# Deep Generative Models

## Lecture 14

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AI Masters

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# Recap of previous lecture

## SDE basics

Let define stochastic process $\mathbf{x}(t)$ with initial condition $\mathbf{x}(0) \sim p_0(\mathbf{x})$:

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w},$$

where $\mathbf{w}(t)$ is the standard Wiener process (Brownian motion)

$$\mathbf{w}(t) - \mathbf{w}(s) \sim \mathcal{N}(0, (t-s)\mathbf{I}), \quad d\mathbf{w} = \boldsymbol{\epsilon} \cdot \sqrt{dt}, \text{ where } \boldsymbol{\epsilon} \sim \mathcal{N}(0, \mathbf{I}).$$

## Langevin dynamics

Let $\mathbf{x}_0$ be a random vector. Then under mild regularity conditions for small enough $\eta$ samples from the following dynamics

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \eta \frac{1}{2} \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t | \boldsymbol{\theta}) + \sqrt{\eta} \cdot \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim \mathcal{N}(0, \mathbf{I}).$$

will comes from $p(\mathbf{x}|\boldsymbol{\theta})$.

The density $p(\mathbf{x}|\boldsymbol{\theta})$ is a **stationary** distribution for the Langevin SDE.

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*Welling M. Bayesian Learning via Stochastic Gradient Langevin Dynamics, 2011*

# Outline

1. Probability flow ODE
2. The worst course overview

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# Probability flow ODE

Stochastic differential equation

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w}$$

Theorem (Kolmogorov-Fokker-Planck)

$$\frac{\partial p(\mathbf{x}, t)}{\partial t} = \operatorname{tr}\left(-\frac{\partial}{\partial \mathbf{x}}\big[\mathbf{f}(\mathbf{x}, t)p(\mathbf{x}, t)\big] + \frac{1}{2}g^2(t)\frac{\partial^2 p(\mathbf{x}, t)}{\partial \mathbf{x}^2}\right)$$

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g^2(t)\frac{\partial}{\partial \mathbf{x}}\log p(\mathbf{x}, t)\right]dt = \tilde{\mathbf{f}}(\mathbf{x}, t)dt$$

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*Song Y., et al. Score-Based Generative Modeling through Stochastic Differential Equations, 2020*

# Probability flow ODE

## Theorem (Kolmogorov-Fokker-Planck)

$$
\begin{aligned}
\frac{\partial p(\mathbf{x}, t)}{\partial t} &= \mathrm{tr}\left(-\frac{\partial}{\partial \mathbf{x}}\big[\mathbf{f}(\mathbf{x}, t)p(\mathbf{x}, t)\big] + \frac{1}{2}g^2(t)\frac{\partial^2 p(\mathbf{x}, t)}{\partial \mathbf{x}^2}\right) = \\
&= \mathrm{tr}\left(-\frac{\partial}{\partial \mathbf{x}}\left[\mathbf{f}(\mathbf{x}, t)p(\mathbf{x}, t) + \frac{1}{2}g^2(t)\frac{\partial p(\mathbf{x}, t)}{\partial \mathbf{x}}\right]\right) = \\
&= \mathrm{tr}\left(-\frac{\partial}{\partial \mathbf{x}}\left[\mathbf{f}(\mathbf{x}, t)p(\mathbf{x}, t) + \frac{1}{2}g^2(t)p(\mathbf{x}, t)\frac{\partial \log p(\mathbf{x}, t)}{\partial \mathbf{x}}\right]\right) = \\
&= \mathrm{tr}\left(-\frac{\partial}{\partial \mathbf{x}}\left[\left(\mathbf{f}(\mathbf{x}, t) + \frac{1}{2}g^2(t)\frac{\partial \log p(\mathbf{x}, t)}{\partial \mathbf{x}}\right)p(\mathbf{x}, t)\right]\right)
\end{aligned}
$$

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*Song Y., et al. Score-Based Generative Modeling through Stochastic Differential Equations, 2020*

# Probability flow ODE

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w}$$

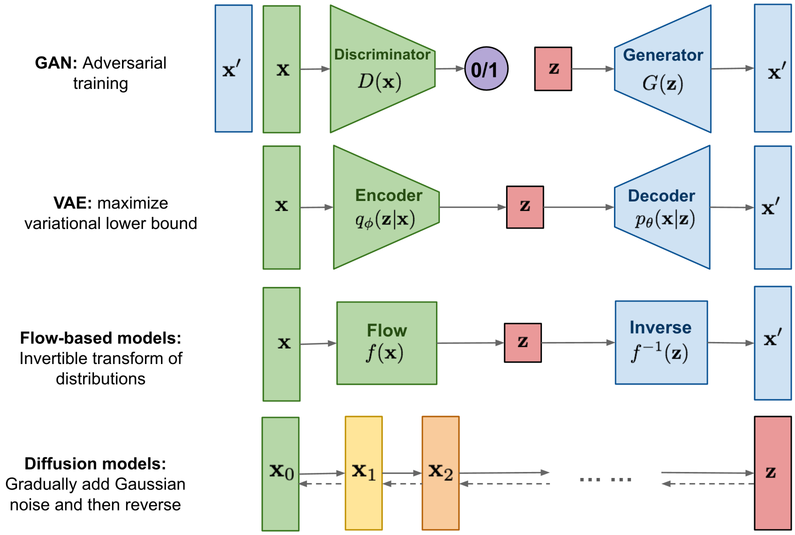
$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g^2(t)\frac{\partial}{\partial \mathbf{x}} \log p(\mathbf{x}, t)\right] dt$$

*Song Y., et al. Score-Based Generative Modeling through Stochastic Differential Equations, 2020*

# Outline

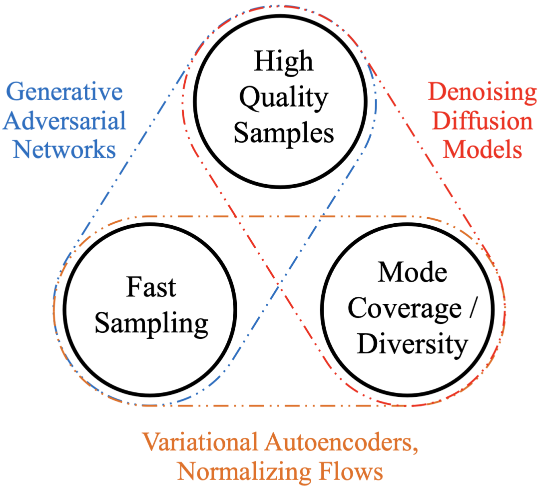
1. Probability flow ODE
2. The worst course overview

# The worst course overview :)




*Weng L. What are Diffusion Models?, blog post, 2021*

# The worst course overview :)




*Xiao Z., Kreis K., Vahdat A. Tackling the generative learning trilemma with denoising diffusion GANs, 2021*

# The worst course overview :)

| Model | Efficient | Sample quality | Coverage | Well-behaved latent space | Disentangled latent space | Efficient likelihood |
|---|---|---|---|---|---|---|
| GANs | ✓ | ✓ | ✗ | ✓ | ? | n/a |
| VAEs | ✓ | ✗ | ? | ✓ | ? | ✗ |
| Flows | ✓ | ✗ | ? | ✓ | ? | ✓ |
| Diffusion | ✗ | ✓ | ? | ✗ | ✗ | ✗ |

*Simon J.D. Prince. Understanding Deep Learning, 2023*

# Summary

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