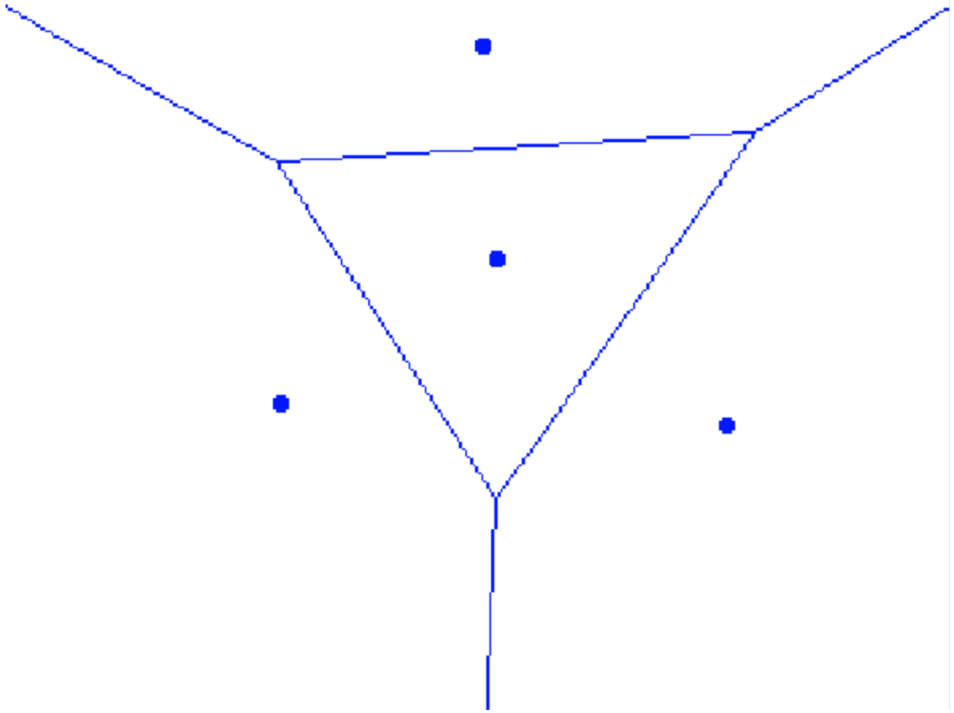
Proximity : Fundamental concepts and algorithms

Summary – Voronoi Diagram

- Voronoi diagram is used to divide a plane with points into separate regions.
- At any point within the diagram, you are closer to the site they contain than any other site, and, at any point along their boundaries, you are equidistant to at least two sites.
- *https://cfbrasz.github.io/Voronoi.html*

Terminologies used in Voronoi diagram

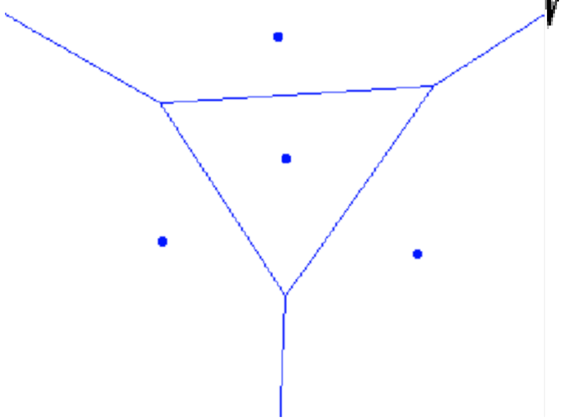
- Voronoi region of a point $V(p_i)$ / Voronoi polygon of a point
- Voronoi edge
- Voronoi vertices
- Voronoi diagram



Voronoi Region [*O'Rourke*]

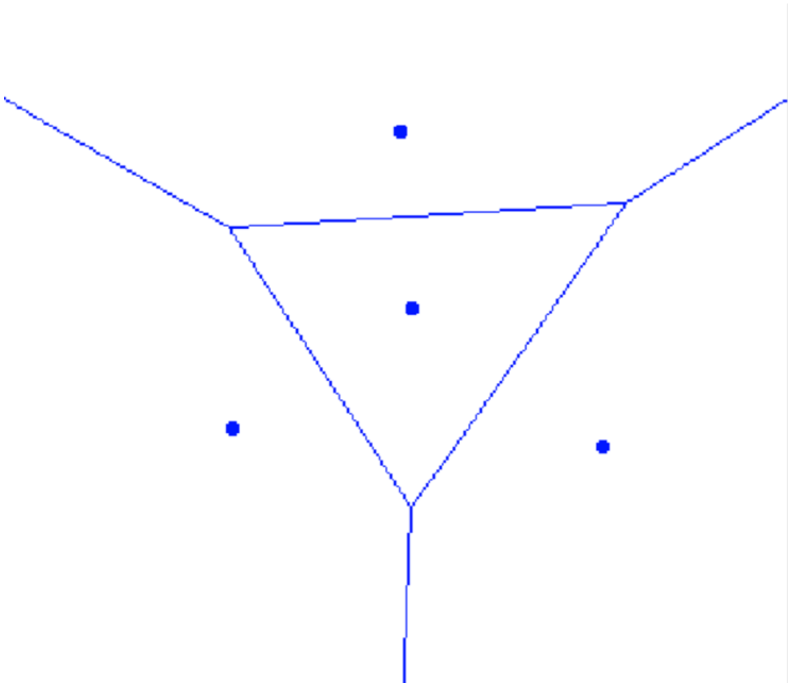
Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of points in the two-dimensional Euclidean plane. These are called the *sites*. Partition the plane by assigning every point in the plane to its nearest site. All those points assigned to p_i form the *Voronoi region* $V(p_i)$. $V(p_i)$ consists of all the points at least as close to p_i as to any other site:

$$V(p_i) = \{x : |p_i - x| \leq |p_j - x| \ \forall j \neq i\}.$$

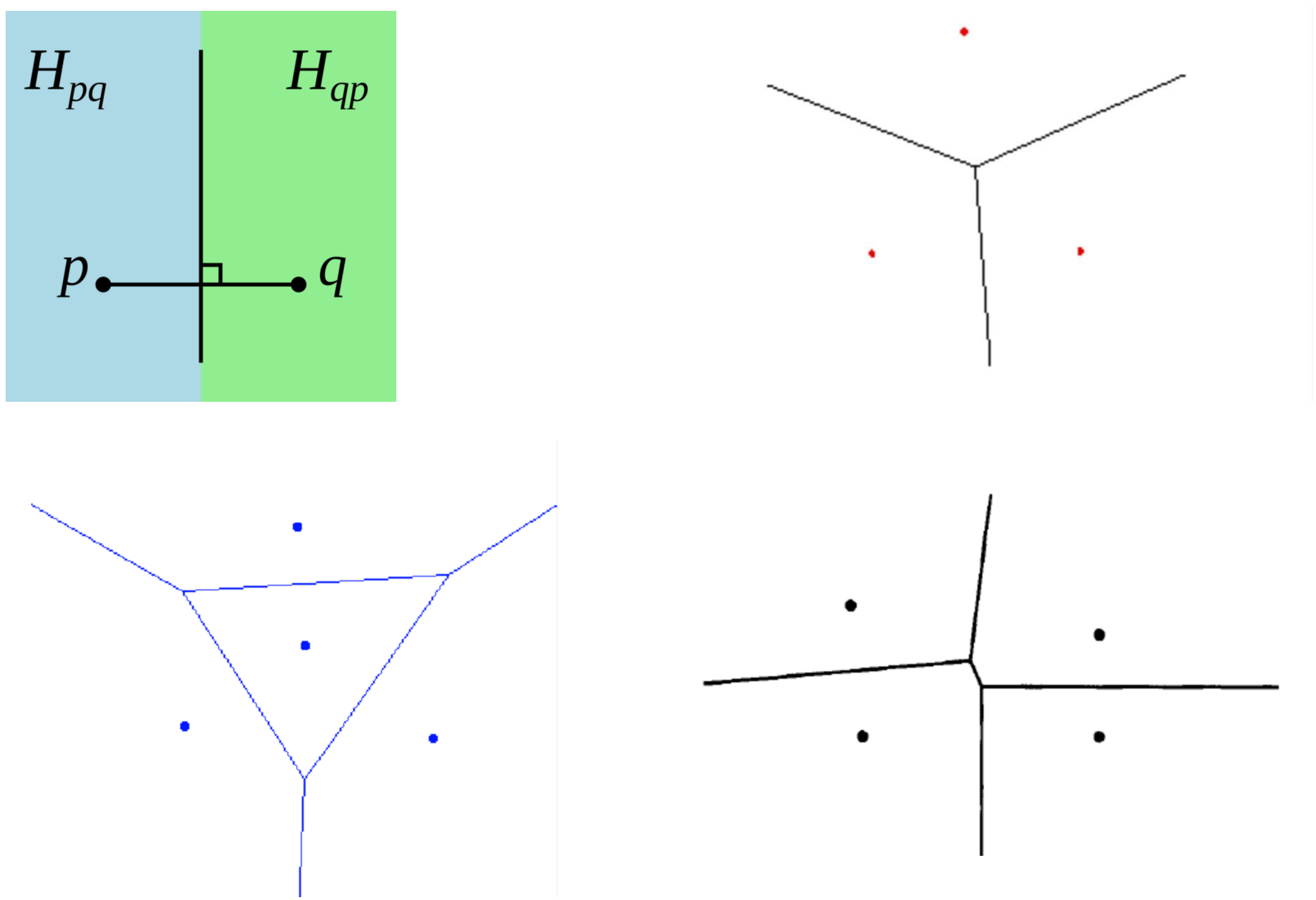


Voronoi Diagram [*O'Rourke*]

Note that we have defined this set to be closed. Some points do not have a unique nearest site, or *nearest neighbor*. The set of all points that have more than one nearest neighbor form the *Voronoi diagram* $\mathcal{V}(P)$ for the set of sites.



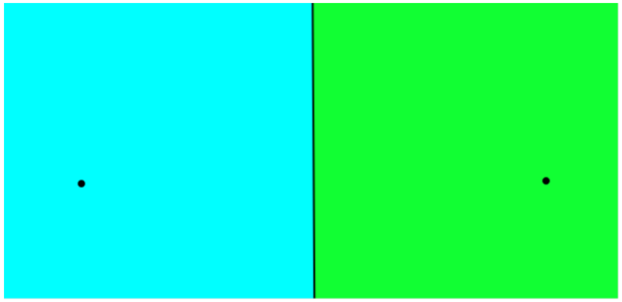
What do we observe ?



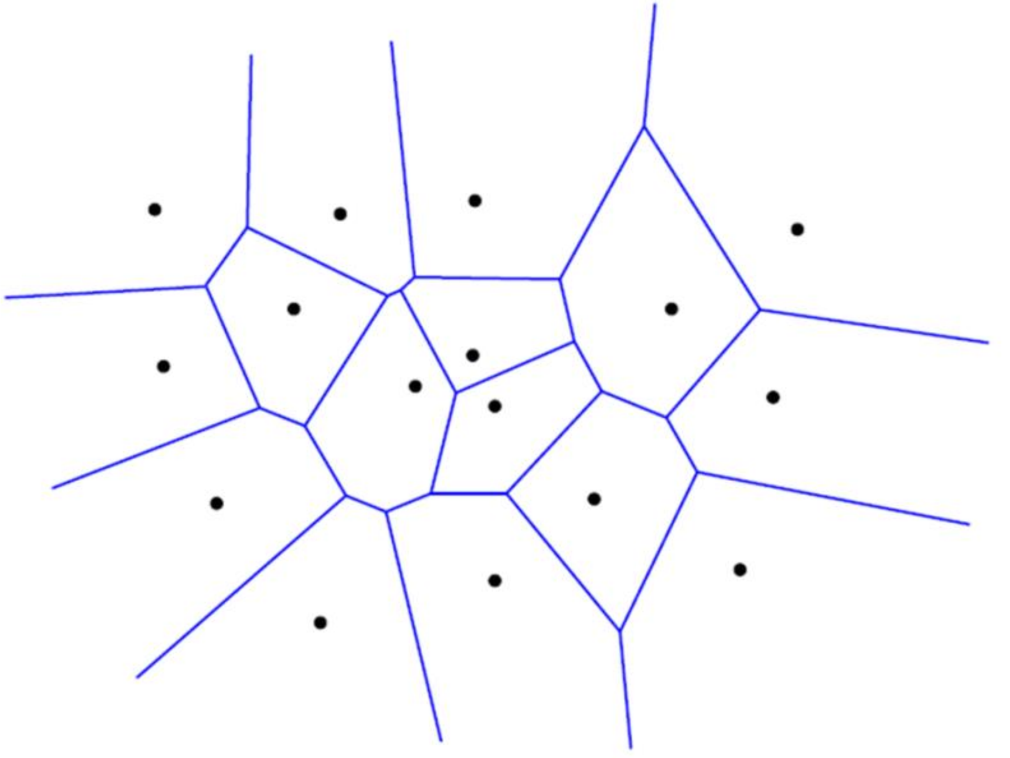
Bisector

- The bisector B_{ij} between the points p_i and p_j play an important role in Voronoi diagram

- V(pi) : all the points that are closer to p_i than to p_j

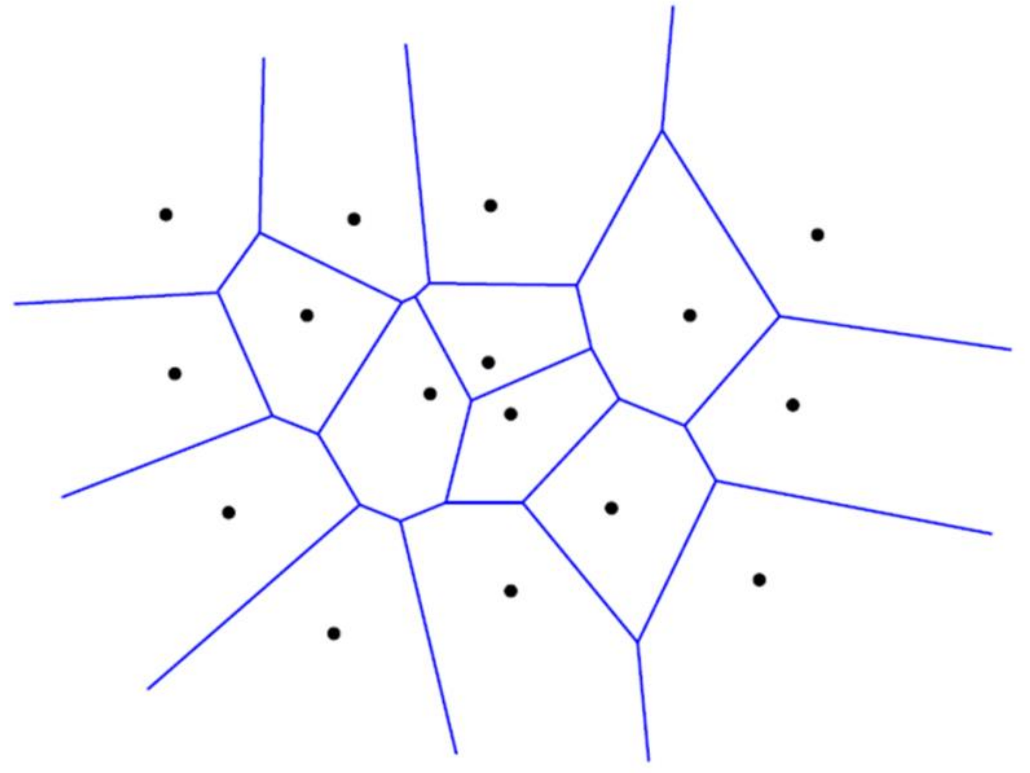


Observations



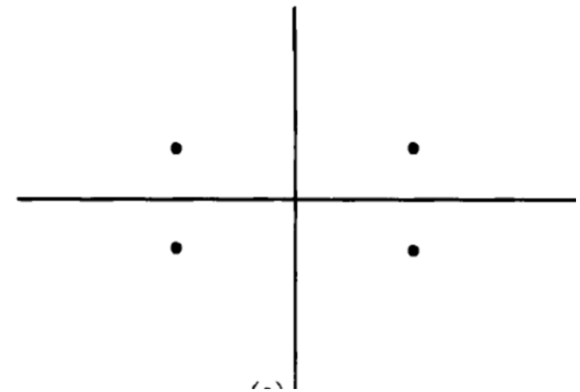
- The Voronoi regions are ………
- The Voronoi regions are convex
- The Voronoi polygons are ………
- The Voronoi polygons are convex
- Because the intersection of any number of half lines is convex

Observations

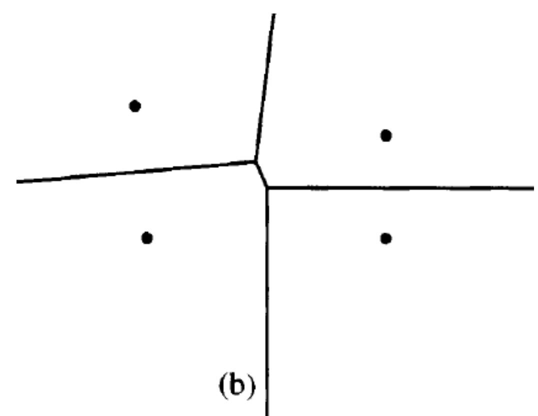


- A point on the interior of a Vononoi edge has ……. nearest points/sites
- A point on the interior of a Vononoi edge has two nearest points/sites
- A voronoi vertex has at most …. nearest sites or at most …. as degree
- A voronoi vertex has at most 3 nearest sites or at most 3 as degree

- Voronoi vertex of a voronoi diagram of four cocircular points has degree as four
- This is a degenerate case



- If we tilt the vertices slightly, we will get a Voronoi diagram



(b)

Applications of Voronoi diagram

Fire Observation Towers

Imagine a vast forest containing a number of fire observation towers. Each ranger is responsible for extinguishing any fire closer to her tower than to any other tower. The set of all trees for which a particular ranger is responsible constitutes the "Voronoi polygon" associated with her tower. The Voronoi diagram maps out the lines between these areas of responsibility: the spots in the forest that are equidistant from two or more towers.

Applications of Voronoi diagram

Image courtesy: http://www.googlemaps/

- Facility Location

Applications of Voronoi diagram

Facility Location

Suppose you would like to locate a new grocery store in an area with several existing, competing grocery stores. Assuming uniform population density, where should the new store be located to optimize its sales? One natural method of satisfying this vague constraint is to locate the new store as far away from the old ones as possible. Even this is a bit vague; more precisely we could choose a location whose distance to the *nearest* store is as large as possible. This is equivalent to locating the new store at the center of the largest empty circle, the largest circle whose interior contains no other stores. The distance to the nearest store is then the radius of this circle.

Applications of Voronoi diagram

Path Planning

Imagine a cluttered environment through which a robot must plan a path. In order to minimize the risk of collision, the robot would like to stay as far away from all obstacles as possible. If we restrict the question to two dimensions, and if the robot is circular, then the robot should remain at all times on the Voronoi diagram of the obstacles. If the obstacles are points (say thin poles), then this is the conventional Voronoi diagram. If the obstacles are polygons or other shapes, then a generalized version of the point Voronoi diagram determines the appropriate path.

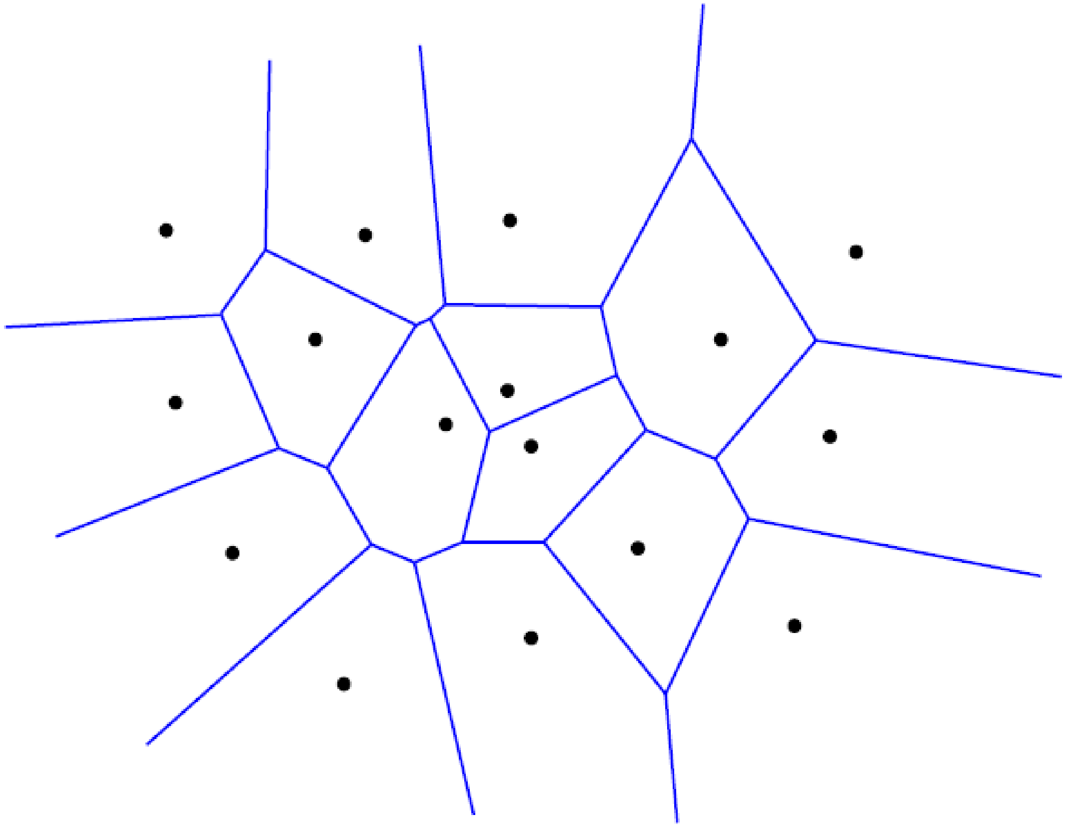
Exercise

- Draw Voronoi diagram of 5 points
- Draw Voronoi diagram of 6 points
- Draw Voronoi diagram of 7 points (of different spacing/ positioning)
- Try different Apps for Voronoi construction where you can add points of your own and understand how the bisectors play a role
- *https://cfbrasz.github.io/Voronoi.html*

VD Properties

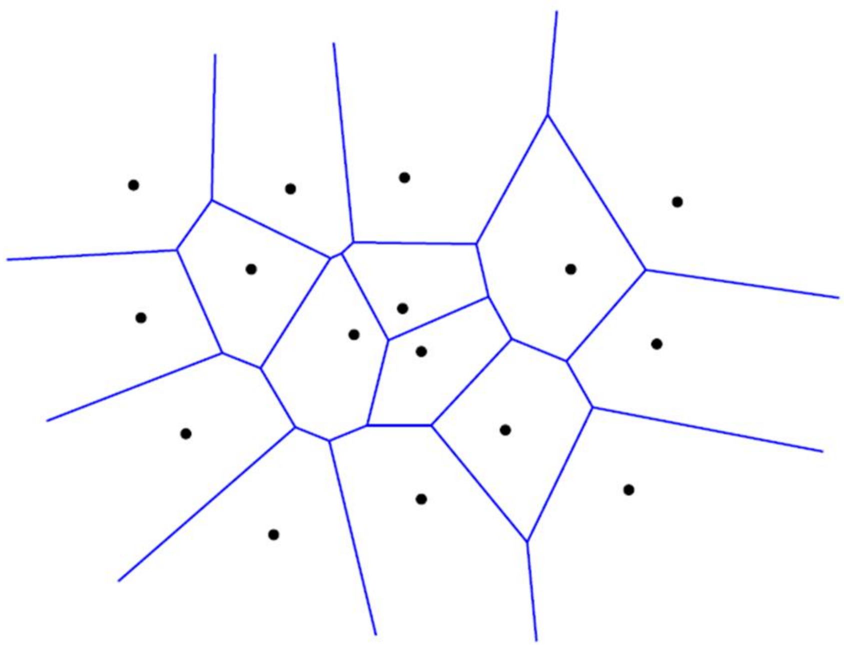
Theorem

- Every vertex of the Voronoi diagram is the common intersection of exactly three edges of the diagram



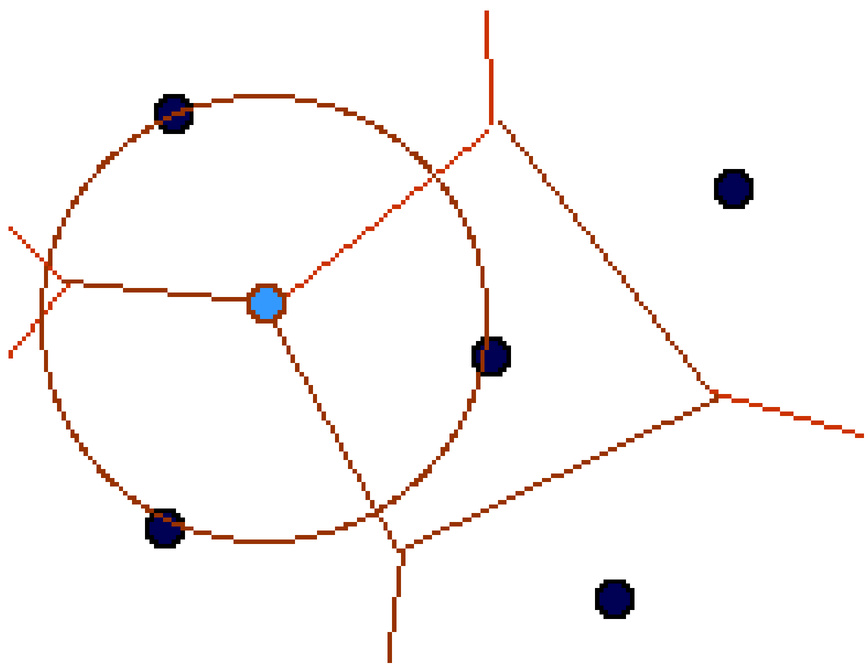
Observation

- Voronoi vertices are centers of circles defined by the three points
- Voronoi diagram is a regular graph of degree three



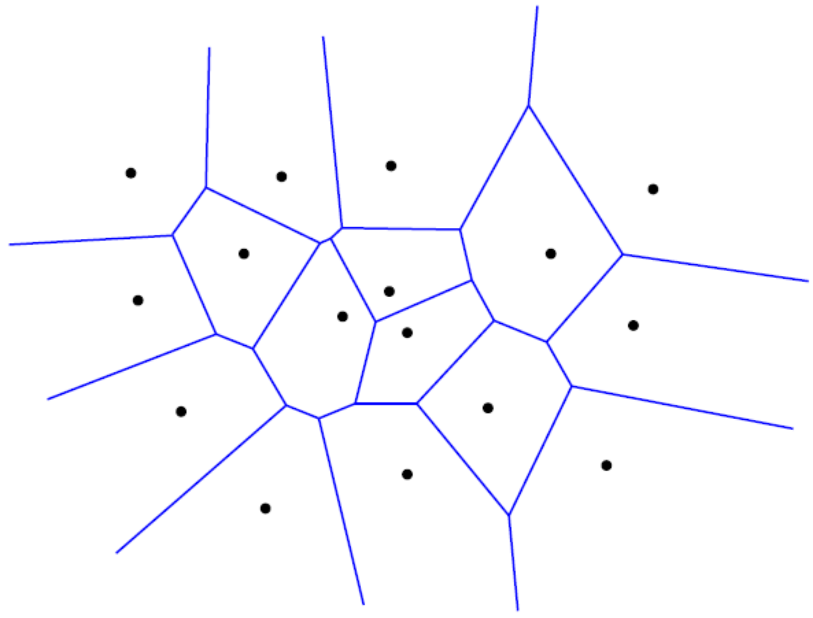
Theorem

- For every vertex v of the Voronoi diagram of S, the circle C(v) contains no other sites of S



Theorem

- Every nearest neighbor of p_i in S defines an edge of the Voronoi polygon V(i)



References

- F.P. Preparata & M.I. Shamos, *Computational Geometry An Introduction,* Springer International Edition, 1985
- J. O Rourke, *Computational Geometry in C,* 2/e, Cambridge University Press, 1998
- *https://cfbrasz.github.io/Voronoi.html* ---- Voronoi Diagram generator

THANK YOU