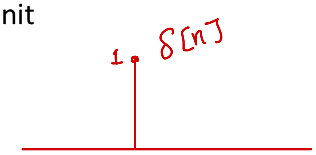
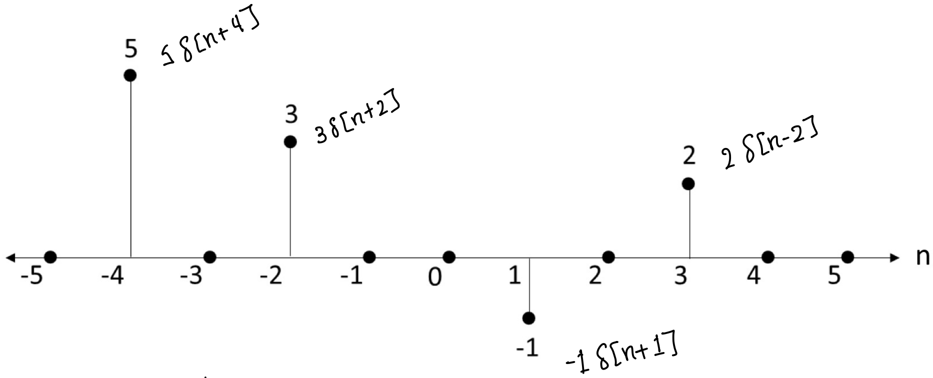
Servando Olvera

Problem 1.1

A sequence $x[n]$ is shown below. Express $x[n]$ as a linear combination of weighted and delayed unit samples.





thus $x[n]$ can be expressed as:

$$\boxed{x[n] = 5\,\delta[n+4] + 3\,\delta[n+2] - \delta[n+1] + 2\,\delta[n-2]}$$

Problem 1.2

For each of the following systems, $y[n]$ denotes the output and $x[n]$ the input. Test each system for linearity and shift-invariance and show the result.

(a) $y[n] = 2x[n] + 3$

$y_1[n] = 2x_1[n] + 3$

$y_2[n] = 2x_2[n] + 3$

$x_3[n] = a \cdot x_1[n] + bx_2[n]$

$y_3[n] = 2x_3[n] + 3$

$y_3[n] = 2\,[ax_1[n] + bx_2[n]] + 3$

$ay_1[n] + by_2[n] = a(2x_1[n]+3) + b(2x[n]+3)$

$2\,[a x_1[n] + b\,x_2[n]] + 3 \neq a(2x_1[n]+3) + b(2x[n]+3)$

for Shift-Invariance

$x[n-n_0] \rightarrow y[n-n_0]$

$y[n] = 2x[n-n_0] + 3$

then

$\Updownarrow$ equivalent

$y[n-n_0] = 2x[n-n_0] + 3$

Shift-Invariant

$\boxed{\text{System is non-linear and shift-Invariant}}$

(b) $y[n] = x[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

$y_1[n] = x_1[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

$y_2[n] = x_2[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

$x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

$y_3[n] = (ax_1[n] + bx_2[n]) \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

$= ax_1[n]\sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right) + bx_2[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

Equivalent

$ay_1[n] + by_2[n] = a\left(x_1[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)\right) + b\left(x_2[n] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)\right)$

$x[n-n_0] \rightarrow y[n-n_0]$

$y[n] = x[n-n_0] \cdot \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right) \not\Leftrightarrow$ Not Equivalent

but

$y[n-n_0] = x[n-n_0] \cdot \sin\left(\frac{2\pi}{7}[n-n_0] + \frac{\pi}{6}\right)$

$\boxed{\text{System is Linear but NOT Shift-Invariant}}$

(c) $y[n] = [x[n]]^2 + 3x[n] - 1$

$y_1[n] = [x_1[n]]^2 + 3x_1[n] + 1$

$y_2[n] = [x_2[n]]^2 + 3x_1[n] + 1$

$x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = [x_3[n]]^2 + 3x_3[n] + 1$

$y_3[n] = [ax_1[n] + bx_2[n]]^2 + 3[ax_1[n] + bx_2[n]] + 1$ $\not\Updownarrow$ Clearly NOT Equivalent NOT linear

$ay_1[n] + by_2[n] = a[[x_1[n]]^2 + 3x_1[n] + 1] + b[[x_2[n]]^2 + 3x_1[n] + 1]$

$x[n-n_0] \rightarrow y[n-n_0]$

so

$y[n] = [x[n-n_0]]^2 + 3x[n-n_0] - 1$

and $\Updownarrow$ Equivalent

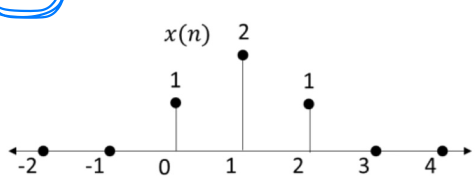
$y[n-n_0] = [x[n-n_0]]^2 + 3x[n-n_0] - 1$

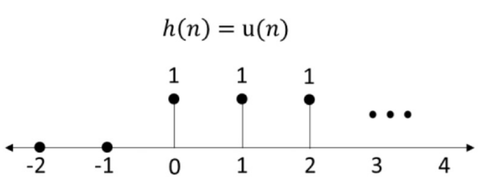
$\boxed{\text{System is non-linear and Shift-Invariant}}$

Problem 1.3

For each of the following pairs of sequences, $x[n]$ represents the input to an LSI system with unit-sample response $h[n]$. Determine each output $y[n]$. Sketch your results.

(a)



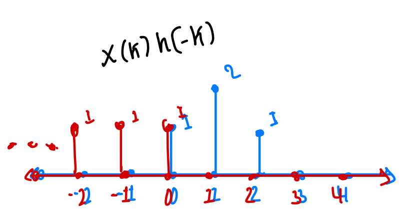


$x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$h[n] = u[n]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

$y[n] = \delta[n] + 3\delta[n-1] + 4u[n-2]$



$y[0] = 1$

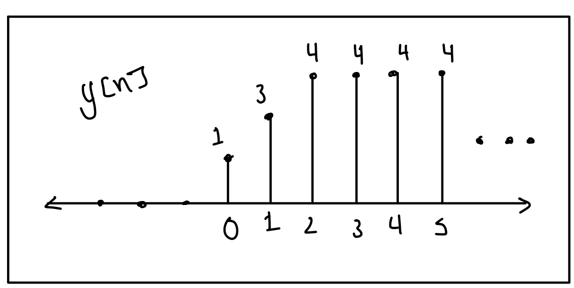
$y[1] = (1)(1) + (2)(1) = 3$

$y[2] = 4$

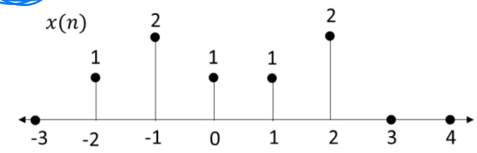
$y[3] = 4$

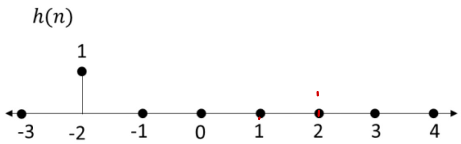
$y[4] = 4$

$\vdots$



(b)



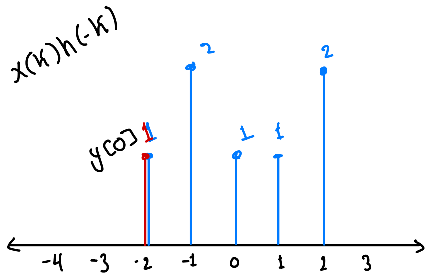


$$x[n] = \delta[n+2] + 2\delta[n+1] + \delta[n] + \delta[n-1] + 2\delta[n-2]$$

$$h[n] = \delta[n+2]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] + \delta[n-3] + 2\delta[n-4]$$



$y[-2] = (1)(0) = 0$

$y[-1] = (2)(0) = 0$

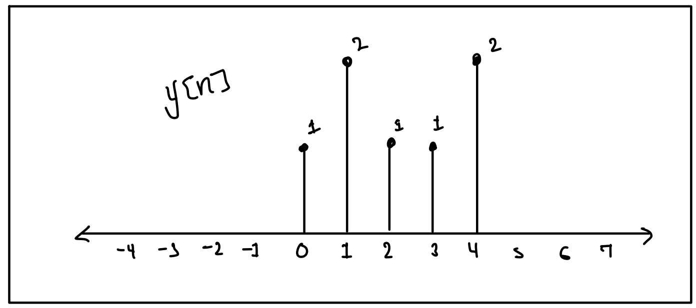
$y[0] = (1)(1) = 1$

$y[1] = (2)(1) = 2$

$y[2] = (1)(1) = 1$

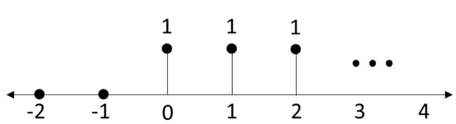
$y[3] = (1)(1) = 1$

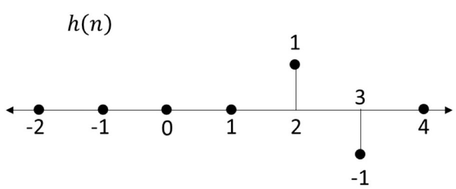
$y[4] = (2)(1) = 1$



(c)

$x(n) = u(n)$

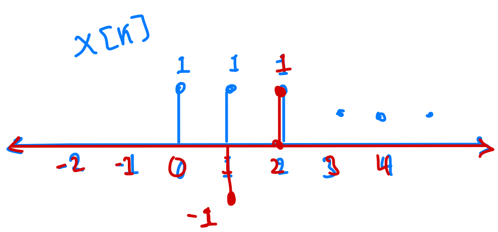




$$x[n] = u[n]$$

$$h[n] = \delta[n-2] - \delta[n-3]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$



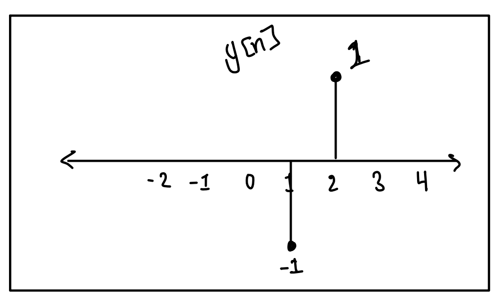
$y[1] = -1$

$y[2] = 1$

$y[3] = 1 - 1 = 0$

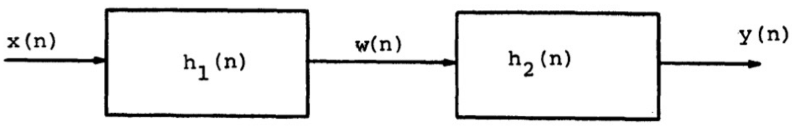
$y[4] = 0 + 0 = 0$

$$y[n] = -\delta[n-1] + \delta[n-2]$$

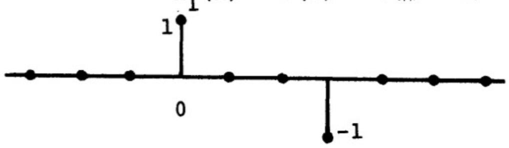


Problem 1.4

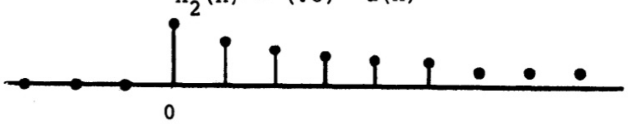
The system shown below contains two linear shift-invariant subsystems with unit sample responses $h_1[n]$ and $h_2[n]$, in cascade.



$h_1(n) = \delta(n) - \delta(n-3)$



$h_2(n) = (.8)^n\, u(n)$



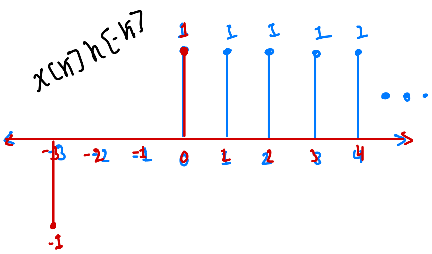
(a) Let $x[n] = u[n]$. Find and sketch $y_a[n]$ by first convolving $x[n]$ with $h_1[n]$ and then convolving that result with $h_2[n]$, i.e.

$x[n] = u[n]$

$h[n] = \delta[n] - \delta[n-3]$

$y_a[n] = [x[n] * h_1[n]] * h_2[n]$

$x[n] * h_1[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$



$y_1[0] = 1$

$y_1[1] = 1$

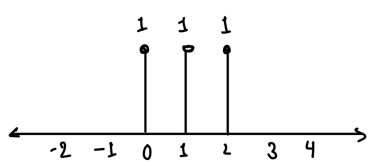
$y_1[2] = 1$

$y_1[3] = -1+1 = 0$

$y_1[4] = -1+1 = 0$

$\vdots$

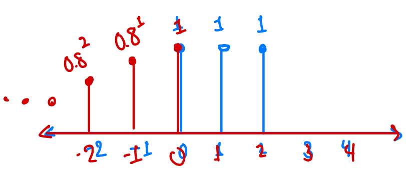
$y_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$



$y_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$h_2[n] = u[n]\,(0.8)^n$

$y_a[n] = y_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} y_1[k]\, h_2[n-k]$



$y_a[0] = 1$

$y_a[1] = 1 + 0.8^1 = 1.8$

$y_a[2] = 1 + 0.8^1 + 0.8^2 = 2.44$

$y_a[3] = 1.954$

$y_a[4] = 1.5616$

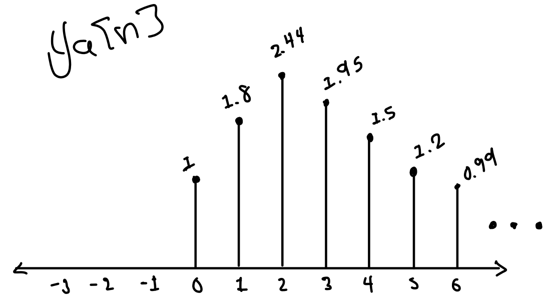
$y_a[5] = 1.2492$

$y_a[6] = 0.99$

$y_a[7] = 0.79$

$y_a[8] = 0.63$

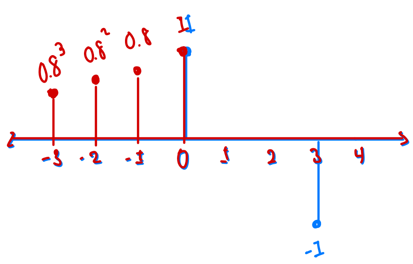
$y_a[n]$



(b) Again let $x[n] = u[n]$. Find and sketch $y_b[n]$ by convolving $x[n]$ with the result of convolving $h_1[n]$ and $h_2[n]$, i.e.

$$y_b[n] = x[n] * [h_1[n] * h_2[n]]$$

$h_1[n] * h_2[n]$



$y_1[0] = 1$

$y_1[1] = 0.8^1 = 0.8$

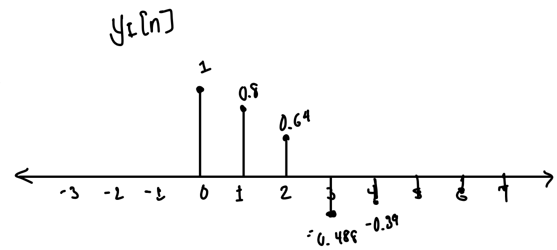
$y_1[2] = 0.8^2 = 0.64$

$y_1[3] = 0.8^3 - 1 = -0.488$

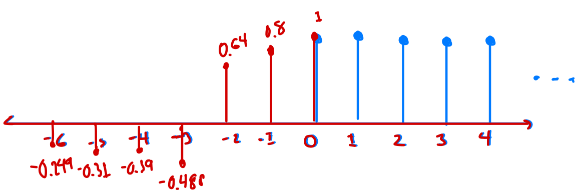
$y_1[4] = -0.39$

$y_1[5] = -0.31$

$y_1[6] = -0.249$



$x[n] * y_1[n]$



$y[0] = 1$

$y[1] = 1 + 0.8 = 1.8$

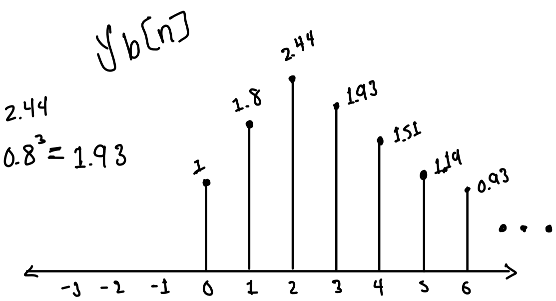
$y[2] = 1 + 0.8 + 0.8^2 = 2.44$

$y[3] = 1 + 0.8 + 0.8^2 - 0.8^3 = 1.93$

$y[4] = 1.51$

$y[5] = 1.19$

$y[6] = 0.93$



(c) What convolution property does this demonstrate?

Given that $y_a[n] \approx y_b[n]$ (Nearly identical), the convolution property being demonstrated is associative.

$$(a \times b)\, c = a \times (b \times c) \quad \checkmark$$

$$[x[n] * h_1[n]] * h_2[n] = x[n] * [h_1[n] * h_2[n]] \quad \checkmark$$