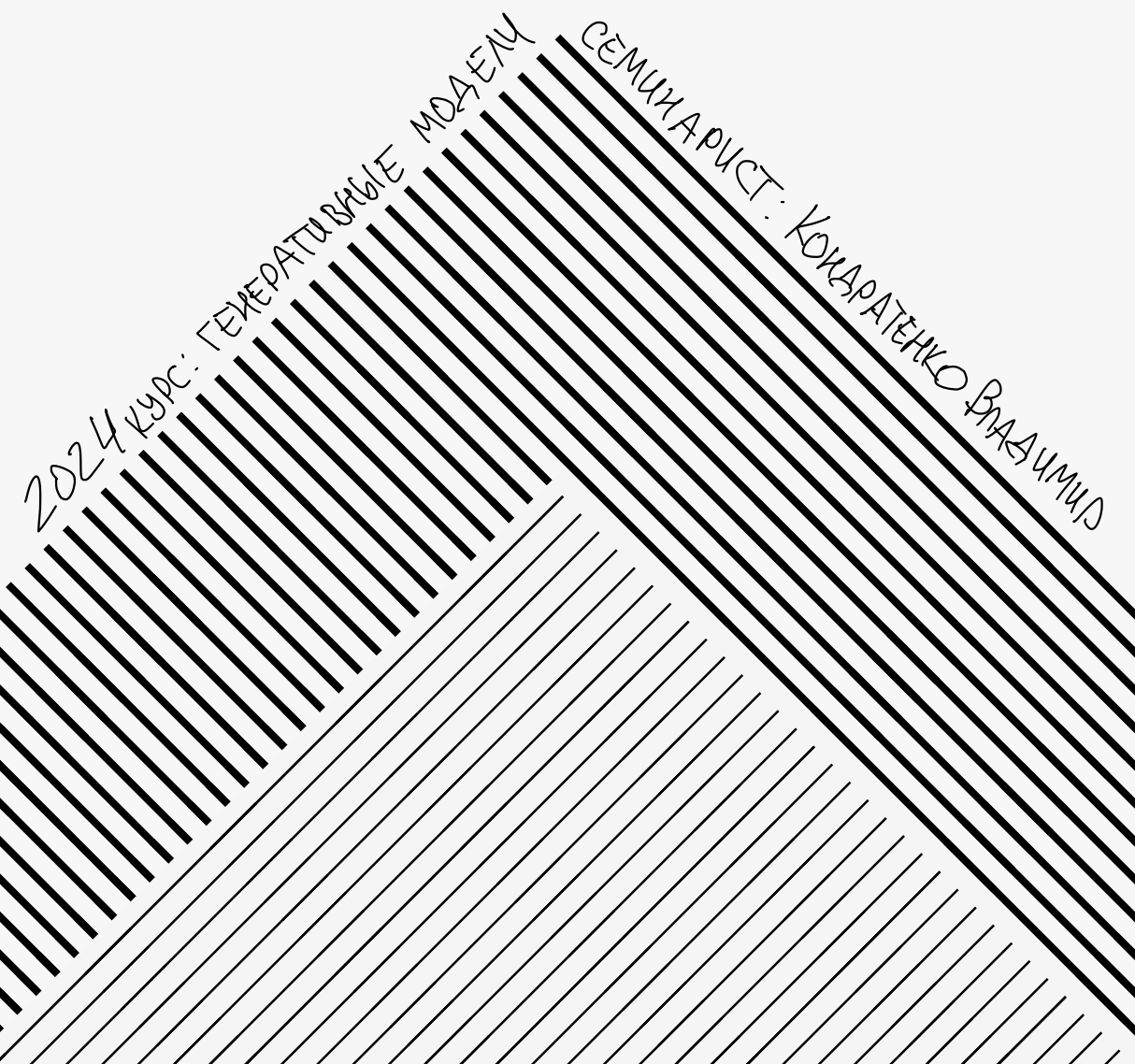


Семинар 5



$$\begin{aligned}
 \underline{P(X|\theta)} &= \sum_{k=1}^K \underbrace{P(X|z, \theta)}_k \cdot P(z) = \sum_{k=1}^K \pi_k \underbrace{N(X|\mu_k, \Sigma_k)}_{N_k(X)}
 \end{aligned}$$

$$\begin{aligned}
 &P(X, z|\theta) \\
 &\quad \uparrow \quad \quad \quad \downarrow \\
 &\text{dim} \quad 1 \dots k \quad \times
 \end{aligned}$$

$$\begin{aligned}
 \underline{P(X, z)_{nk}} &= P(X_n, z=k) = P(X_n | z=k) \cdot P(z=k) = \\
 &= N_k(X) \cdot \pi_k
 \end{aligned}$$

$$\begin{aligned}
 \underline{P(z|X, \theta)} &= \frac{P(X|z) \cdot P(z)}{\underline{P(X)}} = \frac{\pi_k N_k(X)}{\sum_{k=1}^K \pi_k N_k(X)}
 \end{aligned}$$

$$\theta = ? \quad [\mu, \Sigma, \pi]$$

E-max

$$q = P(z=k | x_n, \theta) = \frac{\pi_k N_k(x_n)}{\sum \pi_k N_k(x_n)} = q_{nk}$$
$$z \sim q = (\underbrace{q_1}_{\cancel{q_1}} \dots \underbrace{q_k}_{\cancel{q_k}})$$

M-max

$$\arg \max_{\theta} E_q \log P(x_n, z | \theta) =$$

$$= \sum_{k=1}^k P_{q^*}(z=k) \cdot \log P(x_n, z=k | \theta) =$$

$$= \underbrace{\sum_{k=1}^k q_{nk} \log N_k(x_n) \cdot \pi_k}_A$$

$$\frac{\partial A}{\partial \pi_k} = 0 ; \frac{\partial A}{\partial \mu_k} ; \frac{\partial A}{\partial \Sigma_k}$$

