SEREȘ ARTUR

TEMĂ TS NR 6

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$\Delta(s) = s^3 + 3Ks^2 + (K+2)s + 4$

$K = ?$, pentru ca sistemul să fie stabil

Se impun condițiile:

$1 > 0$

$3K > 0 \Rightarrow K \in (0, \infty)$

$K+2 > 0 \Rightarrow K \in (-2, \infty)$

$4 > 0$

$\Rightarrow K \in (-2, \infty) \cap (0, \infty) = (0, \infty)$

Construim matricea Hurwitz ($n = 3$):

$$\det(H) = \det(H_3) = \begin{vmatrix} 3K & 4 & 0 \\ 1 & K+2 & 0 \\ 0 & 3 & 4 \end{vmatrix}$$

Se impun condițiile de stabilitate:

$\det(H_1) = 3K > 0 \Rightarrow K \in (0, \infty)$

$$\det(H_2) = \begin{vmatrix} 3K & 4 \\ 1 & K+2 \end{vmatrix} = 3K^2 + 6K - 4 > 0$$

$$K_{1,2} = \frac{-6 \pm \sqrt{36 + 4 \cdot 12}}{6} = \frac{-6 \pm \sqrt{84}}{6} = \frac{-6 \pm 9{,}16}{6} \begin{cases} 0{,}527 \\ -2{,}526 \end{cases}$$

K		$-2{,}526$		$0{,}527$	
$f(K)$	+ +	0	- - - -	0	+ +

$\Rightarrow K \in (-\infty; -2{,}526) \cup (0{,}527, \infty)$

$$\det(H_3) = \begin{vmatrix} 3K & 4 & 0 \\ 1 & K+2 & 0 \\ 0 & 3 & 4 \end{vmatrix} = 4(3K^2 + 6K) - 16 > 0$$

$12K^2 + 24K - 16 > 0 \quad | :2$

$6K^2 + 12K - 8 > 0$

$$K_{1,2} = \frac{-12 \pm \sqrt{144 + 4 \cdot 48}}{12} = \frac{-12 \pm \sqrt{336}}{12} = \frac{-12 \pm 18{,}33}{12} \begin{cases} 0{,}527 \\ -2{,}527 \end{cases}$$

$\Rightarrow K \in (-\infty; -2{,}527) \cup (0{,}527; \infty)$

$\Rightarrow K \in (0, \infty) \cap ((-\infty; -2{,}576) \cup (0{,}527, \infty)) \cap ((-\infty, -2{,}527) \cup (0{,}527; \infty))$

$K \in (0{,}527; \infty) \rightarrow$ Sistem Stabil

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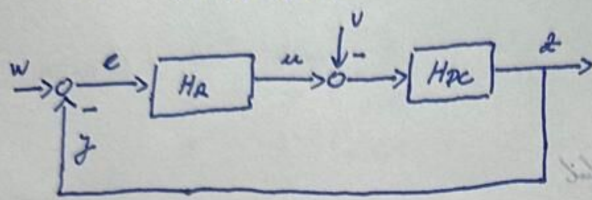
$$H_{PC}(s) = \frac{2}{s^3+4s^2+5s+2}$$



a) Regulator Proporțional (P), $H_R(s)=K$, $K=?$ pentru ca Sistemul să fie Stabil

$$H_{zw}(s) = \frac{H_R(s)\cdot H_{PC}(s)}{1+H_R(s)\cdot H_{PC}(s)} = \frac{H_0(s)}{1+H_0(s)}$$

$$H_{zw}(s) = \frac{K\cdot \frac{2}{s^3+4s^2+5s+2}}{1+K\cdot\frac{2}{s^3+4s^2+5s+2}}$$

Polinomul caracteristic este:

$$\Delta(s) = 1+H_0(s) = \frac{s^3+4s^2+5s+(2K+2)}{s^3+4s^2+5s+2}$$

Ecuația caracteristică este:

$\Delta(s) = s^3+4s^2+5s+(2K+2)$

Pentru ca Sistemul să fie Stabil:

Sunt impuse Condițiile:

$1>0$

$4>0$

$5>0$

$2K+2>0 \Rightarrow K>-1 \Rightarrow K\in(-1,\infty)$

Construim Matricea Hurwitz: $(n=3)$:

$$\det(H) = \det(H_3) = \begin{vmatrix} 4 & 2K+2 & 0 \\ 1 & 5 & 0 \\ 0 & 4 & 2K+2 \end{vmatrix}$$

$\det(H_1) = 4 > 0$

$$\det(H_2) = \begin{vmatrix} 4 & 2K+2 \\ 1 & 5 \end{vmatrix} = 20-2K-2 = 18-2K>0 \Rightarrow K\in(-\infty,9)$$

$$\det(H_3) = \begin{vmatrix} 4 & 2K+2 & 0 \\ 1 & 5 & 0 \\ 0 & 4 & 2K+2 \end{vmatrix} = 20(2K+2)-(2K+2)^2 =$$

$$= 40K+40-4K^2-8K-4$$

$$= -4K^2+32K+36$$

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$-4K^2+32K+36>0 \Rightarrow -K^2+8K+9>0$

$$K_{1,2} = \frac{-8 \pm \sqrt{64+4\cdot 9}}{-2} = \frac{-8 \pm 10}{-2} \left\langle \begin{matrix} -1 \\ 9 \end{matrix} \right.$$

K	-1		9	
$f(K)$	$- - 0$	$+ + + +$	0	$- -$

$\Rightarrow K \in (-1, 9)$

$\Rightarrow K \in (-1, \infty) \cap (-\infty, 9) \cap (-1, 9) = (-1, 9) \rightarrow$ Sistem Stabil

b) Regulator Proportional - integrator (PI), $H_R(s) = K_P + \frac{K_I}{s} = \frac{K_P \cdot s + K_I}{s}$

K_P – coeficientul componentei P.

K_I – coeficientul componentei I.

$$H_{zw}(s) = \frac{H_R(s) \cdot H_{PC}(s)}{1 + H_R(s) \cdot H_{PC}(s)} = \frac{H_0(s)}{1 + H_0(s)}$$

$$H_{zw}(s) = \frac{\left(K_P + \frac{K_I}{s}\right) \cdot \frac{2}{s^3+4s^2+5s+2}}{1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{2}{s^3+4s^2+5s+2}\right)}$$

Polinomul Caracteristic este:

$$\Delta(s) = H_0(s) + 1 = \frac{s^4+4s^3+5s^2+(2K_P+2)s+2\cdot K_I}{s^4+4s^3+5s^2+2s}$$

Ecuația Caracteristică este:

$\Delta(s) = s^4+4s^3+5s^2+(2K_P+2)s+2K_I$

Sunt Impuse Condițiile:

$1>0$

$4>0$

$5>0$

$2K_P+2>0 \Rightarrow K_P \in (-1, \infty)$

$2K_I>0 \Rightarrow K_I \in (0, \infty)$

Construim Matricea Hurwitz $(n=4)$:

$$\det(H) = \det(H_4) = \begin{vmatrix} 4 & 2K_P+2 & 0 & 0 \\ 1 & 5 & 2K_I & 0 \\ 0 & 4 & 2K_P+2 & 0 \\ 0 & 1 & 5 & 2K_I \end{vmatrix}$$

$\det(H_1) = 4 > 0$

$$\det(H_2)=\begin{vmatrix} 4 & 2K_p+2 \\ 1 & 5 \end{vmatrix} = 20-2K_p-2 = 18-2K_p>0$$

$$\Rightarrow K_p \in (-\infty, 9)$$

$$\det(H_3)=\begin{vmatrix} 4 & 2K_p+2 & 0 \\ 1 & 5 & 2K_I \\ 0 & 4 & 2K_p+2 \end{vmatrix} = 20(2K_p+2)-32K_I-(2K_p+2)^2$$

$$= 40K_p+40-32K_I-4K_p^2-8K_p-4$$

$$-4K_p^2+32K_p+36-32K_I>0$$

$$-K_p^2+8K_p+9-8K_I>0$$

$$\left.\begin{aligned} &\Rightarrow K_I < \frac{-K_p^2+8K_p+9}{8} \\ &\text{dar } K_I \in (0,\infty) \end{aligned}\right\} \Rightarrow -K_p^2+8K_p+9>0$$

$$K_{1,2} = \frac{-8 \pm \sqrt{64+4\cdot 9}}{-2} \begin{cases} -1 \\ 9 \end{cases}$$

$$\begin{array}{c|l} K & \qquad -1 \qquad\qquad\qquad 9 \\ \hline f(K) & -\;-\;0\;+\;+\;+\;+\;+\;+\;0\;-\;- \end{array} \Rightarrow K_p \in (-1, 9)$$

$$\det(H_4)=\begin{vmatrix} 4 & 2K_p+2 & 0 & 0 \\ 1 & 5 & 2K_I & 0 \\ 0 & 4 & 2K_p+2 & 0 \\ 0 & 1 & 5 & 2K_I \end{vmatrix}$$

$$= (2K_I)\cdot(-1)^{4+4}\cdot \det\begin{vmatrix} 4 & 2K_p+2 & 0 \\ 1 & 5 & 2K_I \\ 0 & 4 & 2K_p+2 \end{vmatrix}$$

$$= 2K_I\cdot(-K_p^2+8K_p+9)>0$$

dar $K_I \in (0,\infty)$ și $K_p \in (-1,9)$

$$\left.\begin{aligned} &\Rightarrow K_I \in (0,\infty) \cap \left(-\infty, \frac{-K_p^2+8K_p+9}{8}\right) = \left(0, \frac{-K_p^2+8K_p+9}{8}\right) \\ &K_p \in (-1,\infty) \cap (-\infty,9) \cap (-1,9) = (-1,9) \end{aligned}\right\} \text{Sistem Stabil.}$$