

A Comparison of Graph Processing Systems

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Abstract—//TODO: Section

Index Terms—graphs, distributed computing, Galois, Ligra, Polymer, Giraph, Gluon, Gemini

TODOs

- Appendix für Setup Guides - md zu pdf

I. INTRODUCTION

//TODO: Section

This paper makes the following contributions:

- Comparison of several state-of-the-art graph processing frameworks
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-
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II. RELATED WORK

//TODO: Section

III. PRELIMINARIES

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A. Graphs and Paths

An *unweighted graph* is the pair $G = (V, E)$ where the *vertex set* is $V \subseteq \mathbb{N}$ and the *edge set* describes a number of connections or relations between two vertices. Depending on these relations, a graph can be directed or undirected. For a *directed graph* the edge set is defined as

$$E \subseteq \{(x, y) \mid x, y \in V, x \neq y\}$$

and in the *undirected* case, the direction is no longer relevant. Thus, in an undirected graph for each $(x, y) \in E$, it holds $(x, y) = (y, x)$. The size of a graph is defined as the number of edges $|E|$ [1]. Independently of the graph being directed or not, a graph can be *weighted*. In this case a function $w : E \rightarrow \mathbb{R}$ is introduced, that maps an edge to a numerical value, further describing the relation.

A *Path* from starting vertex s to target vertex t is a sequence of vertices

$$P = (x_1, x_2, \dots, x_n) \in V^n$$

with the condition $(x_i, x_{i+1}) \in E$ for each $i \in \{1, \dots, n-1\}$ and $x_1 = s, x_n = t$. Thus we call a target t *reachable* from s if a Path from s to t exists.

B. Single-Source Shortest-Paths

Single-Source Shortest-Paths (SSSP) describes the problem of finding the shortest path from a starting vertex to every other vertex in the input graph. Input to the problem is a weighted graph $G = (V, E)$ and a start vertex $s \in V$. Output is the shortest possible distance from s to each vertex in V . The distance is defined as the sum of edge weights w_i on a path from s to the target. In the case of a unweighted graph, the distance is often described in *hops*, i.e. the number of edges on a path. The most common sequential implementations are Dijkstra's algorithm or BellmanFord [2]–[4].

C. Breadth-First Search

Breadth-first search (BFS) is a search problem on a graph. It requires an unweighted graph and a start vertex as input. The output is a set of vertices that are reachable from the start vertex. In some special cases, a target vertex is also given. In the case of a target being given, the output is true if a path from start to target exists or false otherwise. It is called Breadth-First search because the algorithm searches in a path length-based way. First all paths of length one i.e. all neighbors of the start vertex are checked before checking paths of length two and so on. The search algorithm, where the paths of maximum length are checked first is called Depth-First search.

D. PageRank

//TODO: Aufbau unschön, Formel fehlt PageRank (PR) is a link analysis algorithm that weighs the vertices of a graph, measuring the vertices relative importance within the graph [5]. The algorithm was invented by Sergey Brin and Larry Page, the founders of Google. To this date, Google Search uses PageRank to rank web pages in their search engine results.

This represents a centrality metric of the vertices. The analogy is that the graph represents website pages of the World Wide Web, that are hyperlinked between one another. A website that is more important is likely to receive more links from other websites. PageRank counts the number and quality of links to a page to estimate the importance of a page. The output of PageRank is a percentage for each vertex. This percentage, called the PageRank of the vertex, is the probability with which a web surfer starting at a random web page reaches this webpage (vertex). With a high probability the web surfer uses a random link from the web page they are currently on and with a smaller probability (called damping factor) they jump to a completely random web page.

An optimization to the traditional PageRank implementation is called *Delta-PageRank*. The PageRank score of a vertex is only updated if the relative change of the PageRank is larger than some user-defined delta. This effectively reduces the amount of vertices for which the PageRank has to be recalculated in following iterations.

E. Push and Pull Variants

Many parallel graph algorithms, as well as the three we consider here, are implemented by iterating over vertices, or edges [6]. The respective vertex or edge, which is considered in an iteration, is also called *active*. Applying an operator to this *active* vertex or edge is called an *activity*. This operator considers always only a local so called *neighborhood* to the *active* vertex or edge. To this *neighborhood* belong only vertices and edges, which are in the direct surrounding of the *active* vertex or edge, which allows to parallelise the iteration relatively easy.

In general such an operator can change the whole structure of the *neighbourhood*. But in the following we assume, that an operator changes only *labels* of vertices or edges in the *neighbourhood*. *Labels* of edges also called *weights*. Often this operator can be implemented in two different ways, called *push style* or *pull style*. A *push-style* operator reads the *label* of the *active* vertex or edge and updates the *label* of its *neighborhood*. In contrast the *pull-style* operator reads all values of its *neighborhood* and updates the value of the *active* vertex or edge.

Pull-style operators need less synchronization in parallel implementations, because unlike *pull style* there is only one write and many read operations. Thus locks can be avoided. So *push-style* operators are more efficient, if there are only a few *active* vertices or edges at the same time, or the *neighborhoods* do not overlap, which can not be avoided in general. On the other hand *pull-style* operators are more efficient, if there are many *active* vertices or edges at the same time.

F. Bulk-Synchronous Parallel Model

The Bulk-Synchronous Parallel (BSP) model is a computation model developed by Leslie Valiant [7]. It is commonly used in computation environments with large amounts of synchronous computation.

This model describes components, a communication network between those components and a method of synchronization. The components are capable of performing computations and transactions on *local* memory. Pairs of components can only communicate using messages, thus remote memory access is also only possible in this way. The Messages have a user-defined form and should be as small as possible to keep the network traffic low. **//TODO: Congest Model, log limit in was?** Synchronization is realized through barriers for some or all processes. BSP algorithms are performed in a series of global supersteps. These consist of three steps, beginning with the processors performing local computations concurrently. This step can overlap with the second, the communication between components. Processes can exchange information to

access remote data. Lastly, processes reaching a barrier wait until all other processes have reached the same barrier.

One of the most famous graph processing systems, Pregel [4] is based on the BSP computation model. We include Giraph, an open-source variant of Pregel in our evaluation. Pregel, Giraph and many frameworks similar to those were built to process large graphs reliably (offering fault tolerance) on large MapReduce infrastructures [8]–[10].

G. MapReduce

The MapReduce model is a computation infrastructure developed by Google to reliably handle large data sets on distributed clusters [11].

A user specifies just the two functions Map and Reduce. The system hides the details of parallelization, fault-tolerance, data distribution and load balancing away from the application logic. All of these features are automatically provided. Execution is performed in three phases:

- 1) Map phase: The input data is distributed between a set of Map processes, the Map functionality is specified by the user. Ideally all Map processes run in parallel so the map processes need to be independent. Results from this phase are written into (multiple) intermediate storage points.
- 2) Shuffle phase: The results are grouped according to a key provided by the Map algorithm. Each set of results is then handed to one system for the next phase.
- 3) Reduce phase: Every set of intermediate results is input to exactly one reduce process. The Reduce functionality is again specified by the user and ideally runs in parallel.

Giraph [8] is an example of a system using this framework.

H. Hadoop

//TODO: Section schreiben [12]

IV. FRAMEWORK OVERVIEW

We start with a short overview describing functionality and characteristics of several state-of-the-art graph processing frameworks. All of the following frameworks are part of our testing. **//TODO:**

A. Galois and Gluon

Galois [13] is a general purpose library designed for parallel programming. The system reduces the complexity of writing parallel applications by providing implicitly parallel (unordered or partially ordered) set iterators. These iterators perform operations optimistically, detect arising conflicts and resolve these by invoking inverse methods accordingly. The tasks in the sets can be ordered, the tasks may be executed out of order, but the conflict resolution will ensure semantic equivalence to sequential strictly ordered execution.

The graph analysis subsystem of Galois [14] provides a library of scalable data structures and a topology aware priority scheduler, including optimizations for distributed execution. The scheduler splits the tasks into bags according to a specified partial order, which in turn provide the cores with chunks

of tasks. A global map manages the various bags. Every thread keeps a lazy cache of a portion of the global map, in order to reduce the strain on the global map. Galois includes applications for many graph analytics problems, among these are SSSP, BFS and pagerank. For most of these applications Galois offers several different algorithms to perform these analytics problems and many options e.g. the amount of threads used or policies for splitting the graph. All of these applications can be executed in shared memory systems and, due to the Gluon integration, with a few modifications in a distributed environment

Gluon [15] is a framework written for Galois as a middle-ware for distributed graph analysis applications. It reduces the communication overhead needed in distributed environments by exploiting structural and temporal invariants. Depending on the used graph partitioning policy only a subset of the messages of a naive approach must be sent (structural invariant). Gluon establishes a mapping of local vertex ID's to the order in which the values will be sent/received between the owner and each mirror. A message includes a bit vector where a one in the i -th position means that the according vertex of the established order has been updated. Thus a message only has to include updated values without the need to state the vertex ID (temporal invariants). Gluon is embedded in Galois, but can be integrated in other graph analysis frameworks as well [15].

B. Gemini

Gemini is a framework for distributed graph processing [16]. It was developed with the goal to deliver a generally better performance through efficient communication. While most other distributed graph processing systems achieve very good results in the shared-memory area, they often deliver unsatisfactory results in distributed computing. Furthermore, a well optimized single-threaded implementation often outperforms a distributed system. Therefore it is necessary to not only focus on the performance of the computation but also of the performance of the communication. Gemini tries to bridge the gap between efficient shared-memory and scalable distributed systems. To achieve this goal, Gemini, in contrast to the other frameworks discussed here, does not support shared-memory calculation, but chooses the distributed message-based approach from scratch.

The real bottleneck of distributed systems is not the communication itself, but the extra instructions, as well as memory references and a lower usage of multiple cores compared to the shared memory counterparts. The main reasons for this are (1) the use of hash maps to convert vertex IDs between local and global states, (2) the maintenance of vertex replicas, (3) the communication-bound apply phase in the GAS abstraction, and (4) the lack of dynamic scheduling [16]. Gemini tries to work around all the problems mentioned above, by implementing a message-based system from scratch and getting rid of the extra mapping layer between shared-memory computation and communication. Therefore Ligra's push-pull computation model was adopted and applied to the distributed computation. Furthermore a chunk-based partitioning scheme

was implemented, which allows to partition a graph without a large overhead. Gemini also implements a co-scheduling mechanism to connect the computation and inter-node communication.

Gemini is fairly lightweight and has a clearly defined API between the core framework and the implementations of the individual algorithms. The five already implemented algorithms are Single-Source Shortest-Path (SSSP), Breath-First Search (BFS), PageRank (PR), Connected-Components (CC) and Biconnected-Components (BC).

C. Giraph

Apache Giraph is an example for an open-source system similar to Pregel. Thus, Giraph's computation model is closely related to the BSP model discussed in subsection III-F. This means that Giraph is based on computation units that communicate using messages and are synchronized with barriers [8].

The input to a Giraph computation is always a directed graph. Not only the edges but also the vertices have a value attached to them. The graph topology is thus not only defined by the vertices and edges but also their initial values. Furthermore, one can mutate the graph by adding or removing vertices and edges during computation.

The computation is vertex oriented and iterative. For each iteration step called superstep, the *compute* method implementing the algorithm is invoked on each active vertex, with every vertex being active in the beginning. This method receives messages sent in the previous superstep as well as its vertex value and the values of outgoing edges. With this data the values are modified and messages to other vertices are sent. Communication between vertices is only performed via messages, so a vertex has no direct access to values of other vertices. The only visible information is the set of attached edges and their weights. Supersteps are synchronized with barriers, meaning that all messages only get delivered in the following superstep and computation for the next superstep can only begin after every vertex has finished computing the current superstep. Edge and vertex values are retained across supersteps. Any vertex can stop computing (i.e. setting its state to inactive) at any time but incoming messages will reactivate the vertex. A vote-to-halt method is applied, i.e. if all vertices are inactive or if a user defined superstep number is reached the computation ends. Once calculation is finished, each vertex outputs some local information (e.g. the final vertex value) as result.

In order for Giraph to achieve scalability and parallelization, it is built on top of Apache Hadoop [8]. Hadoop is a MapReduce infrastructure providing a fault tolerant basis for large scale graph processing. Hadoop supplies a distributed file system (HDFS), on which all computations are performed. Giraph is thus, even when only using a single node, running in a distributed manner. Hence, expanding single-node processing to a multi-node cluster is seamless. Giraph uses the Map functionality of Hadoop to run the algorithms. Reduce is only used as the identity function.

Giraph being an Apache project makes it the most actively maintained and tested project in our comparison. While writing

this paper, several new updates were pushed to Giraph's source repository¹.

D. Ligra

//TODO: Rückmeldung 2 umsetzen

Ligra is a lightweight graph processing framework for shared memory machines [3]. It offers a vertex-centric programming interface which can be used to apply a function to each vertex or outgoing edge of a set of vertices in parallel. While doing so the framework can generate a set of active vertices for the next iteration. This abstraction makes it well suited for writing graph traversal algorithms.

When mapping over edges Ligra optimizes algorithms by switching between a push-based and a pull-based approach based on the size of the set. When the size of the set is above a threshold a pull approach is used. If the threshold is not reached a push approach is used.

E. Polymer

Polymer is a Non Uniform Memory Access (NUMA) aware graph-analytics system that inherits the scatter-gather programming interface from Ligra [3]. Key differences are the data layout and access strategies, Polymer implements. The goal is to minimize random and remote memory accesses to improve performance.

The first optimization Polymer applies is data locality and access methods across NUMA nodes [2]. A general design principle for NUMA machines is to partition the input data so that computation can be grouped with the corresponding data on one node. Polymer adopts this and allocates graph data according to the access patterns. It treats a NUMA machine like a distributed cluster and splits work and graph data accordingly between the nodes. Vertices and Edges are partitioned and then allocated across the corresponding memory nodes of the threads, eliminating most remote memory accesses. However, some computation requires vertices to perform computations on edges that are not in the local NUMA-node. For this case, Polymer introduces lightweight vertex replicas that are used to initiate computation on remote edges.

Polymer furthermore has custom storage principles for application-defined data. For such data, the memory locations are static but the data undergo frequent dynamic updates. Due to frequent exchanges of application-defined data between the nodes, remote memory accesses are inevitable. Hence, Polymer allocates application-defined data with virtual addresses, while distributing the actual memory locations across the nodes. Thus, all updates are applied on a single copy of application-defined data.

Runtime states that are dynamically allocated in each iteration would however create overhead due to repeated construction of a virtual address space. These states are thus stored in a custom lock-less (i.e. avoiding contention) lookup table.

Polymer does not only optimize data locality but also its scheduling is custom. Time to synchronize threads on different cores increases dramatically with the growing number of

involved sockets. Inter-node synchronization takes one order of magnitude longer time than intra-node synchronization [2]. Thus, Polymer implements a topology-aware hierarchical synchronization barrier. A group of threads on the same NUMA-node shares a partition of data. This allows them to first only synchronize with threads on the same NUMA-node. Only the last thread of each group synchronizes across groups (i.e. nodes). This behaviour decreases the amount of needed cache coherence broadcasts across the nodes.

Furthermore, Polymer switches between different data structures representing the runtime state, similar to Ligra's behaviour. The main deciding factor to switch is the amount of active vertices relative to an application-defined threshold. Polymer uses a lock-less tree structure representing the active vertices. The leaves use bitmaps, which are efficient for a large proportion of active vertices. When only a small amount of vertices is active, the drawbacks of traversing through sparse bitmaps can be avoided by switching data structures.

Polymer inherits the programming interfaces EdgeMap and VertexMap from Ligra as its main interface.

V. GRAPH FORMATS

Since every framework uses different graph input formats, we supply a conversion tool capable of translating from EdgeList to the required formats.

The two most popular graph databases are those associated with the Koblenz Network Collection (KONECT) [17] and Stanford Network Analysis Project (SNAP) [18]. Data Sets retrieved from one of them can be directly read and translated.

The following sections explain the output formats of our conversion tool.

A. AdjacencyGraph

The AdjacencyGraph and WeightedAdjacencyGraph formats used by Ligra and Polymer are similar to the more popular *compressed sparse rows* format. The format was initially specified for the Problem Based Benchmark Suite, an open source repository to compare different parallel programming methodologies in terms of performance and code quality [19].

The file looks as follows

$$x, n, m, o_1, \dots, o_n, t_1, \dots, t_m$$

where commas are newlines. The files always start with the name of the format i.e. AdjacencyGraph or WeightedAdjacencyGraph in the first line, here shown as x . Followed by n , the number of vertices and m the number of edges in the graph. The o_k are the so-called offsets. Each vertex k has an offset o_k , that describes an index in the following list of the t_i . The t_i are vertex IDs describing target vertices of a directed edge. The index o_k in the list of target vertices is the point where edges outgoing from vertex k begin to be declared. So vertex k has the outgoing edges

$$(k, t_{o_k}), (k, t_{o_k+1}), \dots, (k, t_{o_{k+1}-1}).$$

For the WeightedAdjacencyGraph format, the weights are appended to the end of the file in an order corresponding to the target vertices.

¹<https://gitbox.apache.org/repos/asf?p=giraph.git>

B. EdgeList

The EdgeList format is one of the most commonly used in online data set repositories. The KONECT database uses this format and thus it is the input format for our conversion tool.

An edge list is a set of directed edges $(s_1, t_1), (s_2, t_2), \dots$ where s_i is a vertex ID representing the start vertex and t_i is a vertex ID representing the target vertex. In the format, there is one edge per line and the vertex IDs s_i, t_i are separated with an arbitrary amount of whitespace characters.

For a WeightedEdgeList, the edge weights are appended to each line, again separated by any number of whitespace characters.

C. Binary EdgeList

The binary EdgeList format is used by Gemini. For s_i, t_i some vertex IDs and w_i the weight of a directed edge (s_i, t_i, w_i) , Gemini requires the following input format

$$s_1 t_1 w_1 s_2 t_2 w_2 \dots$$

where s_i, t_i have `uint32` data type and the optional weights are `float32`. Gemini derives the number of edges from the file size, so there is no file header or anything similar allowed.

D. Giraph's I/O formats

Giraph is capable of parsing many different input and output formats. All of those are explained in Giraph's JavaDoc². Both edge- and vertex-centric input formats are possible. One can even define their own input graph representation or output format. For the purposes of this paper, we used an existing format similar to AdjacencyList but represented in a JSON-like manner.

In this format, the vertex IDs are specified as `long` with `double` vertex values and `float` out-edge weights. Each line in the graph file looks as follows

$$[s, v_s, [[t_1, w_{t_1}], [t_2, w_{t_2}] \dots]]$$

with s being a vertex ID, v_s the vertex value of vertex s . The values t_i are vertices for which an edge from s to t_i exists. The directed edge (s, t_i) has weight w_{t_i} .

VI. EVALUATION

//TODO: Einleitung

A. Environment

For testing the graph processing systems, we used 5 machines with two AMD EPYC 7401 (24-Cores) and 256 GB of RAM each. One of those machines was only used as part of the distributed cluster, since it only has 128 GB of RAM. All five machines were running Ubuntu 18.04.2 LTS.

Setup of each framework was performed according to our provided installation guides available in Appendix A. All benchmark cases were initiated by our benchmark script available in our repository. All five frameworks are tested on a single server. Galois, Gemini and Giraph were benchmarked

TABLE I: Size Comparison of the Used Graphs

Graph	# Vertices (M)	# Edges (M)
flickr	0.1	2
orkut	3	117
wikipedia	12	378
twitter	52	1963
rMat27	63	2147
friendster	68	2586
rMat28	121	4294

in on the distributed 5-node cluster as well. Since Galois supports this parameter, we ran multiple tests comparing Galois' performance with different thread counts on a single machine. Furthermore, Galois is a framework capable of utilizing hugepages. We include an evaluation using those on the single node as well. Unless mentioned otherwise, we always show results of each framework utilizing 96 threads (i.e. the maximum on our machines) for the single-node evaluation.

The complete benchmark log files and extracted raw results are available in our repository³.

B. Data Sets

The graphs used in our testing can be seen in detail in Table I. We included a variety of different graph sizes, from relatively small graphs like the flickr graph with 2 million edges up to an rMat28 with 4.2 billion edges. All graphs except the rMat27 and rMat28 are exemplary real-world graphs and were retrieved from the graph database⁴ associated with the Koblenz Network Collection (KONECT)[17]. Both the rMat27 and rMat28 were created with a modified version (we changed the output format to EdgeList) of a graph generator provided by Ligr.

C. Measurements

For every framework, we measured the *execution time* as the time from start to finish of the console command. For the *calculation time*, we tried to extract only the time the framework actually executed the algorithm. Furthermore, the *overhead* is the time difference between execution time and calculation time. This includes time to read the input graph and initialization. For measuring the calculation time, came up with the following:

- For Galois, we extract console log time stamps. Galois outputs "Reading graph complete.". Calculation time is the time from this output to the end of execution. This is not the most reliable way for measuring the calculation times. Not only due to unavoidable buffering in the console output we expect the measured time to be larger than the actual. First, it is not clear that all initialization is in fact complete after reading the graph. Second, we include time in the measurement that is used for cleanup after calculation. However, this method is the only way of retrieving any measurements without introducing custom modifications to the Galois source code.

³<https://github.com/SerenGTI/Forschungsprojekt> // das ist hier unschön.

⁴<http://konect.uni-koblenz.de/>

²<http://giraph.apache.org/apidocs/index.html>

- Polymer outputs the name of the algorithm followed by an internally measured time.
- Gemini outputs a line `exec_time=x`, which was used to measure the calculation time.
- Ligra outputs its time measurement with `Running time : x`.
- Giraph has built in timers for the iterations (supersteps), the sum of those is the computation time.

Each evaluation consisting of graph, framework and algorithm was run 10 times, allowing us to smooth slight variations in the measured times. Later on, we provide the mean values of the individual times as well as the standard deviation where meaningful.

D. Algorithms

The three problems Breadth-first search (BFS), PageRank (PR) and Single-source shortest-path (SSSP) were used to benchmark framework with every graph. We always show the results of PageRank with a maximum of five iterations. For frameworks that support multiple implementations (i.e. PageRank in push and pull modes), we included both in our evaluation.

In detail, the algorithms for each framework are:

- Galois supports all of our tested algorithms, with both a Push and a Pull variant for PageRank available. In the distributed scenario, there are Push and Pull versions for SSSP and BFS available as well.
- Gemini **//TODO:**
- Giraph does not natively supply a BFS algorithm, so in our comparisons a custom implementation is used. For SSSP, slight variations had to be made to the default implementation, to allow us to use different start vertices. For PageRank the supplied implementation is used.
- Ligra **//TODO:**
- Polymer supports SSSP based on BellmanFord, BFS and two implementations of PageRank. The two implementations are a regular PR and a Delta Variant.

VII. RESULTS

//TODO: Rename Section???

A. The frameworks during setup and benchmark

We would like to raise some issues we encountered first while installing and configuring and second while running the different frameworks.

- 1) During setup and benchmark of Gemini, we encountered several bugs in the cloned repository. These include non zero-terminated strings or even missing return statements.
The errors rendered the code as-is unable to perform calculations, forcing us to fork the repository and modify the source code. Our changes can be found in one of our repositories⁵.
- 2) Furthermore, we would like to address the setup of Hadoop for Giraph. It requires multiple edits in `xml`

files that aren't easily automatized. This makes the setup rather time consuming, especially if reconfiguration is needed later on.

- 3) In order for Giraph to run, several Java tasks (the Hadoop infrastructure) have to be constantly running in the background. While we don't expect this to have a significant performance impact on other tasks, it is still suboptimal.
- 4)

On a plus side, setup of the frameworks Polymer and Ligra was straight forward and did not require any special treatment.

B. Single-source Shortest-paths

For the results of our SSSP runs, we first analyze the single-node and distributed performances separately before comparing the two.

1) *Single-node:* Beginning with the single-node performance, Figure 1 shows the average calculation and execution times for SSSP on the different frameworks. Note, that Giraph ran out of memory (>250 GB) for all graphs larger than wikipedia (twitter, rMat27, friendster and rMat28). Thus, the data points on the larger graphs for Giraph are missing in the figures and our evaluation. Also, both Ligra and Polymer failed on rMat28. Because of that, removed rMat28 in this section.

Upon analyzing the calculation time (cf. Figure 1a), most obvious is the fact that Giraph is at least one order of magnitude slower than any other framework. The only exception is Gemini, where Giraph is *only* 4× slower on wikipedia. In general however, Giraph is on average 24× slower than the other frameworks on flickr, 17× on orkut and 12× on wikipedia.

Comparing the other frameworks (i.e. excluding Giraph in the further comparisons), shows them to perform similar on the smaller graphs. There, the average calculation times of the four frameworks (Galois, Gemini, Ligra, Polymer) are 35ms on flickr, 77ms on orkut, 538ms on wikipedia. The four frameworks are close to this average for the smaller graphs. Galois and Gemini being 24ms and 12ms faster than the average. Ligra and Polymer are slower by 27ms and 9ms. Orkut is the graph where all the four frameworks are within 20ms of their average, Galois being the outlier at 19ms slower than average. For wikipedia, Galois and Ligra are close to each other, while Gemini is 560ms (104%) slower than the average of the four frameworks. And Polymer is the fastest framework on wikipedia, being 330ms (61%) faster than the others. For the larger graphs, Polymer has the shortest computation time as well. Gemini is second, taking about 1.9× times the computation time of Polymer on twitter and friendster while being equally fast (8% longer) on rMat27. Galois and Ligra however have much longer computation times compared to Polymer. Galois takes anywhere from 3.3× (twitter) to 4.2× (friendster) the computation time of Polymer. Ligra requires between 3.2× (friendster) and 9.8× (twitter) the computation time of Polymer on the larger graphs. This makes Ligra the slowest Framework on both twitter and rMat27. Galois is the slowest on friendster. However, while especially Galois is comparably slow to Polymer in the computation time, it

⁵<https://github.com/jasc7636/GeminiGraph>

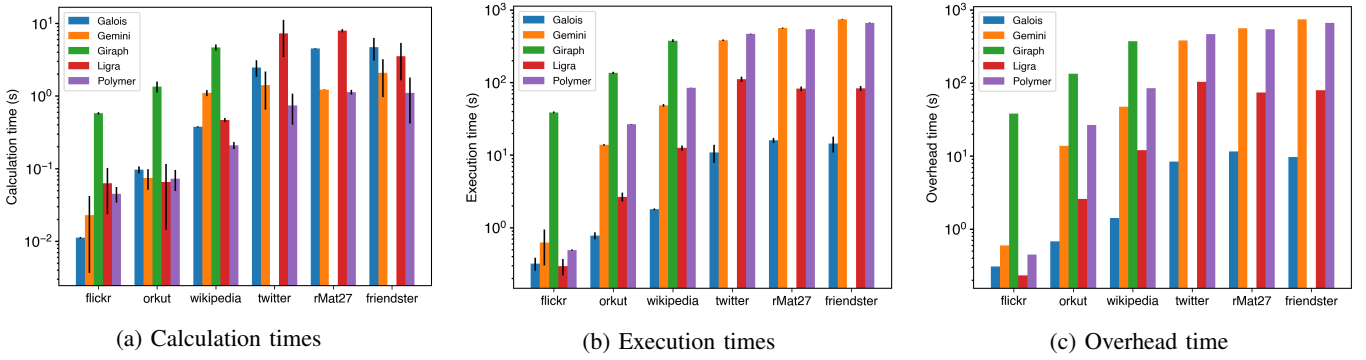


Fig. 1: Average times for SSSP on a single computation node, black bars represent one standard deviation in our testing.

TABLE II: Distributed SSSP Execution Times and Their Relation to Galois Push

Data Set	Galois Push		Galois Pull		Gemin		Giraph	
	×	(s)	×	(s)	×	(s)	×	(s)
flickr	1.0	(2.4)	0.97	(2.4)	1.02	(2.5)	17.46	(42.6)
orkut	1.0	(4.2)	1.28	(5.4)	4.12	(17.3)	14.38	(60.4)
wikipedia	1.0	(14.2)	1.88	(26.6)	5.26	(74.5)	7.11	(100.8)
twitter	1.0	(40.0)	3.56	(142.2)	9.79	(391.1)	8.76	(349.9)
rMat27	1.0	(39.0)	3.98	(154.9)	9.91	(386.0)	14.52	(565.8)
friendster	1.0	(58.5)	2.32	(136.0)	9.72	(568.9)	7.59	(444.0)
rMat28	1.0	(71.5)	5.7	(407.9)	11.07	(792.0)	16.5	(1180.2)

is important to keep in mind that Polymer could not finish computation on rMat28. Meanwhile both Gemini and Galois managed just fine.

The execution times show Galois to be the clear winner on most graphs (cf. Figure 3b). It has the smallest computation times on all graphs except flickr, where Galois is 25ms (8%) slower than Ligra. On the other 5 graphs however, Galois has the smaller execution time by one or sometimes even two orders of magnitude. For example, on orkut Galois requires 0.78s to execute, while the other frameworks require between 2.66s (Ligra) and 26.6s (Polymer). This goes on for the larger graphs, Ligra being second-fastest. Polymer and Gemini close together but both always at least one order of magnitude slower than Galois.

2) *Distributed*: On the distributed cluster, we find similar results as on the single node (cf. Figure 2). Both Galois implementations have significantly smaller execution times compared to Gemini or Giraph on all graphs (cf. Table II). You can see Gemini being worse by at least a factor of 4 compared to Galois Push on all graphs except flickr. Giraph’s execution times in comparison to this are even worse, taking at least $7\times$ longer than Galois Push on all graphs. Comparing the two Galois implementations, we find the calculation and execution times to be similar on smaller graphs and Push being the superior implementation for SSSP on larger data sets. Galois Pull is anywhere from just as fast to $3.5\times$ slower on real-world data sets compared to the Push variant. The synthetic graphs are more extreme. Execution times are close to $4\times$ (rMat27) and $5\times$ (rMat28) longer on Pull. Evidently, Galois Push is the fastest algorithm in our lineup on 6 out of 7 graphs. With the exception being flickr, where Galois Push takes negligibly longer than the Pull counterpart.

When taking a closer look at Giraph, it seems to not cope

well with synthetic data sets. Analyzing the computation times in Figure 2a, we see that it is the fastest framework on our real-world graphs. And that with a considerable margin of other frameworks always taking at least 50% longer (lower bound here is Gemini on flickr) up to Galois Pull needing $18\times$ more time on wikipedia. On both synthetic graphs however, Giraph is actually the slowest to compute. Giraph requires $12\times$ or even $15\times$ the computation time of Gemini on rMat27 or rMat28 respectively. While Giraph’s computation times are very competitive, when comparing the execution times in Figure 2b we see that Giraph is actually the slowest framework on 5 out of 7 graphs. For the other two, namely twitter and friendster, Giraph is second slowest with only Gemini taking longer to complete. Giraph and Gemini’s very long execution times are only due to their overhead being many orders of magnitude larger than Galois overhead (Figure 2c). Overhead for Gemini is greater than that of Galois on every graph. From just a 20% increase on flickr up to friendster, where the overhead is $90\times$ that of Galois Push. For Giraph the overhead times are not as extreme but still generally worse. Even on flickr, Giraph’s overhead time is already $23\times$ that of Galois. On friendster, where Gemini was worst, Giraph *only* requires $73\times$ the overhead time of Galois.

C. Breadth-first search

As with our results for SSSP, for BFS we will begin with our single-node results before looking at the distributed scenario.

1) *Single-Node*: Just like with SSSP, Giraph ran out of memory (>250 GB) on any graph larger than wikipedia. Also, Polymer failed to complete on rMat28. Ligra, that failed during SSSP however completed our benchmark for BFS.

The calculation times provided in Figure 3a show Gemini and Ligra to be comparable in their performance. Their computation times deviate less than 151ms on all graphs except wikipedia and rMat28, with Ligra being the faster framework on most graphs. Ligra is between 2ms (twitter) and 151ms (friendster) faster than Gemini. In turn, Gemini is 17ms faster than Ligra on flickr. Only on wikipedia and rMat28, there is a noticeable difference between the two frameworks. Ligra is 1.2s ($7.8\times$) faster than Gemini on wikipedia and 696ms ($4.4\times$) faster on rMat28.

For the remaining frameworks, we compare to Ligra since it is generally slightly faster than Gemini. Giraph and Polymer

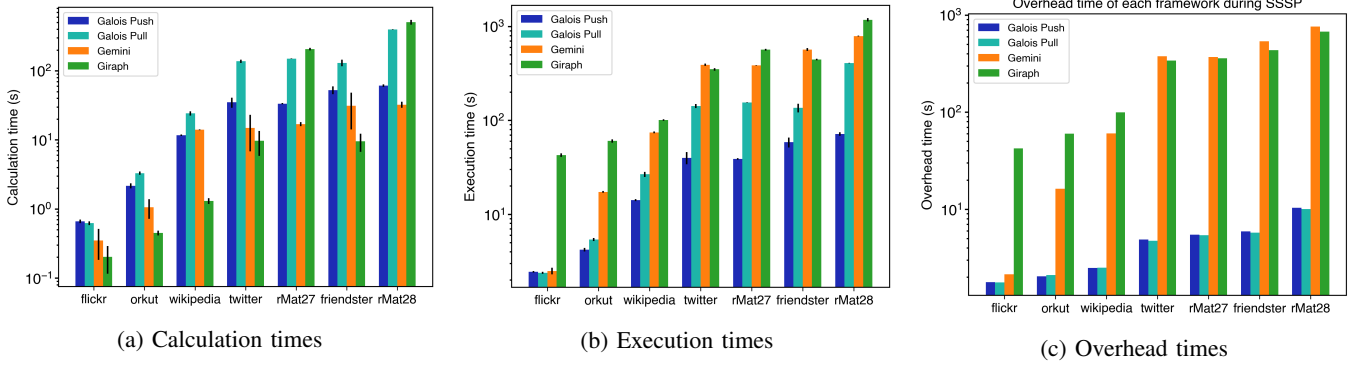


Fig. 2: Average times for SSSP on the distributed cluster, black bars represent one standard deviation in our testing.

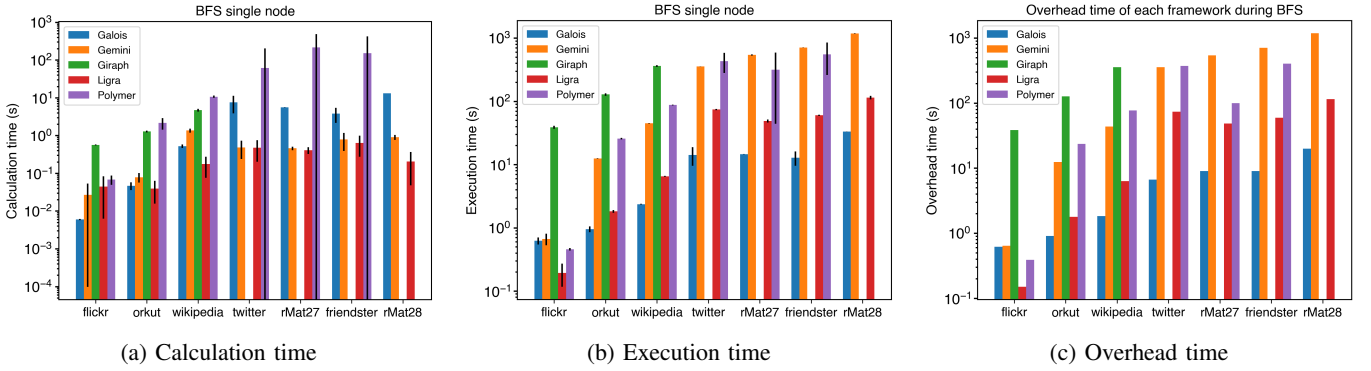


Fig. 3: Average times for BFS on a single computation node, black bars represent one standard deviation in our testing

are one to two orders of magnitude slower than both Ligra or Gemini. The only exception to this is Polymer on flickr, here Polymer is just 24ms (35%) slower than Ligra. Giraph takes between $13\times$ and $32\times$ longer than Ligra on flickr, orkut or wikipedia. As we said, Polymer is comparable to Ligra on flickr, but for the other graphs, its computation times are even longer than those of Giraph. The upside is though, that Polymer managed – contrary to Giraph – to finish computation on some larger graphs. Polymer takes between $55\times$ and $530\times$ longer than Ligra on the graphs larger than flickr. Especially interesting is here the difference between the times of rMat27 and friendster. There are 20% (439M) more edges in friendster, yet the computation for the synthetic rMat27 takes 42% longer.

Galois calculation performance is comparable to Gemini or Ligra on the smaller three graphs (flickr, orkut and wikipedia). Only on the larger graphs is Galois slower than Gemini or Ligra, meanwhile Galois is always faster than Polymer by one order of magnitude.

2) *Distributed*: For both the calculation and the execution times, Breadth-First Search shows similar behaviour as the distributed SSSP test case. This is expected since both are graph traversal algorithms starting in one source vertex. Hence, calculation complexity for each vertex and communication overhead is similar. All measurements can be seen in Figure 4.

The results for calculation time (cf. Figure 4a) show Giraph to have very short calculation times on the real-world graphs, while Giraph’s calculation times on both rMat27 and rMat28 are the worst of all frameworks. Thus, Giraph is fastest on

the three smallest graphs and second fastest on twitter and friendster. On those two graphs, Gemini is fastest with only a small margin between the two frameworks. Just like with SSSP, the Galois implementations have the longest calculation time on the real-world graphs. And Galois is second slowest, on the synthetic graphs.

Comparing the execution times in Figure 4b results again in similar findings to SSSP. While Gemini can compete with Galois on the small flickr graph, moving to larger data sets shows the worse performance of Gemini compared to Galois. Similar to SSSP, Giraph is slowest on all but one graph. Only on friendster is Gemini marginally slower, this was also the case for SSSP. Galois Push is generally faster than the Pull alternative while both Push and Pull versions are faster than Gemini and Giraph across all graphs. This makes Galois Push the clear winner for distributed BFS.

D. PageRank

1) *Single-Node*: Ligra and Polymer support both regular and Delta-PageRank variants. Ligra’s regular PR implementation is faster on 4 of 7 graphs. If the regular version is slower than delta, that is only by a small difference. Explicitly, regular is slower than delta by a range of 6% to 19% on twitter, rMat27 or friendster. For the other graphs, the delta version is slower by a far greater margin of 13% to 68%. Hence, we only show the results of Ligra’s regular PageRank implementation in our evaluation. For Polymer we found the delta version to be faster on all graphs except rMat28. Delta-PR is on average

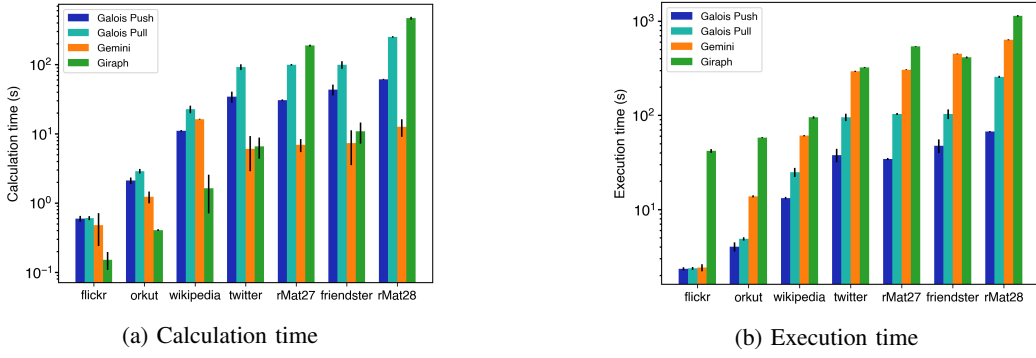


Fig. 4: Average times for BFS on the distributed cluster, black bars represent one standard deviation in our testing

15% faster on the first six graphs, while only being 0.3% slower on rMat28. Thus, the following only shows Polymer’s faster Delta-PR implementation. Giraph required more than the available 250 GB of RAM on any graph larger than wikipedia, hence all of Giraph’s results for the larger graphs are missing here.

The calculation times show some odd behaviour of Galois Push. The required time is less than 1ms, regardless of the graph (cf. Figure 5a). Meanwhile there was no output produced, that would indicate any kind of error. These results would make the calculation times of Galois Push the smallest on all graphs, with a difference of at least one order of magnitude. However, we are very suspicious of these results and thus exclude the calculation time for Galois Push in further comparisons. Because the execution time of Galois Push is always considerably longer than the execution time of Galois Pull (cf. Figure 5a). This leads us to believe that the output that we used for our measurements contains an error.

For the three graphs on which Giraph computed successfully, it is the slowest framework in both calculation and execution times. And that by a difference of three orders of magnitude in the calculation time and one to two orders of magnitude in execution time (cf Figure 5). On the larger graphs (i.e. those, where there is no data for Giraph), Gemini and Ligra are always slowest in execution time. Contrary to this, Galois Pull has the smallest execution times on all graphs except flickr (cf. Figure 5b). Ligra is fastest on flickr, while being second fastest on wikipedia, rMat27 and rMat28. Galois Push is second fastest on orkut and friendster. Interestingly, the execution time for Ligra is at a maximum for twitter. The required time is steadily decreasing with increasing graph size.

2) *Distributed*: The Figure 6 shows our results of PageRank on the distributed cluster. First of all, Giraph was unable to complete the test because it required more than 250GB of RAM for rMat28, thus this result is missing. When comparing the calculation times in Figure 6a to the execution times in Figure 6b, we see similar behaviour of all frameworks. This means that unlike with SSSP or BFS, the calculation times and execution times are similar with respect to the relations of the frameworks to one another. More specifically, there are no overhead outliers like it was the case with Giraph on SSSP.

//TODO: Auswertung fehlt noch.

E. Galois speedup

This section is dedicated to analyzing the speedup behaviour of Galois under the change to two parameters. First we change the thread count Galois is using. Second, a comparison between Galois using hugepages and Galois without hugepages is made. We compare the *calculation speedups* of Galois on the different graphs. Thus, we show the calculation time on any thread count normalized by the calculation time in the single-threaded environment. Beginning with SSSP, followed by BFS and last the comparison for both PR Push and Pull.

1) *Single-Source Shortest-Paths*: Starting with SSSP, we see an algorithm that benefits from many available threads this can be seen in Figure 7. We first look at the speedups without hugepages, seen in Figure 7a. For all larger graphs, speedup is in most cases very close to optimal up to about 8 threads. Twitter has the best speedup overall. It is $2.6\times$ with 2 threads compared to one, $4\times$ with 4, $7.7\times$ with 8 and $9.7\times$ using 16 threads. Behaviour on friendster is similarly good. Here speedup is $1.9\times$ at 2 threads compared to one, $3.5\times$ at 4, $6.1\times$ at 8 threads and $9.7\times$ at 16 threads. Anything above 16 threads however no longer helps decrease the computation time significantly on any graph. Speedup above 16 threads is always less than double the speedup of 16 threads. The maximum measured speedups are $10\times$ (96 threads) for wikipedia, $17\times$ (96 threads) for twitter, $11\times$ (96 threads) for rMat27, $16\times$ (48 threads) for friendster and $19\times$ (40 threads) for rMat28. In some cases increasing thread counts even prolongs calculation time. For example calculation on rMat28 is actually slower with 48 or 96 threads compared to 40 threads. For 40 threads, the speedup is nearly $19\times$, on 48 threads $17\times$ and with 96 threads only $15\times$ compared to one thread. Small graphs, i.e. flickr and orkut neither benefit from more threads nor is the performance significantly held up by synchronization overhead. Performance on flickr can not be sped up at all, with speedup on flickr being very close to 1 for 1 to 8 threads and between $0.7\times$ to $0.9\times$ from 16 to 96 threads. Orkut reaches maximum speedup of $1.6\times$ at 16 threads. However on orkut, the speedup is always greater or equal to 1.

Upon activating the hugepages, we acquired the results seen in Figure 7b. Here, the overall results are similar to the findings without hugepages. Table III shows the mean speedups and variances over the different graphs. We see that

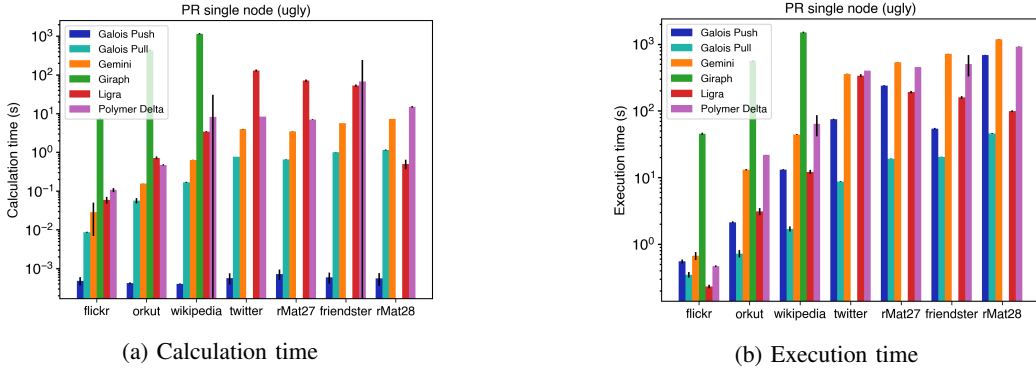


Fig. 5: Average times for PR on a single computation node, black bars represent one standard deviation in our testing

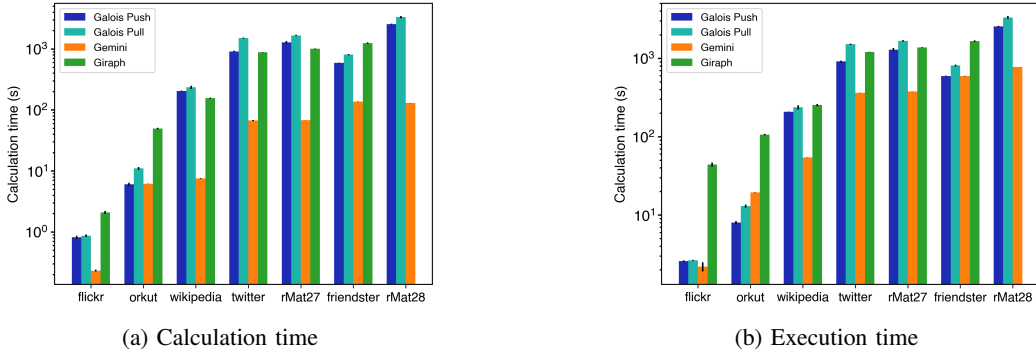


Fig. 6: Average times for PR on the distributed cluster, black bars represent one standard deviation in our testing

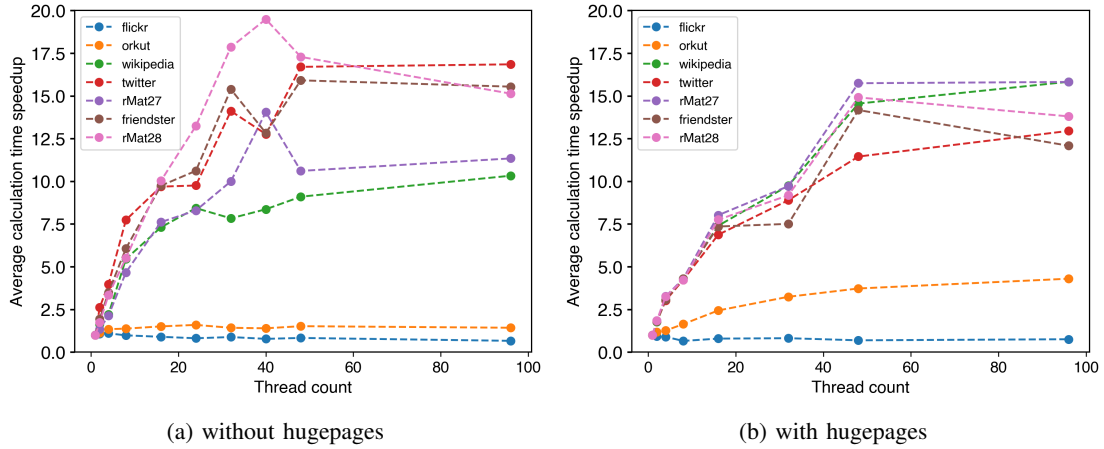


Fig. 7: Calculation time speedups on SSSP for the different graphs with and without hugepages.

the mean speedup is either very similar or slightly reduced by the hugepages. But in all cases, the variance is significantly smaller when using hugepages. This proves a slightly smaller but more reliable speedup with hugepages. Examples for this are on the one hand, orkut that could not reach a speedup above $1.6\times$. Now, with hugepages it is $4.3\times$ faster with 96 threads compared to one thread. On the other hand, twitter reached a speedup of $17\times$ without hugepages and only $13\times$ with.

2) *Breadth-First Search*: For our speedup results on BFS, Figure 8 shows the calculation time speedup of Galois' BFS. On all graphs, the speedup never exceeds $6\times$ even when using

TABLE III: Mean Speedups and Variances for SSSP With and Without HugePages

HP	1		2		4		8	
	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2
w/o	1.0	0.0	1.6	0.2	2.5	1.1	4.5	5.3
w/	1.0	0.0	1.6	0.1	2.5	0.9	3.4	2.1

HP	16		32		48		96	
	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2
w/o	6.7	13.0	9.6	38.3	10.3	41.4	10.2	38.1
w/	5.8	7.3	7.0	10.8	10.7	31.4	10.8	29.9

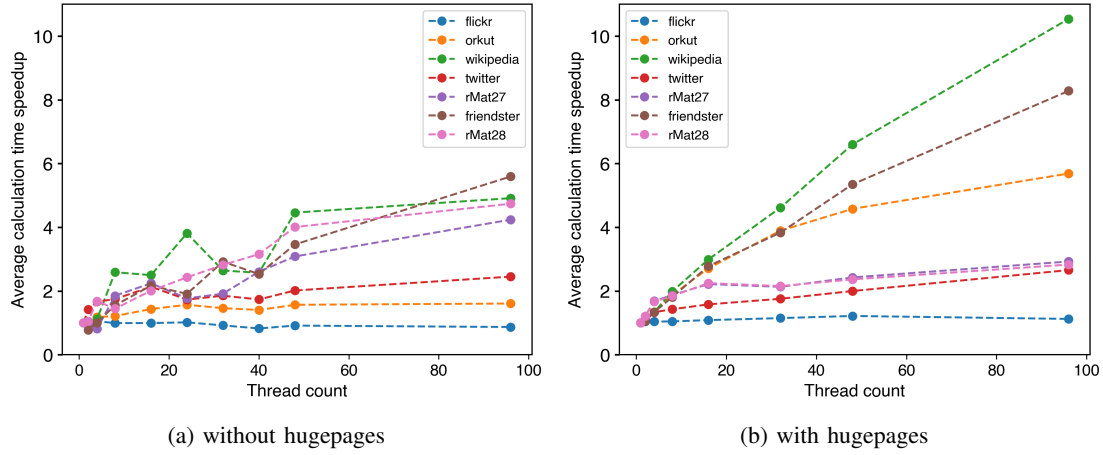


Fig. 8: Calculation time speedups on BFS for the different graphs with and without hugepages.

96 threads. For the smaller graphs (flickr, orkut), we have the same behaviour as on SSSP. Speedup is close to 1 in all cases, with orkut reaching a maximum speedup of $1.6\times$ at 24 threads. On the larger graphs, speedup is possible but only to a very small degree. Wikipedia has a maximum speedup of $4.9\times$ at 96 threads, for twitter it is $2.5\times$ at 96 threads and $5.6\times$ on friendster. The two synthetic data sets reach speedups of $4.2\times$ and $4.7\times$ for rMat27 and rMat28 on 96 threads.

3) *PageRank*: We want to first take a look at the results for PageRank in Pull mode, seen in Figure 9. This is a perfect example for an algorithm that does not benefit from multithreaded computation. Computation is hardly sped up on any graph other than flicker, where the reached maximum is 64%. This maximum is reached at two threads, with speedup steadily declining above that. The rMat28 is the only other graph of one could say computation was sped up at large thread counts. Here we reached a maximum speedup of 31% at 96 threads. All 5 other graphs only reach a speedup greater or equal to 1 in just one or two cases and if so only by a small margin. Computation on Orkut and Twitter reaches a speedup maximum of 12% and 5% at 4 threads, while being less or equal to 1 in all other cases. The wikipedia graph is never sped up. Friendster and rMat27 can be sped up by 6.5% or 10% respectively on 8 threads.

Speedup results on PageRank show odd behaviour in the Galois implementation. There is a significant performance loss on 4, 24 and 40 threads that is far from the expected behaviour. This is most visible for the Push variant seen in Figure 10, we validated the shown results two times. Especially, the speedup for 24 threads is (by interpolating between 16 and 32 threads) expected to be anywhere between 25% and 94%. Actually however, the system does not reach a speedup of more than 4% on any graph, with only rMat27 actually reaching a value greater than 1. On all other graphs, using 24 threads is anywhere from 3% (flickr) to 9% (wikipedia) slower than using just one thread.

Similar yet less pronounced behaviour is observable for Pull in Figure 9. Here especially the values for 24 and 40 threads show a loss in performance. It is most visible on the values for twitter and friendster, where both values drop

significantly compared to the neighbouring 32 and 48 thread results.

//TODO: unfertig We assume the reason for this behaviour to be founded in the hugepages. Galois is recommended to be run with hugepages.

VIII. DISCUSSION

//TODO: Hier zunächst mal ein kurzer Vergleich zwischen Distributed und Single Node wo sinnvoll, dann zwischen den Frameworks und so weiter.

IX. CONCLUSION

//TODO: Section

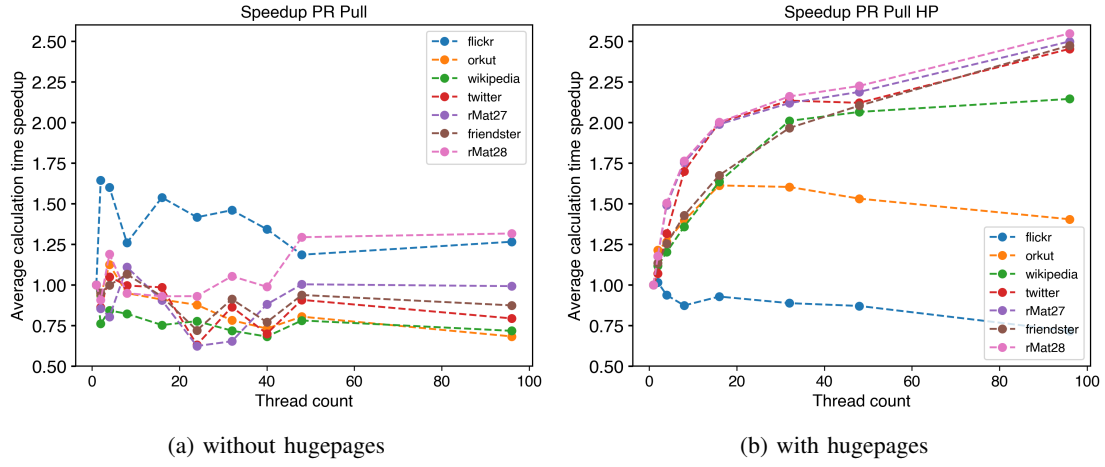


Fig. 9: Calculation time speedups on PR Pull for the different graphs with and without hugepages.

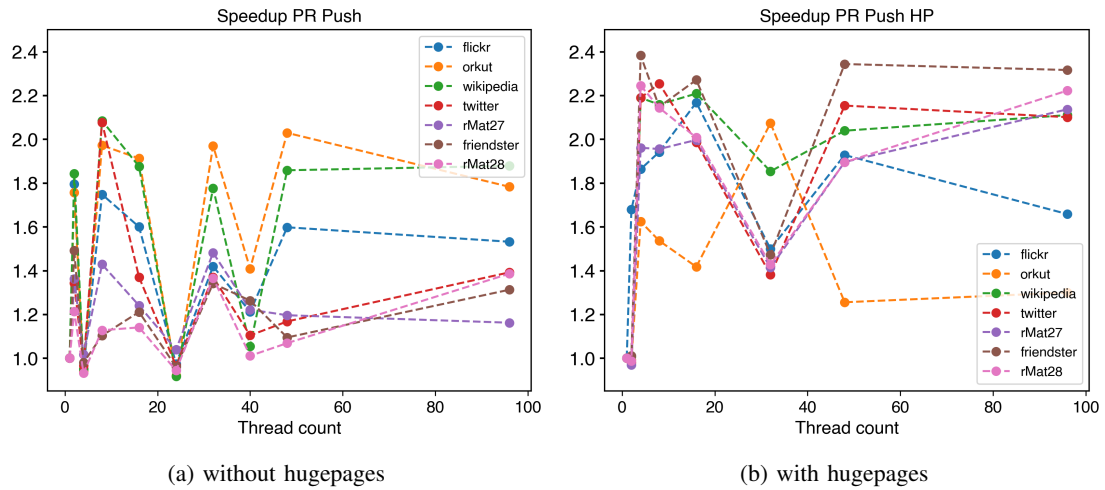


Fig. 10: Calculation time speedups on PR Push for the different graphs with and without hugepages.

ACKNOWLEDGMENTS

//TODO: Beide zusammenfassen

APPENDIX A INSTALLATION GUIDES

The complete installation guides with steps on how to run the same algorithms we used is available in our repository⁶

In case of frameworks like Giraph, where for example no BFS algorithm is provided by the framework itself, the algorithm and a guide on how to include it in the framework can be found in the repository as well.

APPENDIX B CONVERSION TOOL

In order to

We have written a number of conversion tools and installation guides to help users or developers with the use of the tested frameworks.

Our GitHub repository: <http://www.github.com/serengti/Forschungsprojekt>.

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