Servando Olvera

1. $y[n] = x[n] - 2$

- $y_1[n] = x_1[n] - 2$
- $y_2[n] = x_2[n] - 2$

- $x_3[n] = ax_1[n] + bx_2[n]$

- $y_3[n] = x_3[n] - 2$
  $y_3[n] = [ax_1[n] + bx_2[n]] - 2$

- $ay_1[n] + by_2[n] = a[x_1[n]-2] + b[x_2[n]-2]$
  $= ax_1[n] - 2a + bx_2[n] - 2b$

$ax_1[n] + bx_2[n] - 2 \neq ax_1[n] - 2a + bx_2[n] - 2b$

$\boxed{\text{NOT LINEAR}}$

* $y[n-n_0] = x[n-n_0] - 2$

* $x[n-n_0] \rightarrow y_2[n] = x[n-n_0] - 2$

$y_2[n] = y[n-n_0]$

$\boxed{\text{Shift-Invariant}}$

2. $y[n] = (x[n])^2 - 2x[n]$

- $y_1[n] = (x_1[n])^2 - 2x_1[n]$
- $y_2[n] = (x_2[n])^2 - 2x_2[n]$

- $x_3[n] = ax_1[n] + bx_2[n]$

- $y_3[n] = (x_3[n])^2 - 2x_3[n]$
  $y_3[n] = (ax_1[n] + bx_2[n])^2 - 2[ax_1[n] + bx_2[n]]$

$ay_1[n] + by_2[n] = a[(x_1[n])^2 - 2x_1[n]] + b[(x_2[n])^2 - 2x_2[n]]$
$= a(x_1[n])^2 - 2ax_1[n] + b(x_2[n])^2 - 2bx_2[n]$

$(ax_1[n])^2 + 2(ax_1[n] \cdot bx_2[n]) + (bx_2[n])^2 - 2ax_1[n] - 2bx_2[n] \neq a(x_1[n])^2 - 2ax_1[n] + b(x_2[n])^2 - 2bx_2[n]$

$\boxed{\text{NOT LINEAR}}$

* $y[n-n_0] = (x[n-n_0])^2 - 2x[n-n_0]$

* $x[n-n_0] \Rightarrow y_2[n] = (x[n-n_0])^2 - 2x[n-n_0]$

$y_2[n] = y[n-n_0]$

$(x[n-n_0])^2 - 2x[n-n_0] = (x[n-n_0])^2 - 2x[n-n_0]$

$\boxed{\text{Shift-Invariant}}$

3. $y[n] = 2x[n] + x[n-1] - 3x[n-2]$

- $y_1[n] = 2x_1[n] + x_1[n-1] - 3x_1[n-2]$
- $y_2[n] = 2x_2[n] + x_2[n-1] - 3x_2[n-2]$

- $x_3[n = ax_1[n] + bx_2[n]$

- $y_3[n] = 2x_3[n] + x_3[n-1] - 3x_3[n-2]$
  $y_3[n] = 2[ax_1[n] + bx_2[n]] + [ax_1[n-1) + bx_2[n-1] - 3[ax_1[n-2] + bx_2[n-2]]$
  $2ax_1[n] + 2bx_2[n] + ax_1[n-1] + bx_2[n-1) - 3ax_1[n-2] - 3bx_2[n-2]$

Equivalent

- $ay_1[n] + by_2[n] = a[2x_1[n] + x_1[n-1] - 3x_1[n-2]] + b[2x_2[n] + x_2[n-1] - 3x_2[n-2]]$
  $= 2ax_1[n] + ax_1[n-1] - 3ax_1[n-2] + 2bx_2[n] + bx_2[n-1] - 3bx_2[n-2]$

$\boxed{\text{LINEAR}}$

☆ $y[n-n_0] = 2x[n-n_0] + x[n-n_0-1] - 3x[n-n_0-2]$

☆ $x[n-n_0] \rightarrow y_1[n] = 2x[(n-n_0)] + x[(n-n_0)-1] - 3x[n-n_0-2]$

$y_1 = y[n-n_0]$ $\boxed{\text{Shift-Invariant}}$

4.

$$y[n] = \sum_{k=-10}^{0} (x[n] - 2x[n-1])$$

• $y_1[n] = \sum_{n=-10}^{0} (x_1[n] - 2x_1[n-1])$

∘ $y_2[n] = \sum_{n=-10}^{0} (x_2[n] - 2x_2[n-1])$

• $x_3[n] = ax_1[n] + bx_2[n]$

$$y_3[n] = \sum_{-10}^{0} (x_3[n] - 2x_3[n-1])$$

$$= \sum_{-10}^{0} (ax_1[n] + bx_2[n])] - 2[ax_1[n-1] + bx_2[n-1]]$$

$$= a\sum_{-10}^{0} (x_1[n] - 2x_1[n-1]) + b\sum_{-10}^{0} (x_2[n] - 2x[n-1])$$

• $ay_1[n] + by_2[n] = a\sum_{-10}^{0} (x_1 - 2x[n-1] + b\sum_{-10}^{0} (x_2 - 2x(n-1])$

$\boxed{\text{Linear}}$

$* \; y[n-n_0] = \sum_{-10}^{0} (x[n-n_0] - 2x[n-n_0-1]$

$x[n-n_0] \rightarrow y_1[n] = \sum_{-10}^{0} (x[n-n_0] - 2x[n-n_0-1]$

$y_1[n] = y[n-n_0]$

$\boxed{\text{Shift-Invariant}}$

5.

$$S = \sum_{n=-\infty}^{\infty} \left(\frac{5}{3}\right)^n u[n]$$

$S = \sum_{0}^{\infty} \left(\frac{5}{3}\right)^n \Rightarrow$ $\frac{5}{3} > 1$ $\boxed{\text{No closed form}}$

6.

$$S = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[-n]$$

$S = \sum_{-\infty}^{0} \left(\frac{1}{2}\right)^n = \sum_{\infty}^{0} \left(\frac{1}{2}\right)^{-m} \Rightarrow \sum_{0}^{\infty} (2)^n \Rightarrow$ $2 > 1$

$m = -n \quad n = -m$

$\boxed{\text{No closed form Solution}}$

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$$S = \sum_{n=-\infty}^{\infty} \left(3e^{-j\omega}\right)^n u[-n]$$

$$S = \sum_{n=-\infty}^{\infty} \left(3e^{-j\omega}\right)^n u[-n]$$

$$\frac{1}{3} \cdot |\cancel{e^{j\omega}}^{1}| < 1$$

$$S = \sum_{n=-\infty}^{0} \left(3e^{-j\omega}\right)^n \Rightarrow \sum_{0}^{\infty} \left(\frac{1}{3}e^{j\omega}\right)^m = \boxed{\frac{1}{1-\left(\frac{1}{3}e^{j\omega}\right)}}$$

$m=-n,\ n=-m$

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$$S = \sum_{n=0}^{3} e^{j\frac{\pi}{8}n} = e^{0} + e^{j\frac{\pi}{8}} + e^{j\frac{2\pi}{8}} + e^{j\frac{3\pi}{8}}$$

$$\boxed{= 1 + e^{j\frac{\pi}{8}} + e^{j\frac{\pi}{4}} + e^{j\frac{3\pi}{8}}}$$

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$$h[n] = \left(\frac{1}{2}e^{j\omega}\right)^n u[n]$$

Looking at $\sum_{0}^{\infty} \left(\frac{1}{2}e^{j\omega}\right)^n = \frac{1}{1-\frac{1}{2}e^{j\omega}}$, we can observe a closed form solution, therefore the system would be $\boxed{\text{Stable}}$

$$\left.\begin{aligned} h[0] &= \left(\tfrac{1}{2}e^{j\omega}\right)^0 \cdot u[0] \\ h[-1] &= \left(\tfrac{1}{2}e^{j\omega}\right)^{-1} \cdot u[-1] \\ h[1] &= \left(\tfrac{1}{2}e^{j\omega}\right)^{1} \cdot u[1] \end{aligned}\right\}$$

System depends on current inputs only, so system is $\boxed{\text{CAUSAL}}$

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$$h[n] = \cos(\omega n)u[-n-1]$$

$$\left.\begin{aligned} h[0] &= \cos(0)\, u[-1] \\ h[1] &= \cos(\omega)\, u[-2] \end{aligned}\right\}$$

$$h[-1] = \cos(-\omega)\, u[0]$$

at n=-1, System depends on future outputs so system is $\boxed{\text{NON causal}}$

$$h[n] = \cos(\omega n)\, u[-n-1]$$

Since System is being multiplied by cosine, which is bounded from -1 to 1, is is fair to say that system is

$\boxed{\text{STABLE}}$

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$h[n] = u[-n]$

$h[0] = u[0]$
$h[1] = u[-1]$
$h[-1] = u[1]$

System is $\boxed{\text{NON-CAUSAL}}$ since output requires a future output value.

$$\sum_{-\infty}^{\infty} u[-n] \Rightarrow \sum_{-\infty}^{0} 1 = \sum_{0}^{\infty} 1$$

No closed form Solution, Not summable. System is

$\boxed{\text{UNSTABLE}}$

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$h[n] = \left(\frac{1}{2}e^{j\omega}\right)^n u[-n]$

$$\sum_{-\infty}^{0}\left(\tfrac{1}{2}e^{j\omega}\right)^n \Rightarrow \sum_{0}^{\infty}\left(2e^{-j\omega}\right)^m$$

$2>1$, No closed form solution. System is $\boxed{\text{UNSTABLE}}$

$h[0] = \left(\frac{1}{2}e^{j\omega}\right)^0 \cdot u[0]$
$h[1] = \left(\frac{1}{2}e^{j\omega}\right)^1 \cdot u[-1]$
$h[-1] = \left(\frac{1}{2}e^{j\omega}\right)^{-1} \cdot u[1]$

System is $\boxed{\text{NON-CAUSAL}}$ since output requires a future output value.

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$x[n] = \delta[n-3]$, $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$y[n] = \cancel{\sum_{k=-\infty}^{\infty}} \left(\cancel{\delta[k-3]}\right)\left(\left(\tfrac{1}{2}\right)^{n-k} u[n-k]\right) \Rightarrow \left(\tfrac{1}{2}\right)^{n-3} \cdot u[n-3]$$

$k=3$

$$\boxed{y[n] = \left(\tfrac{1}{2}\right)^{n-3} u[n-3]}$$

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$x[n] = \delta[n+1]$, $h[n] = \sin(5\omega n)$

$$y[n] = \cancel{\sum_{k=-\infty}^{\infty}} \left(\cancel{\delta[k+1]}\right)\left(\sin(5\omega(n-k))\right) \Rightarrow \sin(5\omega(n+1))$$

$k=-1$

$$\boxed{y[n] = \sin(5\omega(n+1))}$$

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$x[n] = 2^n u[-n-1]$, $h[n] = \delta[n-1]$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$\cancel{\sum_{k=-\infty}^{\infty}} \left(\cancel{\delta[k-1]}\right)\left(2^{n-k} u[(-n-k)-1]\right) = 2^{n-1} u[(-n-1)-1]$$

$k=1$

$$\boxed{y[n] = 2^{n-1} u[-n-2]}$$

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$x[n] = u[n]$, $h[n] = \left(\frac{1}{3}e^{j\omega}\right)^n$

$\left| \frac{1}{3} \cdot \cancel{|e^{j\omega}|}^{\,1} \right| < 1$

$$y[n] = \sum_{k=-\infty}^{\infty} (u[k]) \left[\left(\frac{1}{3} e^{j\omega} \right)^{n-k} \right] \Rightarrow \sum_{k=0}^{\infty} \left(\frac{1}{3} e^{j\omega} \right)^{n}$$

$$\Rightarrow \frac{1}{1 - \left(\frac{1}{3} \cancel{e^{j\omega}}^{\,1}\right)}$$

$$\Rightarrow \frac{1}{1 - \frac{1}{3}}$$

$$= \frac{1}{2/3}$$

$$\boxed{y[n] \Rightarrow \frac{2}{3}}$$