

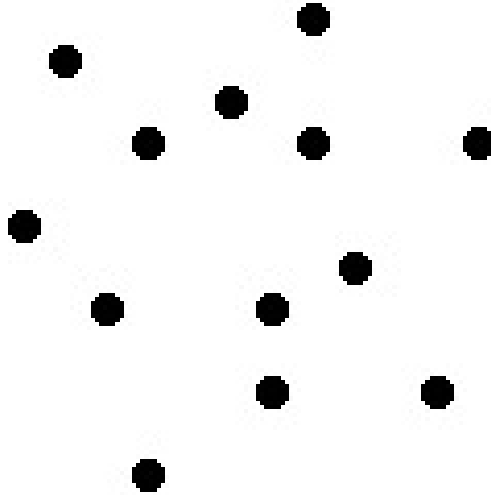
CONVEX HULL

What is a convex hull?

- Word meaning of hull?
- Hull : the outer covering of a fruit or seed / the frame or body of a ship [Wikipedia]
- Outer covering of what??
- A set of points

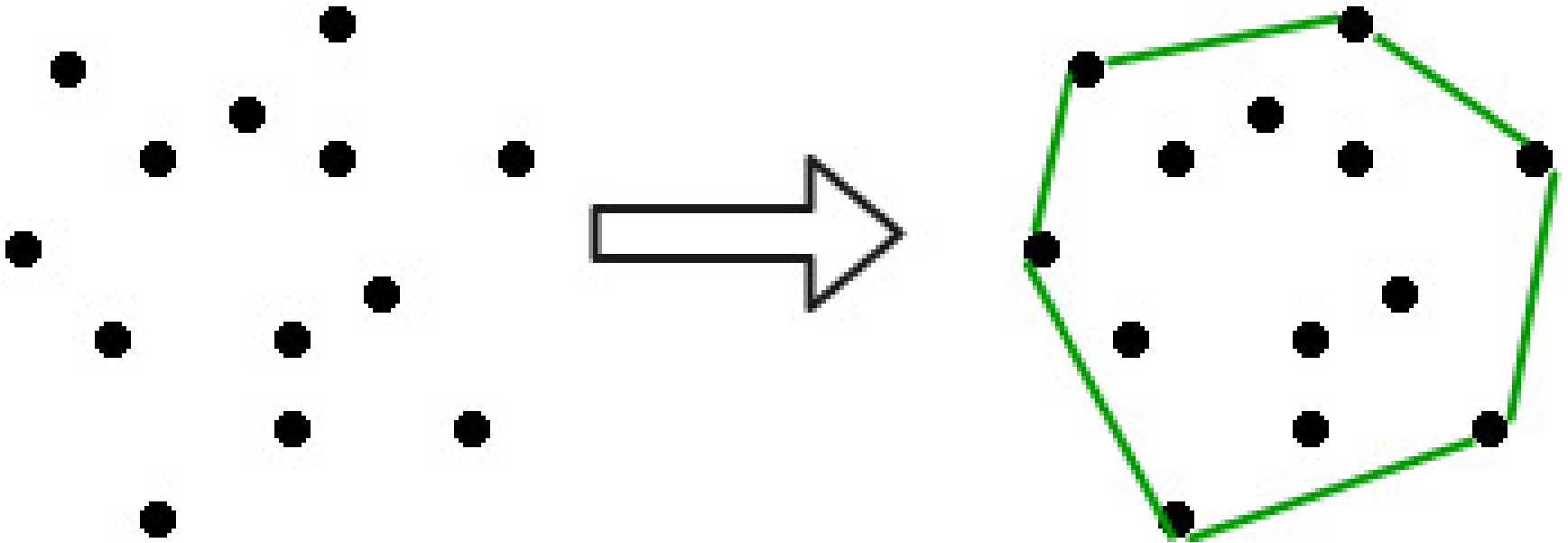
Convex hull

- Outer covering of the set of points



- How many outer coverings can we draw?

Convex Hull (CH)



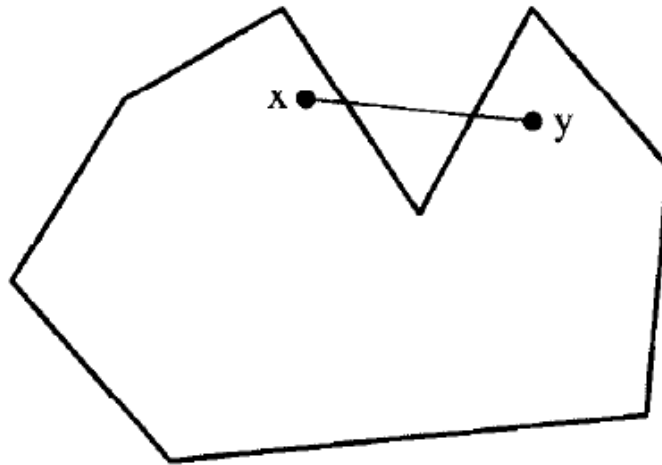
- Closed region including / enclosing all the points

What is special about convex hull?

- A convex hull of a set of points S in the plane is the enclosing convex polygon with:
- Smallest area
- Smallest perimeter

Convex Hull

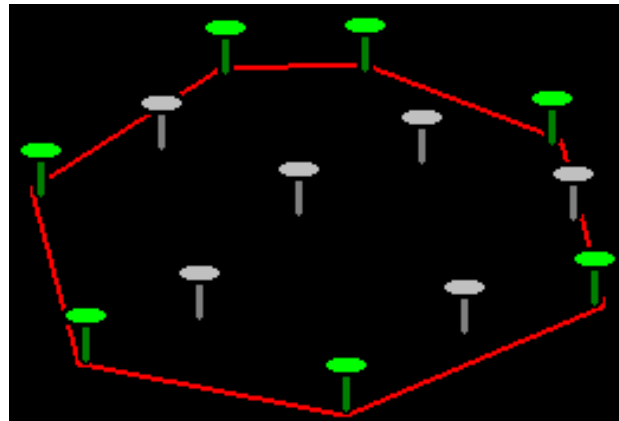
- All the angles are convex



- Any dent implies non-convexity

Visualization of CH

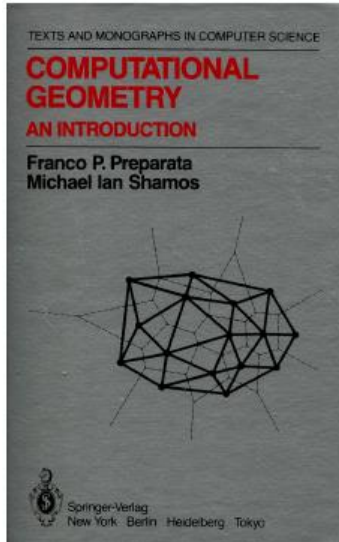
- Imagine that the points are nails sticking out of the plane
- Take an elastic rubber band, stretch it around the nails and let it go
- The rubber band snaps around the nails and assume a shape that minimizes its length



Convex Hull

- One of the most frequently used structure in Computational geometry
- First paper in Computational Geometry – construction of Convex hull by Graham's algorithm in 1972 *
- *Michael Shamos coined the term “computational geometry” in mid of 1970's in his PhD Thesis

- One of the earliest books of Computational Geometry is by Shamos and Preparata in 1985



Michael Shamos



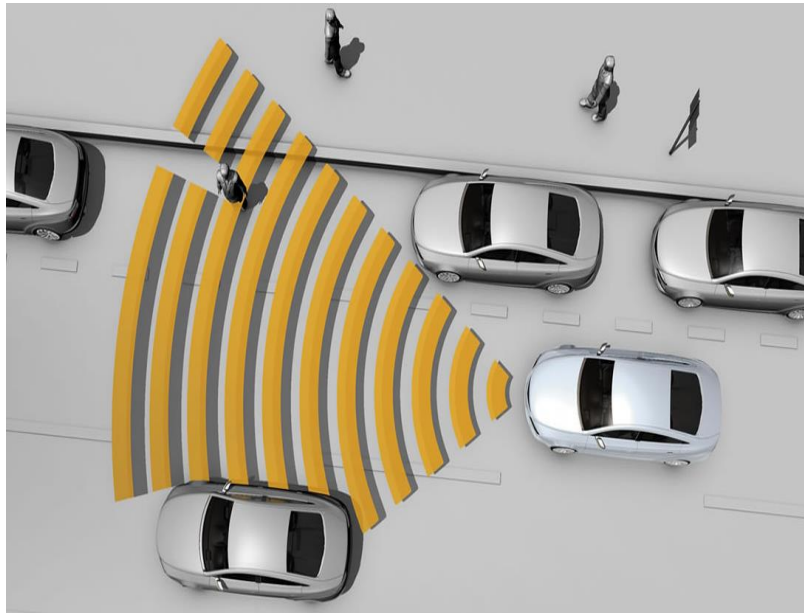
Franco Preparata

- Currently Shamos is a Distinguished Career Professor in the School of Computer Science at Carnegie Mellon University
- **Franco P. Preparata** is a [computer scientist](#), the [An Wang](#) Professor, Emeritus, of Computer Science at [Brown University](#).

Applications of convex hull

Applications :Collision Avoidance

- If CH of a car avoids collision with obstacles, so does the car



- Computation of paths that avoid collision is easier with convex cars
- Hence, CH is used to **plan paths**

Applications: **Nuclear/Chemical Leak Evacuation**

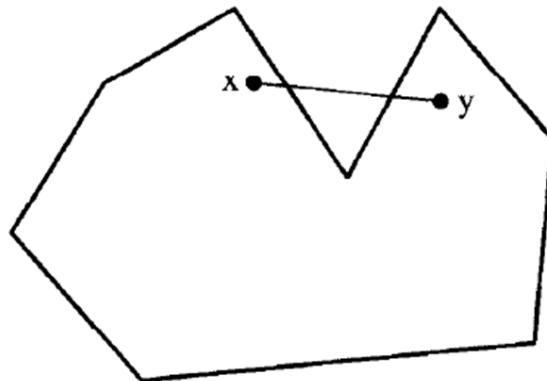
- Imagine a modern city with sensors positioned uniformly all over.
- When a disaster such a chemical leak or nuclear radiation leak, we have to determine the perimeter for immediate evacuation
- Construct the convex hull of those areas with radiation levels (exceeding a certain threshold).

Applications :Tracking Disease Epidemic.

- **Spatial extent of an outbreak in animal epidemics** *Eric Dumonteil, Satya N. Majumdar, Alberto Rosso, and Andrea Zoia* in Proceedings of the National Academy of Sciences of USA, March 12, 2013 110 (11) 4239-4244;
<https://doi.org/10.1073/pnas.1213237110>
- Spatial extent of an epidemic is assessed by computing the convex hull enclosing the infected individuals at a given time

Definition of Convexity

- **Convexity** : A set S is convex if $p \in S$ and $q \in S$ implies that the segment $pq \subseteq S$
- This definition does not specify any dimension, whether S is connected, bounded etc.
- We know that any region with a dent is not convex



Definition of Convex Hull

- *Do you know what is a Half space?*
- *Half space in two dimension is a half plane.*
- *Set of points on or to one side of a line*
- Convex Hull of set of points S is the intersection of all half spaces that contain S
- Exercise: Try the above definition with 3 points, 4 points

More definitions of CH

The convex hull of a finite set of points S in the plane is the smallest convex polygon P that encloses S , smallest in the sense that there is no other polygon P' such that $P \supset P' \supseteq S$.

The convex hull of a finite set of points S in the plane is the enclosing convex polygon P with smallest area.

The convex hull of a finite set of points S in the plane is the enclosing convex polygon P with smallest perimeter.

Standard algorithms for constructing a Convex Hull

References

- J. O Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998)

Thank you