Convex partitioning of a polygon

Another partitioning of P

- Convex Partitioning
- Partition the Polygon into convex pieces
- Is triangulation a convex partitioning?
- Triangulation is a special case of convex partitioning

Objectives of Convex Partitioning

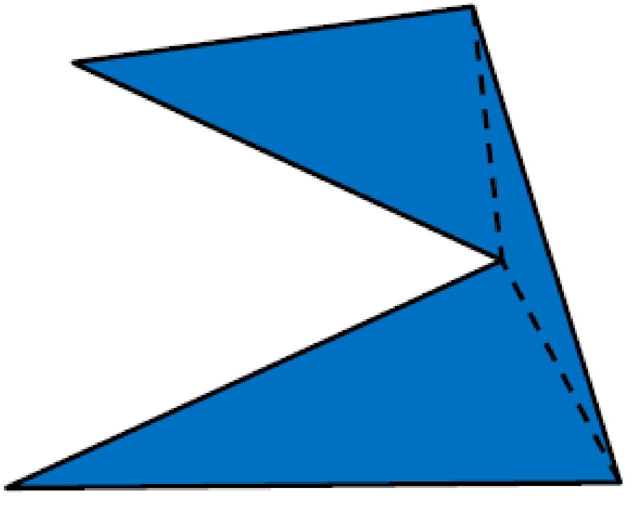
- Partition to as few convex pieces as possible
- Partition as quickly as possible
- Contradictory objectives
- We may have to compromise on the no. of pieces or on the complexity
- **Find an algorithm that produces an optimum partition as quickly as possible**

Two types of partition

- Partition by diagonals
- Partition by segments
- What is the difference between the above two?
- The end points of diagonals must be vertices of P
- The end points of segment need only lie on ∂P

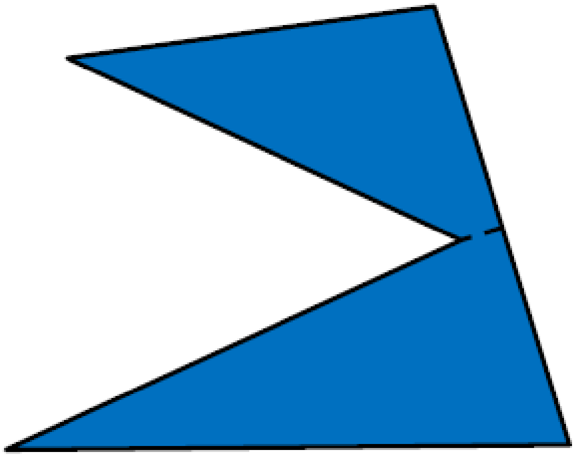
Convex partition by diagonals

- A *convex partition by diagonals* of a polygon P is a decomposition of P into convex polygons obtained by only introducing diagonals.
- **Exercise: Draw a convex partition (by diagonals)**



Convex partition by segments

- A *convex partition by segments* of a polygon P is a decomposition of P into convex polygons obtained by introducing arbitrary segments.
- Exercise: Draw a convex partition of P by segments



Convex Partitioning

- Partitions by segments are more complicated because?
- The end points should be computed
- However, freedom to look beyond the vertices often results in more efficient partition

Convex partition(by segments)

- To evaluate the efficiency of partitions, it is better to have bounds on the partitions
- Theorem (Chazelle) : Let Φ be the fewest number of convex pieces into which a polygon may be partitioned. For a polygon of **r reflex vertices** :

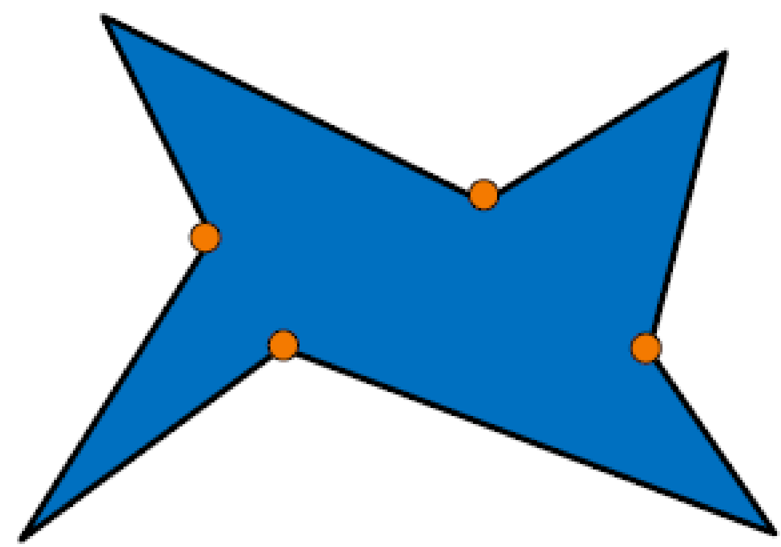
$$\lceil r/2 \rceil + 1 \leq \Phi \leq r + 1$$

- Exercise: Try this theorem empirically with Polygons of 4, 5 and 6 vertices

Convex partition (by segments)

<u>Claim (Chazelle)</u>:
Assume the polygon P has r reflex vertices.
If Φ is the fewest number of polygons required for a convex partition <u>by segments</u> of P then:

$$\lceil r/2 \rceil + 1 \leq \Phi \leq r + 1$$



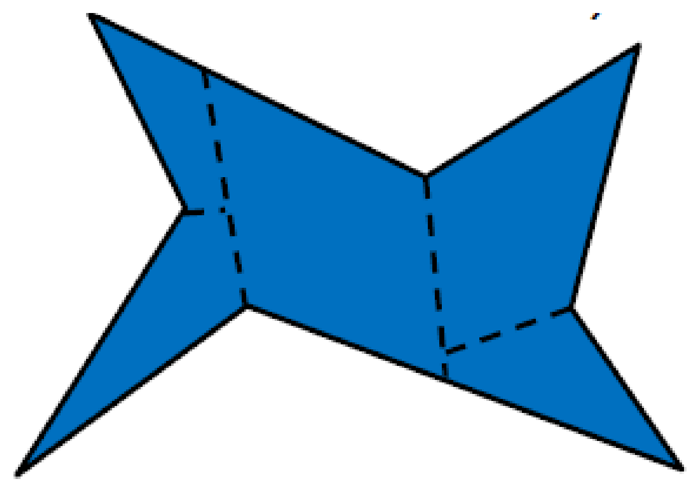
Convex partition (by segments)

Proof ($\Phi \leq r + 1$):

For each reflex vertex, add the bisector.

Because the segment bisects, the reflex angle splits into two convex angles. (Angles at the new vertices have to be $< \pi$.)

Doing this for each reflex vertices, gives a convex partition with $r + 1$ pieces.



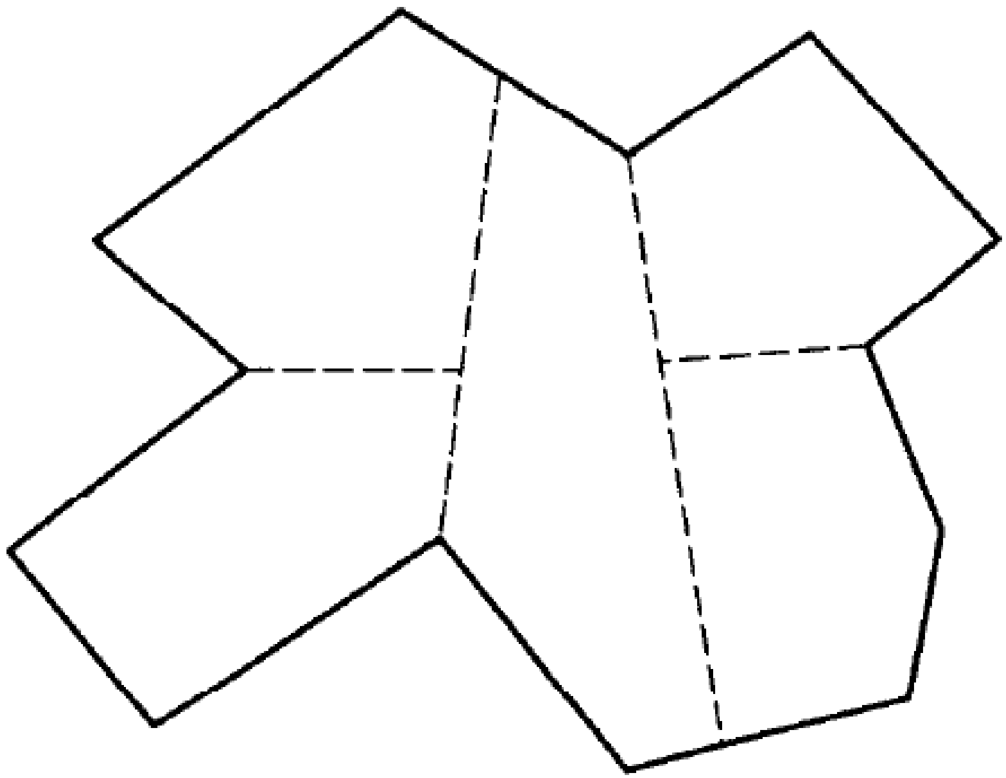
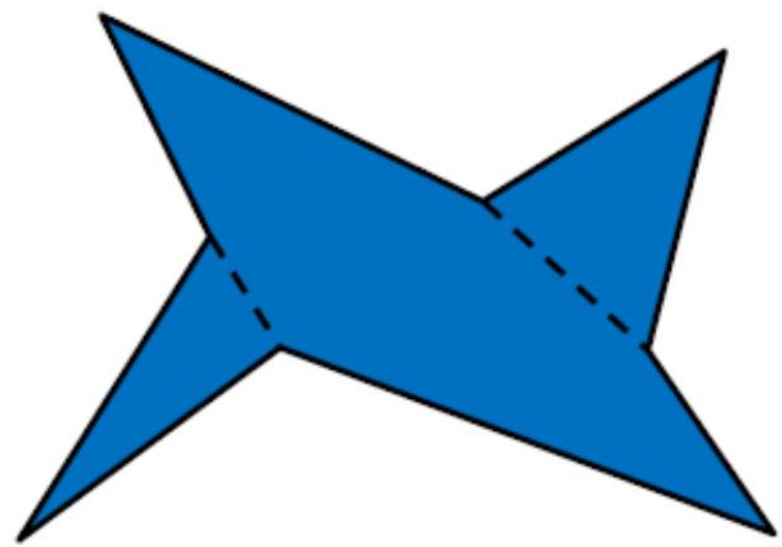


FIGURE 2.10 $r + 1$ convex pieces: $r = 4$; 5 pieces.

Convex partition (by segments)

$$\lceil r/2 \rceil + 1 \leq \Phi$$

Each reflex vertex needs to be split and each introduced segment can split at most two reflex vertices.



Another example

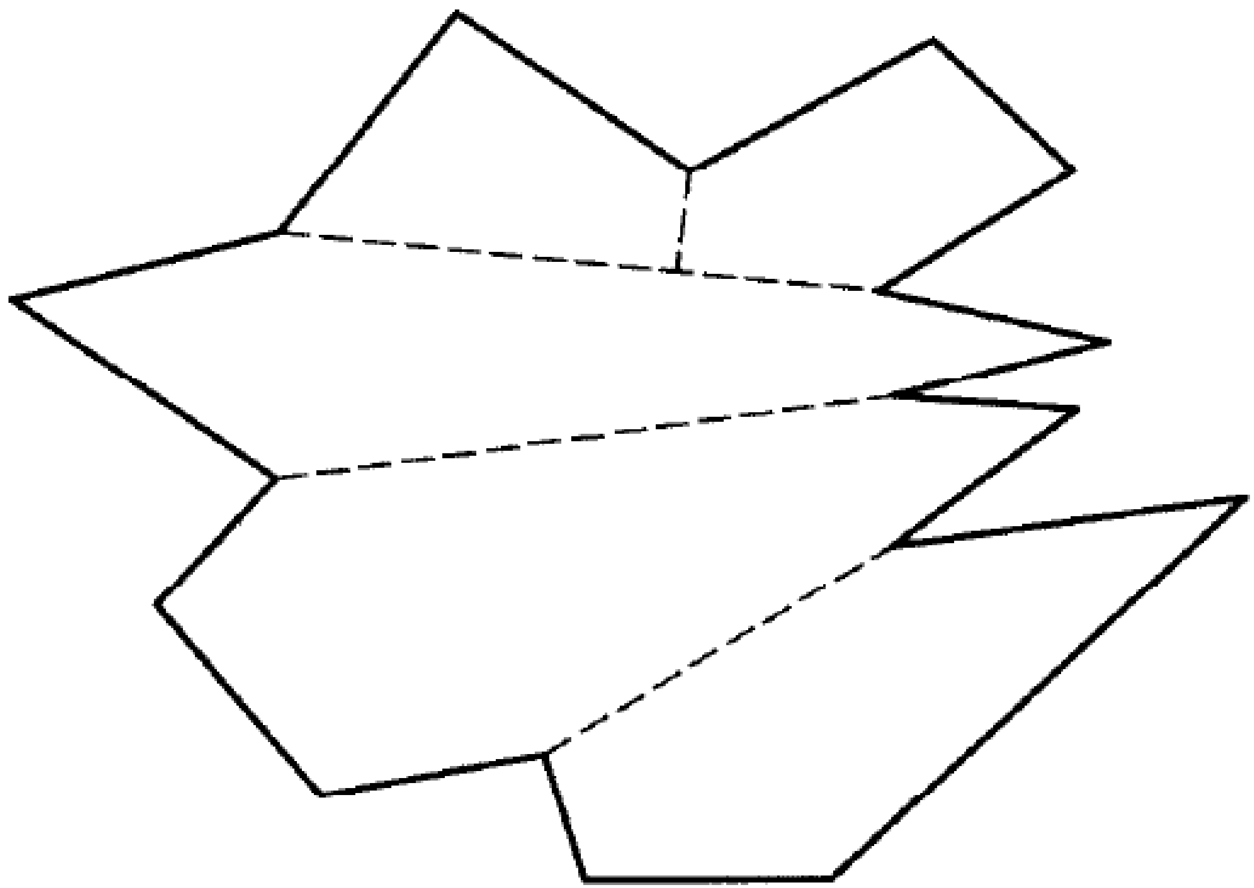


FIGURE 2.11 $\lceil r/2 \rceil + 1$ convex pieces: $r = 7$; 5 pieces.

Exercise

- Check whether this is the optimum partition for the P of previous slide?

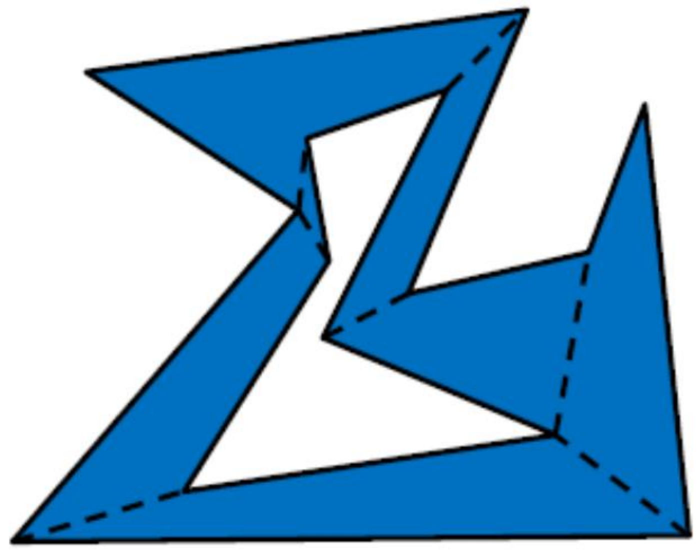
Convex partitioning (by diagonals):

Hertel and Mehlhorn algorithm

- For convex partitioning (by diagonals)
- We need a concept of essential and inessential diagonals

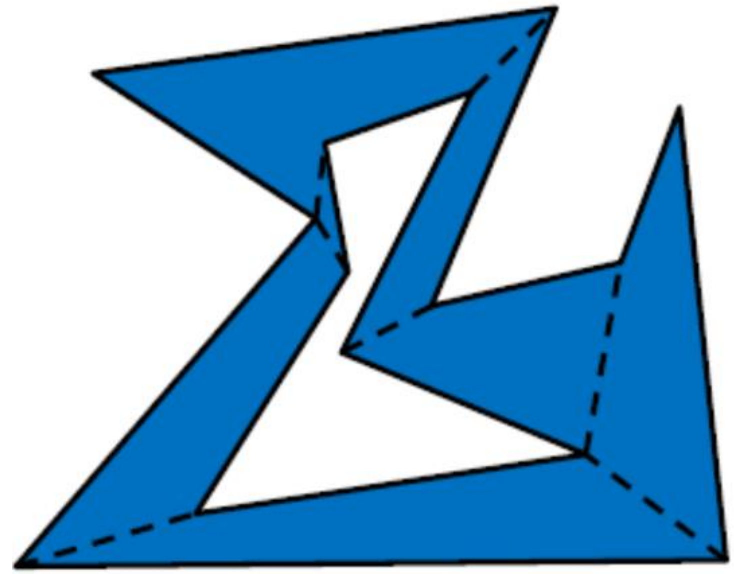
Essential diagonals

A diagonal in a convex partition is *essential for vertex* $v \in P$ if removing the diagonal creates a piece that is not convex at v.



- Note that the example given is not a simple polygon and it is not convex partitioned too(it is used only for explaining what is an essential diagonal)

Inessential diagonals



- What do we observe about essential diagonals?
- Essential diagonal is incident on a reflex vertex
- There can be at most two diagonals essential for any reflex vertex ?
- What is an Inessential diagonal?
- A diagonal is inessential if it is not essential for either of its endpoints

Exercise

- Draw a simple polygon and its convex partitioning with essential diagonals (only)
- Draw a simple polygon and its convex partitioning with both essential diagonals and inessential diagonals
- Draw a simple polygon and its convex partitioning with inessential diagonals (only?)

Hertel and Mehlhorn algorithm

- 1. Start with a triangulation of P
- 2. Remove an inessential diagonal
- 3. Repeat step 2 until P is convex partitioned

- Exercise: Take a simple polygon and perform Hertel and Mehlhorn algorithm on it. Check whether the solution we get is optimum?

References

- J. O'Rourke: Art Gallery Theorems and Algorithms
- J. O Rourke, *Computational Geometry in C,* 2/e, Cambridge University Press, 1998 *)*
- [*https://www.cs.jhu.edu/~misha/Spring16/05.pdf*](https://www.cs.jhu.edu/~misha/Spring16/05.pdf)
  - *From John Hopkins University*

Thank you