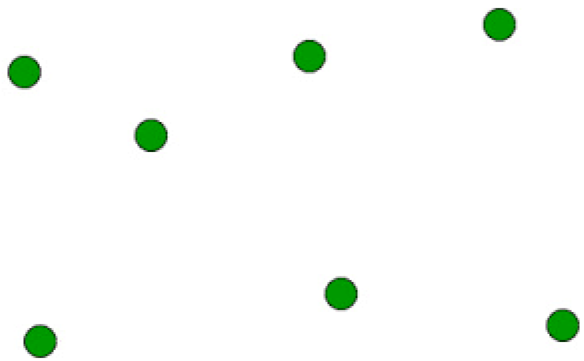
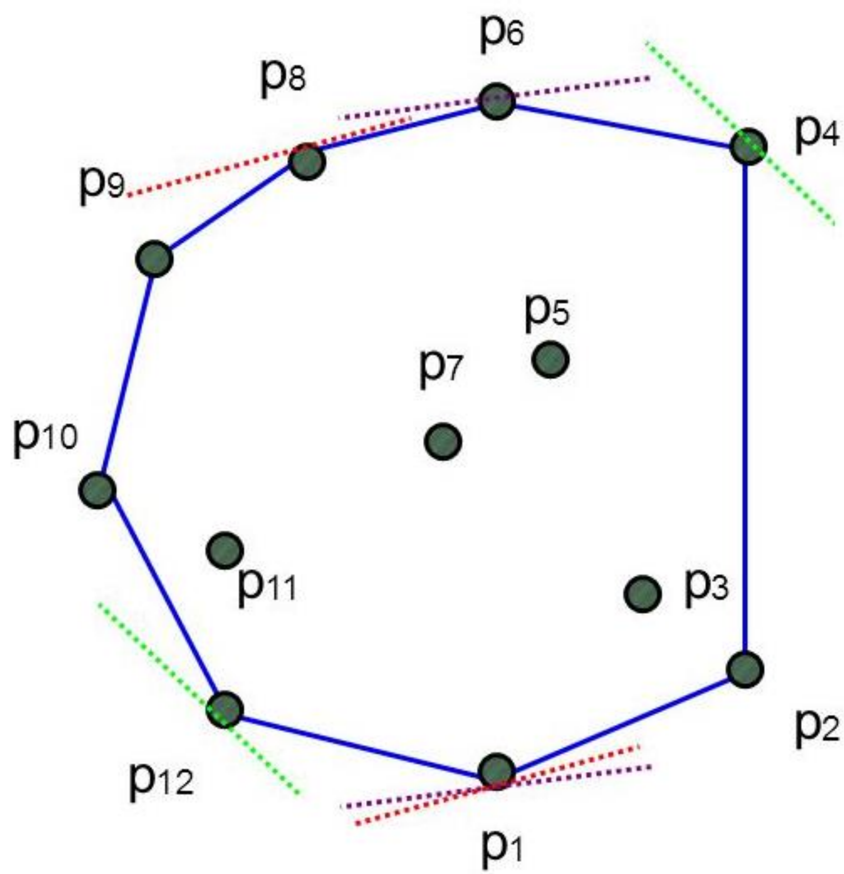
Proximity : Fundamental concepts and algorithms

Diameter of a point set

- **Given n points in a plane, find two that are the farthest apart.**



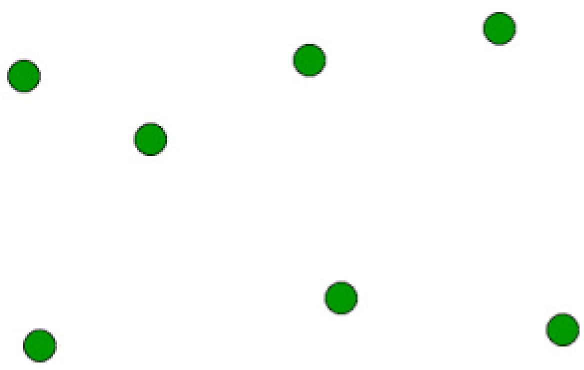
- **Set diameter problem**



p6
p8
p4
p9
p5
p7
p10
p11
p3
p2
p12
p1

Proximity

- **How do we find two closest points?**

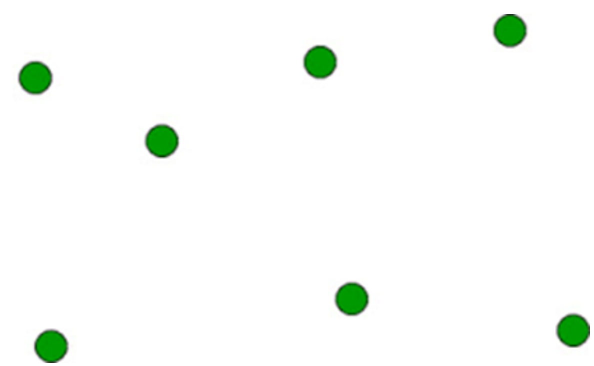


Voronoi Diagram

- The closest pair can be solved by:
- Using Voronoi diagram / The Locus approach

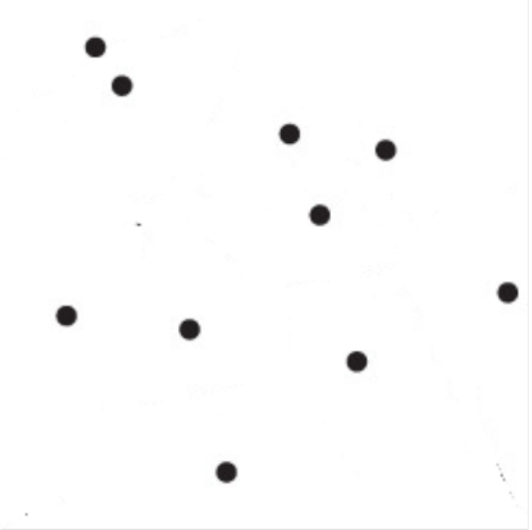
Locus / Loci [*Wikipedia*]

- In geometry, a **locus** (*Latin word for place or location)*
- is a set of all points (commonly, a line, a line segment, a curve or a surface),
- whose location satisfies or is determined by one or more specified conditions

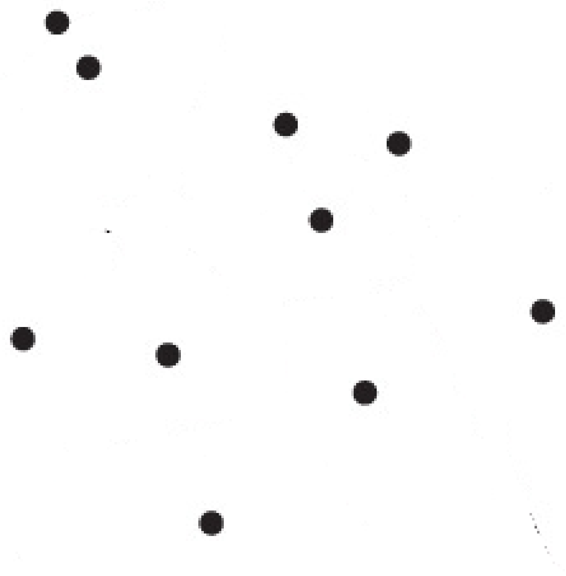


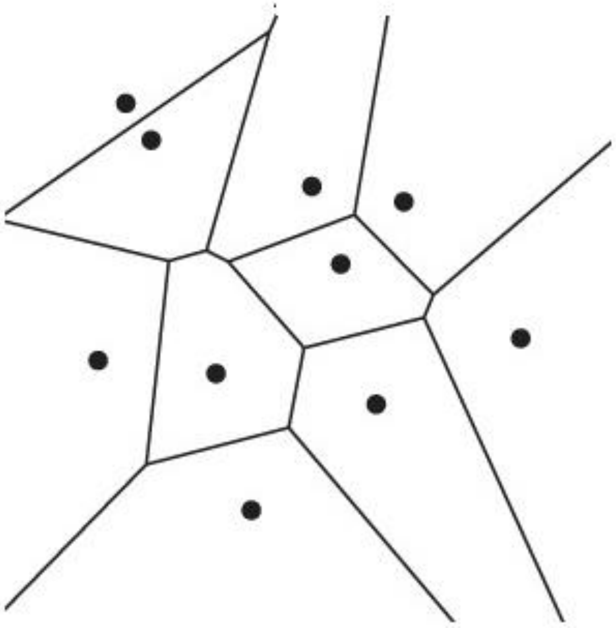
Loci of proximity

- Given a set S of n sites/points in the plane, for each site / point p_i in S what is the locus of points (x,y) in the plane (consider all the infinitely many points in the plane) that are closer to p_i than to any other site/point of S ?



Loci of proximity





- Partition of the plane into regions
- Each region being the locus of the points (x,y) closer to a site of S than to any other site of S

Analyze the structure of the partition

- Only one site in the plane

- No need to partition the plane, as we don't have another point to compare with

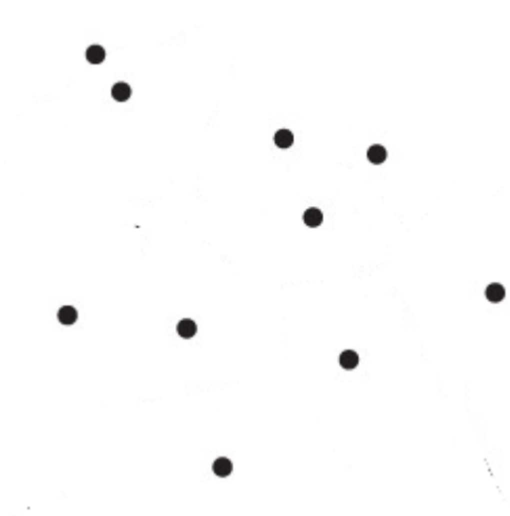
Analyze the structure of the partition

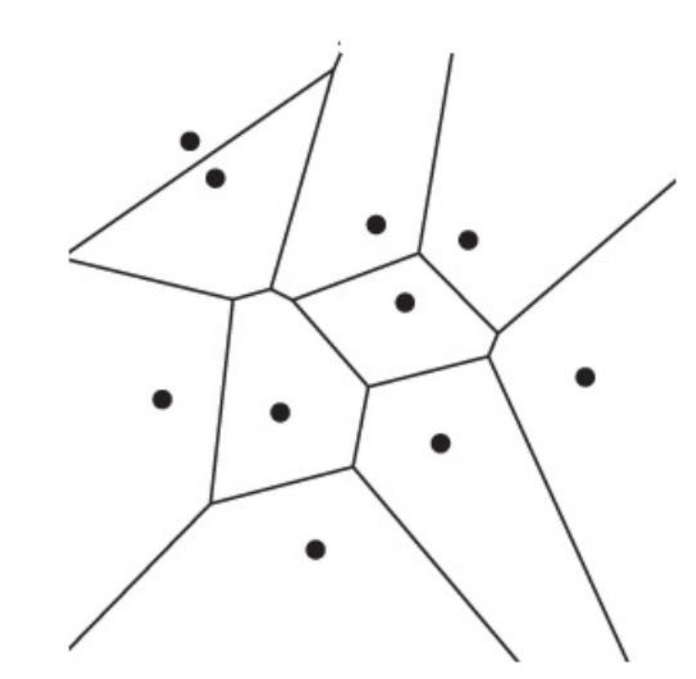
- Two sites in the plane p_i and p_j

- How do we partition the points in such a way that we get a region that is closer to p_i than to p_j ?
- How do we partition the points in such a way that we get a region that is closer to p_j than to p_i ?

Voronoi Diagram

- Voronoi diagram is used to divide a plane with sites/points into separate regions.





- At any point within the diagram, you are closer to the site they contain than any other site, and, at any point along their boundaries, you are equidistant to at least two sites.

Voronoi Diagram

- Known after **Georgy Voronoy** [1868-1908]
- Russian mathematician
- Voronoy is known for
  - Voronoi Diagram
  - Voronoi Iteration
  - Voronoi Formula

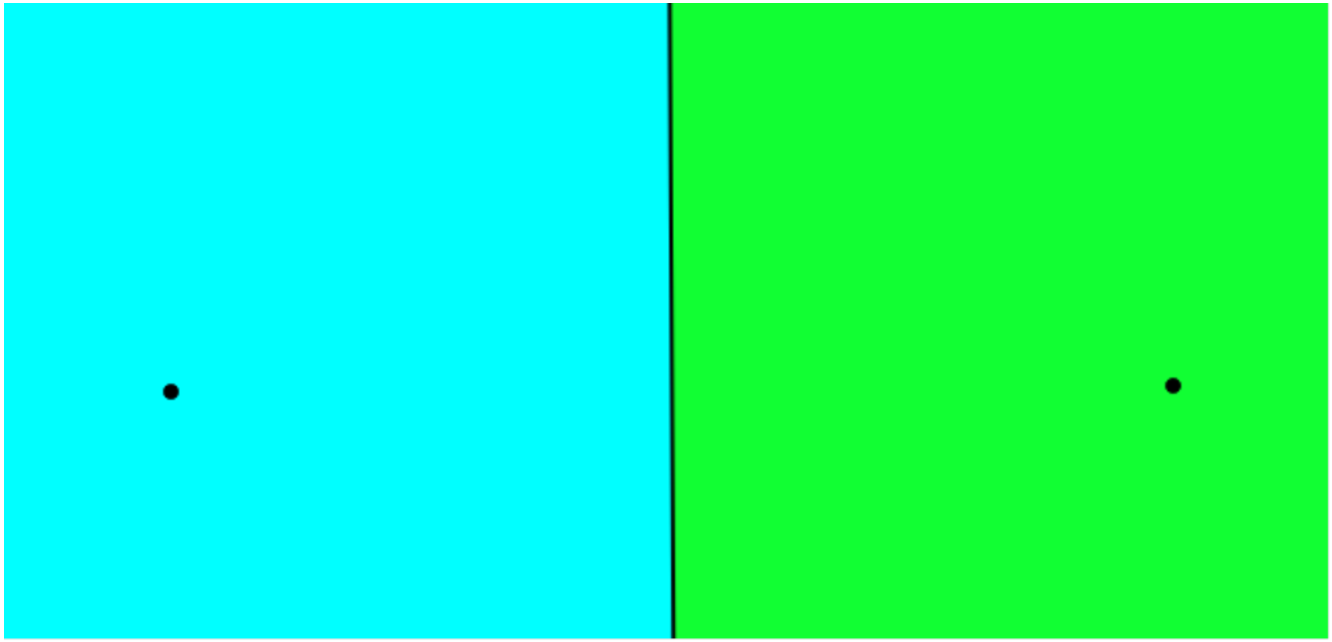
Given the input point set. How to draw Voronoi Diagram?

Draw Voronoi Diagram of two points

Requirement:

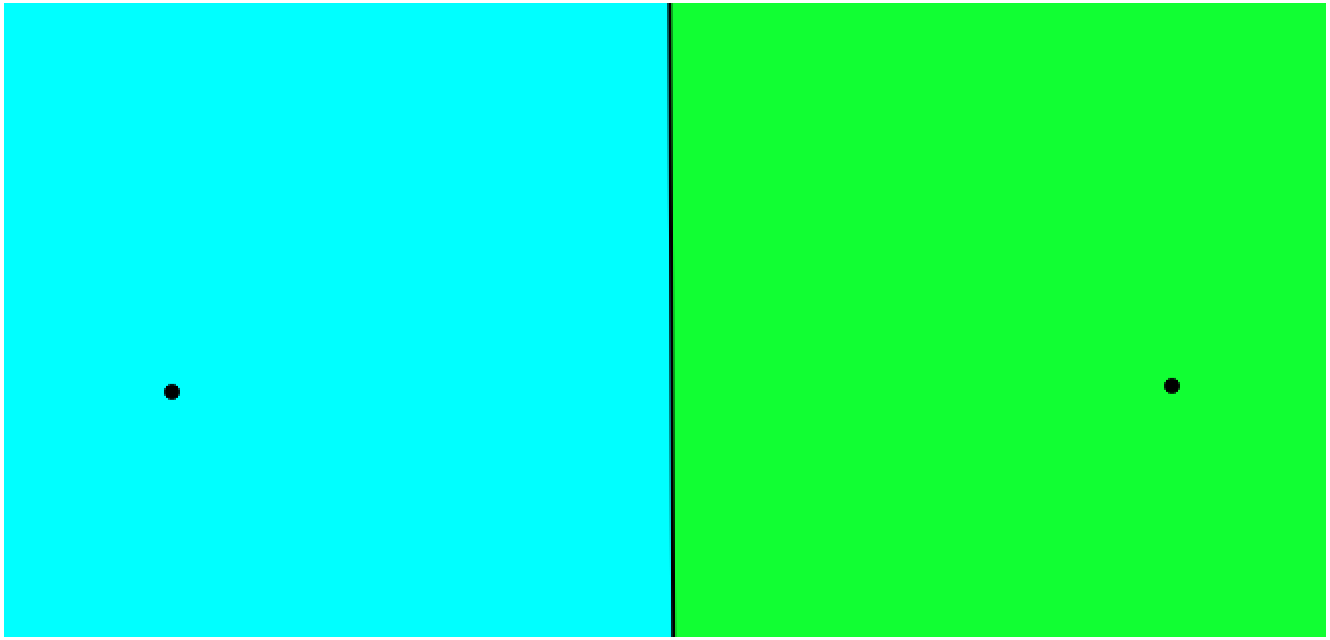
- Two points in the plane p_i and p_j
- We have to partition the points in such a way that we get a region that is closer to p_i than to p_j
- We have to partition the points in such a way that we get a region that is closer to p_j than to p_i

Voronoi diagram of two points in the plane

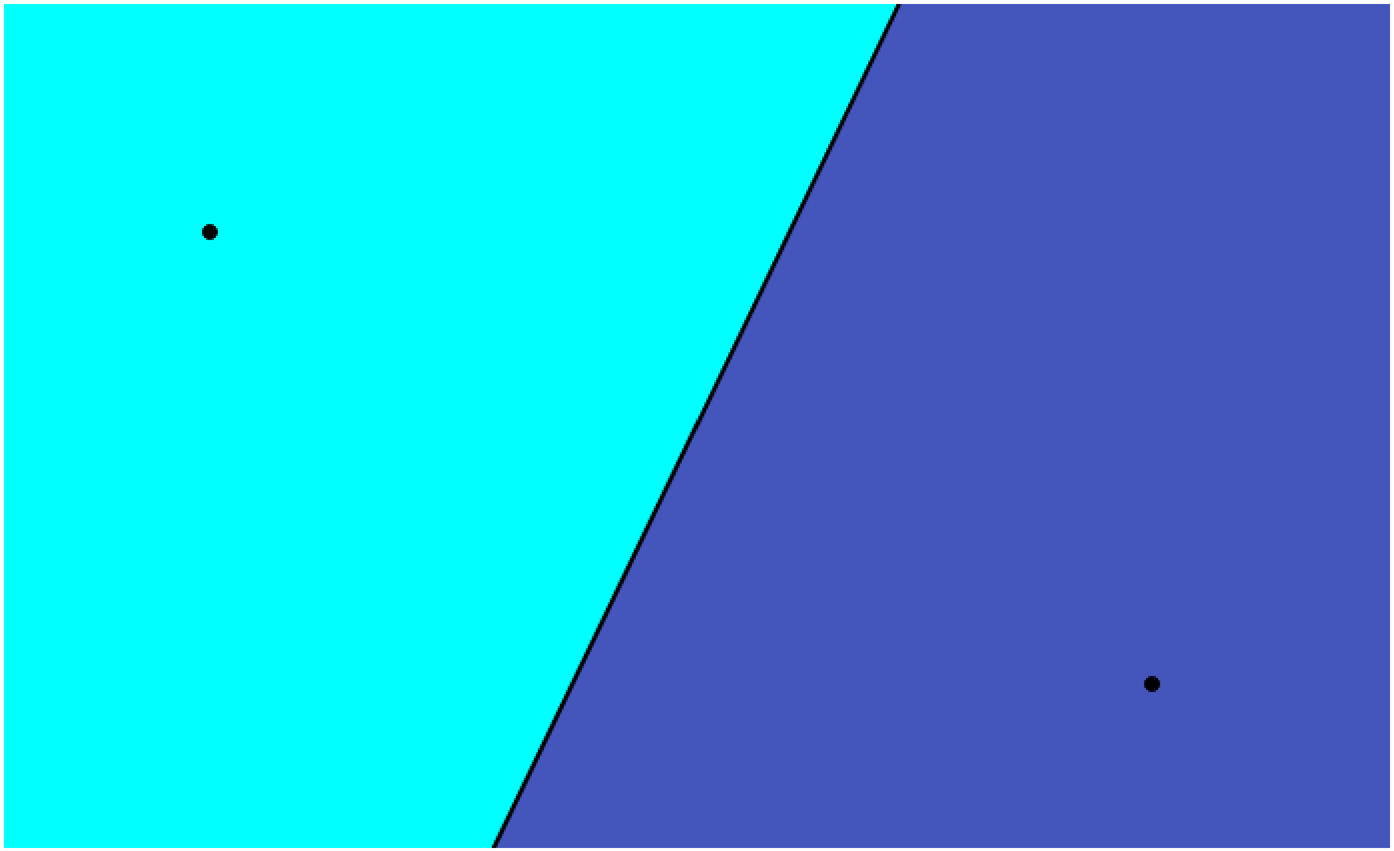


- What did we do here?

Voronoi diagram of two points in the plane



Another instance



Equidistant points on the bisector

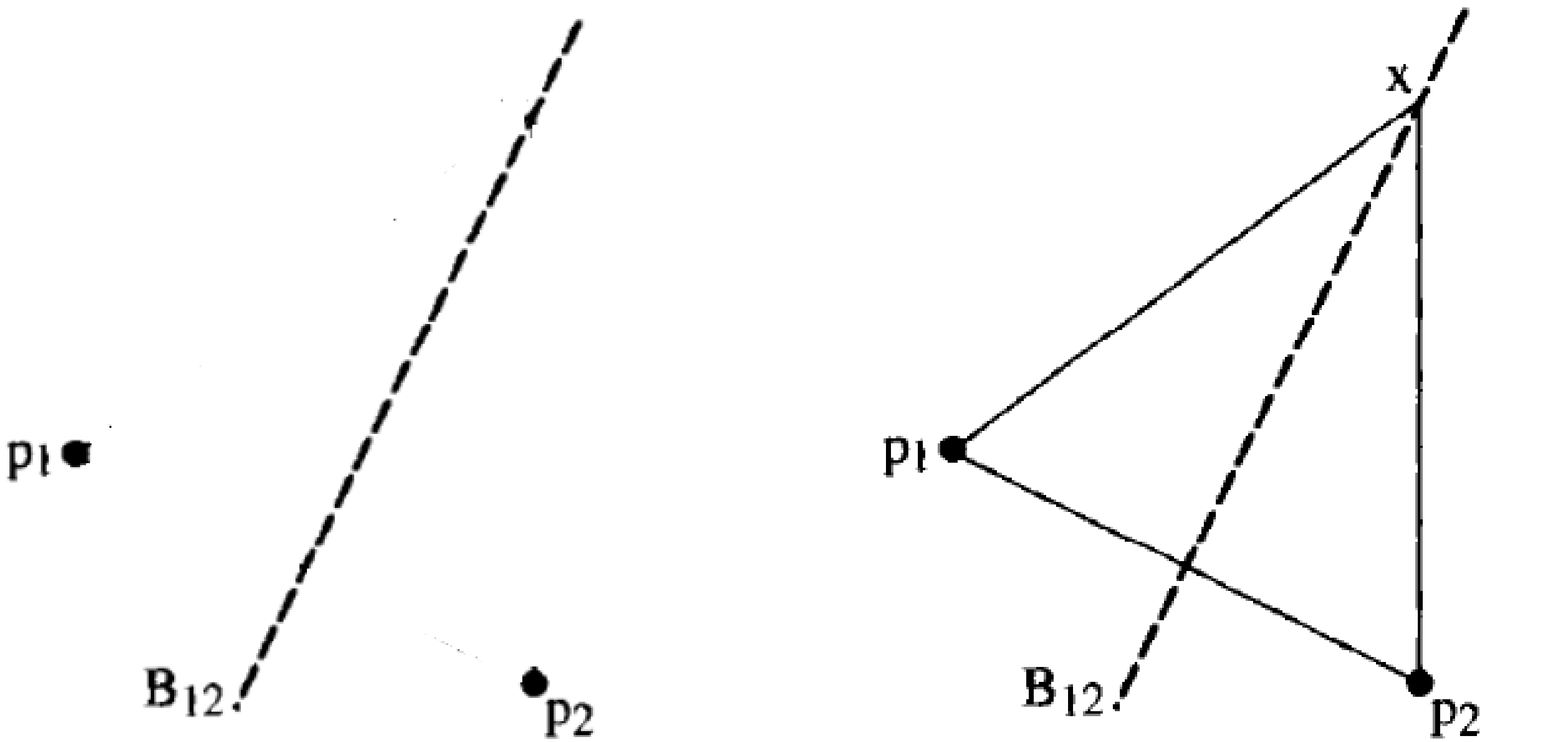
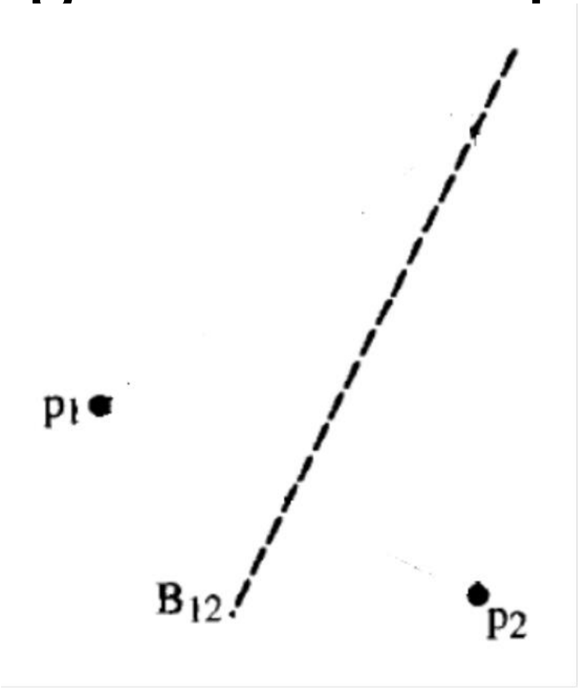


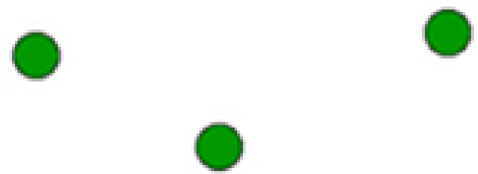
FIGURE 5.2 Two sites: $|p_1 x| = |p_2 x|$.

Voronoi diagram of two points in the plane



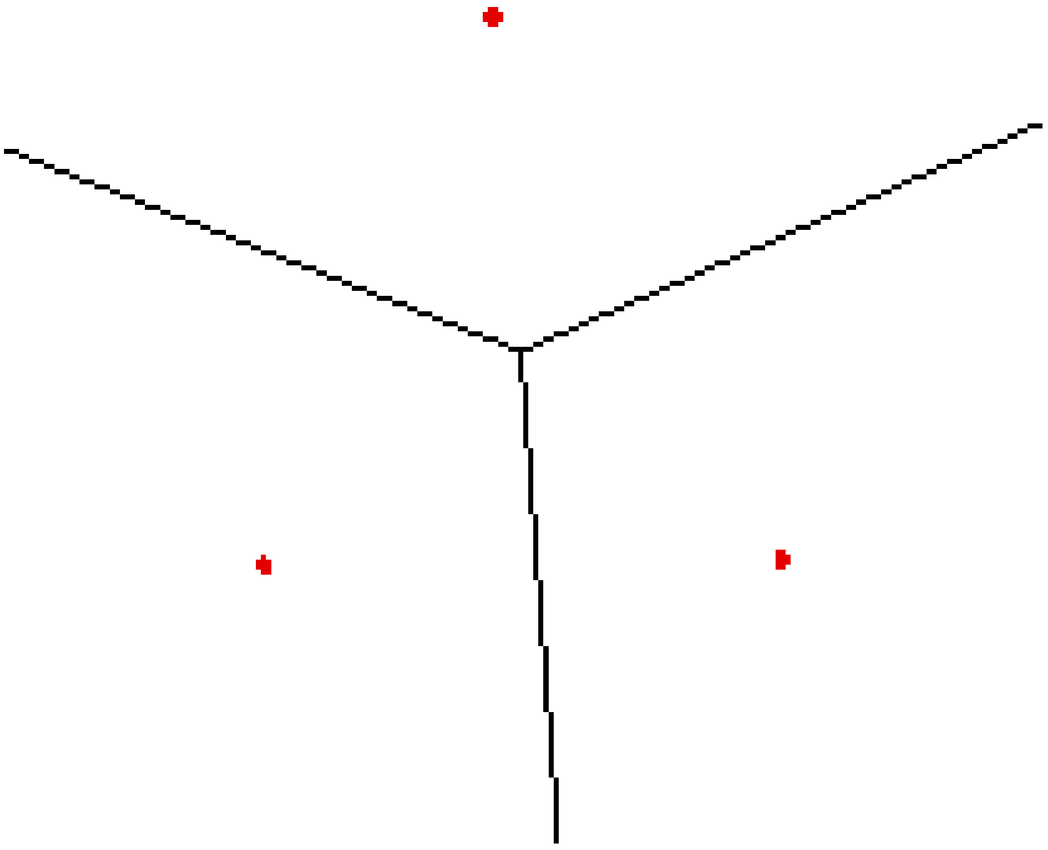
- The set of points closer to p_i is the half line containing p_i that is defined by the perpendicular bisector of the line segment p_i p_j
- The set of points closer to p_j is the half line containing p_j that is defined by the perpendicular bisector of the line segment p_i p_j

Voronoi diagram of three points in the plane

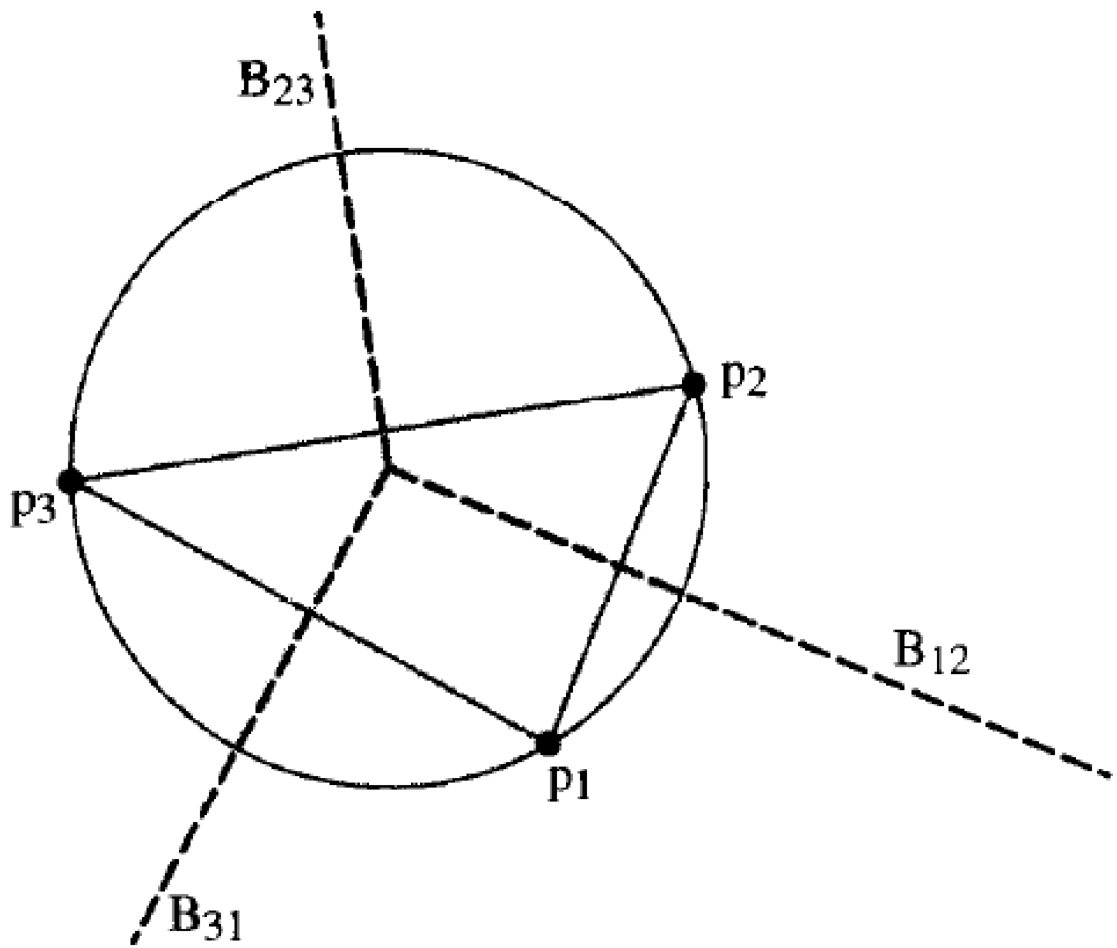


- Draw half line between two points p_i, p_j
- Draw half line between the other two points p_j, p_k
- Draw half line between the other two points p_i, p_k
- Whether the whole half lines are needed?

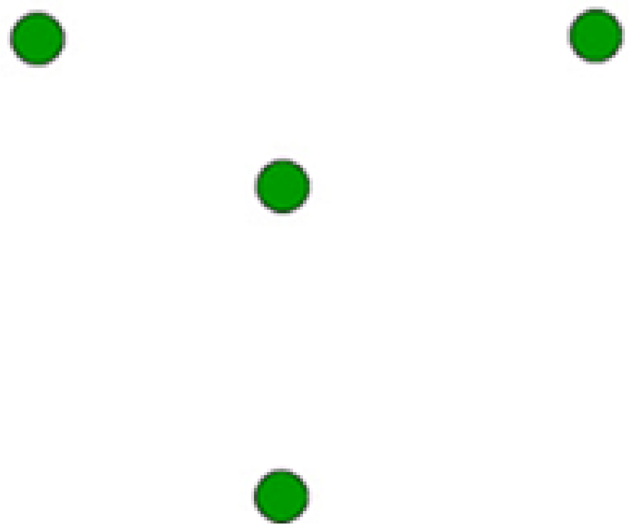
Voronoi diagram of three points



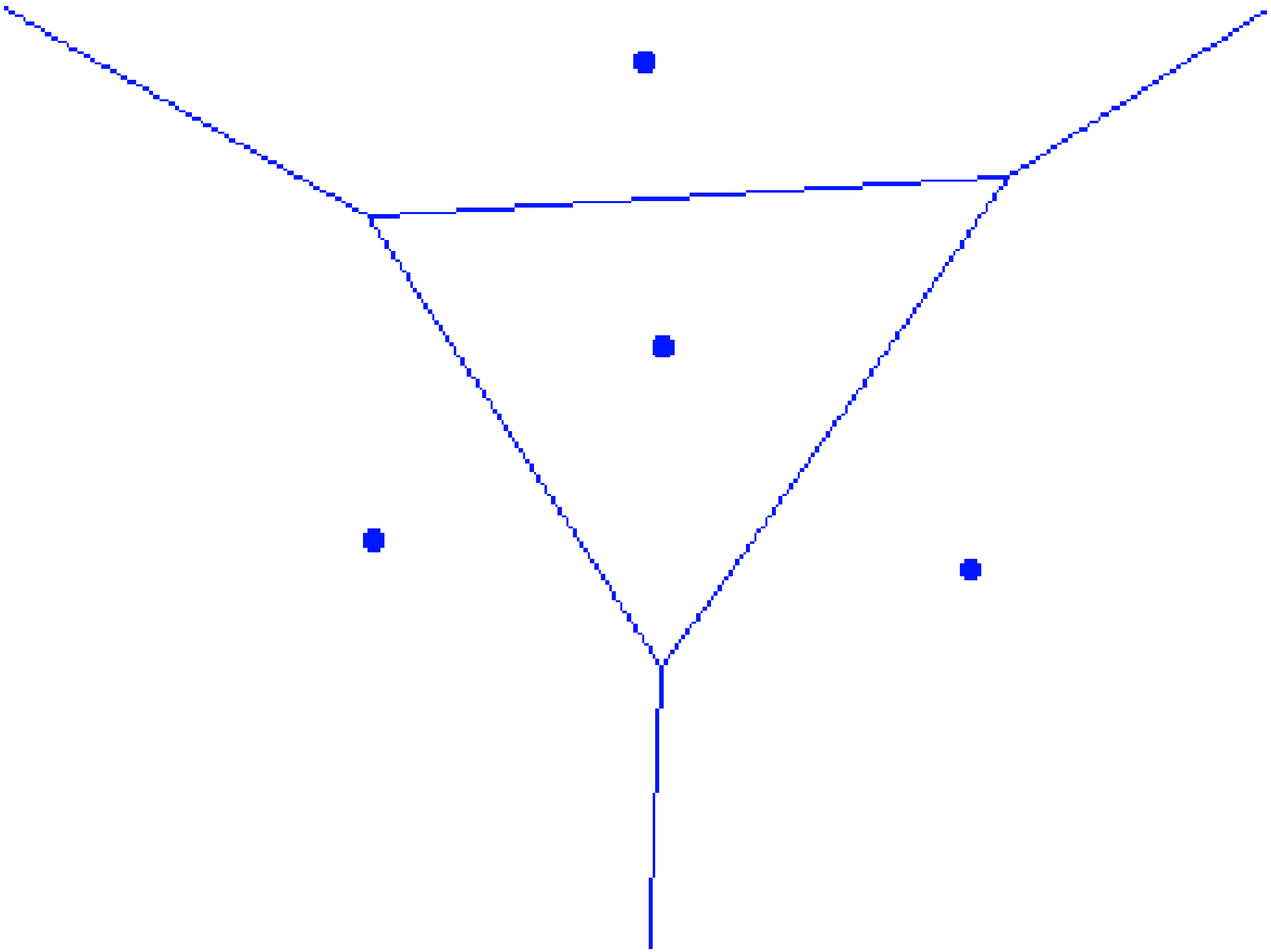
3 points : The circumcenter of Δ



Voronoi diagram of 4 points

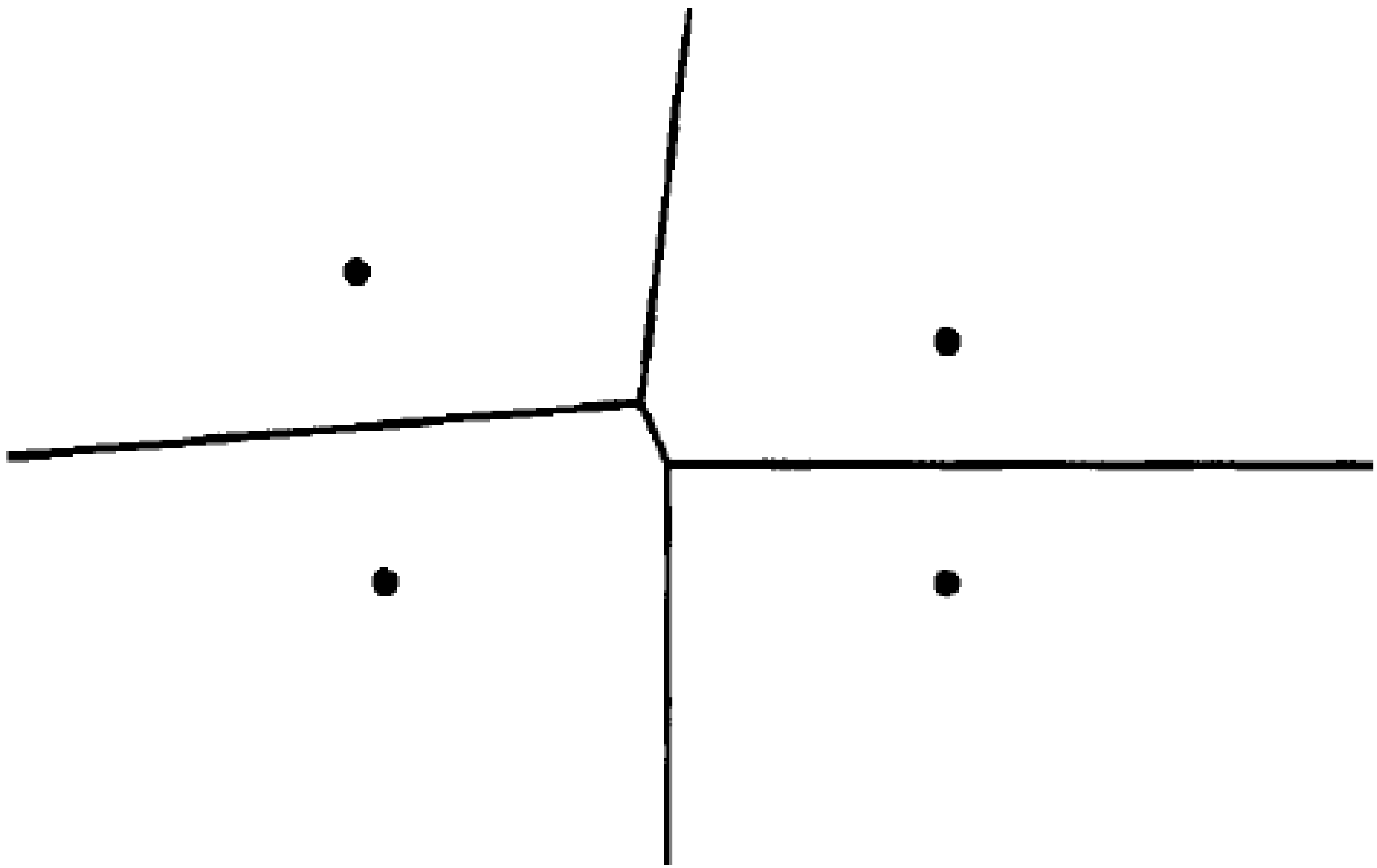


Voronoi diagram of 4 points



Four points

Four points



Summary – Voronoi Diagram

- Voronoi diagram is used to divide a plane with points into separate regions.
- At any point within the diagram, you are closer to the site they contain than any other site, and, at any point along their boundaries, you are equidistant to at least two sites.
- *https://cfbrasz.github.io/Voronoi.html*

References

- F.P. Preparata & M.I. Shamos, *Computational Geometry An Introduction,* Springer International Edition, 1985
- J. O Rourke, *Computational Geometry in C,* 2/e, Cambridge University Press, 1998
- *https://cfbrasz.github.io/Voronoi.html* ---- Voronoi Diagram generator

THANK YOU