The Guggenheim Museum in Bilbao (Image courtesy: BBC.com)

What is the minimum number of guards?

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Placing minimum number of guards

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Art Gallery Problem [Victor Klee, 1973]

- Input : Art Gallery
- Output : Minimum number of guards that can safe-guard or cover the interior walls of the gallery

- The Guggenheim Museum in Bilbao: hard to supervise (Image courtesy: BBC.com)

Input representation

- How do we represent the input / art gallery in geometric terms?
- Polygon

Input representation: Polygon
Formal Definition

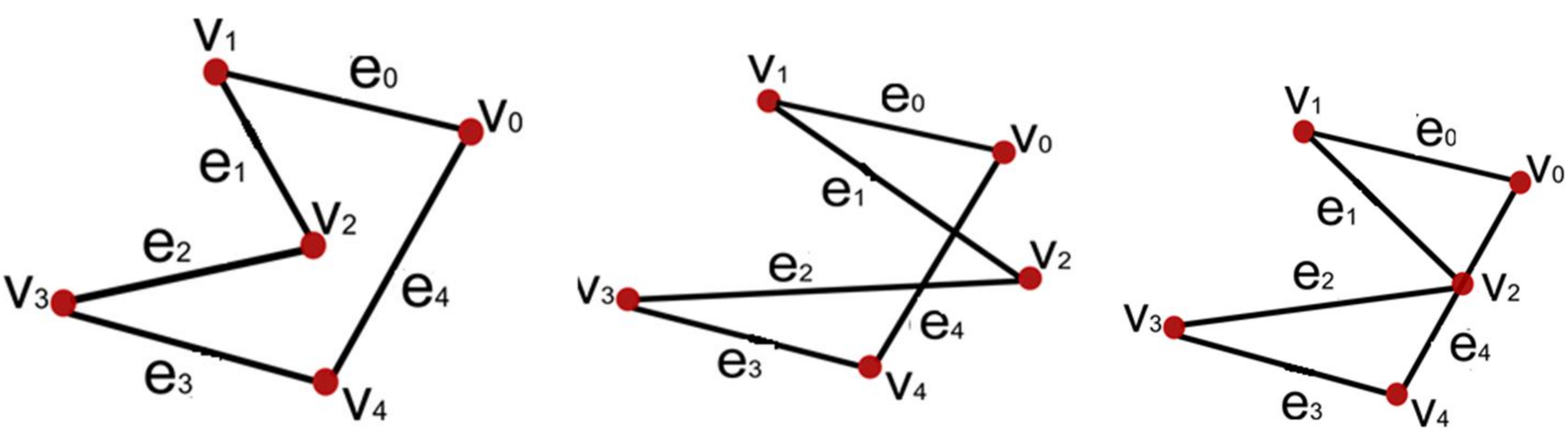
Let $v_0, v_1, v_2, \ldots, v_{n-1}$ be n points in the plane.

Let $e_0 = \overline{v_0 v_1}, e_1 = \overline{v_1 v_2}, \ldots, e_i = \overline{v_i v_{i+1}}, \ldots, e_{n-1} = \overline{v_{n-1} v_0}$

be n segments connecting the points.

Then these segments bound a polygon iff

1. The intersection of each pair of segments adjacent in the cyclic ordering is the single point shared between them: $e_i \cap e_{i+1} = v_{i+1}$, for all $i = 0, \ldots, n-1$.
2. Nonadjacent segments do not intersect: $e_i \cap e_j = \emptyset$, for all $j \neq i + 1$.



Non-simple polygon

- Recall

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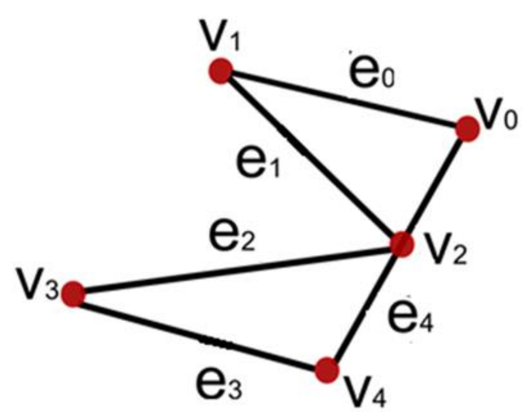
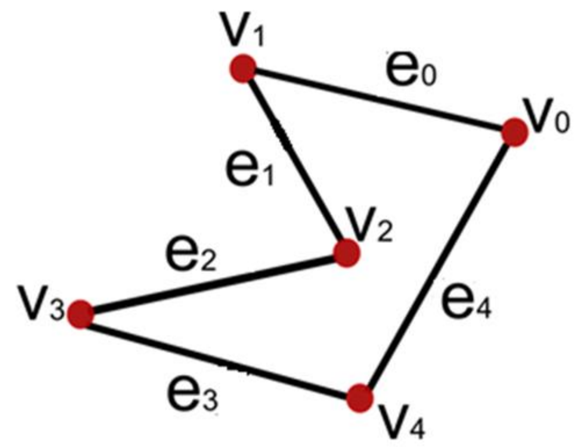
2. Nonadjacent segments do not intersect: $e_i \cap e_j = \emptyset$, for all $j \neq i + 1$.

- e_1 and e_4 intersect
- Condition 2 does not hold



Polygon

- Polygon: region of plane bounded by a finite collection of line segments forming a simple closed curve
- Why is it called a curve?
- Why is it called a closed curve?
- Why is it called a simple closed curve?

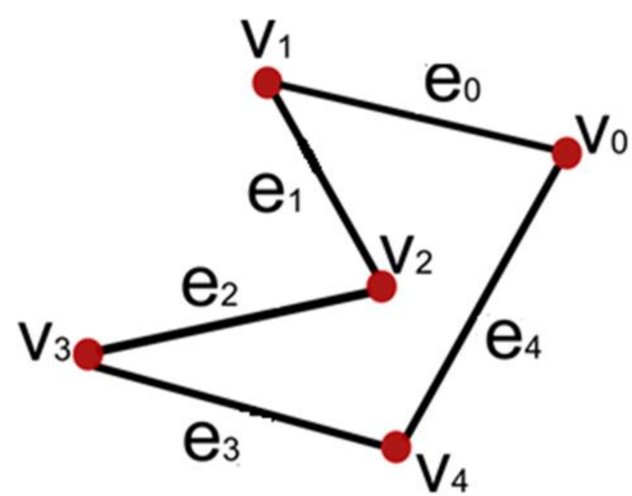


Polygon

- A simple closed curve

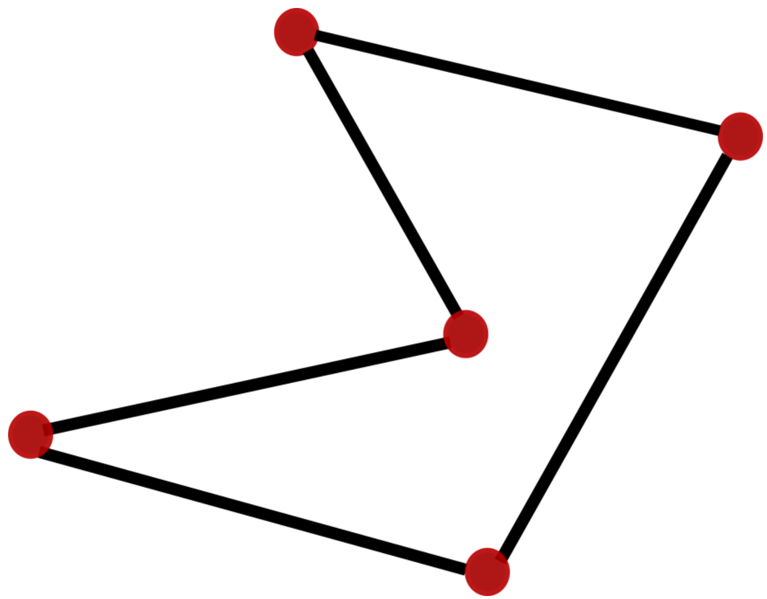
The reason these segments define a *curve* is that they are connected end to end; the reason the curve is *closed* is that they form a cycle; the reason the closed curve is *simple* is that nonadjacent segments do not intersect.

The points v_i are called the *vertices* of the polygon, and the segments e_i are called its *edges*. Note that a polygon of n vertices has n edges.



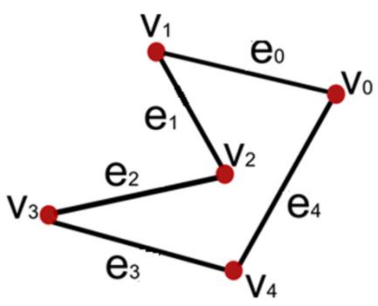
Jordan Curve theorem

Theorem 1.1.1 (Jordan Curve Theorem). *Every simple closed plane curve divides the plane into two components.*



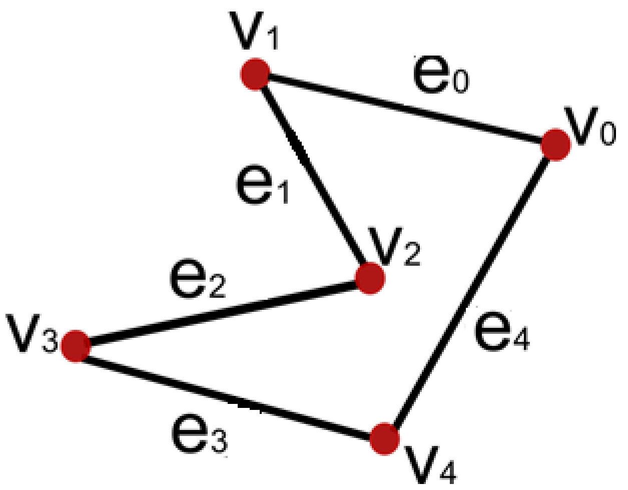
Polygon

- The polygon P divides the plane into two parts:
  - Interior
  - Exterior
  - Exterior is unbounded
  - Interior is bounded
  - Polygon is the region bounded by the line segments including the line segments (boundary)
  - Boundary of P is denoted by ∂P
  - $\partial P \subseteq P$



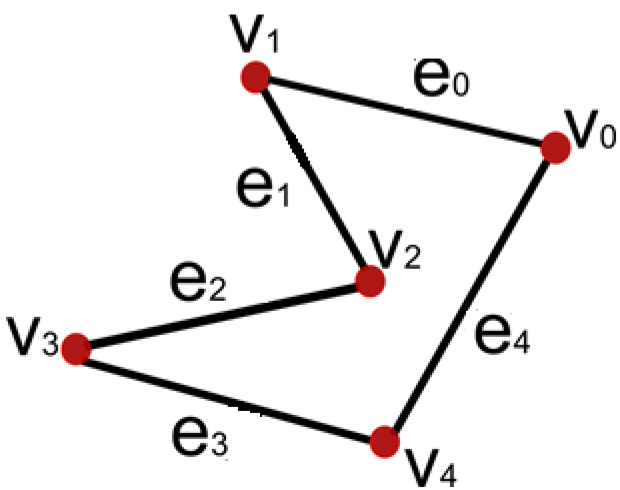
Polygon

- Convention of numbering the vertices of P
  - Counter clockwise order
  - If we walk along the boundary of P, the interior is always to the Left of us



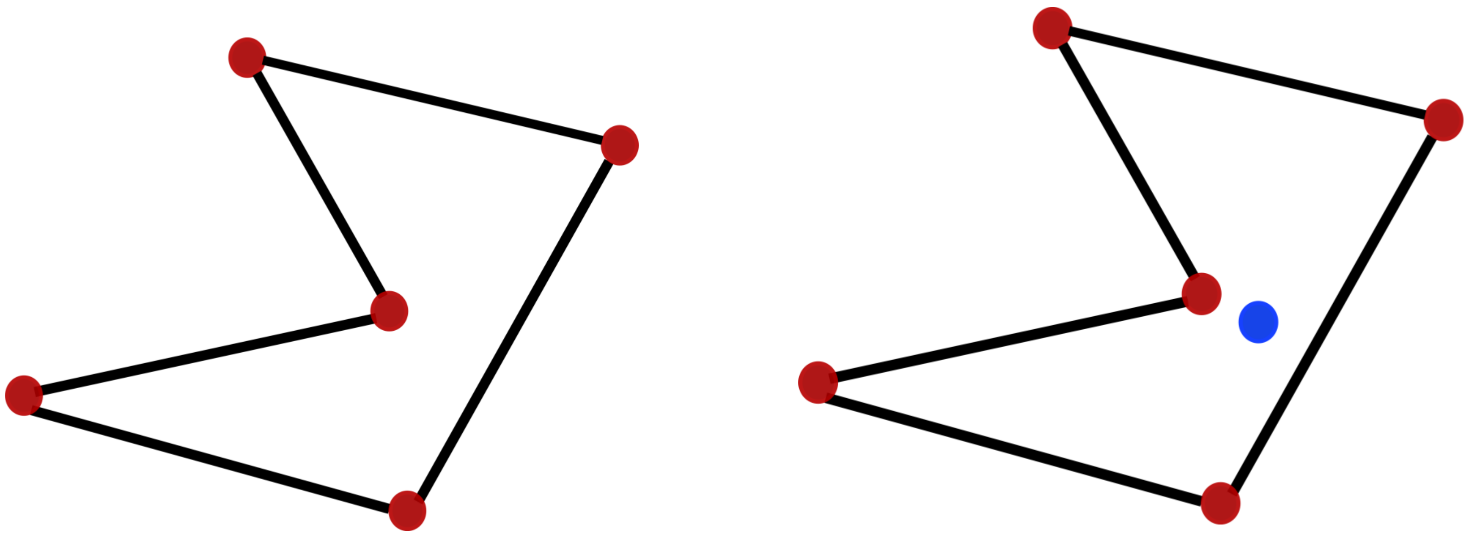
Art Gallery Problem: Geometric problem

- Art gallery: Polygon (*P*)
- Polygon: region of plane bounded by a finite collection of line segments forming a simple closed curve



Placing Guards

- Guard can be placed anywhere interior to the polygon
- Guard can not see through walls
- Guard can turn around 360^o

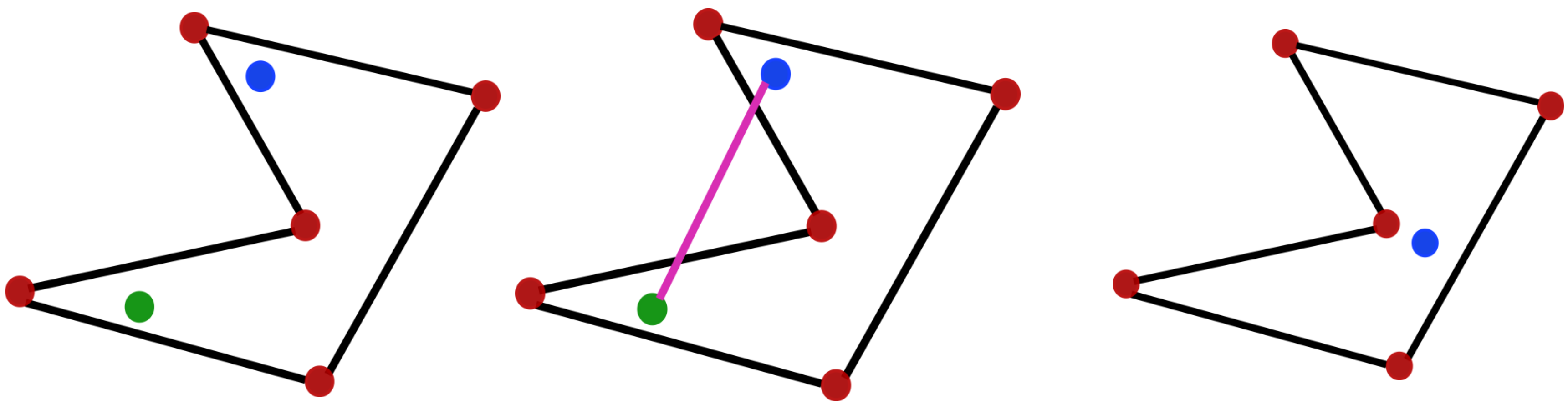


- Guard has 360^o visibility

- Different types of guards : point guard, vertex guard, mobile guard

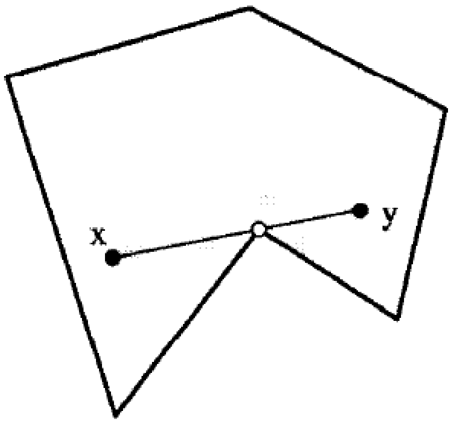
Visibility

- A guard is a point
- Point *x* is visible to point *y* iff the closed segment *xy* is nowhere exterior to the polygon



- Guards themselves do not block the visibility of each other

Visibility



- The vertex is the Grazing contact of line xy
- Definition of visibility which we already know: Point *x* is visible to point *y* iff the closed segment *xy* is nowhere exterior to the polygon
- According to the above definition: *x* is visible to *y*, even though there is a grazing contact between them

Definition of clear visibility

- A vertex can block vision in the case of clear visibility

x has *clear visibility* to y if $xy \subseteq P$ and $xy \cap \partial P \subseteq \{x, y\}$.

Visibility : Covering a polygon

- A set of guards cover a polygon if every point in the polygon is visible to some guard
- What we have to do is: Given a simple Polygon *P* with *n* vertices, compute the minimum number of guards which cover *P*

Reference

- J. O Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998

Thank you