1.

1.1.

() ( ). , .

1.1. — .

, , ...

1. .
2. , , , .
3. , (?).

, . ?

1. $v \ll c$.
2. $\Delta l \gg \frac{\hbar}{p}$ — .
3. $m\varphi \ll mc^2$, φ — .
4. $t \gg t_{BB}$.

1.2. — , .

1.3. — , .

1.1 ((?)). , ?

1.2.

$$\mathbf{r}(t) = \{x(t), y(t), z(t)\};\ \{\mathbf{r}_i(t)\}_{i=\overline{1,N}}$$

1.2.1. .

$\mathbf{r}(t)$ — ( ).

$$\begin{aligned}
\mathbf{r} &= r\mathbf{e}_r = \sqrt{x^2+y^2+z^2}\mathbf{e}_r, \\
\mathbf{e}_r &= \frac{\mathbf{r}}{r} = \left\{\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right\}; \\
\mathbf{v} &= \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \{\dot{x}, \dot{y}, \dot{z}\}, \\
\mathbf{v} &= v\boldsymbol{\tau}.
\end{aligned}$$

.

$$q=(q_1,q_2,q_3), x=X(q,t), y=Y(q,t), z=Z(q,t) \Rightarrow$$
$$q(t) \Rightarrow \mathbf{r}(t)=\mathbf{rq}(t),t=\{X(q(t),t),Y(q(t),t),Z(q(t),t)\};$$
$$\mathbf{v}(t)=\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}=\sum\frac{\partial\mathbf{r}}{\partial q_i}\dot{q}_i+\frac{\partial\mathbf{r}}{\partial t};$$
$$\mathbf{v}_i=\left\{\frac{\partial X}{\partial q_i}\dot{q}_i,\frac{\partial Y}{\partial q_i}\dot{q}_i,\frac{\partial Z}{\partial q_i}\dot{q}_i\right\}.$$

$$\mathbf{v}=\sum v_i\mathbf{e}_i;(e_i,e_j)=\delta_{ij}\Rightarrow v^2=v_1^2+v_2^2+v_3^2$$

1.2.1 ( ). $q=(\varphi,\rho,z)$,

$$\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \\ z, \end{cases}$$

$$\mathbf{v}=\frac{\mathrm{d}}{\mathrm{d}t}\{\rho\cos\varphi,\rho\sin\varphi,z\}\Rightarrow$$

$$\begin{cases} \mathbf{v}_\varphi=\rho\dot{\varphi}\underbrace{\{-\sin\varphi,\cos\varphi,0\}}_{\mathbf{e}_\varphi} \\ \mathbf{v}_\rho=\dot{\rho}\underbrace{\{\cos\varphi,\sin\varphi,0\}}_{\mathbf{e}_\rho} \\ \mathbf{v}_z=\dot{z}\{0,0,1\}, \end{cases}$$

,

$$(\mathbf{e}_\rho,\mathbf{e}_\varphi)=0.$$

.

$$\mathbf{a}\overset{\text{def}}{=}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}=\{\ddot{x},\ddot{y},\ddot{z}\}$$
$$\mathbf{a}=\frac{\mathrm{d}}{\mathrm{d}t}(v\boldsymbol{\tau})=\dot{v}\boldsymbol{\tau}+\underbrace{v\dot{\boldsymbol{\tau}}}_{=\frac{v^2}{R}\mathbf{n}}$$
$$\mathbf{n}=\frac{\dot{\mathbf{r}}}{\dot{\boldsymbol{\tau}}}$$
$$|\dot{\boldsymbol{\tau}}|=\frac{v}{R}$$
$$[\boldsymbol{\tau},\mathbf{n}]=\mathbf{b}-\ -\quad .$$

1.2.2. .

, , .

0. + ( ).

1. : , .

, - : . :

$$\begin{cases} t=t' \\ \mathbf{r}'=\mathbf{r}+\mathbf{u}t, \qquad \mathbf{u}=const. \end{cases} \tag{1.1}$$

(1.1) , .

2. . «» .

$$\ddot{\mathbf{r}}_i = \mathbf{f}_i(\mathbf{r}_1, \dots, \mathbf{r}_N, \mathbf{v}_1, \dots, \mathbf{v}_N, t)$$

$6N$ — .

$$\begin{cases} \mathbf{r}_i(t_0) = \mathbf{r}_{i0} \\ \mathbf{v}_i(t_0) = \mathbf{v}_{i0}, \end{cases}$$

, , : , .

:

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i\left(\{\mathbf{r}_i\}, \{\dot{\mathbf{r}}_i\}, t\right). \tag{1.2}$$

, (1.2) , m_i — , .

:

$$\begin{cases} m_1 a_1 = F \\ m_2 a_2 = F \end{cases} \Rightarrow \frac{m_1}{m_2} = \frac{a_2}{a_1}, \qquad \begin{cases} m a_1 = F_1 \\ m a_2 = F_2 \end{cases} \Rightarrow \frac{F_1}{F_2} = \frac{a_1}{a_2}.$$

.

$$\mathbf{p}_i \stackrel{\text{def}}{=} m_i \mathbf{v}_i \qquad — ,$$

$$\mathbf{M}_i \stackrel{\text{def}}{=} m_i [\mathbf{r}_i, \mathbf{v}_i] \qquad — ,$$

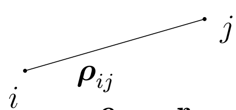
$$\mathbf{N}_i \stackrel{\text{def}}{=} [\mathbf{r}_i, \mathbf{F}_i] \qquad — .$$

1.2.2. . $m(t)$ — , , : — , . $\mathbf{F}$, : $m(\dot{\mathbf{r}})$ — , , $m(\ddot{\mathbf{r}})$ — , $F(\dddot{\mathbf{r}})$ — — , , , , — .

3. . , .

$$\mathbf{F}_i = \mathbf{F}_i^{(e)}(t, \mathbf{r}_i) + \sum \mathbf{F}_{ij}^{(i)}(t, \mathbf{r}_i, \mathbf{r}_j),$$

( ), — . « » , i- . , , , $\mathbf{r}_i, \mathbf{r}_j$ $\mathbf{r}_k$ — , . :



$$\boldsymbol{\rho}_{ij} = \mathbf{r}_j - \mathbf{r}_i$$

$$\begin{cases} \mathbf{F}_{ij}^{(i)} + \mathbf{F}_{ji}^{(i)} = 0 \\ [\boldsymbol{\rho}_{ij}, \mathbf{F}_{ij}] = 0 \end{cases} \tag{1.3}$$

1.2.3. , , , : , , , ...

1.1. (a)

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i^{(e)} = \mathbf{F}^{(e)} \Rightarrow \left(\sum m_i\right) \cdot \ddot{\mathbf{R}} = \mathbf{F} = \mathbf{F}^{(e)}, \mathbf{R} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i},$$

, .

1.2.4. .

1.2.5. , , ...

(b)

$$\mathbf{N} = \sum \mathbf{N}_i = \sum \mathbf{N}_i^{(e)},$$

, , . ?

$$[\mathbf{r}_i, \mathbf{F}_{ij}] + [\mathbf{r}_j, \mathbf{F}_{ji}] = [\mathbf{r}_i - \mathbf{r}_j, \mathbf{F}_{ij}] = 0,$$

(1.3), , — , .

1.2.3. .

.

$$\mathbf{p} \stackrel{\text{def}}{=} \sum m_i \mathbf{v}_i \Rightarrow$$
$$\dot{\mathbf{p}} = \mathbf{F} = \mathbf{F}^{(e)},$$

II , — III .

$$\mathbf{p} = const \Leftarrow \sum \mathbf{F}_i^{(e)} = 0,$$

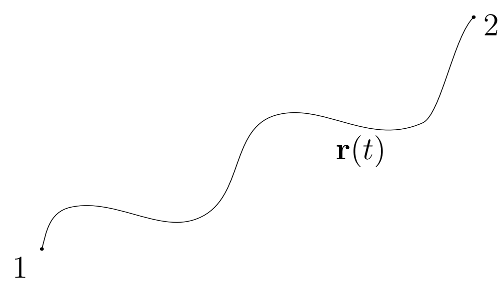
III .

.

$$\mathbf{M} \stackrel{\text{def}}{=} \sum m_i [\mathbf{r}_i, \mathbf{v}_i];$$
$$\dot{\mathbf{M}} = \sum m_i \big\{ [\mathbf{r}_i, \dot{\mathbf{v}}_i] + [\dot{\mathbf{r}}_i, \mathbf{v}] \big\} = \mathbf{N} = \mathbf{N}^{(e)};$$
$$\begin{cases} \sum \mathbf{N}_i^{(e)} = 0, \\ \text{III} \end{cases} \Rightarrow \mathbf{M} = const$$

.

$$m\ddot{\mathbf{r}} = \mathbf{F} \qquad | \cdot \mathbf{v} \Rightarrow$$
$$m\mathbf{v}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\left(v_x \dot{v}_x + v_y \dot{v}_y + v_z \dot{v}_z\right) = \frac{m}{2} \cdot 2 \frac{\mathrm{d}}{\mathrm{d}t}\left(v_x^2 + v_y^2 + v_z^2\right) = \frac{\mathrm{d}T}{\mathrm{d}t} \Rightarrow$$
$$T = \frac{mv^2}{2}$$



. 1: .

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \mathbf{F} \cdot \mathbf{v}$$
$$\int_{t_1}^{t_2} \frac{\mathrm{d}T}{\mathrm{d}t}\, dt = T(2) - T(1) = \int_{t_1}^{t_2} \mathbf{F} \cdot \underbrace{\mathbf{v}\, dt}_{d\mathbf{r}} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\, d\mathbf{r} = \int_1^2 dA$$

1.4. , $U(\mathbf{r}, t)$.

$$\mathbf{F} = -\frac{\mathrm{d}U}{\mathrm{d}\mathbf{r}} = \left\{ -\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z} \right\} = -\nabla U$$
$$rot\, \mathbf{F} \equiv 0 \Leftrightarrow [\nabla, \mathbf{F}] = 0,$$

. ,

$$\oint \mathbf{F}(\tau, r)\, d\mathbf{r} = 0$$

1.5.

$$\frac{\partial U}{\partial t} = 0 \Rightarrow \quad (\ ).$$

1.6.

$$dA = 0,$$

, (, a).

$$(\mathbf{F}, \mathbf{v}) = 0$$

1.7. :

$$dA < 0$$

1.2.6.

$$\mathbf{F} = -k(t, \mathbf{r}, \mathbf{v}) \cdot \mathbf{v}, \quad k > 0$$

$\mathbf{F}$, , :

$$\mathbf{F} = -\nabla U(\mathbf{r}, t) + \mathbf{F}_g + \mathbf{F}_d$$

:

$$T + U = E.$$

:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{\partial U}{\partial \mathbf{r}}\mathbf{v} + \mathbf{F}_d \cdot \mathbf{v}; \tag{1.4}$$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial \mathbf{r}}\dot{\mathbf{r}}; \tag{1.5}$$

(1.4) (1.5):

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial U}{\partial t} + \mathbf{F}_d \cdot \mathbf{v},$$

, , , .

.

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i \qquad | \cdot \mathbf{v}_i, \sum_i \quad i = 1, N$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \sum_{i=1}^{N} \mathbf{F}_i \mathbf{v}_i; \quad T = \sum_{i=1}^{N} \frac{m_i v_i^2}{2}$$

$$\mathbf{F}_i = \mathbf{F}_i^{(e)} + \sum_{i \neq j} \mathbf{F}_{ij}$$

$$F_i^{(e)} = -\frac{\partial U^{(e)}(\mathbf{r}_i)}{\partial \mathbf{r}_i} + \mathbf{F}_{g\,i}^{(e)} + \mathbf{F}_{d\,i}^{(e)},$$

, . :

$$\sum \mathbf{F}_i^{(e)} \cdot \mathbf{v}_i = -\sum \frac{\partial U^{(e)}}{\partial \mathbf{r}_i} \cdot \dot{\mathbf{r}}_i + \sum \mathbf{F}_{d\,i}^{(e)} \cdot \mathbf{v}_i, \tag{1.6}$$

$U^{(e)}$:

$$U^{(e)} = \sum_{i=1}^{N} U_i^{(e)}(\mathbf{r}_i),$$

$$\frac{\mathrm{d}U^{(e)}}{\mathrm{d}t} = \sum \frac{\partial U^{(e)}}{\partial \mathbf{r}_i} \cdot \dot{\mathbf{r}}_i + \frac{\partial U^{(e)}}{\partial t}. \tag{1.7}$$

(1.6) (1.7) :

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(T + U^{(e)}\right) = \frac{\partial U^{(e)}}{\partial t} + \sum \mathbf{F}_{d\,i}^{(e)} \cdot \mathbf{v}_i.$$

$$U_{ij} = U_{ij}(|\mathbf{r}_j - \mathbf{r}_i|) = U_{ij}\rho_{ij} \Rightarrow U_{ij} = U_{ji};$$

$$\mathbf{F}_i = -\frac{\partial U_{ij}}{\partial \boldsymbol{\rho}_{ij}}\frac{\partial \boldsymbol{\rho}_{ij}}{\partial \mathbf{r}_i} = +\frac{\partial U_{ij}(\boldsymbol{\rho}_{ij})}{\partial \boldsymbol{\rho}_{ij}}, \mathbf{F}_j = -\frac{\partial U_{ij}}{\partial \boldsymbol{\rho}_{ij}}\frac{\partial \boldsymbol{\rho}_{ij}}{\partial \mathbf{r}_j} = -\frac{\partial U_{ij}(\boldsymbol{\rho}_{ij})}{\partial \boldsymbol{\rho}_{ij}};$$

$$\mathbf{F}_{ij} = -\frac{\partial}{\partial \mathbf{r}_i} U_{ij}(\mathbf{r}_i, \mathbf{r}_j);$$

$$\sum_{\substack{i,j \\ i \neq j}} \mathbf{F}_{ij} \cdot \mathbf{v}_i = \sum \underbrace{(\mathbf{v}_i - \mathbf{v}_j)}_{-\dot{\boldsymbol{\rho}}_{ij}} \frac{\partial U_{ij}}{\partial \boldsymbol{\rho}_{ij}} = -\frac{\mathrm{d}}{\mathrm{d}t} U^{(i)};$$

$$U^{(i)} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} U^{(i)}(\boldsymbol{\rho}_{ij}) \Rightarrow$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(T + U^{(e)} + U^{(i)}\right) = \frac{\partial U^{(e)}}{\partial t} + \sum \mathbf{F}_{d\,i}^{(e)}}$$

$\ldots E = const$:

1. / .

2. .

3. : . $U^{(i)}(|\mathbf{r}_i - \mathbf{r}_j|)$.

1.2.7.

$$E = -e\nabla\varphi + \frac{e}{c}[\mathbf{v}, \mathbf{B}]$$

.

$$I(t, \mathbf{r}_i, \mathbf{v}_i) = const$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial U}{\partial t} + \vec{v} \cdot \vec{F}_d$$

; .

$\Rightarrow$.

1.2.8.

$$\begin{cases} \mathbf{F} = \frac{\alpha}{x^2+y^2}\boldsymbol{\tau}, \\ \boldsymbol{\tau} = \{-y, x, 0\}, \\ x = R\cos\omega t, \\ y = R\sin\omega t, \\ z = 0 \end{cases} \Rightarrow$$

$$A = \frac{\alpha}{R^2}\int_0^T (\boldsymbol{\tau}, \mathbf{v})\,dt = \frac{\alpha}{R^2}\int_0^T (R^2\omega\sin^2\omega t + R^2\omega\cos^2\omega t)\,dt = \frac{\alpha}{R^2}R^2\omega\int_0^T dt = \alpha\omega T = 2\pi\alpha$$

1.3.

.

1.8. — .

$$|\vec{r}_i|, |\vec{v}_i| < const$$

1.9. .

$$f(t) \to \langle f\rangle = \frac{1}{\tau}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} f(t')\,dt',$$

τ — .

1.3.1. .

$$m\ddot{x} + \kappa x = 0;$$
$$F_x = -\kappa\dot{x};$$
$$x = A\cos\omega t + \varphi;$$
$$U = \frac{\kappa x^2}{2}, \quad \langle U\rangle = \frac{1}{4}\kappa A^2.$$

, «»:

$$F = -\mu\dot{x},$$
$$m\ddot{x} + \mu\dot{x} + kx = 0.$$

1.10 ( ).

$$\tau_0 = \frac{f}{\dot{f}}$$

1.11 ( ). $f(x_1, x_2, \ldots, x_n)$ k,

$$f(\alpha x_1, \ldots, \alpha x_n) = \alpha^k f(x_1, \ldots, x_n) \ \forall\alpha.$$

1.2 (( )). , $f(x_1, x_2, \ldots, x_n)$ k

$$\sum_{i=1}^{n} \frac{\partial f}{\partial x_i} x_i = kf. \tag{1.8}$$

. , (1.8) $\alpha = 1$:

$$\frac{\partial}{\partial\alpha} f(\alpha\mathbf{x}) = \frac{\partial f(\alpha\mathbf{x})}{\partial\mathbf{x}} \left.\frac{\mathrm{d}(\alpha x)}{\mathrm{d}\alpha}\right|_{\alpha=1} = \frac{\partial f}{\partial x} x,$$

— kf.

$\varphi(\alpha) = \alpha^{-k} f(\alpha x_1, \alpha x_2, \ldots, \alpha x_n)$ $x_1, x_2, \ldots, x_n$ α:

$$\varphi'(\alpha) = -k\alpha^{-k-1} f(\alpha x_1, \alpha x_2, \ldots, \alpha x_n) + \alpha^{-k} \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} x_i.$$

,

$$\sum_{i=1}^{n} \frac{\partial f(\alpha\mathbf{x})}{\partial(\alpha x_i)} x_i = kf(\alpha x_1, \alpha x_2, \ldots, \alpha x_n),$$

, $\varphi'(\alpha)$, , $\varphi'(\alpha) = const$, , $\alpha = 1$: $\varphi(1) = f(x_1, \ldots, x_n)$. ,

$$\alpha^k \varphi(\alpha) = f(\alpha x_1, \alpha x_2, \ldots, \alpha x_n) = \alpha^k f(x_1, \ldots, x_n). \qquad \square$$

. . , ? ( ), : (: , —).

1.3 ( ). .

1. $|\vec{r}_i|, |\vec{v}_i| < \infty$, .
2. $\mathbf{F} = -\boldsymbol{\nabla} U(\vec{r})$ — , , .
3. $U(\alpha\vec{r}) = \alpha^k U(\vec{r})$ — k.
4. $\tau = T$, $\tau \gg \tau_0$, $\tau \sim \tau_0$ $\underset{\tau_0}{\Delta T} \ll T$, , , , , .

, :

$$\langle T\rangle \approx \frac{k}{2} \langle U\rangle.$$

:

$$\langle T\rangle \approx \frac{k}{k+2} E,$$
$$\langle U\rangle \approx \frac{2}{k+2} E.$$

.

$$T=\frac{mv^2}{2}=\frac{1}{2}\mathbf{pv}=\frac{1}{2}\mathbf{p\dot r}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\mathbf{pr}\right)-\frac{1}{2}\mathbf{\dot p r}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\mathbf{pr}\right)-\frac{1}{2}$$

$$\langle T\rangle=-\frac{1}{2}\langle\mathbf{Fr}\rangle+\frac{1}{2\tau}\mathbf{pr}\Big|_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} \tag{1.9}$$

(1.9) . , 4 «» — .

$\tau=T$ T — , , .

$\tau\to\infty$.

$\tau\gg\tau_0$. $\vec p\cdot\vec r$. , « »: $v\sim\frac{r}{\tau_0},\vec p\cdot\vec r\sim\tau_0 pv,\frac{1}{2}\frac{\tau_0}{\tau}pv=\frac{\tau_0}{\tau}T$. .

$\tau\sim\tau_0$, : $\frac{1}{2}\frac{\tau_0}{\tau}\Delta T$, .

; 2 3:

$$\begin{cases}\langle T\rangle\approx-\frac{1}{2}\langle\mathbf{Fr}\rangle\\ \mathbf{F}\cdot\mathbf{r}\overset{(2)}{=}-\frac{\partial U}{\partial\mathbf{r}}\mathbf{r}\overset{(3)}{=}-kU\end{cases}\Rightarrow\langle T\rangle\approx\frac{k}{2}\langle U\rangle$$

□

1.3.2. $U=-\frac{\alpha}{r}\Rightarrow k=-1,\langle T\rangle=-E,\langle U\rangle=2E$ — , .

1.3.3 ( ). .

2. $\mathbf{F}=-\boldsymbol{\nabla}U(\mathbf{r},t)+\mathbf{F}_d$

4. $\tau\gg\tau_0,\ \ \tau\sim\tau_0\ll\tau_T,\ \tau_T=\tau_0\frac{T}{\Delta T}$.

5. $\tau_0\ll\tau_E\ (\Delta E\ll E)$

. , :

$$\mathbf{F}\cdot\mathbf{r}=\frac{\partial U}{\partial\mathbf{r}}\mathbf{r}+\mathbf{F}_d\mathbf{r}=-kU+\mathbf{F}_d\mathbf{r},$$

;

$$\frac{E}{\tau_E}\sim\frac{U}{\tau_U}+\frac{r}{\tau_0}F_d\Rightarrow$$

$$rF_d\sim\frac{\tau_0}{\tau_E}E\sim\frac{\tau_0}{\tau_E}U\Rightarrow$$

$$\langle T\rangle\approx\frac{k}{2}\langle U\rangle,\langle T\rangle\approx\frac{k}{k+2}\langle E\rangle.$$

□

1.3.4. $m\ddot x+\mu\dot x+kx=0$

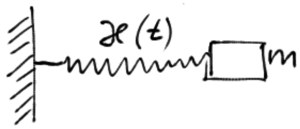
$$\frac{\mathrm{d}E}{\mathrm{d}t}=-\mu\dot x^2=-\frac{2\mu}{m}T=\frac{m\dot x^2}{2},$$

$$k=2\Rightarrow\langle-\rangle\frac{\mu}{m}\langle E\rangle\Rightarrow$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\langle E\rangle=-\frac{\mu}{m}\langle E\rangle}$$

$$\langle E\rangle=\mathcal{E}\exp\left\{-\frac{\mu}{m}t\right\}$$

1.3.5. $\tau_\varkappa \gg \dfrac{2\pi}{\omega}, \omega^2 = \dfrac{\varkappa}{m}$



$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial U(tx)}{\partial t} = \frac{\partial U}{\partial \varkappa}\frac{\mathrm{d}\varkappa}{\mathrm{d}t} = \frac{\dot{\varkappa} x^2}{2},$$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle = \left\langle \frac{\dot{\varkappa} x^2}{2} \right\rangle,$$

$$U = \frac{\varkappa x^2}{2}, \quad \left\langle \frac{\varkappa x^2}{2} \right\rangle = \frac{\varkappa}{2}\left\langle x^2 \right\rangle, \quad \tau_T \gg \tau_0, \quad \varkappa(t) \quad ,$$

$$\langle U \rangle = \frac{1}{2}\langle E \rangle \quad ,$$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle \overset{?}{=} \frac{\mathrm{d}}{\mathrm{d}t}\langle E \rangle = \frac{\dot{\varkappa}}{\varkappa}\langle U \rangle = \frac{\dot{\varkappa}}{2\varkappa}\langle E \rangle \Rightarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\langle E \rangle = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\ln\varkappa = \frac{\mathrm{d}}{\mathrm{d}t}\ln\sqrt{\varkappa} \Rightarrow$$

$$\langle E \rangle = E_0\sqrt{\frac{\varkappa(t)}{\varkappa_0}} \Rightarrow$$

$$\boxed{\frac{\langle E \rangle}{\omega(t)} \simeq const}$$

1.4 ( ).

. , , :

$$\langle T \rangle \approx -\frac{1}{2}\sum_{i=1}^{N}\mathbf{F}_i\mathbf{r}_i, \quad — \quad .$$

$$\mathbf{F}_i^{(e)} = -\frac{\partial}{\partial \mathbf{r}}U^{(e)},$$

$$\mathbf{F}_{ij}^{(i)} = -\frac{\partial U_{ij}^{(i)}}{\partial \mathbf{r}_i} = \frac{\partial U_{ij}}{\partial \rho_{ij}},$$

$$\sum_{i=1}^{3N}\mathbf{F}_i^{(e)}\mathbf{r}_i = -\sum_{i=1}^{N}\frac{\partial U^{(e)}}{\partial \mathbf{r}_i}\mathbf{r}_i = -k^{(e)}U^{(e)},$$

$$\sum_{\substack{i=1\\ i\neq j}}^{N}\mathbf{F}_{ij}^{(i)}\mathbf{r}_i = -\sum_{i\neq j}\frac{1}{2}\frac{\partial U_{ij}^{(i)}}{\partial \boldsymbol{\rho}_{ij}}\boldsymbol{\rho}_{ij} = -k^{(i)}U^{(i)}$$

$$\boxed{\langle T \rangle = \frac{k^{(i)}}{2}\left\langle U^{(i)} \right\rangle + \frac{k^{(e)}}{2}\left\langle U^{(e)} \right\rangle}$$

$$E = T + U^{(i)} + U^{(e)}$$

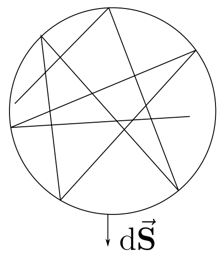
□

1.3.6 ( ).

$$\boldsymbol{\rho}_{ij} = const \Rightarrow U^{(i)} = const, k^{(i)} = 0$$

$$\boxed{\langle T \rangle = \frac{k^{(e)}}{2} \left\langle U^{(e)} \right\rangle}$$

1.3.7 ( ).



$$\langle T \rangle = -\frac{1}{2} \left\langle \sum_i \mathbf{F}_i \mathbf{r}_i \right\rangle = \frac{p}{2} \int\limits_{S_V} \mathbf{r}\,\mathrm{d}\mathbf{S} = \frac{3}{2} pV$$

$$\mathbf{F}_i = -p\,\mathrm{d}\mathbf{S}_i\,, U_i^{(i)} = 0$$

$$\oint \mathbf{r}\,\mathrm{d}\mathbf{S} = \int\limits_0^{2\pi} \int\limits_0^{\pi} a a^2 \sin\theta\,\mathrm{d}\theta = 4\pi a^3 = 3V$$

$$\boldsymbol{\nabla} \cdot \mathbf{r} = \frac{\partial x_i}{\partial x_i} \Rightarrow \oint \mathbf{r}\,\mathrm{d}\mathbf{S} = \int \boldsymbol{\nabla} \cdot \mathbf{r}\,\mathrm{d}V = 3 \int \mathrm{d}V = 3V.$$

1.3.8 ( ). — . , . ?

$$U^{(e)} = 0,$$
$$U_{ij}^{(i)} = -\frac{\alpha}{\rho_{ij}} \Rightarrow k^{(i)} = -1,$$
$$\langle T \rangle = -\frac{1}{2} \left\langle U^{(i)} \right\rangle,$$

$$\langle T \rangle = \frac{3}{2} k \frac{m}{\mu} T_\star,$$
$$\left\langle U^{(i)} \right\rangle = -G \frac{m^2}{R} \Rightarrow$$

$$T_\star \sim \frac{m}{R} \sim R^2 \rho.$$

2.

:

$$m_i\ddot{\vec{r}}_i = \vec{F}_i,\ i = \overline{1, N},$$

, , .

2.1. — $\{\vec{r}_i, \vec{v}_i\}$.

$$m_i\ddot{\vec{r}}_i = \vec{F}_i^{(a)} + \vec{R}_i$$

, — . $\vec{F}_i^{(a)}$ — ( ), $\vec{R}_i$ — ( ).

2.1. .

$$f(t, \{\vec{r}_i\}, \{\vec{v}_i\}) = 0,$$

— .

$$f(\dots) \geqslant 0 \text{— , .}$$

; , -. ...[1]

$$f(t, \{\vec{r}_i\}) = 0 \Rightarrow$$

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{N} \frac{\partial f}{\partial \vec{r}_i}\vec{v}_i = 0. \tag{2.10}$$

, , , , (2.10). ()($\not t$), ()(t) .

2.1.1.

$$\boxed{\text{WILL BE COMPLEMENTED}}$$

[1]. [1], c. 204, [2], . 201.

2.2.

.

2.2.1.

(s =1) . x q. ( ):

$$L(t,x,\dot{x}) = \frac{1}{2}\alpha(x)\dot{x}^2 + \beta(x)\dot{x} - U(x),$$

, α, β — .

1. $\frac{\partial L}{\partial t} = 0$.

2. — , , , β, . .

$$\beta\dot{x} = \frac{\mathrm{d}}{\mathrm{d}t}\int \beta \,\mathrm{d}x\,.$$

2.2 ( ). — — . $x = x_0 = const$ ($\dot{x} = \ddot{x} = \dots = 0$).

3. , $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x}$.

$$\frac{\partial L}{\partial \dot{x}} = \alpha(x)\dot{x} \Rightarrow \alpha\ddot{x} + \alpha'\dot{x}^2 = \frac{\partial L}{\partial x} = \frac{1}{2}\alpha'\dot{x}^2 - U' \Rightarrow$$

$$\alpha(x)\dot{x} + \frac{1}{2}\frac{\partial \alpha}{\partial x}\dot{x}^2 = -\frac{\partial U}{\partial x} \Rightarrow \frac{\partial U(x_0)}{\partial x} = 0.$$

x_0:

$$\frac{\partial U(x_0)}{\partial x} = 0$$

x_0:

$$x = x_0 + q \Rightarrow \alpha(x) = \alpha(x_0) + \dots; \quad \alpha(x_0) = m$$
$$U(x) = U(x_0) + U'(x_0)q + \frac{1}{2}U''(x_0)q^2 + \approx \frac{1}{2}kq^2; \quad k = U''(x_0) \Rightarrow$$
$$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$$

, «», . , , , : . , , , . : k q , , . :

$$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2 = \frac{m}{2}(\dot{q}^2 - \omega^2 q^2), \quad \omega^2 = k/m,$$

$$\ddot{q} + \omega^2 q = 0$$

, . :

$$q = Ce^{\lambda t} \Rightarrow \lambda^2 Ce^{\lambda t} + \omega^2 Ce^{\lambda t} = 0$$
$$\Rightarrow \lambda^2 + \omega^2 = 0 \Leftrightarrow \lambda = \pm i\omega \Rightarrow$$
$$q = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$$

,

$$q \in \mathbb{R} \Rightarrow C_2 = C_1^* \Leftrightarrow q = C_1 e^{i\omega t} + \dots = 2\,\mathrm{Re}\,C_1 e^{i\omega t} = \mathrm{Re}\,Ce^{i\omega t}, \quad C \in \mathbb{C}$$
$$C = ce^{i\varphi} - \quad \Rightarrow q = c\cos(\omega t + \varphi)$$

2.1. $\hat{D}$ — ,

$$\hat{D}q = 0, \quad \hat{D} \in \mathbb{R},$$

$X \in \mathbb{C},\ \hat{D}X = 0,$

$$\begin{cases} \hat{D}(\operatorname{Re} X) = 0 \\ \hat{D}(\operatorname{Im} X) = 0 \end{cases}$$

.

$$\hat{D}(\operatorname{Re} X + i \operatorname{Im} X) = 0 \Rightarrow$$
$$\hat{D}(\operatorname{Re} X) + i\hat{D}(\operatorname{Im} X) = 0$$

$\square$

$$q = Ce^{\lambda t}$$
$$\ddot{q} + \omega^2 q = Ce^{\lambda t}(\lambda^2 + \omega^2) = 0 \Rightarrow q = Ce^{i\omega t} \Rightarrow q = \operatorname{Re} Ce^{i\omega t}$$
$$\omega^2 > 0 \quad q = c\cos(\omega t + \varphi) = a \sin \omega t + b \cos \omega t,$$

:

$$C = ce^{i\varphi}.$$

ω , k , m , k «» , . .

$$\omega^2 < 0 \quad \lambda = \pm\sqrt{|\omega|} \in \mathbb{R}$$
$$q = c_1 e^{\lambda t} + c_2 e^{-\lambda t} = a \operatorname{sh} \lambda t + b \operatorname{ch} \lambda t \underset{H/w}{\overset{?}{=}} c \operatorname{sh}(\lambda t + \varphi) \underset{H/w}{\overset{?}{=}} \tilde{c} \operatorname{ch}(\lambda t + \varphi)$$
$$\omega = 0 \quad q = c_1 t + c_2 \quad \ddot{q} = 0$$

. $(L = \frac{m}{2}(\dot{q}^2 - \omega^2 q^2),\ H = \frac{m}{2}(\dot{q} + \omega^2 q^2) = const)$.

2.2.1 (.).

$$\ddot{q} + \omega_0 q + 2\gamma\dot{q} = 0,$$

.

$$q = Ce^{\lambda t} \Rightarrow (\underbrace{\lambda^2 + \omega_0^2 + 2\gamma\lambda}_{0})Ce^{\lambda t} = 0$$
$$\lambda = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} = -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2},$$

, .

$$q = ce^{-\lambda t} \cos\left(\sqrt{\omega_0^2 - \gamma^2} t + \varphi\right),$$

. H/w $\omega_0 = \gamma,\ \omega_0 < \gamma,\ \omega_0 > \gamma$ ( ).

2.2.2 (, .).

$$\ddot{q}+\omega^2 q = f(t)$$
$$\dot{q}+i\omega t = a(t)e^{i\omega t}, \tag{2.11}$$
$$a(t)\in\mathbb{C}\Rightarrow q(t)=\frac{1}{\omega}\operatorname{Im} ae^{i\omega t}$$

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{q}+i\omega t\right)=(\dot{a}+i\omega a)e^{i\omega t}\\ (2.11)*i\omega: \quad -i\omega\dot{q}+\omega^2 q=-i\omega a e^{i\omega t}\end{cases} \Rightarrow \ddot{q}+\omega^2 q=\dot{a}e^{i\omega t}=f(t)\Rightarrow a(t)=\int^t f(t)e^{-i\omega t}\,\mathrm{d}t$$

, $a(t)$.

2.2 (.). (.) $\int\limits_{-\infty}^{+\infty} fe^{-i\omega t}\,\mathrm{d}t = F,\ \ H(+\infty)-H(-\infty)=\frac{m}{2}|F|^2,$

F — .

. H/w □

2.1.

$$\ddot{q}+2\gamma\dot{q}+\omega_0^2 q = f(t)$$

, : $f(t)=A\cos\omega t\Rightarrow q(t)=?$.

2.2.2.

$$L=\sum_{i,j}\frac{1}{2}\alpha_{ij}(x)\dot{x}_i\dot{x}_j+\sum_i\beta_i(x)\dot{x}_i-U(x)$$

. , x — — $U(x)$:

$$x=x_0\equiv const\Leftrightarrow\frac{\partial U(x_0)}{\partial x_i}=0,\quad \forall i=\overline{1,s}$$

$$x=x_0+q\Rightarrow\alpha_{ij}(x)\approx m_{ij}=\alpha_{ij}(x_0)$$
$$U(x)\approx\sum_{i,j}\frac{1}{2}k_{ij}q_iq_j;\quad k_{ij}=\frac{\partial^2 U(x_0)}{\partial x_i\partial x_j}$$

m_{ij}, k_{ij}?

1. m_{ij} $(m_{ij}=mji)$, α — , , m .
2. $k_{ij}=k_{ji}:$, ( ).

. , . ,

$$\sum_i\beta_i(x_0)\dot{x}_i=\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum\beta_i(x_0)x_i\right),$$
$$\beta_i(x)\approx\cancel{\beta_i(x_0)}+\sum_j\frac{\partial\beta_i(x_0)}{\partial x_j}q_j;\quad g_{ij}=\frac{\partial\beta_i(x_0)}{\partial x_j},$$

:

$$\boxed{L=\sum_{i,j=1}^{s}\left\{\frac{1}{2}m_{ij}\dot{q}_i\dot{q}_j+g_{ij}q_j\dot{q}_i-\frac{1}{2}k_{ij}q_iq_j\right\}}. \tag{2.12}$$

2.2.3 ($g_{ij} = 0$). (2.12) , , $g_{ij} = 0$:

$$L = \sum_{i,j=1} \left\{ \frac{1}{2} m_{ij} \dot{q}_i \dot{q}_j - \frac{1}{2} k_{ij} q_i q_j \right\}.$$

, : , . , . .

2.3.

$$\left\{ \begin{array}{l} .(+) \\ . \end{array} \right. \Rightarrow \ !$$

$\exists \quad a_{ik} \mid q_i = \sum_i a_{ik}\theta_k, \dot{q}_i = \sum a_{ik}\dot{\theta}_k.$

$$L = \sum_k^s \{\frac{1}{2} m_k \dot{\theta}_k^2 - \frac{1}{2} k_k \theta_k^2\} = \sum_k \frac{m_k}{2} \{\dot{\theta}_k^2 - \omega_k^2 \theta_k^2\}, \tag{2.13}$$

$\omega_k^2 = k_k/m_k$, s , . (2.13):

$$\ddot{\theta}_k + \omega_k^2 \theta_k = 0 \Rightarrow$$
$$\theta(t) = C_k \cos(\omega t + \varphi_k) \approx \operatorname{Re} C_k e^{i\omega_k t}$$

2.3. $\{\omega_k\}$ — $\omega_k, \ k = \overline{1, s}$.

2.4. $\{\theta_k\}$ — .

?

2.5. $\theta_k = 0, \ \ k = k^* \Rightarrow$

$$q_j = a_{jk^*} \theta_{k^*}(t)$$

.

:

$$q_j(t) = \sum_{k=1}^s a_{jk} \theta_k(t).$$

2.2.1. , . , , , .

2.2.2. , (2.13) , , $\omega_k^2 > 0 \ \forall k$, , , .

(2.13):

$$\frac{\partial L}{\partial \dot{q}_k} = \{\frac{1}{2} m_{ik} \dot{q}_i + \frac{1}{2} m_{kj} \dot{q}_j + g_{kj} q_j\} = \sum_i \{m_{ik} \dot{q}_i + g_{ki} q_i\},$$
$$\frac{\partial L}{\partial q_k} = \sum \{g_{ik} \dot{q}_i - k_{ik} q_i\} \overset{!!!!!}{\Rightarrow}$$
$$\frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0 \Rightarrow \sum_i \{m_{ij} \ddot{q}_i + (g_{ij} - g_{ji}) \dot{q}_i + k_{ij} q_i\} = 0.$$

2.2.3. $G_{ij} = -G_{ji}$.

$$q_i = \operatorname{Re} C_i e^{\lambda t},$$

$$\operatorname{Re} \sum_i \{m_{ij}\lambda^2 + G_{ij}\lambda + k_{ij}\} C_i e^{\lambda t} = 0 \Leftrightarrow$$
$$\sum_i \{m_{ij}\lambda^2 + G_{ij}\lambda + k_{ij}\} C_i = 0. \tag{2.14}$$

, :

$$\det\left(m_{ij}\lambda^2 + G_{ij}\lambda + k_{ij}\right) = 0 - .$$

. $P_{2s}(\lambda) = 0$, , λ — , λ^* — , $P_{2s} \in \mathbb{R}$, . , , , , $P_s(\lambda^2) = 0$. — . , . λ — , $-\lambda$:

$$\begin{gathered} \det\left(m_{ij}(-\lambda)^2 + G_{ij}(-\lambda) + k_{ij}\right) \Leftrightarrow \\ \det\left(m_{ij}\lambda^2 + G_{ji}\lambda + k_{ij}\right) \Leftrightarrow \\ \det\left(m_{ji}\lambda^2 + G_{ji}\lambda + k_{ji}\right), \end{gathered}$$

m_{ji} k_{ji} , , , λ — — , . , $-\lambda$ — , ,

$$P_s(\lambda^2) = 0.$$

:

$$\lambda \longrightarrow \lambda^*, -\lambda, -\lambda^*.$$

, , - , . (2.14). : C_j^*, ,

$$C_i C_j^* = (c_i' + ic_i'')(c_j' - ic_j'') = (\underbrace{c_i'c_j' + c_i''c_j''}_{S_{ij}}) + i(\underbrace{c_i''c_j' - c_i'c_j''}_{A_{ij}}),$$

:

$$\sum_{i,j} \boxed{m_{ij}S_{ij}\lambda^2 + iG_{ij}A_{ij}\lambda + k_{ij}S_{ij} = 0}$$

1. $k_{ij}(+); G_{iJ} = 0 \Rightarrow \lambda^2 = -\frac{k_{ij}S_{ij}}{m_{ij}S_{iJ}} < 0 \Rightarrow \lambda = \pm i\omega$ — , .
2. $(+)k_{ij} \Rightarrow \lambda^2 > 0 \Rightarrow \pm\lambda \Rightarrow c_1 e^{\lambda t} + c_2 e^{-\lambda t}$ — .
3. , . $k_{ij}(+) \& G_i \neq 0 \Rightarrow \lambda^2 < 0$, .

, . .

$$\begin{aligned} P_s(\lambda^2) = 0 \Rightarrow \{\lambda_k^2\} k = \overline{1, s} \\ \lambda_k^2 < 0 \Rightarrow \lambda_k = \pm i\omega_k \\ q_j^{(k)} = \operatorname{Re} C_{jk} e^{i\omega_k t} \end{aligned}$$

$$\sum_{i=1}^{s}\left(m_{ij}\lambda^2+G_{ij}\lambda+k_{ij}\right)C_i=0 \quad j=\overline{1,s}\Rightarrow C_{jk}=a_{jk}e^{i\varphi_{jk}}B_k$$

$$\forall B_k=b_ke^{i\varphi_{0_k}}$$

q, :

$$q_j=\sum_{k=1}^{s}q_j^{(k)}=\sum_k \operatorname{Re} b_k a_{jk}e^{i\omega_k t+i\varphi_{jk}+i\varphi_{0_k}}$$

$\varphi_{jk}=0,\ \ G=0$ ( ),

$$q_j=\sum_k a_{jk}\underbrace{\operatorname{Re} B_k e^{i\omega_k t+i\varphi_{0_k}}}_{\theta_k(t)},$$

.

,

$$q_j=\sum_k \operatorname{Re}\left\{B_kC_{jk}e^{i\omega_k t}\right\}.$$

.

2.2.4 (). - $z=h(x,y)$. , $x=y=0$ $\min h(x,y)$,

$$z=h(x,y)=\frac{x^2}{2\rho_1^2}+\frac{y^2}{2\rho_2^2}+\dots \quad \rho_{1,2}\text{ — }.$$

:

$$L=T-U=\frac{m}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-mg\left(\frac{x^2}{2\rho_1^2}+\frac{y^2}{2\rho_2^2}\right), \tag{2.15}$$

$\dot{z}^2$, , (2.15)

$$L=\frac{m}{2}\left\{\dot{x}^2-\Omega_1^2x^2+\dot{y}^2-\Omega_2^2y^2\right\}, \quad \Omega_{1,2}=\frac{g}{\rho_{1,2}^2}.$$

$$x=a\cos(\Omega_1 t+\varphi_1),$$
$$y=b\cos(\Omega_2 t+\varphi_2), \quad .$$

2.2.5 ( ). x, y, , , , :

$$\mathbf{v}_{co}=[\boldsymbol{\Omega},\mathbf{r}]\Rightarrow$$

$$\begin{cases} v_x=\dot{x}-\Omega y\\ v_y=\dot{y}+\Omega x\end{cases} \Rightarrow L=\frac{m}{2}\left\{(\dot{x}-\Omega y)^2+(\dot{y}+\Omega x)^2-\Omega_1^2x^2-\Omega_2^2y^2\right\},$$

, ( ), :

$$L=\frac{m}{2}\{\dot{x}^2+\dot{y}^2+2\Omega(x\dot{y}-y\dot{x})-(\underbrace{\Omega_1^2-\Omega^2}_{\widetilde{\Omega}_1^2})x^2-(\underbrace{\Omega_2^2-\Omega^2}_{\widetilde{\Omega}_2^2})y^2\}.$$

$$\left.\begin{array}{l}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}}=m\ddot{x}-m\Omega\dot{y}=\frac{\partial L}{\partial x}=m\Omega\dot{y}-m\widetilde{\Omega}_1^2x\\ \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{y}}=m\ddot{y}+m\Omega\dot{x}=\frac{\partial L}{\partial y}=-m\Omega\dot{x}-m\widetilde{\Omega}_2^2y\end{array}\right\}\Rightarrow$$

$$\begin{cases}\ddot{x}-2\Omega\dot{y}+\widetilde{\Omega}_1^2x=0\\ \ddot{y}+2\Omega\dot{x}+\widetilde{\Omega}_2^2y=0\end{cases}$$

$$x=C_1e^{i\Omega t}$$

$$y=C_2e^{i\Omega t}$$

$$\begin{cases}\left(-\omega^2+\widetilde{\Omega}_1^2\right)C_1-2\Omega i\omega C_2=0\\ 2\Omega i\omega C_1+\left(-\omega^2+\widetilde{\Omega}_2^2\right)C_2=0,\end{cases}\tag{2.16}$$

(2.16) , C_1 C_2 ,

$$\boxed{\left(\omega^2-\widetilde{\Omega}_1^2\right)\left(\omega^2-\widetilde{\Omega}_2^2\right)=4\Omega^2\omega^2}.$$

ω. ,

$$\omega\ll\widetilde{\Omega}_1,\widetilde{\Omega}_2.$$

, :

$$H=const\quad H=\sum\frac{1}{2}\left\{m_{iJ}\dot{q}_i\dot{q}_j+k_{ij}q_iq_j\right\}.$$

$$H/w\quad\frac{\mathrm{d}H}{\mathrm{d}t}=0\Leftrightarrow P_s(\lambda^2)=0\ \ \lambda\ \ -\lambda-\ \ .$$

2.2.3.

. :

$$\mathbf{F}_i=-\sum_j\mu_{ij}\mathbf{v}_j,$$

, , , $R(t,\{\mathbf{r}_i\},\{\mathbf{v}_i\}$, , :

$$\mathbf{F}_i=-\sum_j\mu_{ij}\mathbf{v}_j=-\frac{\partial R}{\partial \mathbf{r}_i}\quad R=\frac{1}{2}\sum_{i,j}\mu_{ij}\mathbf{v}_i\mathbf{v}_j=\frac{1}{2}\gamma_{iJ}\dot{q}_i\dot{q}_j,$$

$$Q_j=\sum_{i=1}^N\mathbf{F}_i\frac{\partial \mathbf{r}_i}{\partial q_j}=-\sum_{i=1}^N\frac{\partial R}{\partial \mathbf{v}_i}\frac{\partial \mathbf{v}_i}{\partial \dot{q}_j}=-\frac{\partial R}{\partial \dot{q}_j}=-\sum_{i=1}^n\gamma_{ij}\dot{q}_i,$$

, ( ):

$$\gamma_{ij}=\gamma_{ji}.$$

:

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j}-\frac{\partial L}{\partial q_j}+\frac{\partial R}{\partial \dot{q}_j}=0}.$$

$$H/w \Rightarrow \sum_{j=1}^{s} \{m_{ij}\dot{q}_j + (G_{ij} + \gamma_{ij})\,\dot{q}_j k_{ij} q_j\} = 0,$$

G_{ij} — ( ), ,
$P_s(\lambda^2) = 0$; $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$; λ_{ij} — (, $P_{2s}(\lambda) = 0$ — , , , , « », $\frac{\mathrm{d}H}{\mathrm{d}t} = \sum Q_j\dot{q}_j = -\sum \frac{\partial R}{\partial \dot{q}_j}\dot{q}_j = -2R$,
, , , , ,

$$e^{\lambda t};\ \lambda = \lambda' + i\lambda'' \Rightarrow$$
$$e^{\lambda' t}\cos(\lambda'' t + \varphi),$$

, — , λ λ^* ( ), λ $-\lambda$ — .

2.3.

2.3.1.

, , . , , ( ). , , . .
— " ".

$$L(t, q, \dot{q}, q = (q_1, \ldots, q_s), s = 3N - k,$$

$$Q = 0,$$

$$\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0\right).$$

2.6. $\{q\}$ — .

$q(t)$ — , — , . . — « ».

$$q_1 = q(t_1),$$
$$q_2 = q(t_2).$$

, , ? , , () .

2.3.2. -

- , , .

$$q(t) \xrightarrow{S} \mathbb{R}$$

. ,

$$S[q(t)] \to \mathbb{R}.$$

.

$$\boxed{S[q(t)] = \int L(t, q, \dot{q}(t))\,\mathrm{d}t},$$

( ). , .

2.1.

1. $q(t_1) = q_1, q(t_2) = q_2,$

2. $\exists M: \; |q_1 - q_2| < M$,

$q(t)$, , $S[q(t)] \to min$ — , , .

2.2 ( ). $q(t_1) = q_1, q(t_2) = q_2$, , $S[q(t)] \to stat$ — , .

( ) $q(t) + \delta q(t), \;\; \delta q(t_1) = 0, \; \delta q(t_2) = 0, \; \forall \delta q(t).$

$$S[q(t) + \delta q(t)] - S[q(t)] \geqslant 0$$

, . ,

$$\delta q(t) = \alpha \cdot h(t),$$
$$h(t_1) = h(t_2) = 0,$$

$$S(\alpha) = S[q(t) + \alpha \cdot h(t)] \geqslant,$$

... . $\alpha = 0$

$$S(\alpha) \to \frac{\partial S}{\partial \alpha} = 0 \mid_{\alpha=0}.$$

$$\frac{\partial}{\partial \alpha} \int\limits_{t_1}^{t_2} L(t, q + \alpha h, \dot{q} + \alpha \dot{h})\,\mathrm{d}t = \int\limits_{t_1}^{t_2} \left\{ \frac{\partial L}{\partial q} h + \frac{\partial L}{\partial \dot{q}} \dot{h} \right\}_{|\alpha=0} \mathrm{d}t =$$

$$= \int\limits_{t_1}^{t_2} \left\{ \frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{q}} \right) \right\} h(t)\,\mathrm{d}t + \underbrace{\left. \frac{\partial L}{\partial \dot{q}} h \right|_{t_1}^{t_2}}_{=0} =$$

$$= \int\limits_{t_1}^{t_2} \sum_{j=1}^{s} \left\{ \frac{\partial L}{\partial q_j} - \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{q}_j} \right) \right\} h_j(t) dt = 0 \qquad \forall h_j(t). \Rightarrow$$

$$\forall \, j = 1, s \quad \boxed{\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0},$$

, , (, , — ,).

, , ,

$$S[q + \delta q] - S[q] = \int\limits_{t_1}^{t_2} \left\{ L(t, q + \delta q, \dot{q} + \delta \dot{q}) - L(t, q, \delta q) \right\} \mathrm{d}t\,,$$

,

$$S[q + \delta q] - S[q] = \int\limits_{t_1}^{t_2} \left\{ \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} + \ldots \right\} \mathrm{d}t =,$$

$$= \int\limits_{t_1}^{t_2} \left\{ \frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}} \right\} \delta q \,\mathrm{d}t + \ldots.$$

, $\delta S[q, \delta q]$.

$$\delta S = 0 \Longleftrightarrow \text{ - II } .$$

,

$$\text{-e II} \Longrightarrow \begin{cases} \delta S = 0, \\ \delta^2 S \geqslant 0 \end{cases} \quad , \ q_1, q_2.$$

2.3.1 (, -). , $\frac{d}{dt}\frac{\partial T}{\partial \dot q} - \frac{\partial T}{\partial q} = Q$. $T(t, q, \dot q)$, — $\delta S = \int\limits_{t_1}^{t_2} Q \, \mathrm{d}t$,

$$Q = \sum_{j=1}^{s} Q_j \delta q_j = \sum_{i=1}^{N} \mathbf{F}_i^{(a)} \delta \mathbf{r}_i = \delta A^{(a)},$$

,

$$S[q(t)] = \int\limits_{t_1}^{t_2} \left(T + A^{(a)}\right) \mathrm{d}t \, .$$

, - , ,

$$S = \int\limits_{t_1}^{t_2} (T - U) \, \mathrm{d}t \, .$$

H/w , , , , .

2.3.3.

? ...

,

$$L = \frac{1}{2}(\dot q^2 - \omega^2 q.$$

:

$$\ddot q = -\omega^2 q,$$
$$q = a \sin(\omega t + \varphi).$$

$q + \delta q$,

$$S[q + \delta q] - S[q]. \tag{2.17}$$

$t_1 = 0$, $t_2 = \tau$, $\tau \neq nT/2$, q_1 q_2, , , $\tau = nT/2$ $(T = \frac{2\pi}{\omega})$, , , .

2.7. — , ( ) .

2.3.1 ( ). , , , .

$$\begin{cases} \delta q(0) = 0 \\ \delta q(\tau) = 0, \forall \delta q(t) \end{cases} \Rightarrow \delta q(t) = \sum_{n=1}^{\infty} a_n \sin\left(n \frac{\pi}{\tau} t\right), \ \forall a_n \in \mathbb{R}.$$

(2.17)

$$S[q+\delta q]-S[q]=\int\limits_0^\tau \frac{1}{2}\left((\dot q+\delta\dot q)^2-\omega^2(q+\delta q)^2-\dot q^2+\omega^2 q\right)\mathrm{d}t=$$
$$=\int\limits_0^\tau \frac{1}{2}\left\{\underbrace{2\dot q\delta\dot q-2\omega^2 q\delta q}_{\dot q\delta\dot q+\ddot q\delta q=\frac{\mathrm{d}}{\mathrm{d}t}(\dot q\delta q)}+\delta\dot q^2-\omega^2\delta q^2\right\}\mathrm{d}t=$$
$$=\int\limits_0^\tau \frac{1}{2}\sum_{n=1}^{\infty}a_n^2\left\{(n\frac{\pi}{\tau})^2\cos^2(n\frac{\pi}{\tau}t)-\omega^2\sin^2(\frac{n\pi}{\tau}t)\right\}\mathrm{d}t=$$
$$=\frac{\pi}{4}\sum_{n=1}^{\infty}a_n^2(n^2\Omega^2-\omega^2),$$

, $\Omega>\omega$, , $\tau<T/2$. , , [2].
. , ([3]) .
, , .
, , .

2.3.4.

? , .
, — , . — .

1. $L(t,q,\dot q)$, $L'=L+\frac{\mathrm{d}}{\mathrm{d}\Phi}(t,q)$. L

$$S[q]=\int_{t_1}^{t_2}L\,\mathrm{d}t\to\delta S[q,\delta q]=0\Leftrightarrow.\quad q(t).$$

L'

$$S'[q]=\int_{t_1}^{t_2}L'\,\mathrm{d}t=S[q]+\Phi(t,q)|_{t_1}^{t_2}\to\delta S'=\delta S\,,$$

, q t.

2. .

$$L(t,q,\dot q)\to$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot q}-\frac{\partial L}{\partial q}=0\overset{q^*=\varphi(t,q)}{\longrightarrow}\qquad q^*(t)?$$
$$\left(\frac{\mathrm{d}\tilde\varphi}{\mathrm{d}t}=\frac{\partial\tilde\varphi}{\partial t}+\frac{\partial\tilde\varphi}{\partial q^*}\dot q^*,\quad q=\tilde\varphi(t,q^*)\right)$$
$$S[q]=\int_{t_1}^{t_2}L\,\mathrm{d}t\equiv\int_{t_1}^{t_2}\underbrace{L\left(t,\tilde\varphi(t,q^*),\frac{\mathrm{d}\tilde\varphi}{\mathrm{d}t}\right)}_{L^*(t,q^*,\dot q^*)}\mathrm{d}t=S^*[q^*(t)]$$
$$\delta S[q]=\delta S^*[q^*]=0$$

---

[2], , . [3], §5, . 7.
[3] , .

2.3.2. $L = L_0 + L_1 + L_2$, .

3. .

$$L(t,q,\dot q) \to . \ \to \begin{cases} q^* = \varphi(t,q) \\ t^* = \psi(t,q) \end{cases} \to ?\ \text{-}\ q^*(t^*)$$

$$\begin{cases} q = \tilde\varphi\,(t^*, q^*) \\ t = \widetilde\psi(t^*, q^*) \end{cases}$$

$$S[q(t)] = \int_{t_1}^{t_2} L(t,q,\dot q)\,\mathrm{d}t = \int_{t_1^*}^{t_2^*} \underbrace{L\left(\widetilde\psi, \tilde\varphi, \frac{\mathrm{d}\tilde\varphi}{\mathrm{d}\widetilde\psi}\right)\frac{\mathrm{d}\widetilde\psi}{\mathrm{d}t^*}}_{L^*\left(t^*, q^*, \frac{\mathrm{d}q^*}{\mathrm{d}t^*}\right)}\mathrm{d}t^* = S^*[q^*(t^*)],$$

,

$$\delta S \equiv \delta S^*,$$

, L^* , , «**».

$$\left.\begin{aligned} \mathrm{d}q &= \tfrac{\partial\tilde\varphi}{\partial t^*}\,\mathrm{d}t^* + \tfrac{\partial\tilde\varphi}{\partial q^*}\,\mathrm{d}q^* \\ \mathrm{d}t &= \tfrac{\partial\widetilde\psi}{\partial t^*}\,\mathrm{d}t^* + \tfrac{\partial\widetilde\psi}{\partial q^*}\,\mathrm{d}q^* \end{aligned}\right\} \Rightarrow L^* = L\left(\widetilde\psi, \tilde\varphi, \frac{\frac{\partial\tilde\varphi}{\partial t^*} + \frac{\partial\tilde\varphi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}t^*}}{\frac{\partial\widetilde\psi}{\partial t^*} + \frac{\partial\widetilde\psi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}t^*}}\right)\cdot\left(\underbrace{\frac{\partial\widetilde\psi}{\partial t^*} + \frac{\partial\widetilde\psi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}t^*}}_{\frac{\mathrm{d}t}{\mathrm{d}t^*}}\right) \tag{2.18}$$

2.3.3. !

2.3.2. ( ).

$$\mathbf{p} = m\mathbf{v} = \frac{\partial L}{\partial \mathbf{v}} \Rightarrow L = \frac{mv^2}{2}.$$

$$\left.\begin{aligned} H &= \mathbf{p}\cdot\mathbf{v} - L = L \\ H &= T \end{aligned}\right\} \Rightarrow L = T\ ()$$

( ). H/w

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \frac{\partial L}{\partial \mathbf{v}} \Rightarrow L = -mc^2\sqrt{1 - v^2/c^2}$$

$$H = \mathbf{p}\cdot\mathbf{v} - L = \frac{mv^2 + mc^2}{\sqrt{\cdots}} = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$H = T \Rightarrow L \neq T\ (.)$$

2.3.5. ( )

, , — , , — , , .
. , .

2.4 ( ). , , .
,

$$\begin{aligned} q^* &= \varphi(t, q, \alpha) \\ t^* &= \psi(t, q, \alpha), \end{aligned}$$

,

1. $q = q^*$, $t = t^*$ $\alpha = 0$.
2. $|\alpha| < \varepsilon$ $\tilde\varphi(t^*, q^*, \alpha) = q$, $\widetilde\psi(t^*, q^*, \alpha) = t$.
3. (.) Φ-[4] $L^*\left(t^*, q^*, \frac{\mathrm{d}q^*}{\mathrm{d}t^*}\right) = L\left(t^*, q^*, \frac{\mathrm{d}q^*}{\mathrm{d}t^*}\right) + \frac{\mathrm{d}}{\mathrm{d}t^*}\Phi(t^*, q^*, \alpha)$. [5]

---

[4] L^* (3).

[5] $S[q] = S^*[q^*] = S[q^*] + \Phi|_{t_1^*}^{t_2^*}$.

$$I(t,q,\dot q)\left\{\sum_{j=1}^{s}\frac{\partial L}{\partial \dot q_j}\frac{\partial \varphi_j}{\partial \alpha}-H\frac{\partial \psi}{\partial \alpha}+\frac{\partial \Phi}{\partial \alpha}\right\}_{\alpha=0}=const.$$

2.3.3 ( ).

$$x^*=x+\alpha, \Phi=0\Rightarrow I=\frac{\partial L}{\partial \dot x}\left(\frac{\partial X^*}{\partial x}\right)_{\alpha=0}=p_x.$$

2.3.4 ().

$$\begin{cases} x^*=x\cos\alpha+y\sin\alpha \\ y^*=-x\sin\alpha+y\cos\alpha, \end{cases}\qquad \Phi=0\Rightarrow I=p_x\frac{\partial x^*}{\partial \alpha}+p_y\frac{\partial y^*}{\partial \alpha}=yp_x-xp_y=M_z$$

2.3.5 ( ). $z^*=z+\frac{h}{2\pi}\alpha,\ \ I=M_z+p_z\frac{h}{2\pi}$.

2.3.6.

$$t^*=t+\alpha\Rightarrow I=-H.$$

2.3.7 ( ).

$$\begin{cases} x^*=x-\alpha t \quad (\alpha=u) \\ t^*=t \end{cases}\qquad \Rightarrow \frac{\partial x^*}{\partial \alpha}=-t,$$

$$L=\frac{m}{2}\dot x^2,\ \dot x=\dot x^*+\alpha,$$
$$L^*=L(\dot x^*+\alpha)=\underbrace{\frac{m}{2}\dot x^*}_{L^*(\dot x^*)}+\underbrace{m\alpha\dot x^*+\frac{m}{2}\alpha^2}_{\frac{\mathrm d}{\mathrm dt}\left(\alpha m x^*+\frac{m\alpha^2}{2}t\right)}\Rightarrow$$
$$\Rightarrow I=p_x(-t)+mx=const\ (=0),$$
$$p_x=m\frac{x}{t}.$$

. Φ- $\alpha\to 0$.

$$q^*=\varphi(t,q,\alpha)=q+\alpha\cdot Q+\dots \qquad\qquad Q_j=\left.\frac{\partial \varphi_j}{\partial \alpha}\right|_{\alpha=0}$$
$$t^*=\psi(t,q,\alpha)=t+\alpha\cdot T+\dots \qquad\qquad T=\left.\frac{\partial \psi}{\partial \alpha}\right|_{\alpha=0}.$$

,

$$\frac{\mathrm d q^*}{\mathrm d t^*}=\frac{\dot q+\alpha\dot Q+\dots}{1+\alpha\dot T+\dots}=\dot q+\alpha(\dot Q-\dot q\dot T)+\dots.$$

(3) L^*

$$L^*=L\left(t^*-\alpha T,q^*-\alpha Q,\frac{\mathrm d q^*}{\mathrm d t^*}-\alpha(\dot Q-\dot q\dot T)\right)\cdot\left(1-\alpha\dot T\right)=$$
$$=L(t^*,q^*,\frac{\mathrm d q^*}{\mathrm d t^*})-\alpha\Big\{\underbrace{\frac{\partial L}{\partial t}}_{=-\dot H T}T+\underbrace{\frac{\partial L}{\partial q}Q}_{\dot p Q}+\frac{\partial L}{\partial \dot q}\left(\dot Q-\dot q\dot T\right)+L\dot T\Big\}=$$
$$=L(*)-\alpha\left\{-\frac{\mathrm d}{\mathrm dt}(HT)+\frac{\mathrm d}{\mathrm dt}(pQ)\right\}+\dots, \tag{2.19}$$

$\frac{\partial L}{\partial \dot q}\dot Q = p\dot Q$, $-\frac{\partial L}{\partial \dot q}\dot q\dot T + L\dot T = \dot T(L - \dot q p) = -H\dot T$. Φ-, L^* (2.3.5)

$$L^* = L(*) + \underbrace{\frac{\mathrm{d}}{\mathrm{d}t^*}\Phi(t^*, q^*, \alpha)}_{\frac{\mathrm{d}}{\mathrm{d}t}\left(\alpha \left.\frac{\partial \Phi}{\partial \alpha}\right|_{\alpha=0} + \dots\right)}.$$

,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{pQ - HT + \left.\frac{\partial \Phi}{\partial \alpha}\right|_{\alpha=0}\right\} = 0,$$

. □

2.3.6. ?

$$L(t, q, \dot q) \Rightarrow S[q(t)] = \int_{t_1}^{t_2} L\left(t, q(t), \dot q(t)\right)\mathrm{d}t$$

$$. \Leftrightarrow \begin{cases} \delta S = 0 \\ \delta q(t_1) = \delta q(t_2) = 0 \end{cases} \quad () \quad \oplus \quad \begin{cases} q_1, q_2 \ , \ . , \\ \quad () \\ S \to min\ (max) \end{cases}$$

, , ?

1. .

2. .

$$\begin{cases} q^* = q + \alpha \cdot Q(t, q) + O(\alpha^2) \\ t^* = t + \alpha \cdot T(t, q) + O(\alpha^2) \end{cases} \quad + \Phi\text{-}\ L^* = L\left(t^*, q^*, \frac{\mathrm{d}q^*}{\mathrm{d}t^*}\right) + \frac{\mathrm{d}}{\mathrm{d}t^*}\Phi(t^*, q^*, \alpha),$$
$$\left(t \ \ q \quad , \Phi = \Phi_0 + \alpha \cdot \Xi + O(\alpha^2)\right),$$

$$\frac{\partial L}{\partial \dot q}Q + H \cdot T + \Xi = const \ \ . . .$$

2.3.4.

$$\begin{cases} q^* = \varphi(q, t) \\ t^* = \psi(q, t) \\ \dots \end{cases} \Rightarrow \begin{cases} Q = \left.\frac{\partial \varphi}{\partial \alpha}\right|_{\alpha=0}, \\ T = \left.\frac{\partial \psi}{\partial \alpha}\right|_{\alpha=0}, \\ \Xi = \left.\frac{\partial \Phi}{\partial \alpha}\right|_{\alpha=0} \end{cases}$$

2.3.7. .

$$L = \frac{mv^2}{2},$$

, , $L = T - U$, , $L(t, q, \dot q)$, . , .

1. . . $\Rightarrow L(\cancel{t}, \cancel{\mathbf{r}}, \mathbf{v}) = L(v^2)$. , $L(v^2)$, :

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial v^2}\frac{\partial v^2}{\partial \mathbf{v}} = \frac{\partial L}{\partial v^2} \cdot 2\mathbf{v} = const\ (\mathbf{r} - .) \Rightarrow v = const, \mathbf{v} = const$$

$$H = \mathbf{p} \cdot \mathbf{v} - L = 2v^2 \frac{\partial L(v^2)}{\partial v^2} - L = const\ (t - .),$$

:)

2. , - , , , -, .

$$\begin{cases} t^* = t \\ r^* = \mathbf{r} - \mathbf{u} \cdot t;\ \mathbf{u} = \alpha \cdot \mathbf{n}_{const} \end{cases} \xrightarrow{Th.\,Noether} \begin{cases} T = 0 \\ Q = -\mathbf{n} \cdot t \end{cases} \Rightarrow \boxed{-\frac{\partial L}{\partial v^2} 2(\mathbf{n} \cdot \mathbf{v})t = F(t, \mathbf{r})} \frac{\mathrm{d}}{\mathrm{d}t} \rightarrow$$

$$\frac{\partial L}{\partial v^2} \cdot 2(\mathbf{n} \cdot \mathbf{v}) = \frac{\partial F(t, \mathbf{r})}{\partial t} + \frac{\partial F(t, \mathbf{r})}{\partial \mathbf{r}}\mathbf{v},$$

, , ,

$$\frac{\partial L}{\partial v^2} \cdot 2(\mathbf{n} \cdot \mathbf{v}) = \frac{\partial F(t, \mathbf{r})}{\partial t} + \frac{\partial F(t, \mathbf{r})}{\partial \mathbf{r}}\mathbf{v} = a + (\mathbf{b}, \mathbf{n}) \overset{\mathbf{b} = \mathbf{n} \cdot m}{=} m(\mathbf{n}, \mathbf{v})$$

$$(H/w \quad (\mathbf{n}, \mathbf{v}) = a + (\mathbf{b}, \mathbf{v}) \Rightarrow a = 0, \mathbf{b} = \mathbf{n})$$

, , , :

$$\frac{\partial L}{\partial v^2} = \frac{m}{2} \Rightarrow L = \frac{mv^2}{2} + const.$$

,

$$L = \frac{mv^2}{2} - U(t, \mathbf{r}, \mathbf{v}) \dots,$$

.

, ,

$$\begin{cases} \mathbf{r}^* = \frac{\mathbf{r} - \mathbf{u}t}{\sqrt{1 - u^2/c^2}} \\ t^* = \frac{t - (\mathbf{u}, \mathbf{r})/c^2}{\sqrt{1 - u^2/c^2}}. \end{cases} \overset{\mathbf{u} = \alpha \cdot \mathbf{n}}{\Longrightarrow} \boxed{\begin{cases} \mathbf{r}^* = \mathbf{r} - \alpha \mathbf{n} \cdot t + \dots \\ t^* = t - \alpha(\mathbf{n}, \mathbf{r})/c^2 + \dots \end{cases}}.$$

- , . , ,

$$\mathbf{p} = 2\mathbf{v}\frac{\partial L}{\partial t^2}; H = 2v^2 \frac{\partial L}{\partial v^2} - L \Rightarrow \mathbf{p} = const, \mathbf{v} = const, v = const.$$

$L(v^2)$,

$$\mathbf{p}Q - H \cdot T = -F(t, \mathbf{r})$$

$$-2(\mathbf{n}, \mathbf{v})\frac{\partial L}{\partial v^2} \cdot t + \left(2v^2 \frac{\partial L}{\partial v^2} - L\right) \frac{(\mathbf{n}, \mathbf{r})}{c^2} = F(t, \mathbf{r}) \ \Bigg| \frac{\mathrm{d}}{\mathrm{d}t}$$

$$(\mathbf{n}, \mathbf{v}) \left\{ \left(2v^2 \frac{\partial L}{\partial v^2} - L\right) \frac{1}{c^2} - 2\frac{\partial L}{\partial v^2} \right\} = \underbrace{\frac{\partial F}{\partial t}}_{=0} + \underbrace{\frac{\partial F}{\partial \mathbf{r}}}_{\parallel \mathbf{n}}\mathbf{v} = (\mathbf{n}, \mathbf{v}) \cdot \underbrace{\frac{L_0}{c^2}}_{const}$$

$$-\frac{\partial L}{\partial v^2}\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{2c^2}(L - L_0)$$

$$\frac{\partial \ln(L-L_0)}{\partial v^2} = \frac{1}{2}\frac{1}{v^2-c^2} = \frac{1}{2}\frac{\partial}{\partial v^2}\ln\left|v^2-c^2\right|$$

$$L = L_0 + A\sqrt{c^2-v^2} \to \frac{mv^2}{2} \quad v \to 0 \quad \stackrel{cA=-mc^2}{\Rightarrow} \boxed{L = -mc^2\sqrt{1-v^2/c^2}+mc^2} \Rightarrow$$

$$H = \frac{mc^2}{\sqrt{1-v^2/c^2}} \neq L,$$

$L \neq T,$; $\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}}$.

2.4.

( )

2.4.1.

$$U = \mathcal{E},\ U = \frac{q}{C},\ \mathcal{E} = -L\dot{I} = -L\ddot{q} \Rightarrow$$

$$L\ddot{I} + \frac{1}{C}q = 0 \ - \quad \omega^2 = \frac{1}{LC},$$

$$L\ddot{I} + \frac{1}{C}I = 0 \ \ |\cdot CL \Longrightarrow CL^2\ddot{I} + LI = 0.$$

, , ,

$$\mathcal{L} = \frac{1}{2}L\dot{q}^2 - \frac{1}{2C}q^2 = T - U : \begin{cases} T = \frac{1}{2}LI^2 - \quad , \\ U = \frac{1}{2C}q^2 - \quad . \end{cases}$$

(CL^2):

$$\mathcal{L} = \frac{1}{2}CL^2\dot{I}^2 - \frac{1}{2}LI^2 = T - U, \begin{cases} T = \frac{1}{2}CU^2, \\ U = \frac{1}{2}LI^2, \end{cases}$$

, , . , . H/w

2.4.2.

$$\mathcal{L} = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2} - \mathrm{mg}\,x$$

$$\mathcal{L} = \frac{1}{2}L(x)\dot{q}^2 - \frac{1}{2C(x)}q^2,$$

$$\left(C(x) = \frac{S_C}{4\pi x}, L(x) = 4\pi\frac{N^2 S_L}{L-x}\right)$$

$$\mathcal{L} = \mathcal{L} + \mathcal{L}$$

.

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathcal{L}}{\partial\dot{x}} = m\ddot{x} = \frac{\partial\mathcal{L}}{\partial x} = -kx - mg + \frac{1}{2}\frac{\partial L}{\partial x}\cdot\dot{q}^2 - \frac{q^2}{2}\frac{\partial}{\partial x}\frac{1}{C}\,[6]$$

. q $\dot{q}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathcal{L}}{\partial\dot{q}} = \frac{\mathrm{d}}{\mathrm{d}t}(L(x)\dot{q}) = L\ddot{q} + \frac{\partial L}{\partial x}\dot{x}\dot{q} = \frac{\partial\mathcal{L}}{\partial q} = \frac{q}{C}.$$

[6] $-\frac{q^2}{2}\frac{4\pi}{S_C} = -2\pi\sigma q = Eq -$. , , .

3.

3.1.

$\ddot{x} = F(t, x, \dot{x})$.[7]

3.1.1.

$$\ddot{x} = F(x, \dot{x}) \Rightarrow \begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

$$\begin{cases} \dot{x} = 0 \\ \dot{y} = 0 \end{cases} \Leftrightarrow \begin{cases} f(x_0, y_0) = 0 \\ g(x_0, y_0) = 0. \end{cases}$$

$$\begin{cases} x = x_0 + \xi \\ y = y_0 + \eta \end{cases} \Rightarrow \begin{cases} \dot{xi} = \alpha\xi + \beta\eta \\ \dot{\eta} = \gamma\xi + \delta\eta, \end{cases} \quad \alpha = \frac{\partial f}{\partial x}\bigg|_{x_0, y_0}, \beta = \frac{\partial f}{\partial y}\bigg|_{x_0, y_0}, \ldots, \delta = \frac{\partial g}{\partial y}\bigg|_{x_0, y_0}.$$ [8]

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} e^{\lambda t} \Rightarrow \begin{cases} \lambda a = \alpha a + \beta \\ \lambda b = \gamma a + \delta b \end{cases} \Rightarrow \det \begin{pmatrix} \alpha - \gamma & \beta \\ \gamma & \delta - \lambda \end{pmatrix} = 0 \Leftrightarrow$$

$$\Leftrightarrow (\lambda - \alpha)(\lambda - \delta) = \gamma\beta,$$

$$\lambda_{1,2} = \frac{\alpha + \beta}{2} \pm \sqrt{\left(\frac{\alpha - \delta}{2}\right)^2 - \gamma\beta}.$$

1. $\lambda = \pm i\omega$ — , , . — .
2. $\lambda p \pm i\omega$ — , p , [9], . «» «».
3. $\lambda \in \mathbb{R}, \lambda_1 \cdot \lambda_2 > 0$ — — . .
4. $\lambda \in \mathbb{R}, \lambda_1 \cdot \lambda_2 < 0$ — , .

H/w ?

3.1.1.

$$\ddot{x} = f(x, v), v = 0, m = 1$$

$$\begin{cases} \dot{x} = v \\ \dot{v} = f_0(x) - \mu \cdot v = -\frac{\partial U(x)}{\partial x} - \mu \cdot x \end{cases}$$

$$\frac{\partial U(x_0)}{\partial x} = 0; \frac{\partial^2 U(x_0)}{\partial x^2} = U'' \neq 0; \begin{cases} x = x_0 + \xi \\ v = \eta \end{cases} \Rightarrow$$

$$\begin{cases} \dot{\xi} = v \\ \dot{v} = -U'' \cdot \xi - \mu \cdot v \end{cases} \Rightarrow \det \begin{pmatrix} -\lambda & 1 \\ -U'' & -\lambda - \mu \end{pmatrix} = 0 \Rightarrow \boxed{\lambda^2 + \mu\lambda + U'' = 0} \Rightarrow$$

$$\lambda_{1,2} = -\frac{\mu}{2} \pm \sqrt{\frac{\mu^2}{4} - U''}$$

, , , , .

---

[7] $x = 1;\ \dim x = 1, s = 1$

[8] .

[9] $p < 0$.

3.1.2.

, .

$$L(x,\dot{x}) - \quad .$$

$$\frac{\partial L}{\partial t} = 0 \Rightarrow H(x,\dot{x}) = const \quad .$$

— $H(x,\dot{x})$.

$$H(x,\dot{x}) = E = const \Rightarrow \dot{x} = V(x,E)$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = V(x,E) \Rightarrow \mathrm{d}t = \frac{\mathrm{d}x}{V(x,E)} \Rightarrow \boxed{t = \int^{x} \frac{\mathrm{d}x}{V(x,E)},}$$

E, , , $V(x,E)$ $V_i(x,E)$, i .

1. . .   . .: , 1970.

2. . .   . .: , 1966.

3. . .   . .: , 1980.