Other ways of Polygon Partitioning

Polygon Partitioning

- Partitioning the Polygon to :
- Triangles
- Monotone polygons
- Trapezoids
- Monotone mountains
- Convex polygons

Why do we need these many different partitioning?

- Triangulation is of $O(n^2)$ time—(Go through the reading exercise)
- We need to speed up the triangulation algorithm
- The different partitions have applications of their own
- For example, convex partition is preferred:
  - Computations are easier on convex polygons
  - One application of convex partition is character recognition

Triangulation algorithm

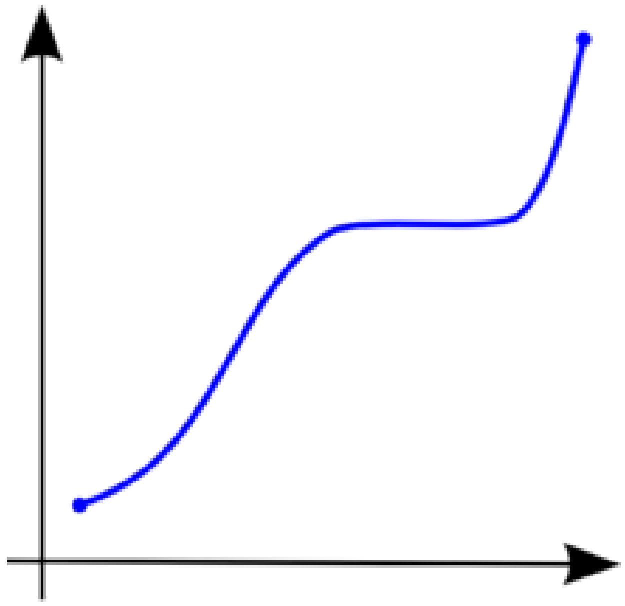
- Suppose we go for partitioning P in to triangles by adding diagonals
- First we need to check whether the line segment added is a diagonal
- For that, we have to check :
- Whether the line segment is intersecting with any other edge of the T
- Whether the line segment is completely interior to P
- Is it possible : ***to find a line segment is a diagonal*** in linear time?

Diagonal in sub-linear time

- Now we are trying to check whether a line segment is a diagonal in sub-linear time
- An algorithm whose execution time, f(n), grows slower than the size of the problem n, but only gives an approximate or probably correct answer.
- For that, first the polygon is divided in to monotone pieces
- What do we mean by monotonicity?
- Have you heard about monotonically increasing or monotonically decreasing functions?

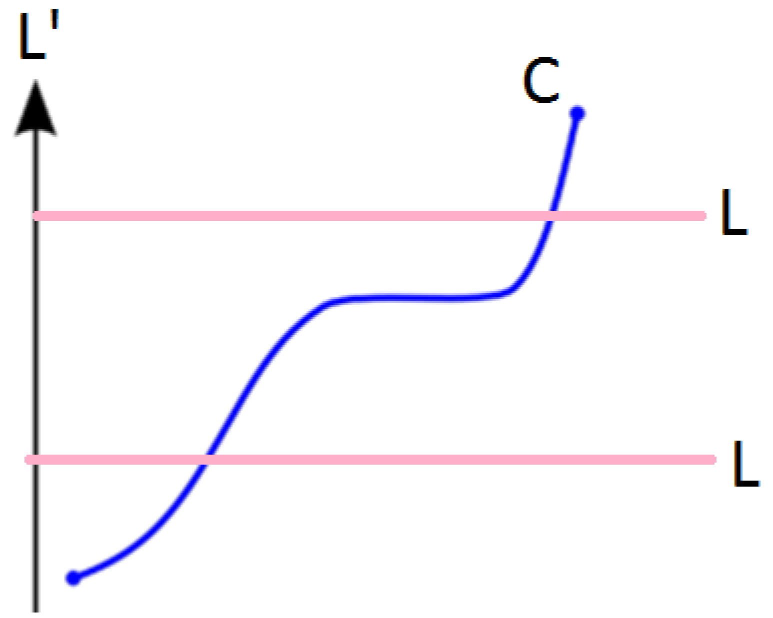
Monotonically increasing function

- A monotonically increasing function should not decrease

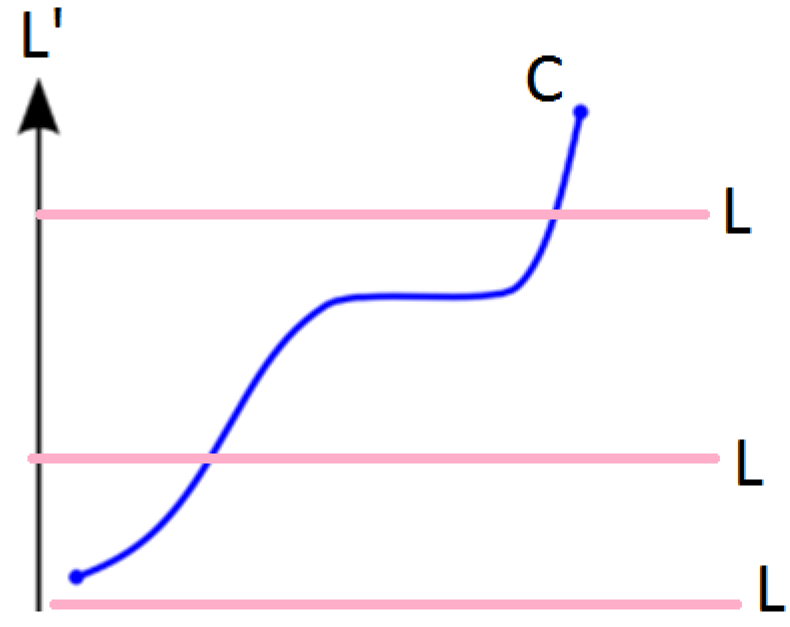


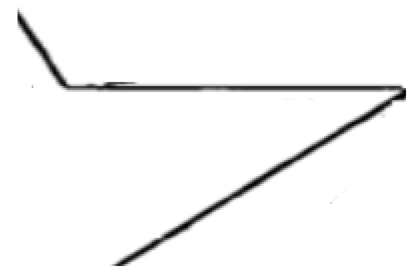
Monotonicity

- Monotonicity is defined with respect to a line
- What is **monotonicity of a polygonal chain?**
- A polygonal chain C is **strictly monotone** with respect to L’ if every line L orthogonal to L’ meets C in at most one point



Monotonicity



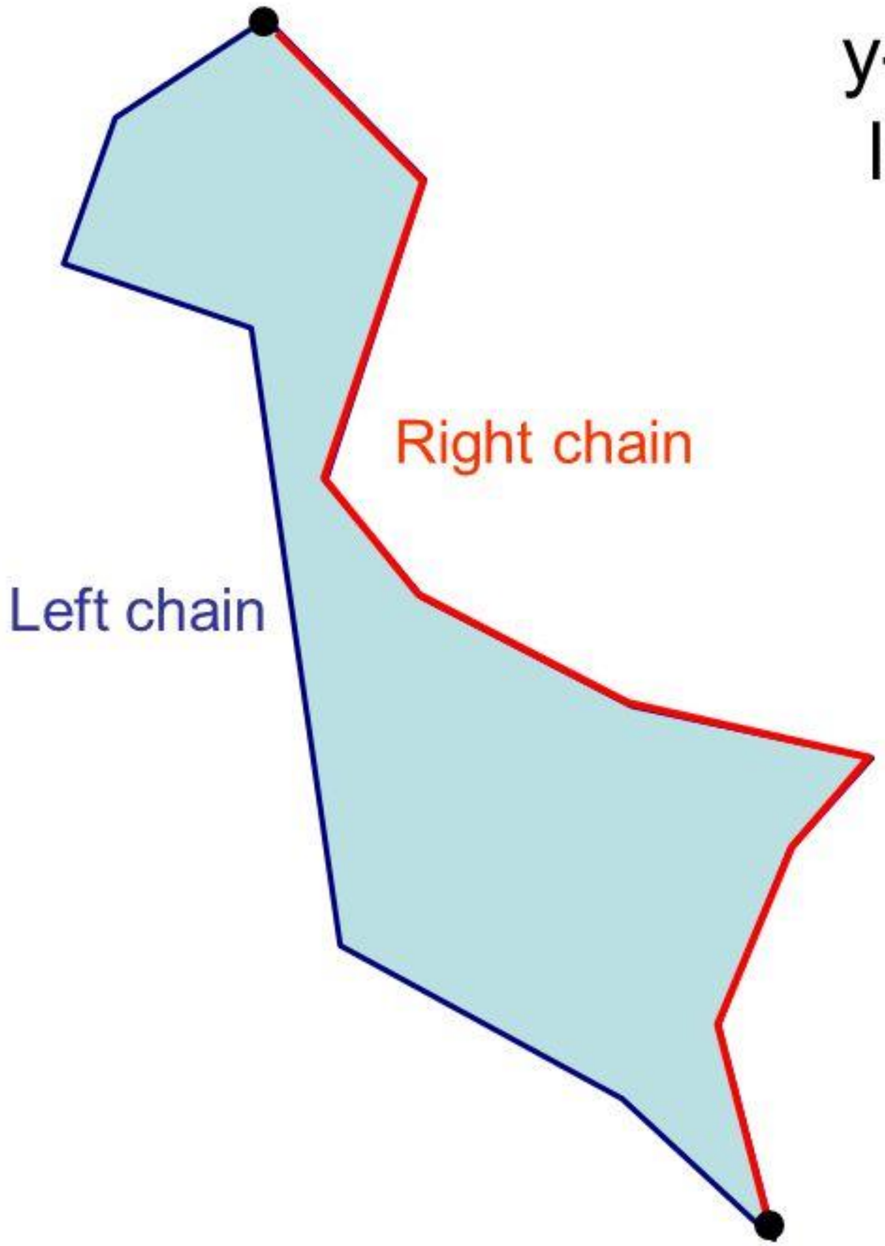


- L ∩ C is either empty or a single point
- A polygonal chain C is **monotone** if L ∩ C is either empty, a single point or a single line segment

A monotone polygon

- A polygon P is said to be monotone with respect to line L if ∂P can be split in to two polygonal chains such that each chain is monotone with respect to L
- The two chains share a vertex at either end
- Draw a monotone polygon

y-monotone polygon: left and right chains



We will also assume that the polygon is strictly y-monotone, i.e. it is y-monotone and has no horizontal edges. Additionally, you may assume that no two vertices have the same y-coordinate

Exercise

- Draw a polygon which is strictly monotone

- Draw a polygon which is monotone

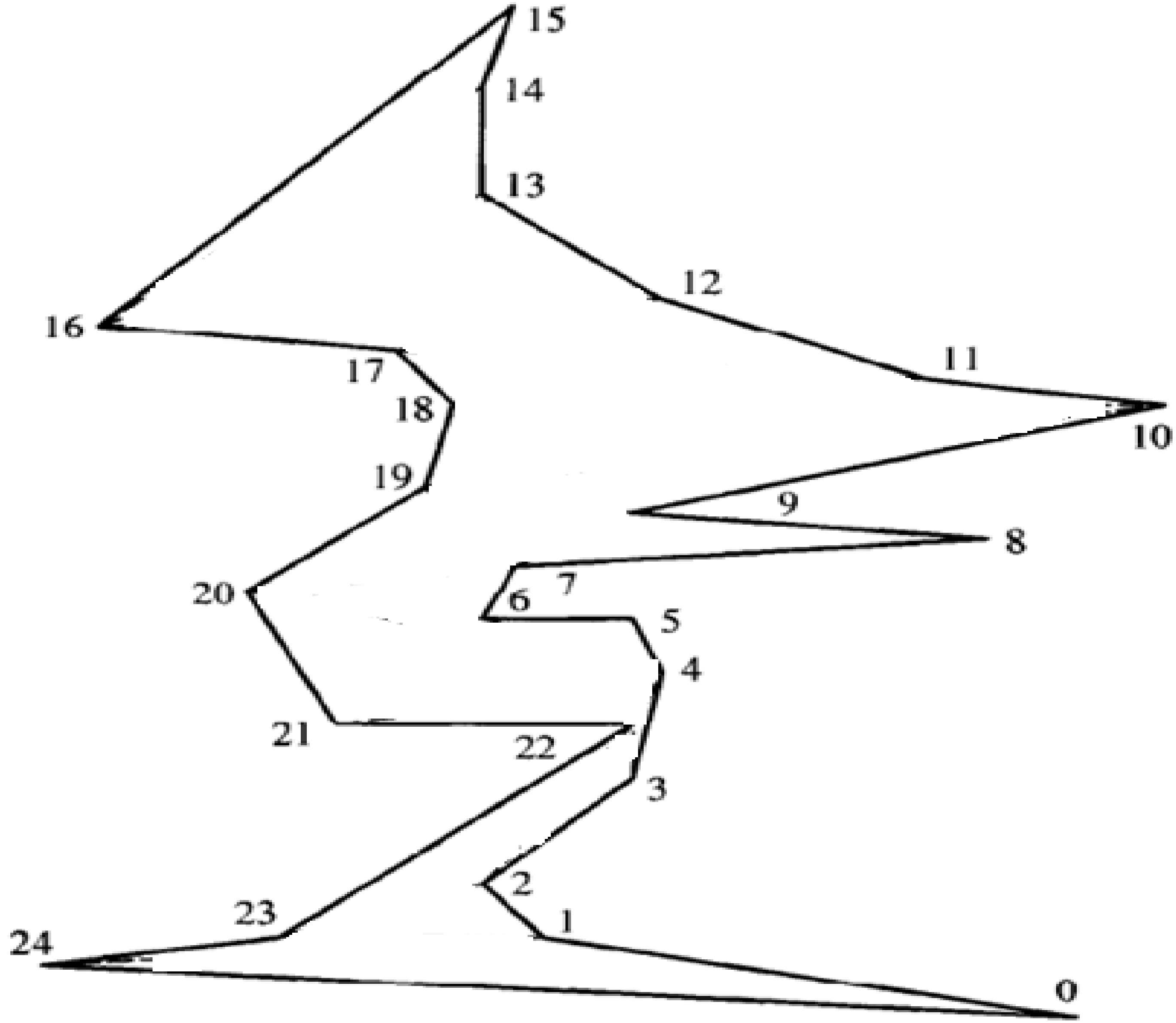
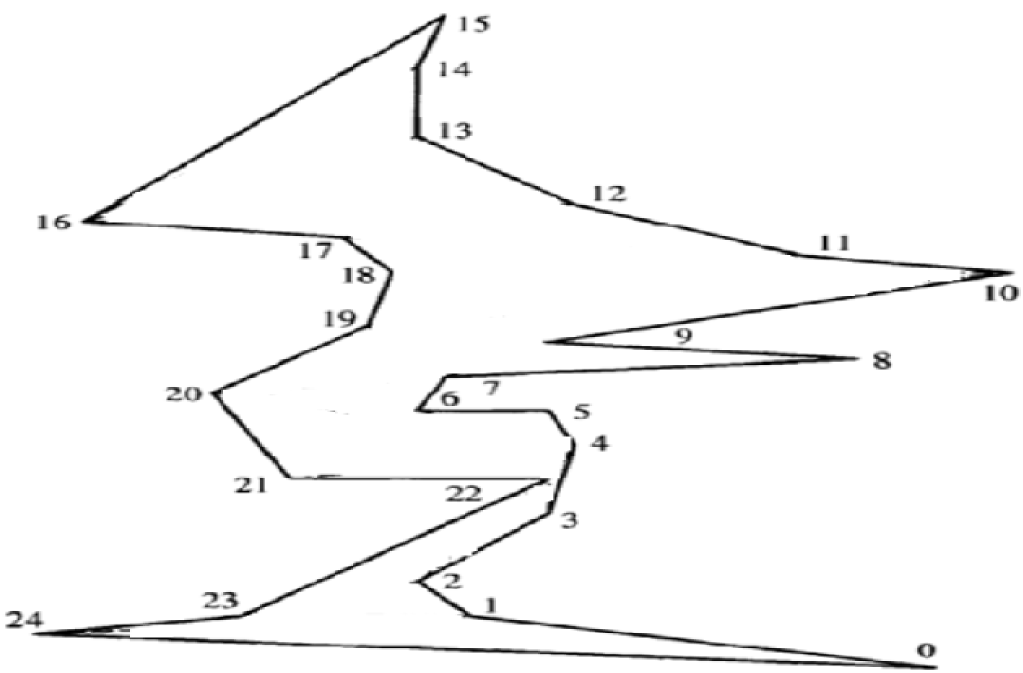


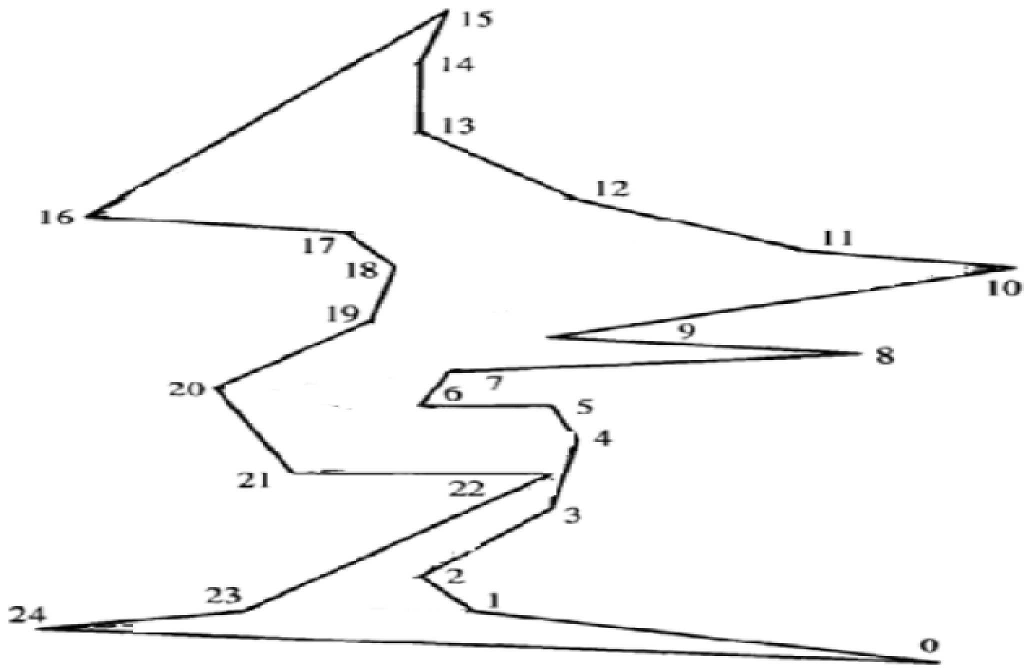
FIGURE 2.1 A polygon monotone with respect to the vertical.

The monotone chains

- A = (v_0, v_1, …., v_{15})
- B = (v_{15}, v_{16}, …., v_{24}, v_0)
- These chains strictly monotone?
- Neither chains are strictly monotone



The monotone chains



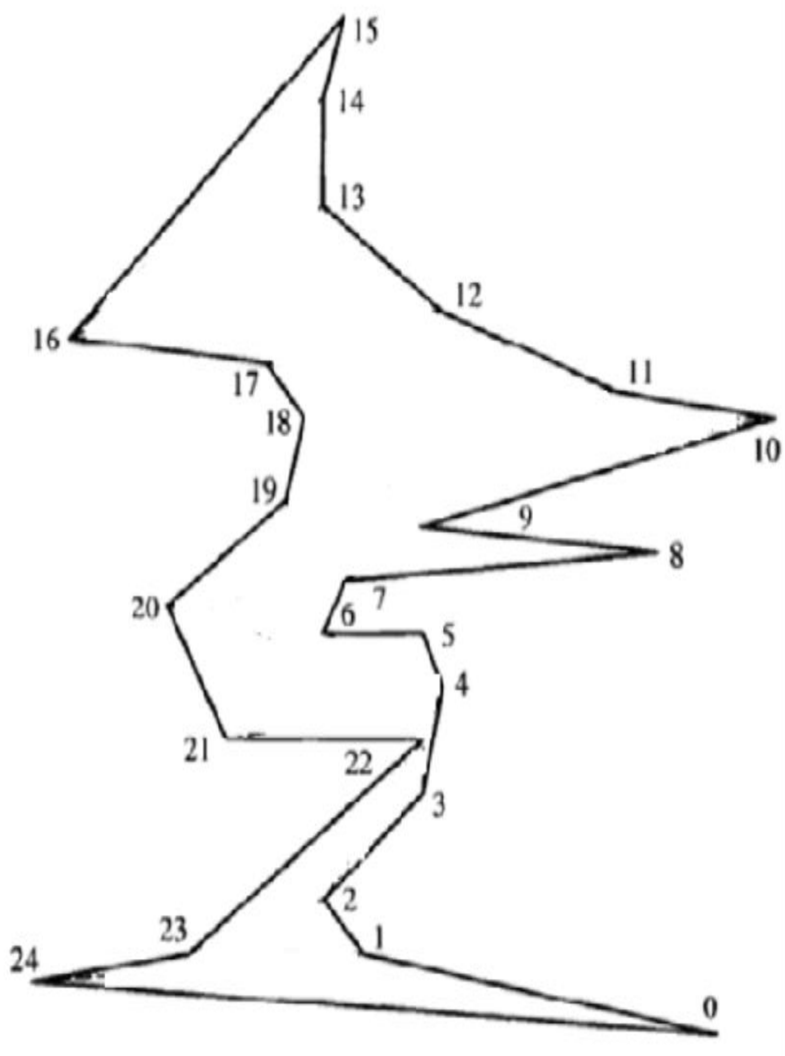
- Because edges (v_5, v_6) and (v_{21}, v_{22}) are horizontal
- Some polygons are monotone with respect to many lines

Exercise

- Draw a polygon which is monotone with respect to two lines

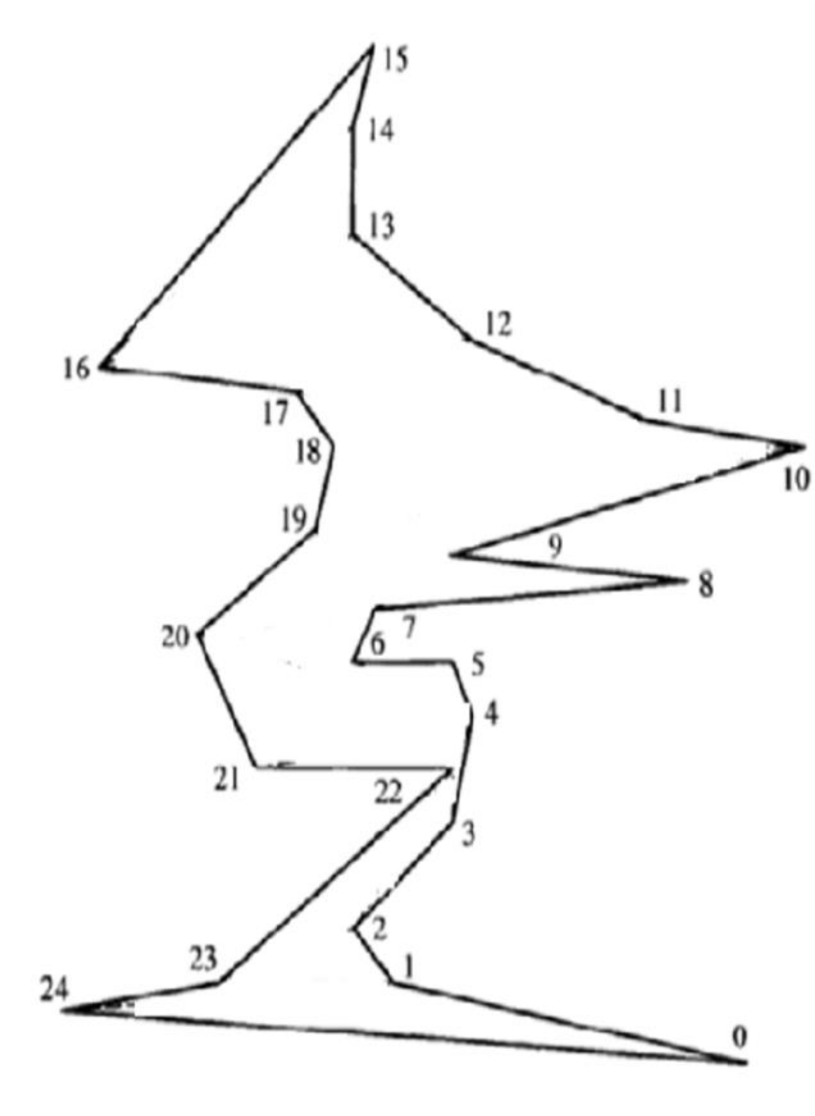
Properties of monotone polygons

- The vertices in each chain of a monotone polygon are sorted with respect to the line of monotonicity
- Let the line of monotonicity be Y axis
- The vertices in a P can be sorted by y coordinate in linear time
- How do we do that?



Sort the monotone polygon

- Find the highest vertex
- Find the lowest vertex
- Partition the boundary in to two chains
- The vertices in each chain are sorted with respect to y
- Two sorted list of vertices can be merged in linear time to produce one list sorted by y coordinate



Local characteristic of monotonicity

- P is monotone if it is monotone locally w.r.t. each vertex except highest & lowest
- For example take v_{11}
- v_{11} is monotone locally
- In other words, v_{11} has one adjacent vertex with higher y value and other adjacent vertex with smaller y value than v_{11}

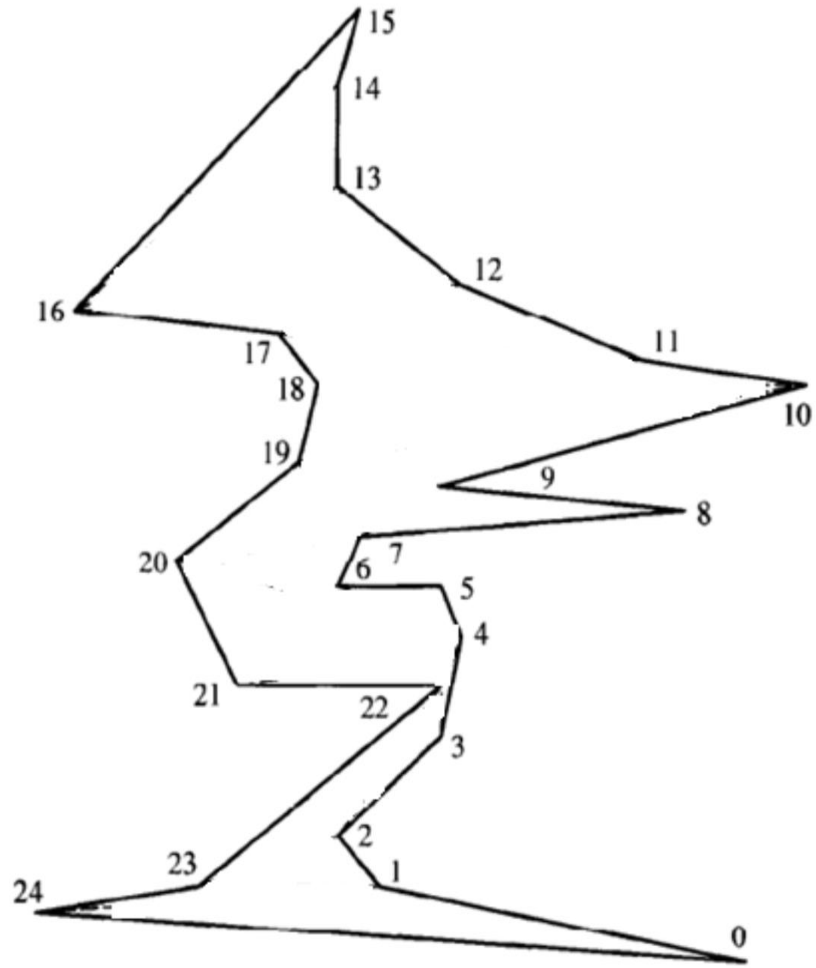


FIGURE 2.1 A polygon monotone with respect to the vertical.

What breaks monotonicity?

- Exercise:
  - (i) Draw a non-monotone chain
  - (ii) Draw a non-monotone polygon

- What is the local characteristic of non- monotonicity ?

References

- J. O'Rourke: Art Gallery Theorems and Algorithms
- J. O’Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998

Thank you