Семинар 1

2024 курс: Генеративные модели

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СЕМИНАР 1

$X \sim U(0, \theta)$

$[0; \theta]$

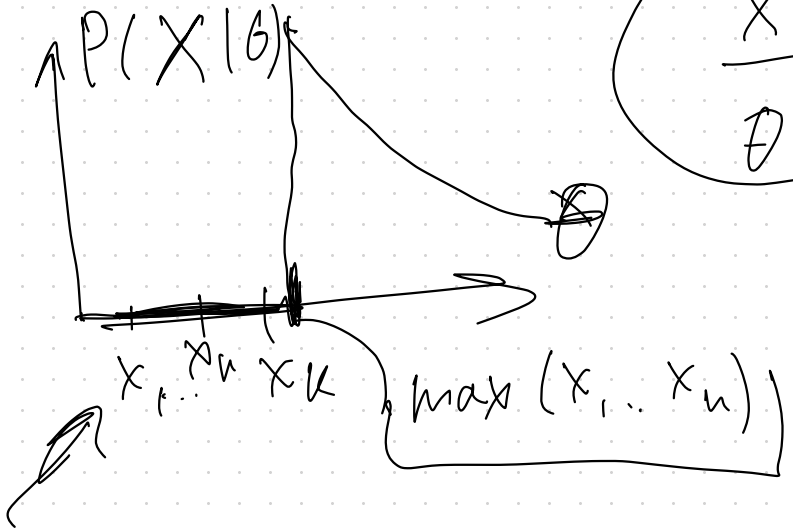
$$F_{U(0,\theta)}(x) = P(x < X) = \frac{x}{\theta}$$

$U[0; \theta]$

$x \in [0, \theta]$ 0

$X_{1 \dots n} \sim U(0; \theta)$

$\frac{x^n}{\theta^n}$



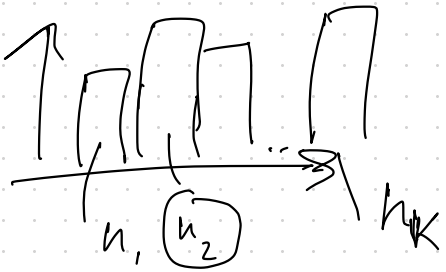
$$\hat{\theta}_{ML} = \max(x_1 \dots x_n)$$

$$F_{\hat{\theta}_{ML}}(x) = P(\hat{\theta}_{ML} < x) = P(\max(x_1 \dots x_n) < x)$$

$$= P([x_1 < x] \cap [x_2 < x] \dots \cap [x_n < x]) =$$

$$= \prod_{i=1}^{n} P(x_i < x) = \left(\frac{x}{\theta}\right)^n$$

$x_1 \dots x_n$



$$\log P(x|\theta) = \sum_{k=1}^{K} n_i \log \theta_i$$

$$\sum_{k=1}^{K} \theta_i = 1$$

$$L = \sum_{k=1}^{K} n_i \log \theta_i + \lambda\left(\sum_{k=1}^{K} \theta_i - 1\right)$$

$$\frac{\partial L}{\partial \theta_i} = \begin{cases} \frac{n_i}{\theta_i} + \lambda = 0 \\ \sum_{i=1}^{K} \theta_i = 1 \end{cases} \Rightarrow \boxed{\theta_i = -\frac{n_i}{\lambda}}$$

$$1 = -\sum_i \frac{n_i}{\lambda} = -\frac{N}{\lambda}$$

$$\Rightarrow 1 = -\frac{N}{\lambda} \Rightarrow \lambda = -N$$

$$\boxed{\hat{\theta}_{ML} = \frac{n_i}{N}}$$

$P(+|3) = 0{,}01$

$P(+|\bar{3}) = 0{,}9$

$P(\bar{3}) = 0{,}001$

$$P(3|+) = \frac{P(+|3) \cdot P(3)}{P(+) = P(+|3) \cdot P(3) + P(+|\bar{3}) \cdot P(\bar{3})} \approx 0{,}917$$

$P(3|+,+)$ $\quad P(+,+|3) = P(+|3) \cdot P(+|3)$

$\sim 0{,}109$