Other ways of Polygon Partitioning

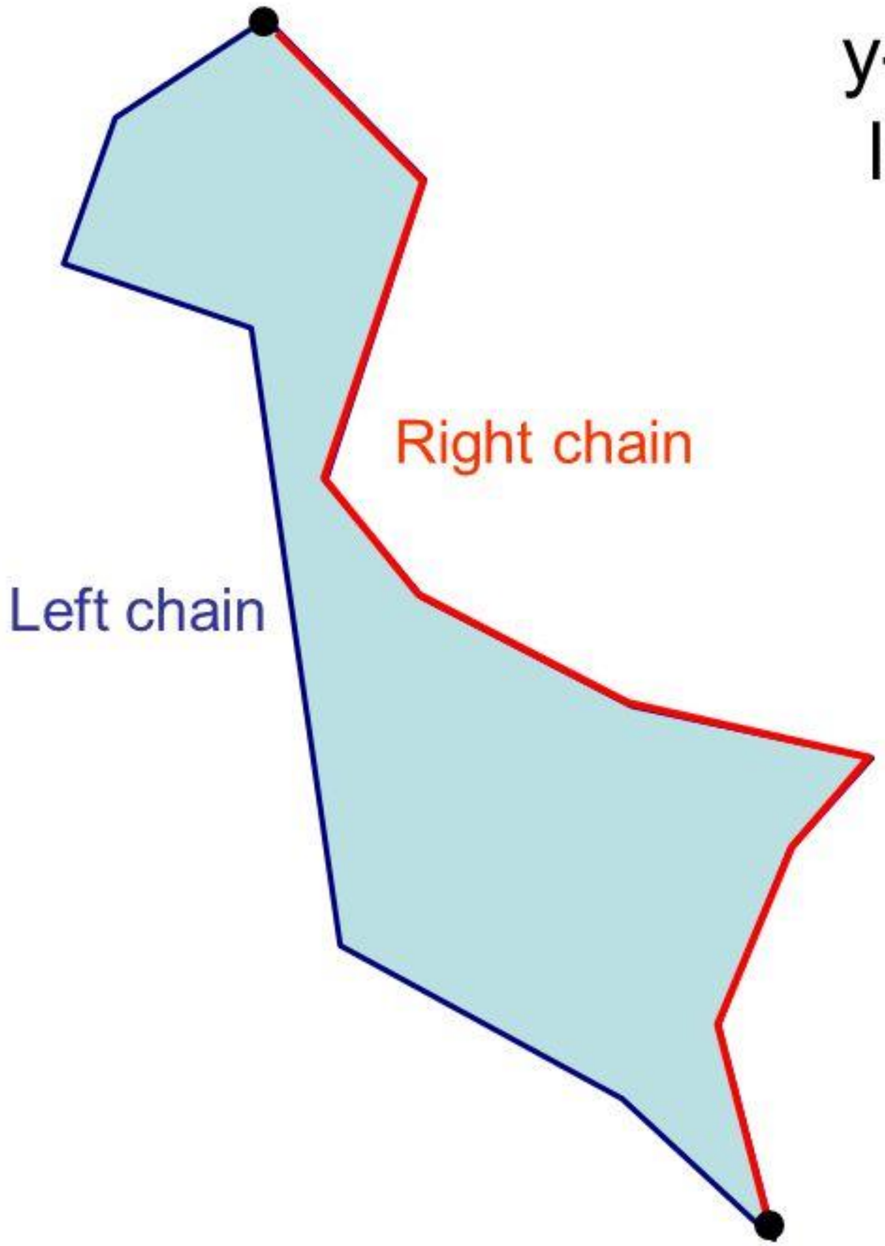
Polygon Partitioning

Partitioning the Polygon to Monotone polygons

A monotone polygon

- A polygon P is said to be monotone with respect to line L if ∂P can be split in to two polygonal chains such that each chain is monotone with respect to L
- The two chains share a vertex at either end

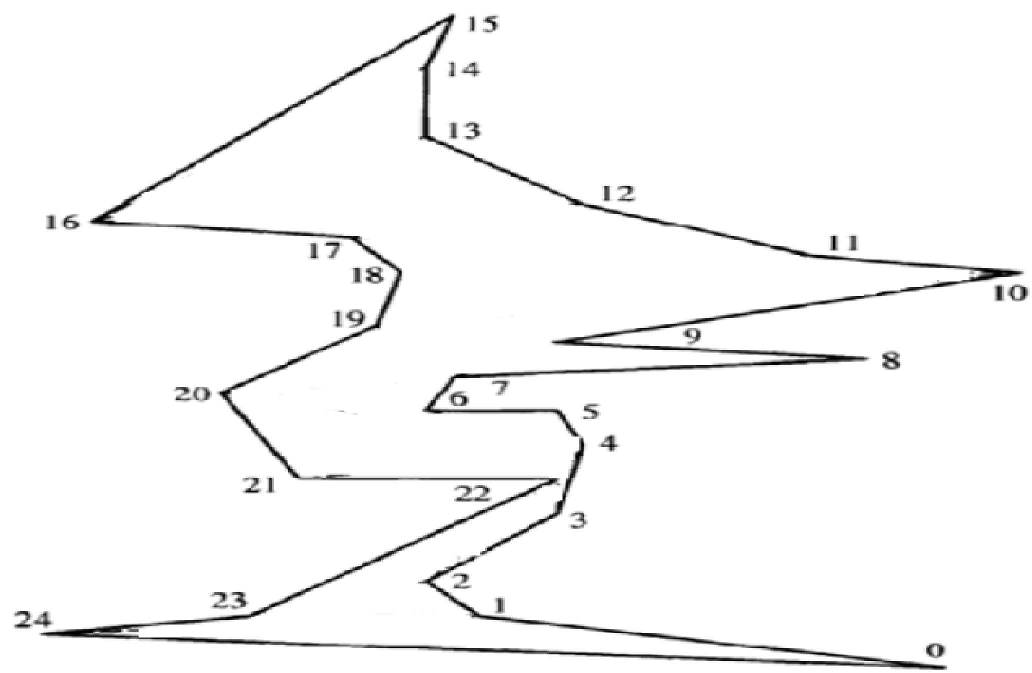
y-monotone polygon: left and right chains



We will also assume that the polygon is strictly y-monotone, i.e. it is y-monotone and has no horizontal edges. Additionally, you may assume that no two vertices have the same y-coordinate

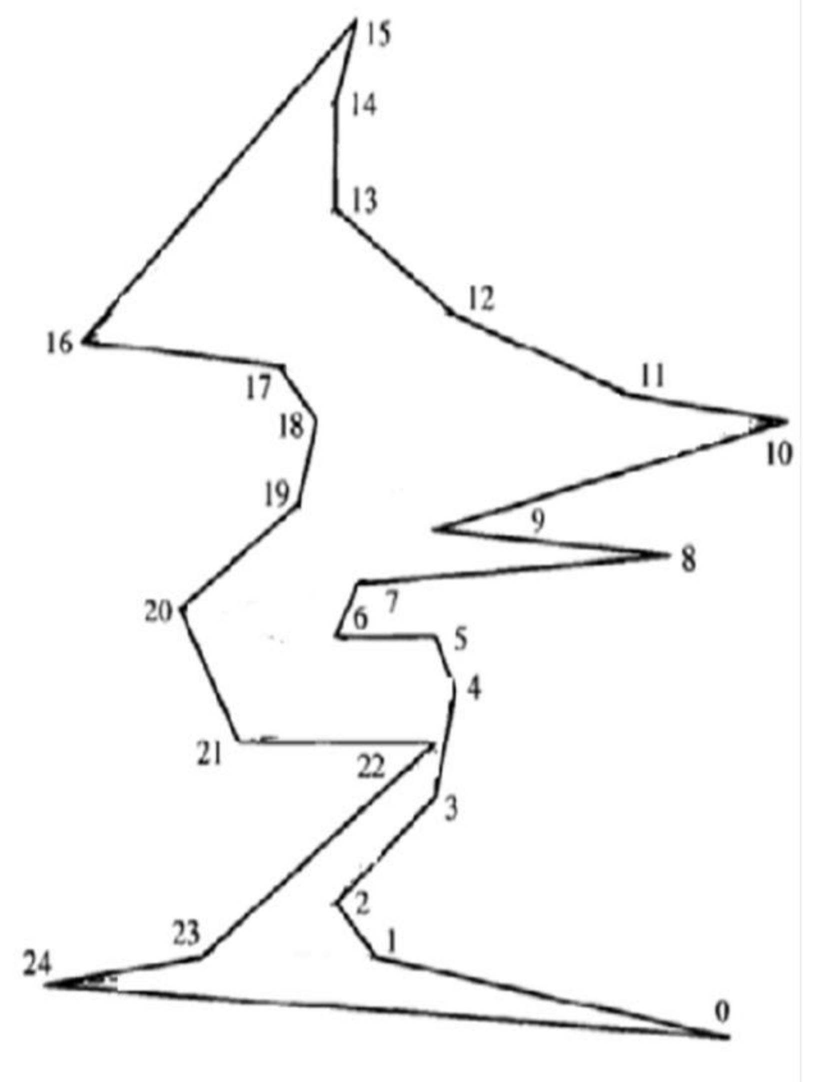
The monotone chains

- $A = (v_0, v_1, \ldots., v_{15})$
- $B = (v_{15}, v_{16}, \ldots., v_{24}, v_0)$



Sort the monotone polygon

- Find the highest vertex
- Find the lowest vertex
- Partition the boundary in to two chains
- The vertices in each chain are sorted with respect to y
- Two sorted list of vertices can be merged in linear time to produce one list sorted by y coordinate



Local characteristic of monotonicity

- P is monotone if it is monotone locally w.r.t. each vertex except highest & lowest
- For example take v_{11}
- v_{11} is monotone locally
- In other words, v_{11} has one adjacent vertex with higher y value and other adjacent vertex with smaller y value than v_{11}

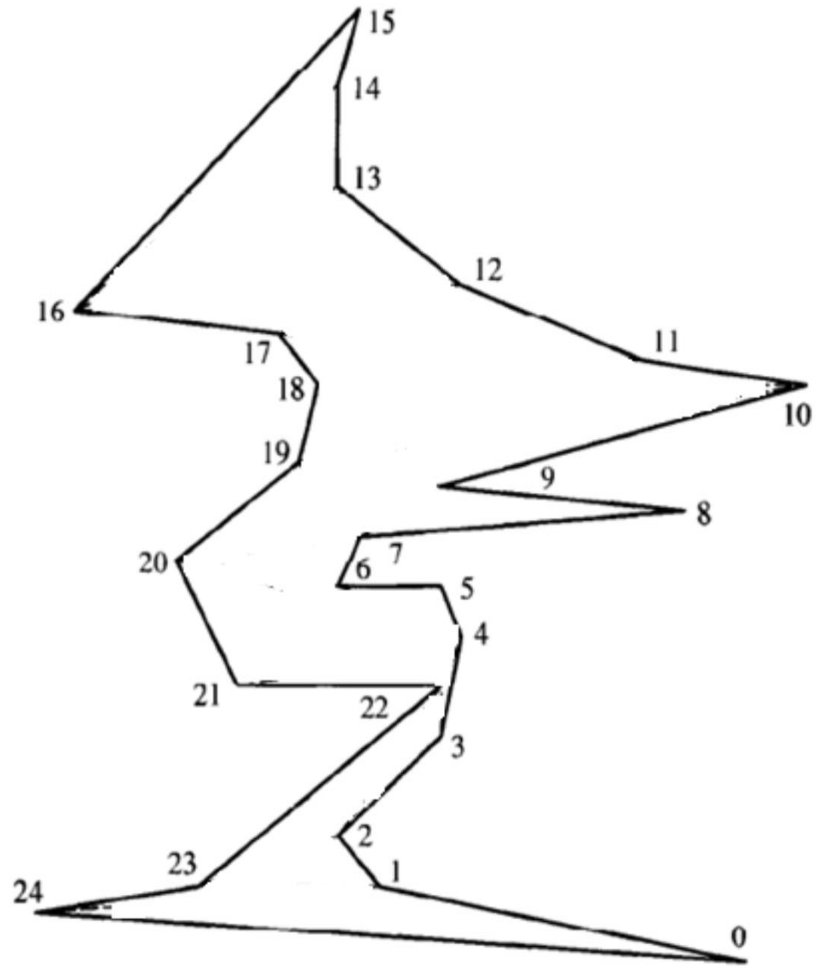
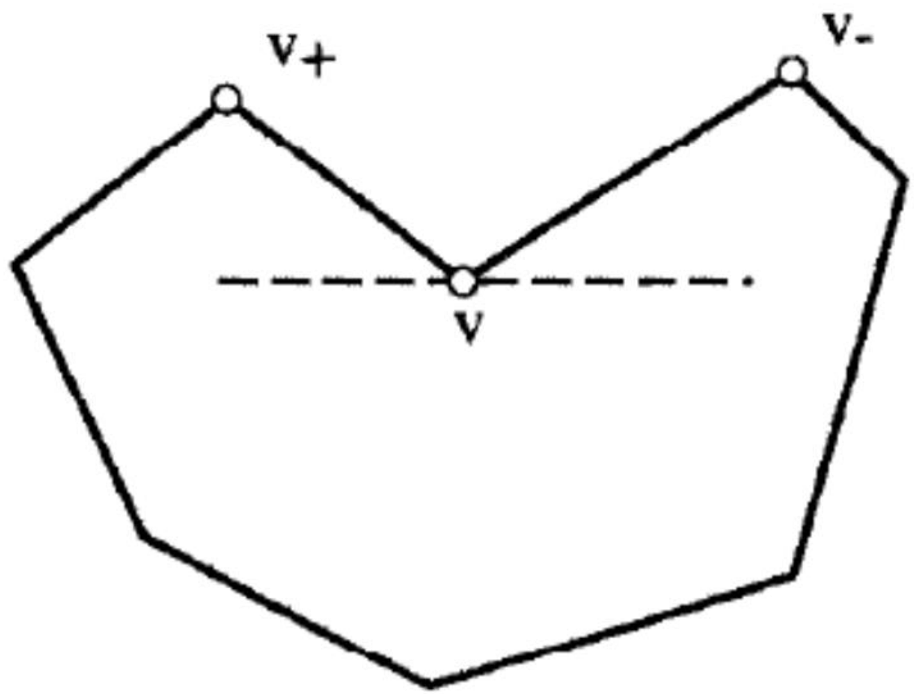


FIGURE 2.1 A polygon monotone with respect to the vertical.

What breaks monotonicity?

- What is the local characteristic of
  non- monotonicity ?
- If there is a reflex vertex (v) whose both the adjacent vertices are above v or both the adjacent vertices are below v, then there is a non-monotonicity locally
- Interior cusp

Interior cusps



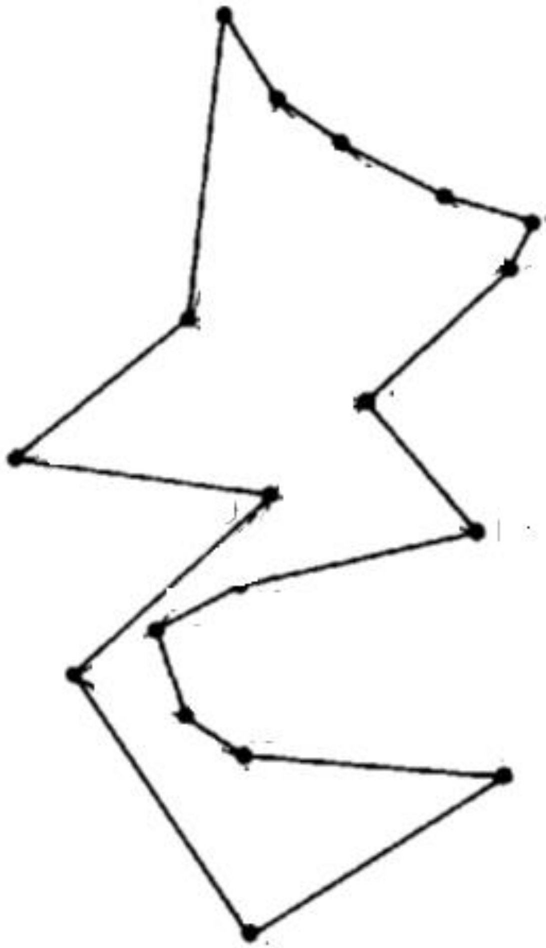
- Lemma : If a polygon has no interior cusps, then it is monotone

Triangulation of a monotone polygon

Triangulation of a monotone polygon

- Given a monotone polygon P
- Claim: To triangulate P by adding a diagonal, we need not check whether it is a diagonal

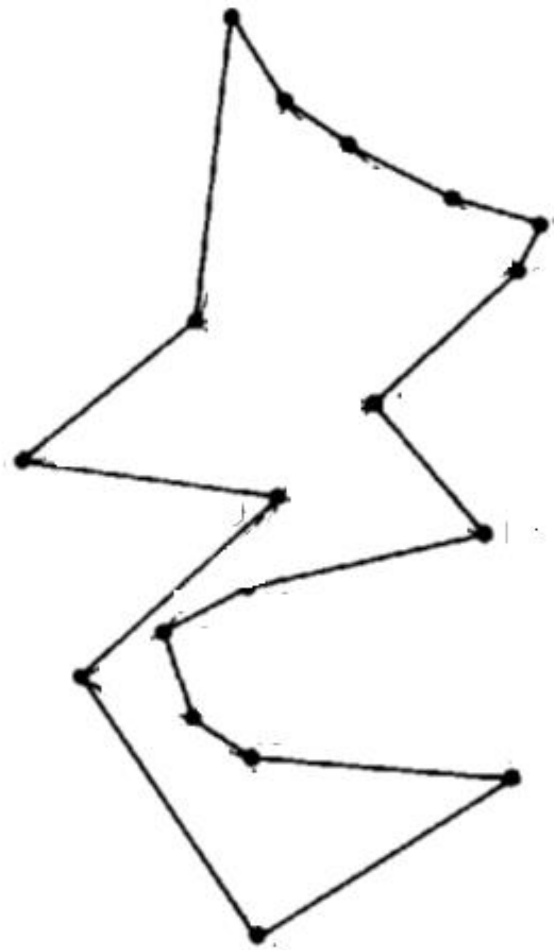
Triangulation of a monotone polygon

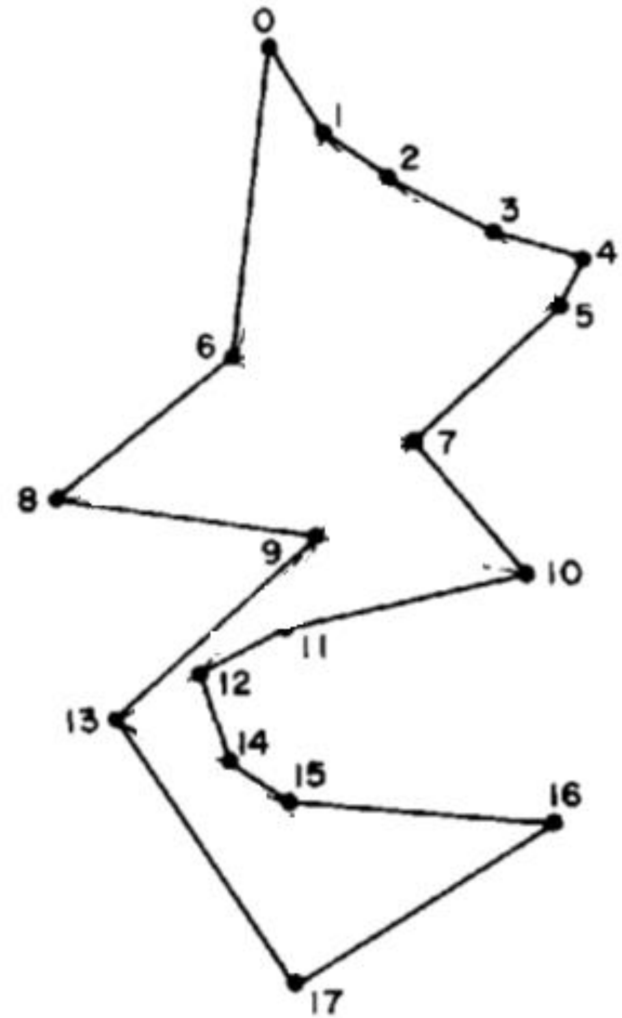


Algorithm: Initial steps

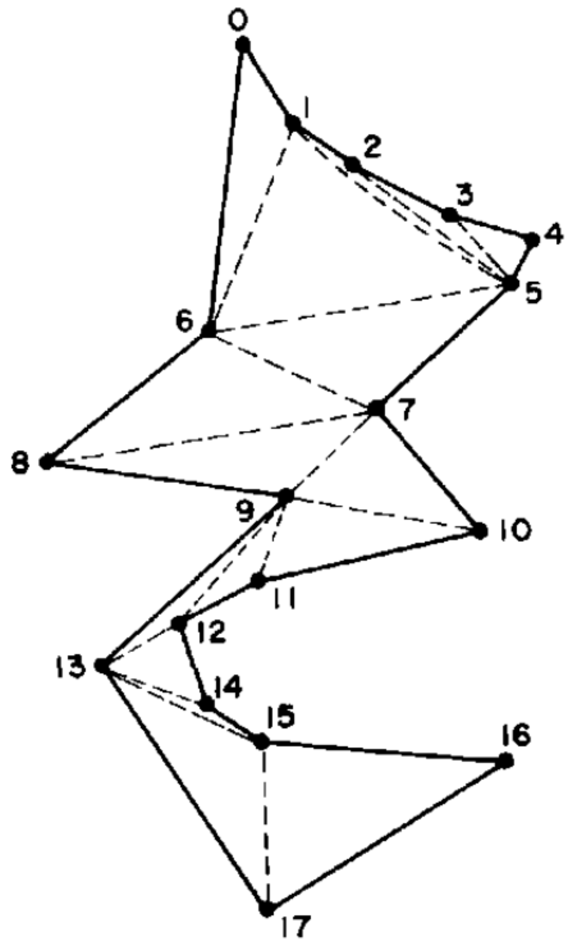
- Step 1: Sort the vertices of each chain of P
- Step 2: Merge the vertices of the left and right monotone chains of the polygon

Input and Output : For the initial steps



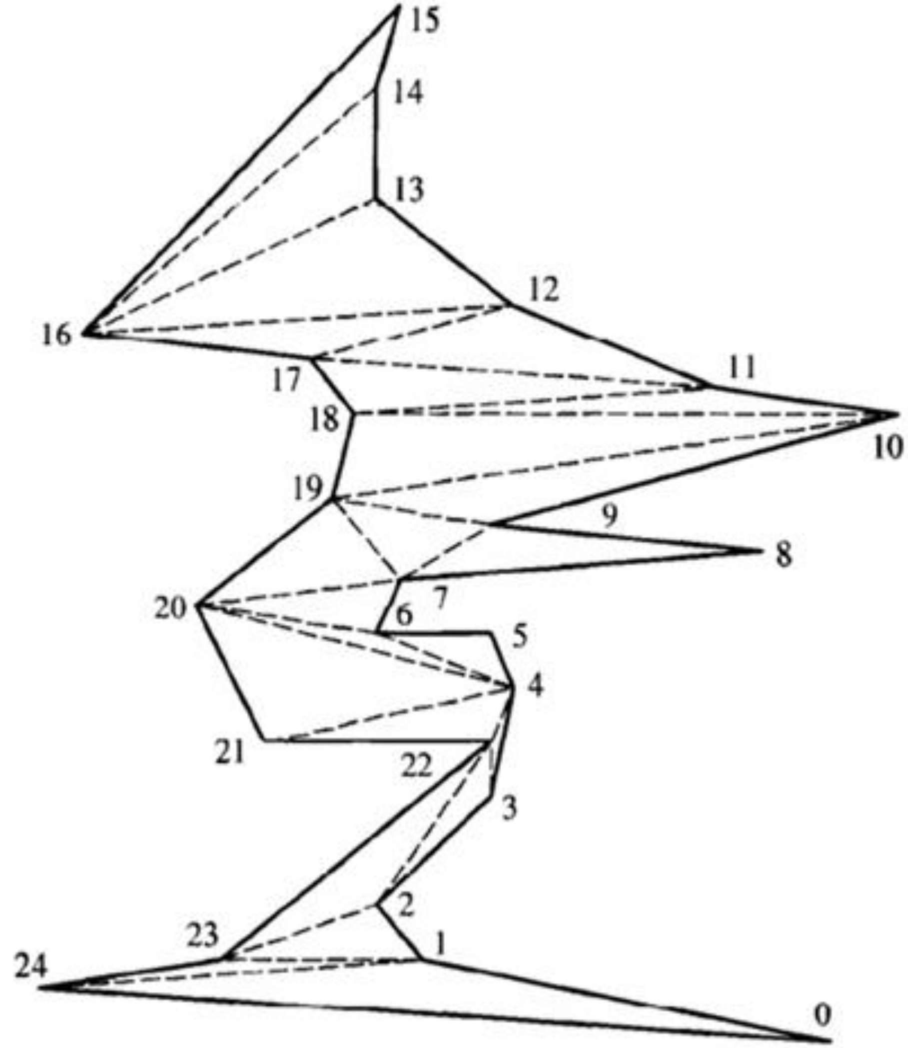


Idea of the algorithm



- Sort the vertices from top to bottom (in linear time)
- Cut off the triangles in a greedy fashion ie. at each step, the first available triangle is removed

Important step- Iterate:



- For each vertex v, connect v to all the vertices above it and visible via a diagonal, and remove the top portion of the polygon thereby triangulated, continue with the next vertex below v

Summarizing : Triangulation of a monotone polygon - Outline

- 1. For each vertex v , connect v to all the vertices above it and visible via a diagonal
- 2. Remove a top portion of the P (ie. The triangles formed with respect to a vertex)
- 3. Continue the above two steps with the next vertex

What do we observe?

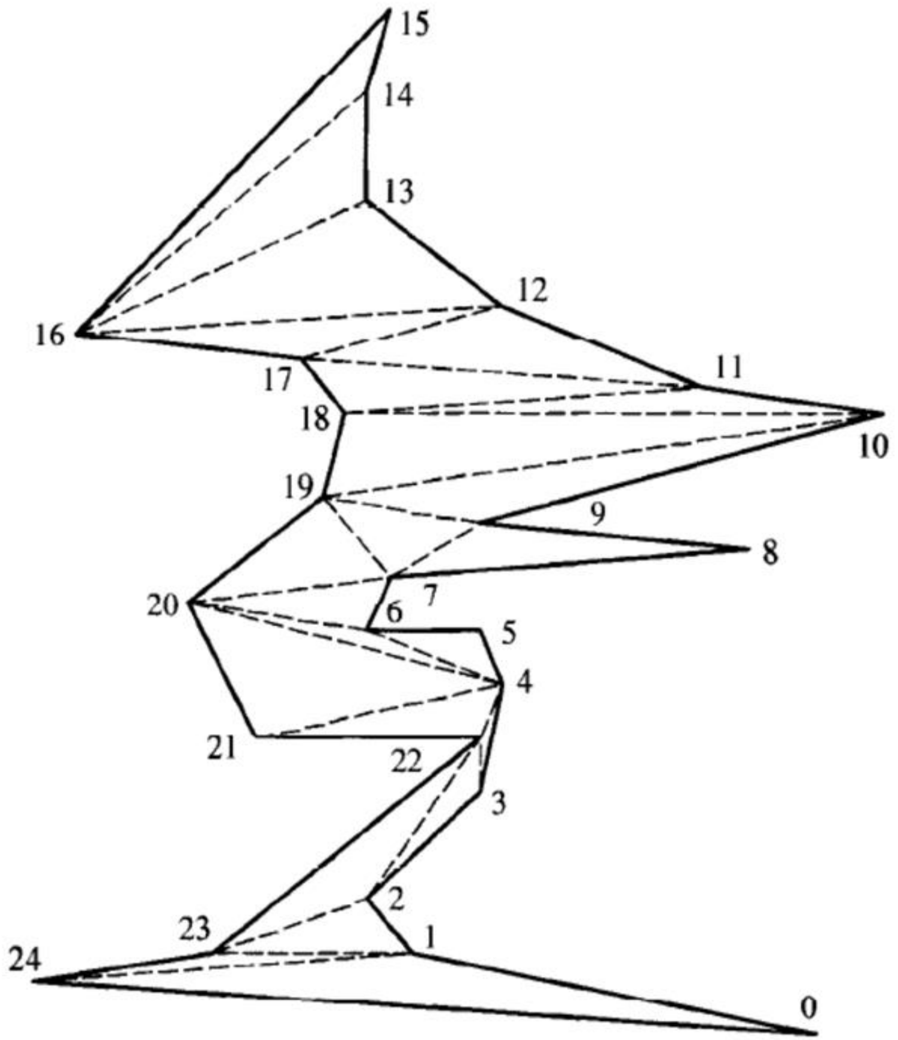
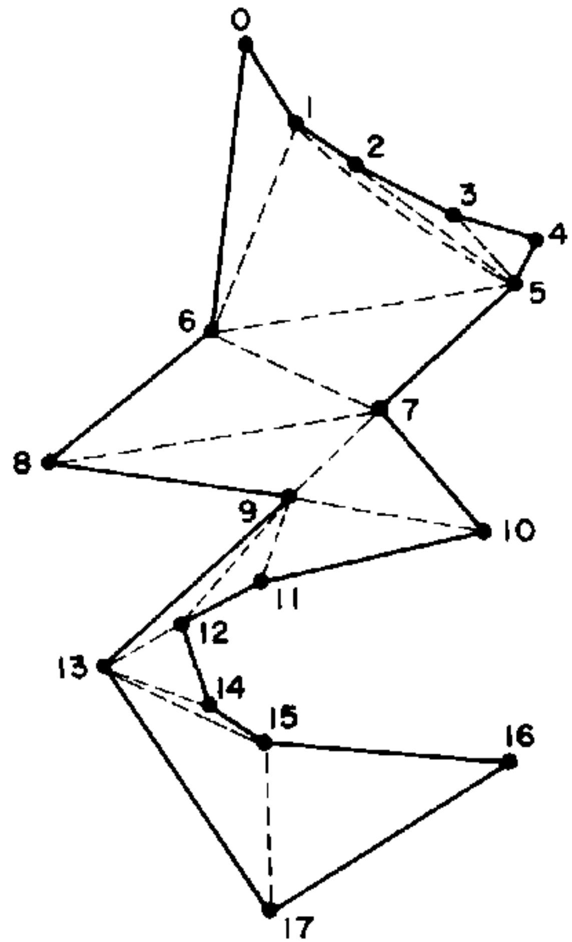
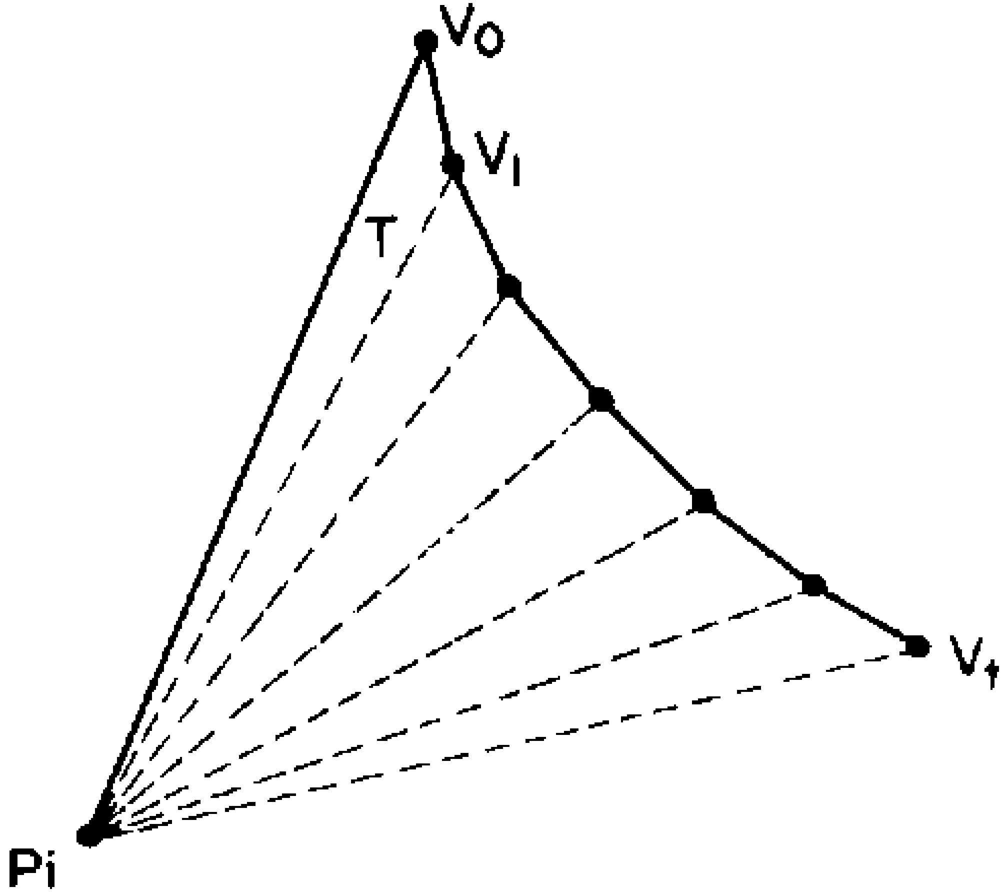


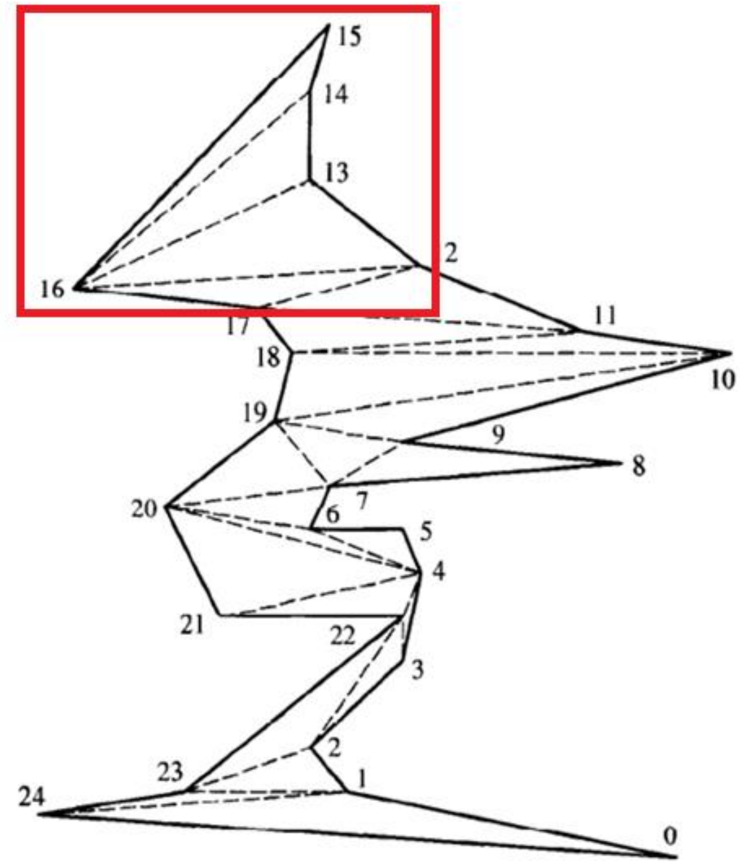
FIGURE 2.1 A polygon monotone with respect to the vertical.

What do we observe?



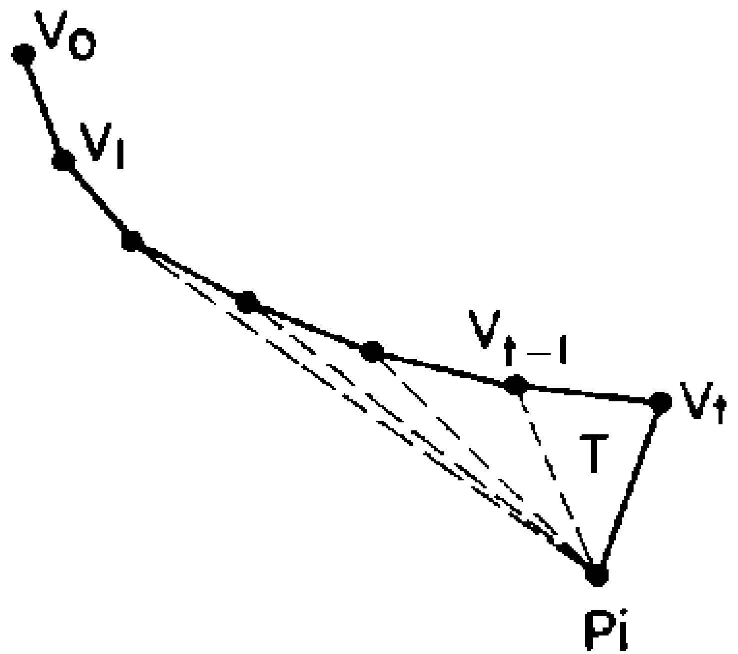
To join the vertices to form triangles: Case-1

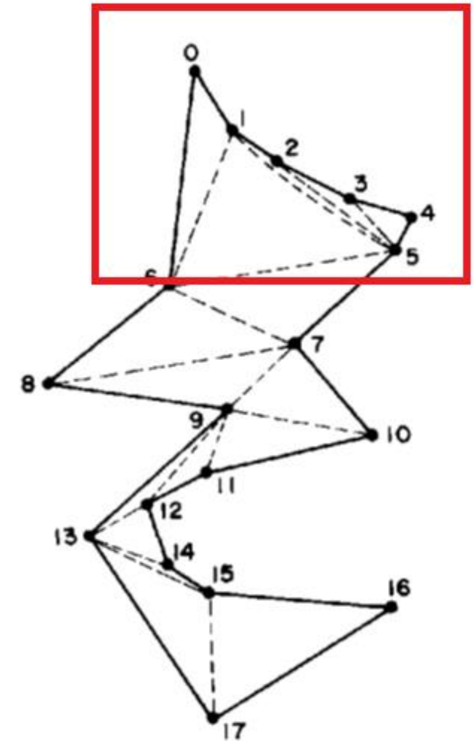




To join the vertices to form triangles

- Case-2 :





References

- J. O’Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998
- J. O'Rourke: Art Gallery Theorems and Algorithms

Thank you