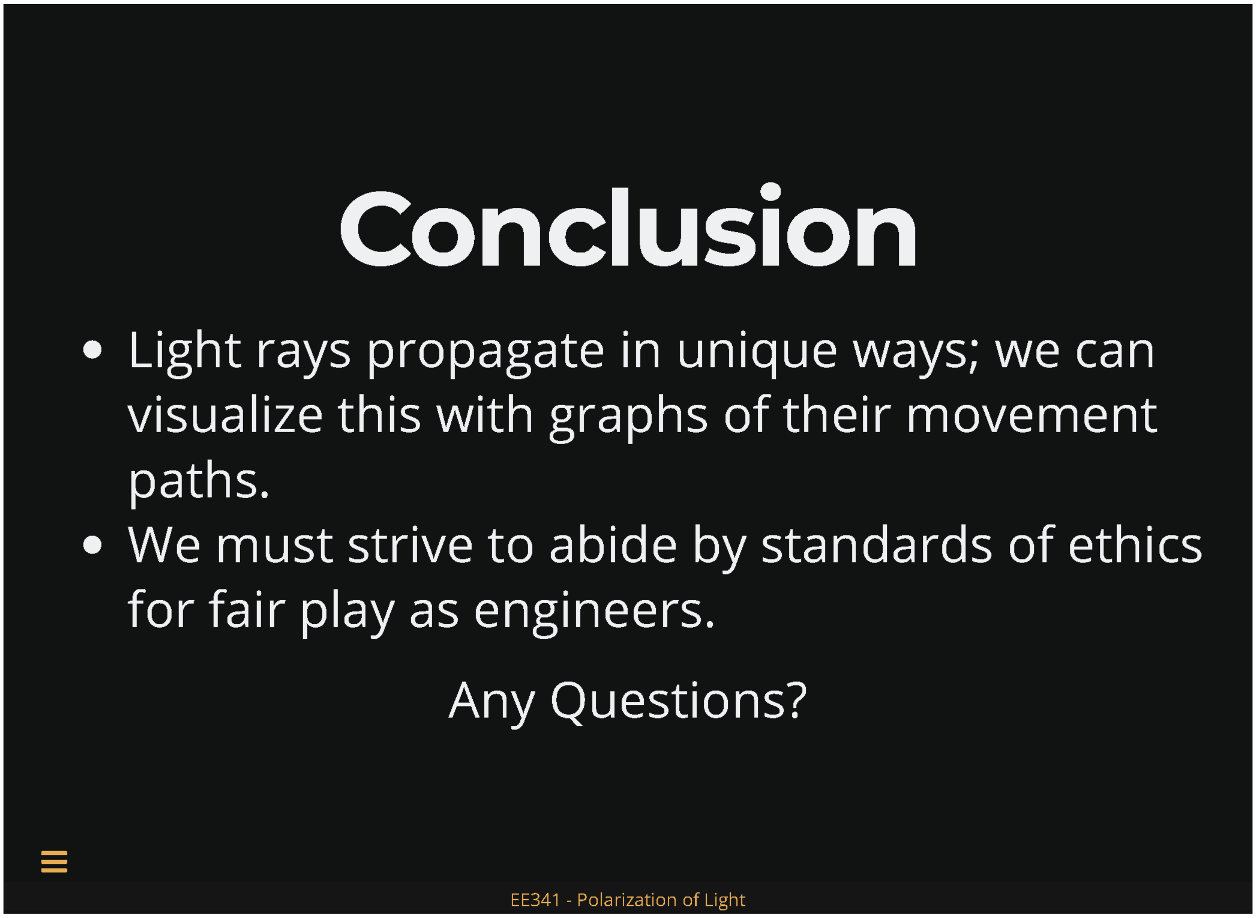

Tough Moral Dilemma?
From the IEEE Code of Ethics...
EE341 - Polarization of Light


Avoid Conflict of Interests
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist
EE341 - Polarization of Light


Correct Errors
7. to seek, accept, and offer honest criticism of technical work, **to acknowledge and correct errors**, and to credit properly the contributions of others
EE341 - Polarization of Light


Avoid Injuries
9. to avoid injuring others, their property, reputation, or employment by false or malicious action
EE341 - Polarization of Light


It’s not a Trolley Problem
Fork in the Road
Credit:
https://gavinortlund.files.wordpress.com/2013/09/fo
in-the-road.jpg
EE341 - Polarization of Light


Conclusion
Light rays propagate in unique ways; we can visualize this with graphs of their movement paths.
We must strive to abide by standards of ethics for fair play as engineers.
Any Questions?
EE341 - Polarization of Light

Credit

...where credit's due.

- NowForSomethingCompletelyDifferent: https://en.wikipedia.org/wiki/And_Now_for_Som
- Stronghold: https://en.wikipedia.org/wiki/Strongh
- TwoRoadsDiverged: https://gavinortlund.files.wo in-the-road.jpg
- PolarizationOfLight: https://www.physicsclassroo 1/Polarization
- EthicalCaseStudy: https://www.scu.edu/ethics/fo areas/more/engineering-ethics/engineering-ethi with-you/

EE341 - Polarization of Light

Exams

E01

Not included in this report.

E02

Original MATLAB Code

Test 02: Light Polarization, Draft I

Elijah T. Rose (elirose)

March 11, 2020

Unfortunately, I was not able to internalize polarities and these problems within 2 days. I will continue working on it and attempt to turn in another revision after spring break – afterall, this is a learning assessment, not a punitive one.

Function File

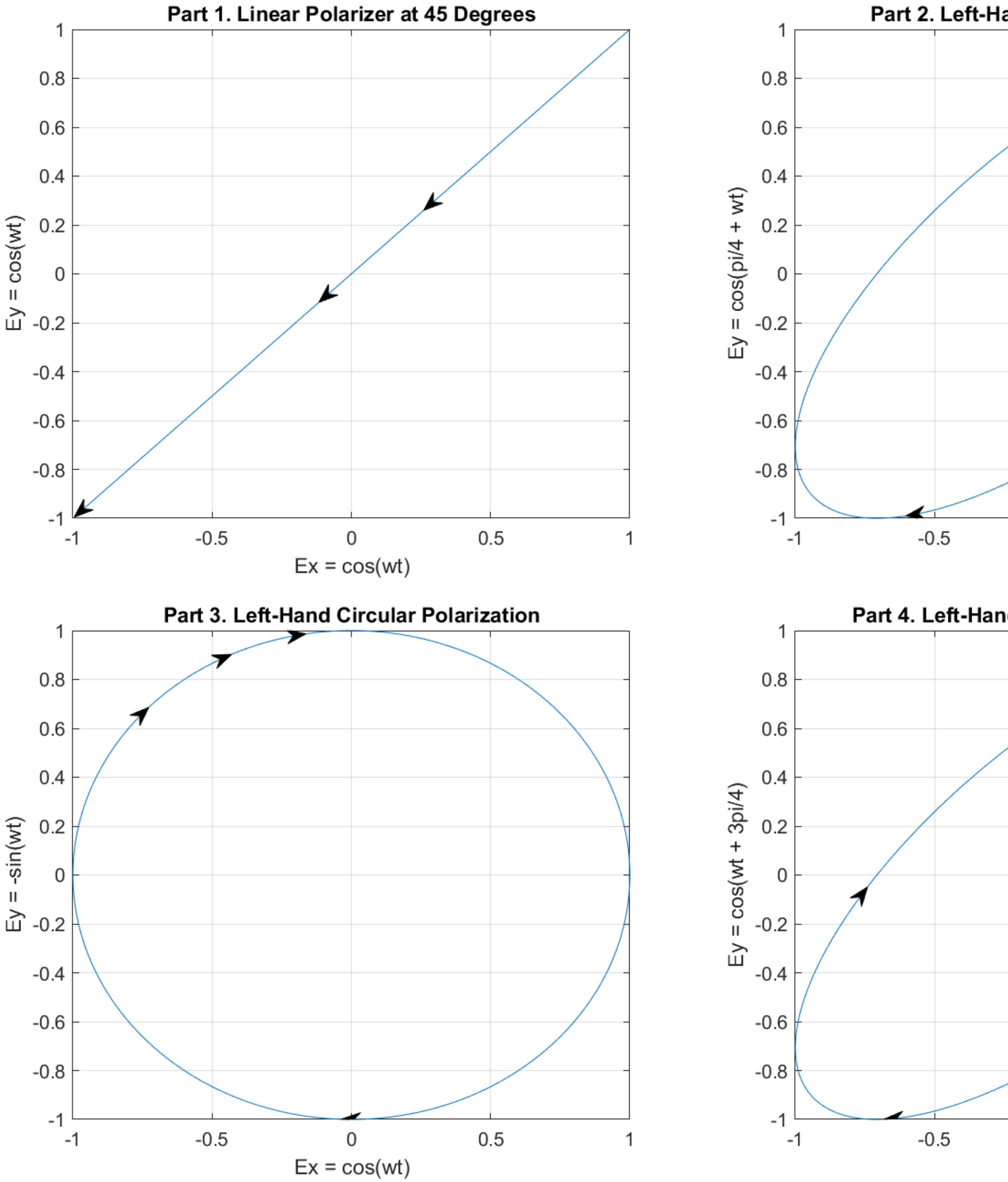
```
function GraphPolarizer(xFun, yFun, varargin)
    t = 0:0.01:50;
    x=xFun(t);
    y=yFun(t);
    plot(x,y)


    %% Arrow Annotation
    % Dynamically generates directional arrows around the curve.
    for i = 0:0
        %Pt1 = randi(length(t)-1);
        Pt1 = round(length(t)/8);
        if (Pt1 == 0)
            Pt = Pt + 1;
        end
        Pt2 = Pt1 + 1;
        Pa = [x(Pt1) y(Pt1)];
        Pb = [x(Pt2) y(Pt2)];
        Vba = Pb - Pa;
        a = annotation('arrow', [0 0], [0 0]);
        set(a, 'parent', gca);
        set(a, 'position', [Pa(1) Pa(2) Vba(1) Vba(2)])
    end

    %% Labels Galore
    % Labelling things using the variables.
    grid
    narg = length(varargin);
    title("Part 1");
    xlabel("Ex = "+func2str(xFun)+"(wt)");
    ylabel("Ey = "+func2str(yFun)+"(wt)");
    switch narg
        case 1
            title("Part " + varargin(1));
        case 2
            title("Part " + varargin(1));
            xlabel("Ex = "+ varargin(2));
        case 3
            title("Part " + varargin(1));
```

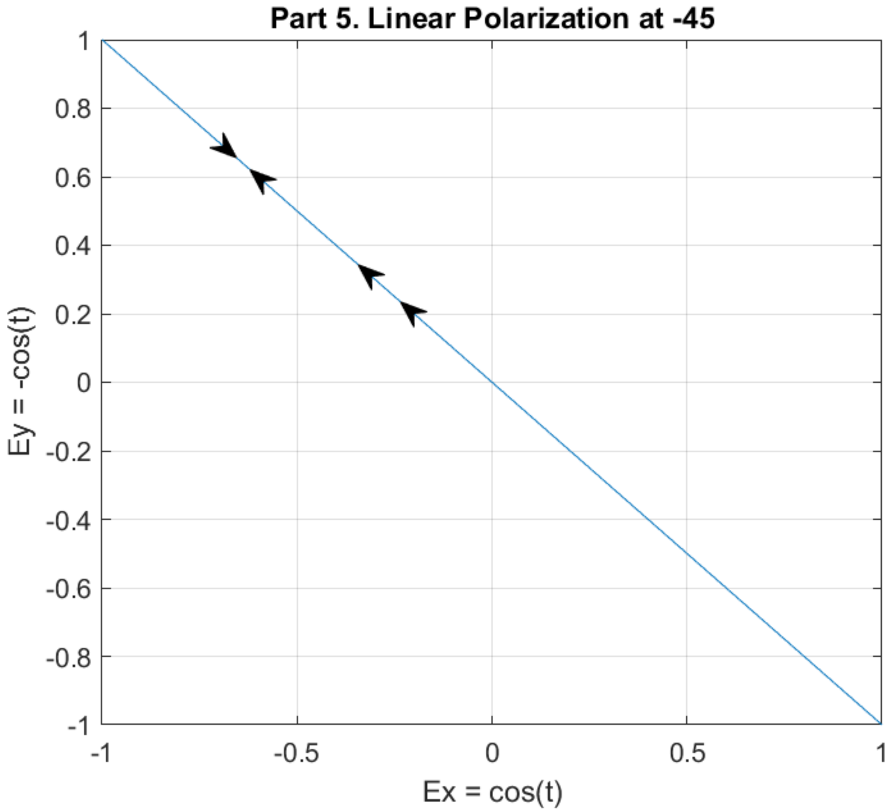
```
            xlabel("Ex = "+ varargin(2));
            ylabel("Ey = "+ varargin(3));
    end

    %% Saving
    persistent callCount
    if isempty(callCount)
        callCount = 0;
    end
    callCount = callCount + 1;

    saveas(gcf, "Part_" + int2str(callCount) + ".png");
end

%% Mistakes
    % And this is why I bloody HATE MatLab. Look at this absolute
    % NIGHTMARE to get an frickin arrow to show up because lord forbid
    % they allow annotation to accept real coordinates or quiver to have
    % adjustable graphics. Good grief. Thanks to marsei
    % https://stackoverflow.com/questions/18776172/...
    % in-matlab-how-do-i-change-the-arrow-head-style-in-quiver-plot
    % for this unholy abdomination of a workaround.

%      xmin = min(x)
%      xmax = max(x)
%      xrange = xmax - xmin
%      ymin = min(y)
%      ymax = max(y)
%      yrange = ymax - ymin
%      Pan = [(Pa(1)-xmin)/xrange (Pa(2)-ymin)/yrange]
%      Pbn = [(Pb(1)-xmin)/xrange (Pb(2)-ymin)/yrange]
%      quiver(Pa(1), Pa(2), Vba(1), Vba(2), 0, 'MaxHeadSize',50);
%      ah = annotation('arrow','headStyle','cback1','HeadLength',...
%           headLength,'HeadWidth',headWidth);
%      set(ah,'parent',gca);
```

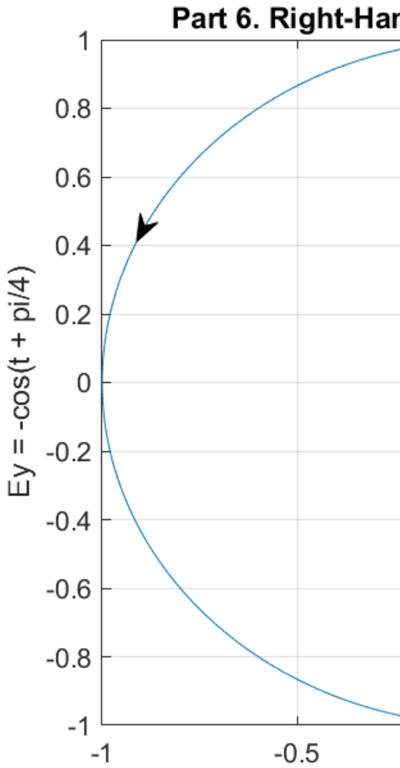
Running File

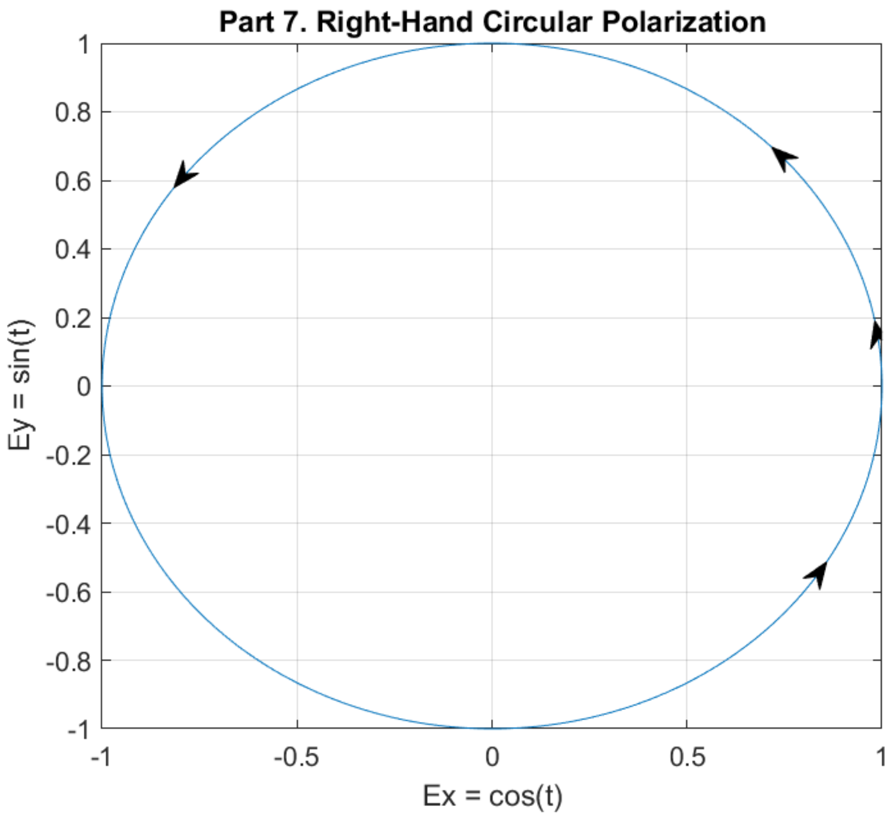
```
clear
clc

GraphPolarizer(@cos,@cos,"1. Linear Polarizer at 45 Degrees");


GraphPolarizer(@cos,@cosQuarter, "2. Left-Hand Elliptical Polarizer at 45",...
    "cos(wt)", "cos(pi/4 + wt)");

GraphPolarizer(@cos,@nSin, "3. Left-Hand Circular Polarization",...
    "cos(wt)", "-sin(wt)");


GraphPolarizer(@cos,@cosTripleQuarter, "4. Left-Hand Elliptical Polarization at -45",...
    "cos(wt)", "cos(wt + 3pi/4)");
```

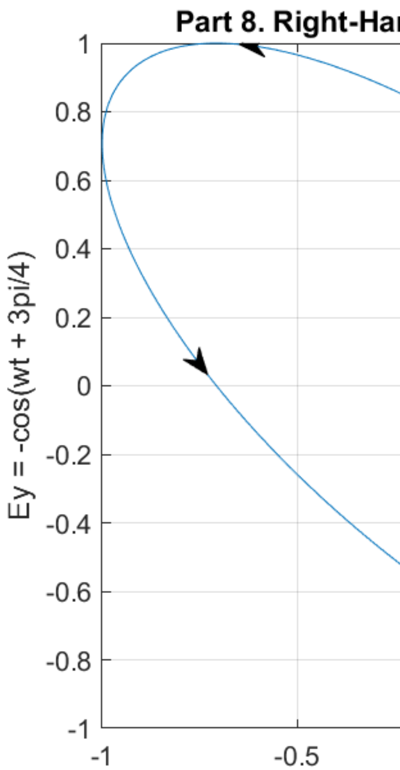
```
% Q: Why do we use wt above, thent here?
GraphPolarizer(@cos,@nCos, "5. Linear Polarization at -45",...
    "cos(t)", "-cos(t)");

GraphPolarizer(@cos,@nCosQuarter, "6. Right-Hand Elliptical Polarization at -45",...
    "cos(t)", "-cos(t + pi/4)");


GraphPolarizer(@cos,@sin, "7. Right-Hand Circular Polarization",...
    "cos(t)", "sin(t)");

GraphPolarizer(@cos,@nCosTripleQuarter, "8. Right-Hand Elliptical Polarization at 45",...
    "cos(wt)", "-cos(wt + 3pi/4)");

GraphPolarizer(@cos,@cos,"9. Linear Polarizer at 45 Degrees");

function result = cosQuarter(t)
     result = cos(t + pi/4);
end

function result = nSin(t)
    result = -sin(t);
end

function result = cosTripleQuarter(t)
     result = cos(t + pi/4);
end

function result = nCos(t)
    result = -cos(t);
end

function result = nCosQuarter(t)
     result = -cosQuarter(t + pi/4);
end

function result = nCosTripleQuarter(t)
     result = -cosTripleQuarter(t);
end
```

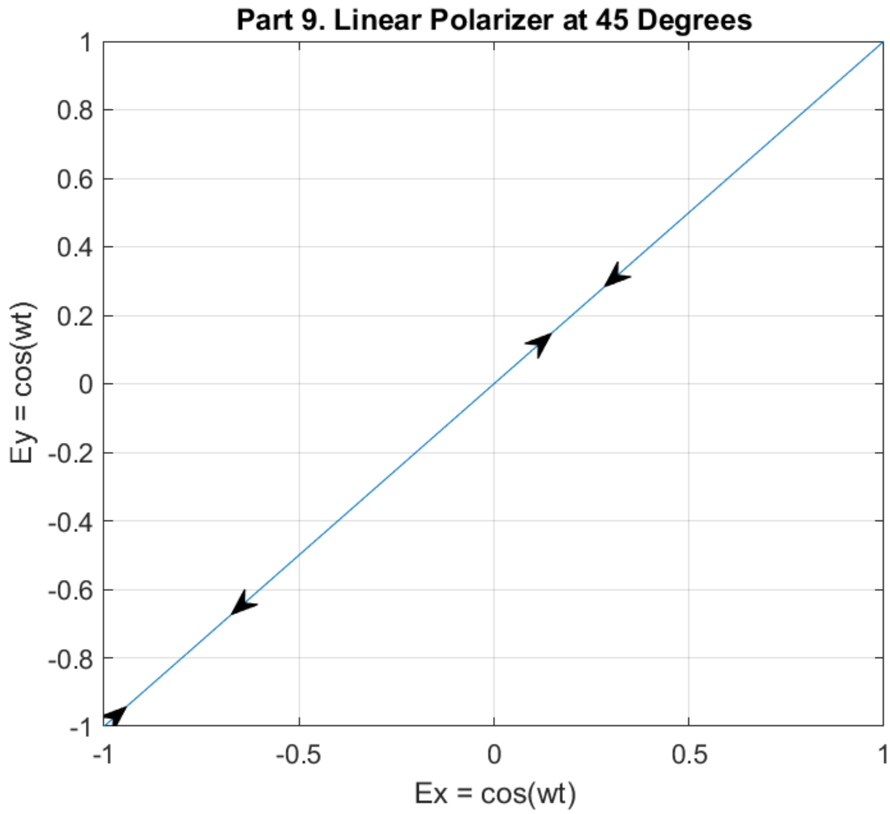

Part 1. Linear Polarizer at 45 Degrees
Ey = cos(wt)
Ex = cos(wt)
Part 2. Left-Ha
Ey = cos(pi/4 + wt)
Part 3. Left-Hand Circular Polarization
Ey = -sin(wt)
Ex = cos(wt)
Part 4. Left-Hand
Ey = cos(wt + 3pi/4)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0.5


Part 5. Linear Polarization at -45
Ey = -cos(t)
Ex = cos(t)


Part 6. Right-Har
Ey = -cos(t + pi/4)


Part 7. Right-Hand Circular Polarization
Ey = sin(t)
Ex = cos(t)


Part 8. Right-Har
Ey = -cos(wt + 3pi/4)


Part 9. Linear Polarizer at 45 Degrees
Ey = cos(wt)
Ex = cos(wt)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

Improved MATLAB Code

Appendix

Acknowledgements