1\. a) $h[n] = \left(\frac{2}{3}\right)^n u[n]$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} \left(\frac{2}{3}\right)^n u[n]\, e^{-jwn}$$

$$\left|\frac{2}{3}\right| \cdot \left|\cancel{e^{-jw}}^{\,1}\right| < 1$$

$$= \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \left(\frac{2}{3} e^{-jw}\right)^n$$

$$\boxed{H(e^{jw}) = \frac{1}{1 - \left(\frac{2}{3} e^{-jw}\right)}}$$

b) $h[n] = \delta[n-10] + \delta[n+10]$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n-10]\, e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n+10]\, e^{-jwn}$$

$$X(e^{jw}) = e^{-j10w} + e^{j10w}$$

$$= 2\left[\frac{e^{j10w} + e^{-j10w}}{2}\right]$$

$$\boxed{= 2\cos(10w)}$$

$m = -n \quad -m = n$

c) $h[n] = 10^n u[-n]$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} 10^n u[-n]\, e^{-jwn} \Rightarrow \sum_{n=-\infty}^{0} 10^n \cdot e^{-jwn}$$

$$\sum_{m=\infty}^{0} 10^{-m} \cdot e^{jwm} \Rightarrow \sum_{m=0}^{\infty} \left(\frac{e^{jw}}{10}\right)^m$$

$$\left|\frac{e^{jw}}{10}\right| < 1$$

$$\boxed{H(e^{jw}) = \frac{1}{1 - \left(\frac{e^{jw}}{10}\right)}}$$

2. a) $x[n] = (1.1)^n u[-n]$

$$H(e^{jw}) = \sum_{-\infty}^{\infty} (1.1)^n u[-n] = \sum_{-\infty}^{0} (1.1)^n e^{-jwn} = \sum_{0}^{\infty} \left(\frac{1}{1.1}\right)^m e^{jwm}$$

By inspection, it is obvious that the sequence will converge and thus it has a fourier transform.

b) $\delta[n] + \left(\frac{1}{5}\right)^n u[n-2]$

$$= \sum_{n=-\infty}^{\infty} 1 + \sum_{n=-\infty}^{\infty} \left(\frac{1}{5}\right)^n u[n-2] e^{jwn}$$

$$1 + \sum_{n=2}^{\infty} \left(\frac{1}{5}\right)^n e^{jwn}$$

By inspection, it is obvious that the sequence will converge and thus it has a fourier transform.

c) $x[n] = u[n]$

By inspection, it is obvious that the sequence will not converge thus it does not have a fourier transform.