

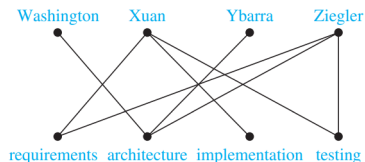
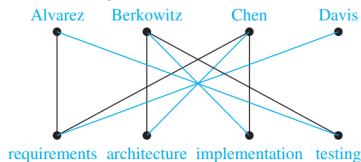
CS2006D DISCRETE STRUCTURES

Renjith P.



SPECIAL GRAPHS

- ★ Sometimes a graph has the property that its **vertex set** can be divided into **two** disjoint subsets such that each edge connects a vertex in one of these subsets to a vertex in the other subset
- ★ Job Assignments

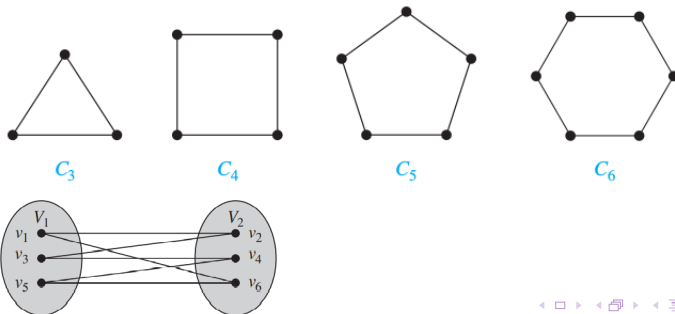


- ★ The graph representing marriages between men and women in a village
- ★ Time slot - event allocation
- ★ Scheduling Jobs to machines
- ★ Objects and actions in a AR environment



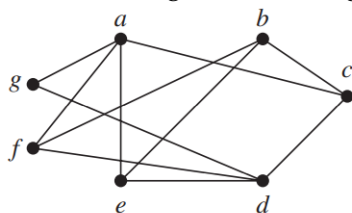
BIPARTITE GRAPHS

- ★ A simple graph G is called **Bipartite** if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair (V_1, V_2) a bipartition of the vertex set V of G .
- ★ An even **cycle** C_n , n is even, is a bipartite graph
- ★ C_3, C_5, \dots are not bipartite

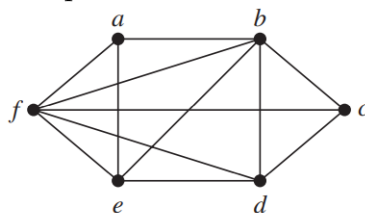


BIPARTITE GRAPHS

★ Which among the following are bipartite



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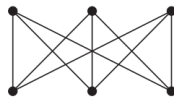
A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color

COMPLETE BIPARTITE GRAPHS

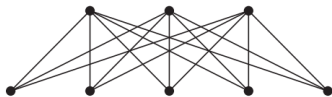
- ★ A **complete bipartite graph** $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.
- ★ The complete bipartite graphs $K_{2,3}$, $K_{3,3}$, $K_{3,5}$, and $K_{2,6}$



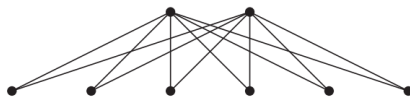
$K_{2,3}$



$K_{3,3}$



$K_{3,5}$

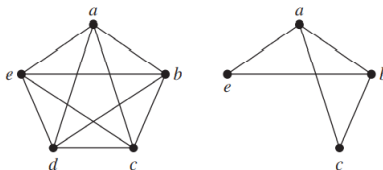


$K_{2,6}$

SUB-GRAPHS

A **subgraph** of a graph $G (V, E)$ is a graph $H (W, F)$, where $W \subseteq V$ and $F \subseteq E$

A subgraph H of G is a **proper subgraph** of G if $H \neq G$.

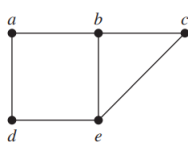


Let $G (V, E)$ be a simple graph. The **subgraph induced** by a subset W of the vertex set V is the graph (W, F) , where the edge set F contains an edge in E if and only if both endpoints of this edge are in W

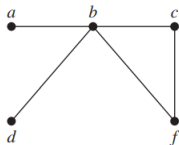
- ★ When we remove a vertex v and all edges incident to it from $G (V, E)$, we produce a subgraph, denoted by $G - v$

UNION OF TWO GRAPHS

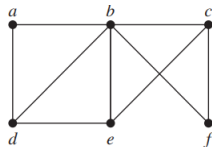
The union of two simple graphs $G_1 (V_1, E_1)$ and $G_2 (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_1 \cup G_2$



G_1



G_2

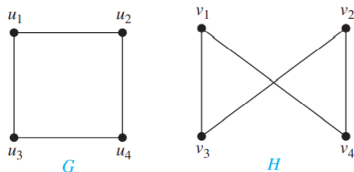


$G_1 \cup G_2$

- ★ The **complementary graph** \overline{G} of a simple graph G has the same vertices as G . Two vertices are adjacent in \overline{G} if and only if they are not adjacent in G

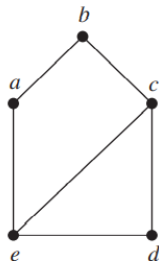
GRAPH ISOMORPHISM

The simple graphs $G_1 (V_1, E_1)$ and $G_2 (V_2, E_2)$ are **isomorphic** if there exists a **one-to-one** and **onto** function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2 , for all a and b in V_1 . Such a function f is called an isomorphism.

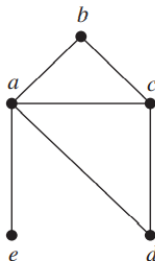


- ★ Two simple graphs that are not isomorphic are called **nonisomorphic**
- ★ Isomorphic simple graphs must have the same number of vertices
- ★ Isomorphic simple graphs must have the same number of edges
- ★ In addition, the degrees of the vertices in isomorphic simple graphs must be the same

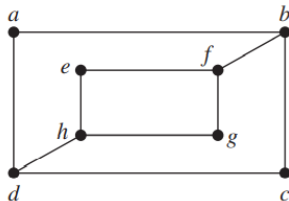
GRAPH ISOMORPHISM - SOME PRACTICE QUESTIONS



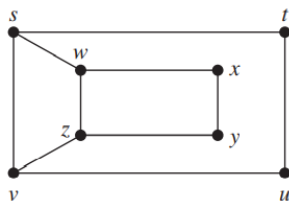
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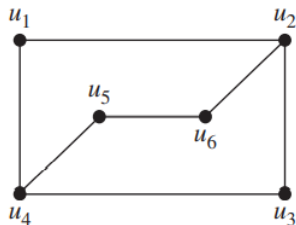


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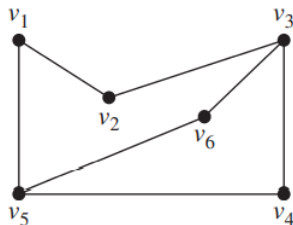


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GRAPH ISOMORPHISM - SOME PRACTICE QUESTIONS



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A simple graph G is called self-complementary if G and \overline{G} are isomorphic (P_3, C_5)

GRAPHIC SEQUENCE

- ★ The **degree sequence** of a graph is the list of vertex degrees, usually written in nonincreasing order, as $d_1 \geq \dots \geq d_n$
- ★ Every graph has a degree sequence
- ★ A graphic sequence is a list of nonnegative numbers that is the degree sequence of some simple graph
- ★ Not all sequences are graphic
- ★ For a graph G ,
min degree: $\delta(G)$ or simply δ
max degree: $\Delta(G)$ or simply Δ

Havel Hakimi Theorem

For $n > 1$, an integer list d of size n is graphic if and only if d' is graphic, where d' is obtained from d by deleting its largest element Δ and subtracting 1 from its Δ next largest elements

Havel Hakimi Theorem

For $n > 1$, $d_1 \geq \dots \geq d_n$ is graphic if and only if
 $d_2 - 1 \geq d_3 - 1 \geq \dots d_{d_1+1} - 1 \geq d_{d_1+2} \geq \dots \geq d_n$ is graphic

★ Which of the following are graphic sequences?

1 (5,5,4,3,2,2,2,1)

2 (5,5,4,4,2,2,1,1)

3 (5,5,5,3,2,2,1,1)

4 (5,5,5,4,2,1,1,1)

Every simple graph with at least two vertices has two vertices of equal degree

- ★ A simple graph is called **regular** if every vertex of this graph has the same degree.
- ★ A regular graph is called **n-regular** if every vertex in this graph has degree n .
- ★ For which values of n are these graphs regular?
a) K_n b) C_n c) W_n d) Q_n

PRACTICE QUESTIONS

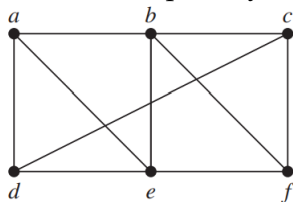
- ★ For which values of n are these graphs bipartite?
a) K_n b) C_n c) W_n d) Q_n
- ★ Find the degree sequence of each of the following graphs.
a) K_5 b) C_6 c) W_4 d) Q_3
- ★ How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2?
- ★ Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.
- ★ How many vertices does a regular graph of degree four with 10 edges have?
- ★ Describe each of these graphs. a) $\overline{K_n}$ b) $\overline{K_{m,n}}$
- ★ Prove or disprove: Complement of a regular graph is regular.
- ★ If G is a simple graph with 15 edges and \overline{G} has 13 edges, how many vertices does G have?

PRACTICE QUESTIONS

- ★ If the simple graph G has v vertices and e edges, how many edges does \overline{G} have?
- ★ If the degree sequence of the simple graph G is 4, 3, 3, 2, 2, what is the degree sequence of \overline{G} ?
- ★ If the degree sequence of the simple graph G is d_1, d_2, \dots, d_n , what is the degree sequence of \overline{G} ?
- ★ How many nonisomorphic simple graphs are there with n vertices, when n is
a) 2? b) 3? c) 4?

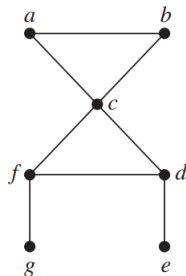
PATHS AND RELATED TERMINOLOGY

- ★ A **walk** is defined to be an alternating sequence of vertices and edges of a graph, $v_0, e_1, v_1, e_2, \dots, v_{n-1}, e_n, v_n$, where v_{i-1} and v_i are the endpoints of e_i for $i = 1, 2, \dots, n$
- ★ A **closed walk** is a walk with the same end vertices
- ★ A **trail** is used to denote a walk that has no repeated edge
- ★ A closed trail is called a **circuit**
- ★ A **path** is a trail with no repeated vertices
- ★ We denote a path by its vertex sequence $x_0, x_1, \dots, x_{n-1}, x_n$

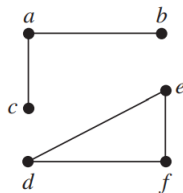


CONNECTED GRAPH

- ★ An undirected graph is called **connected** if there is a path between every pair of distinct vertices of the graph
- ★ An undirected graph that is not connected is called **disconnected**
- ★ There is a simple path between every pair of distinct vertices of a connected undirected graph



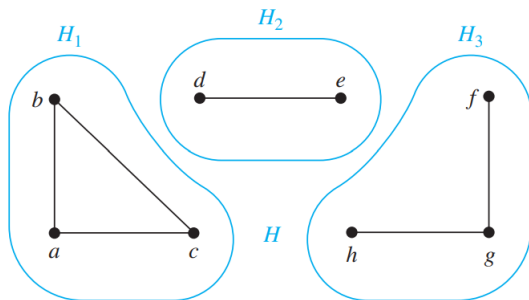
G_1



G_2

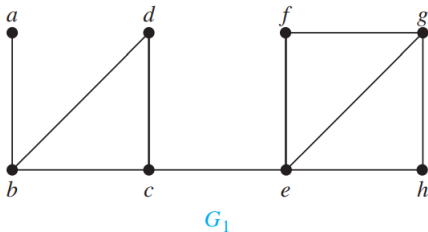
COMPONENTS OF A GRAPH

- ★ A **connected component** of a graph G is a connected subgraph of G that is not a proper subgraph of another connected subgraph of G
- ★ A connected component of a graph G is a maximal connected subgraph of G



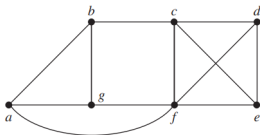
CUT VERTEX AND CUT EDGE

- ★ A vertex whose removal produces a graph with more connected components than in the original graph is called a **cut vertex** (or **articulation point**).
- ★ The removal of a cut vertex from a connected graph produces a subgraph that is not connected
- ★ An edge whose removal produces a graph with more connected components than in the original graph is called a **cut edge** or **bridge**
- ★ If the graph has more than two vertices, then at least one end point of a bridge is a cut vertex



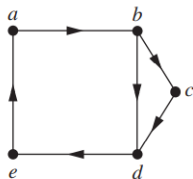
NONSEPARABLE GRAPHS

- ★ Not all graphs have cut vertices
The complete graph K_n , where $n \geq 3$, has no cut vertices
- ★ Connected graphs without cut vertices are called **nonseparable graphs**
- ★ A graph is 2-connected, or **biconnected**, if it is nonseparable and has at least three vertices
- ★ We measure the graph connectivity based on the minimum number of vertices that can be removed to disconnect a graph
- ★ A subset V' of the vertex set V of G (V, E) is a **vertex cut**, or **separating set**, if $G - V'$ is disconnected
- ★ We define the **vertex connectivity** of a noncomplete graph G , denoted by $\kappa(G)$, as the minimum number of vertices in a vertex cut

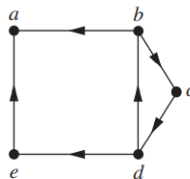


CONNECTEDNESS IN DIRECTED GRAPHS

- ★ A directed graph is **strongly connected** if there is a path from a to b and from b to a whenever a and b are vertices in the graph
- ★ A directed graph is **weakly connected** if there is a path between every two vertices in the underlying undirected graph



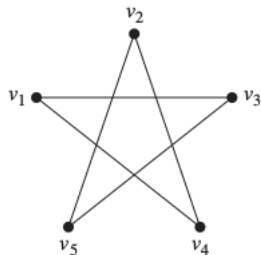
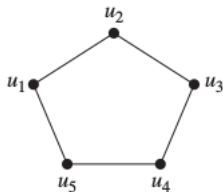
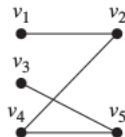
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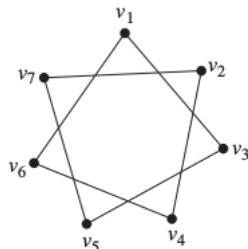
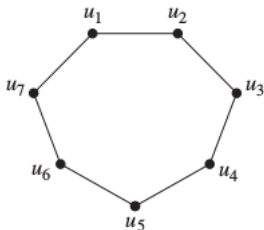
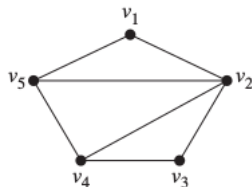
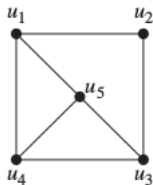
PRACTICE QUESTIONS

★ Isomorphic or not



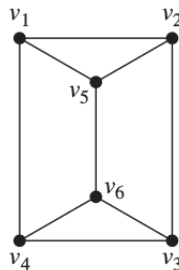
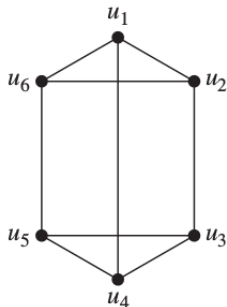
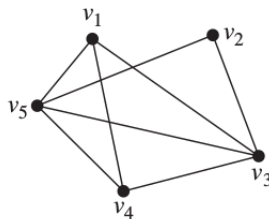
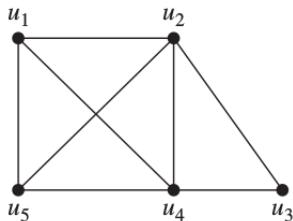
PRACTICE QUESTIONS

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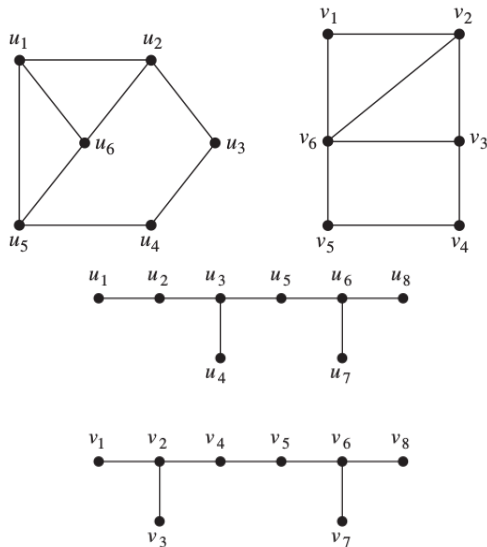
PRACTICE QUESTIONS

★ Isomorphic or not



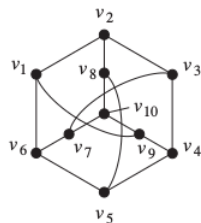
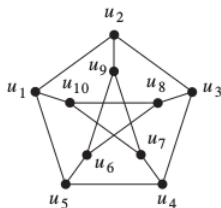
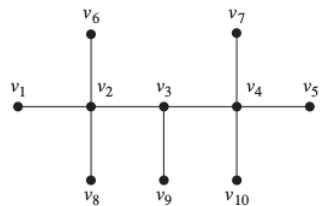
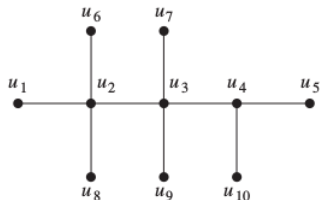
PRACTICE QUESTIONS

★ Isomorphic or not



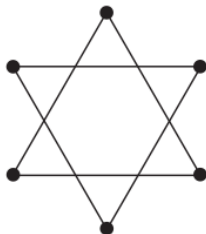
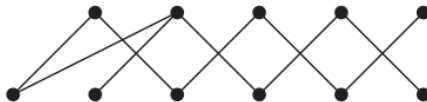
PRACTICE QUESTIONS

★ Isomorphic or not



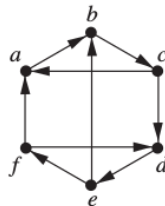
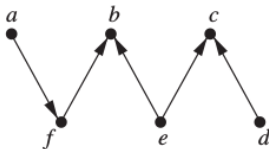
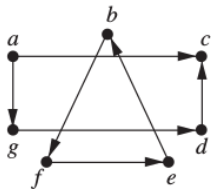
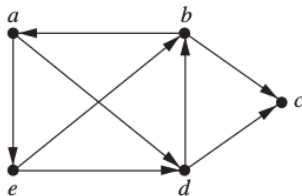
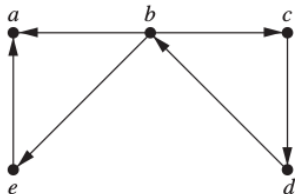
PRACTICE QUESTIONS

★ Connected or not



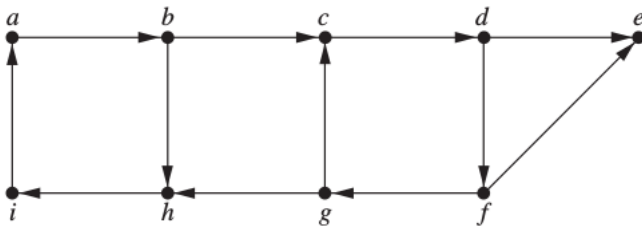
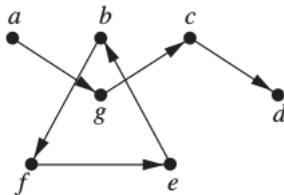
PRACTICE QUESTIONS

★ Strongly Connected or Weakly Connected



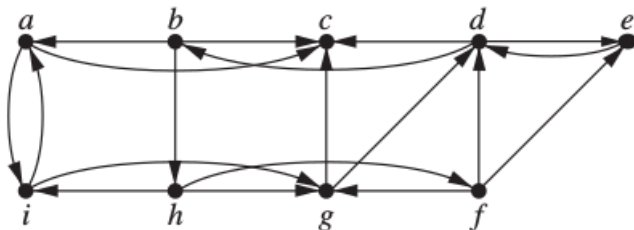
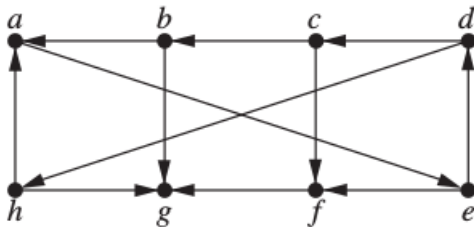
PRACTICE QUESTIONS

★ Strongly Connected or Weakly Connected



PRACTICE QUESTIONS

★ Strongly Connected or Weakly Connected



PRACTICE QUESTIONS

★ Cut vertices and Cut edges

