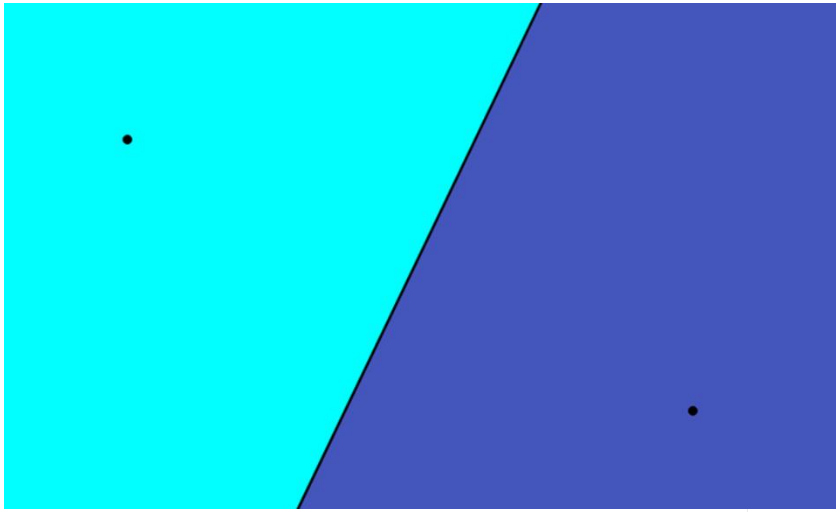
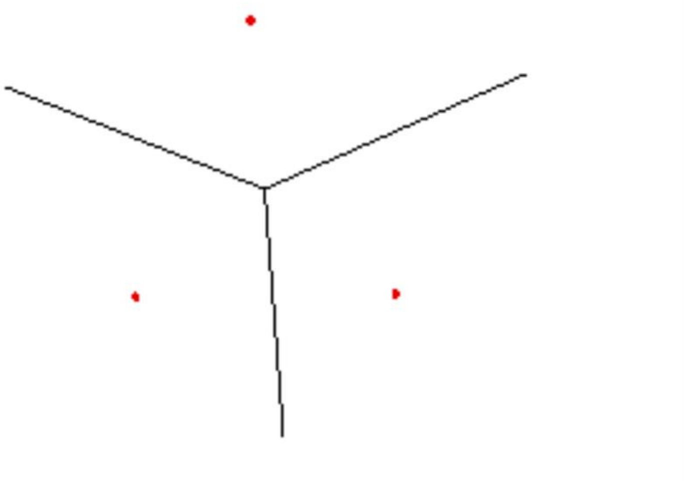
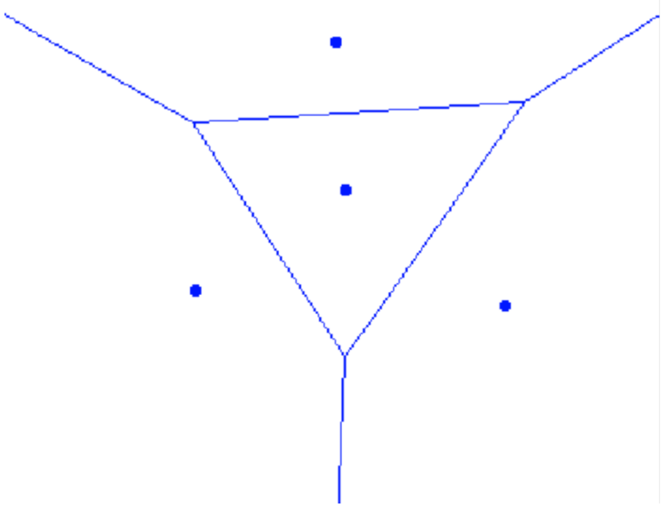
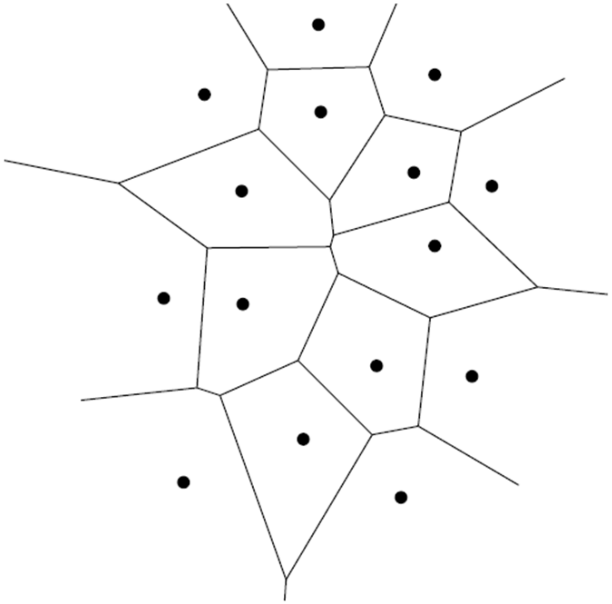
Farthest Voronoi diagram

Closest Voronoi Diagrams



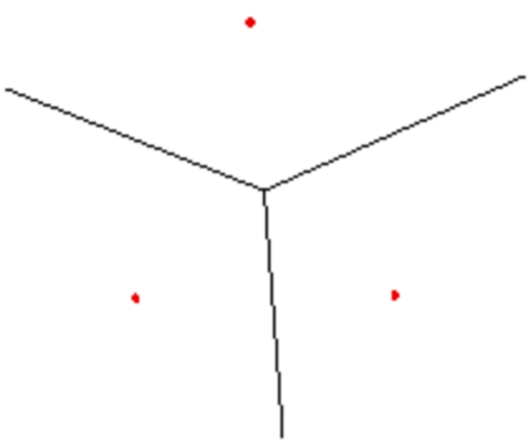






Voronoi Diagram

- Closest Voronoi diagram of a set of sites *S* decomposes the plane into regions around each site $s_i \in S$ such that each point within the region around s_i is closer to s_i than to any other site in *S*.

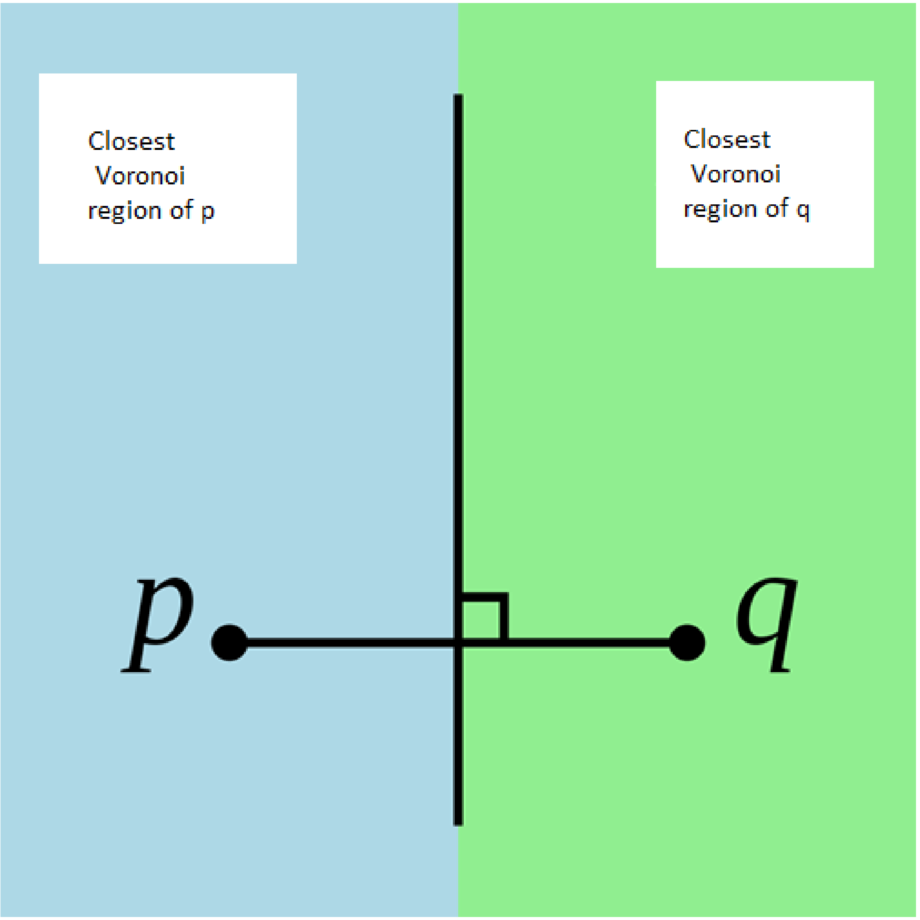


- Farthest Voronoi diagram of a set of sites *S* decomposes the plane into regions such that each point in the region of s_i is farther to s_i than to any other site in *S*

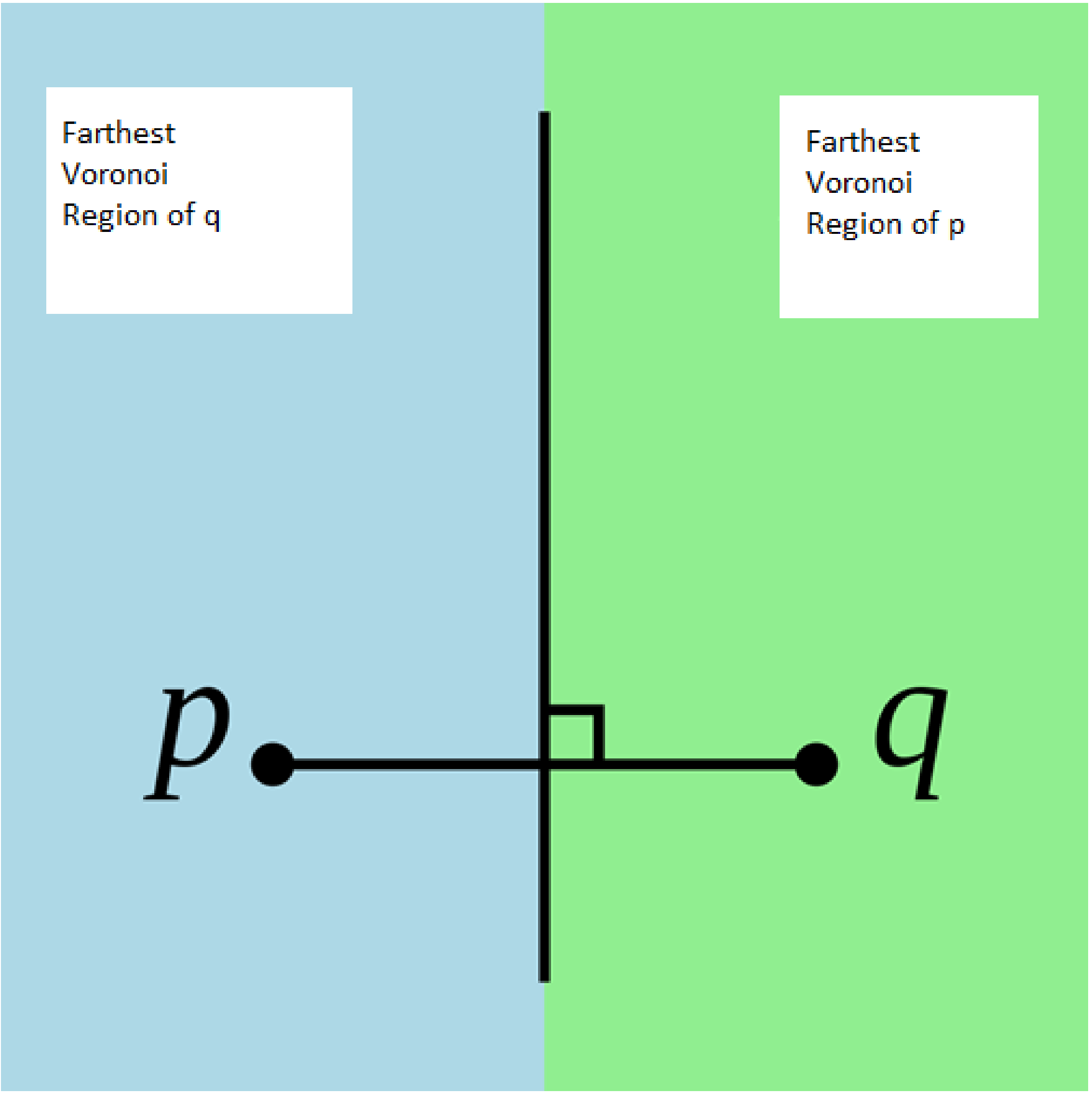
Exercise

- Draw closest Voronoi diagram of two points
- Draw farthest Voronoi diagram of two points

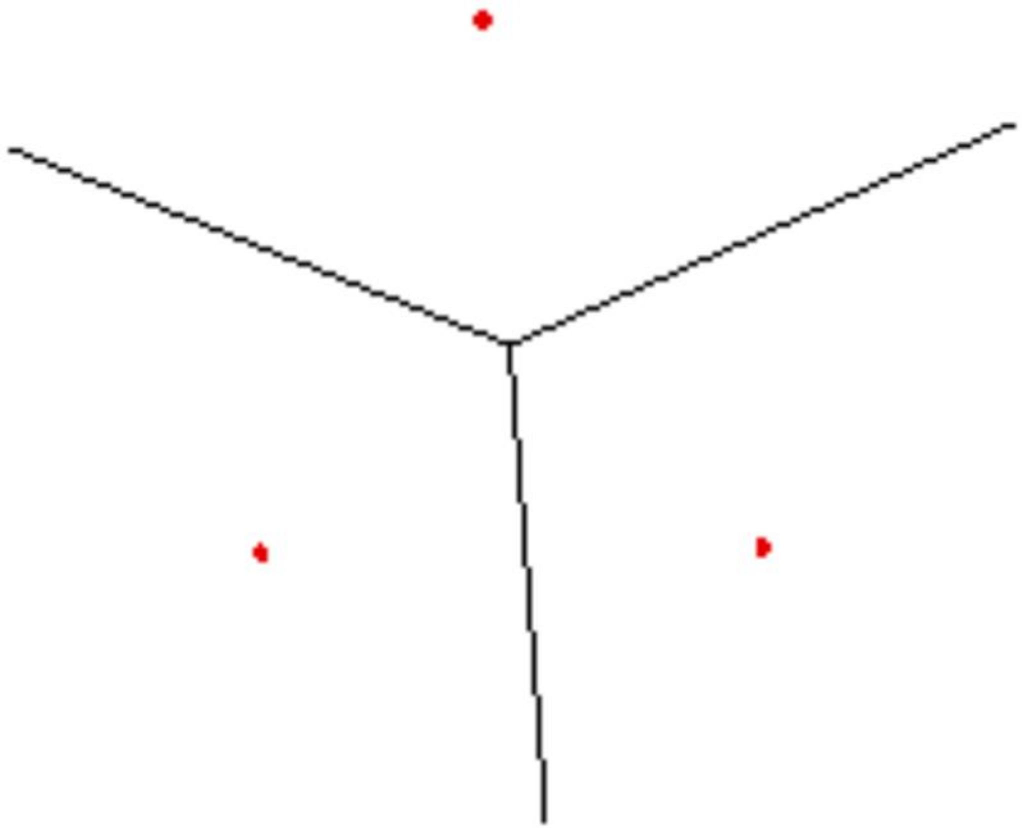
Closest Voronoi diagram



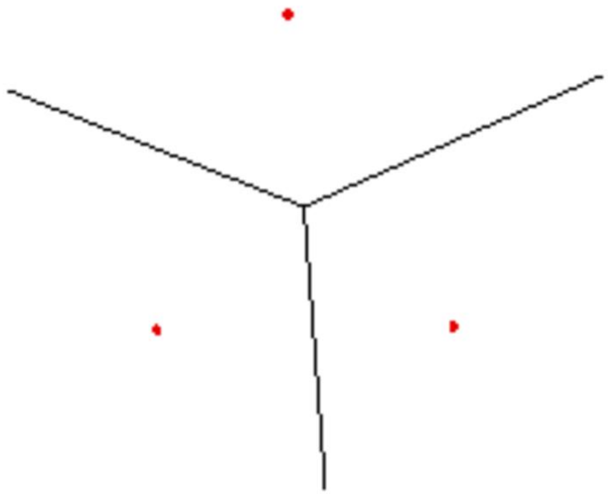
Farthest Voronoi diagram of two points

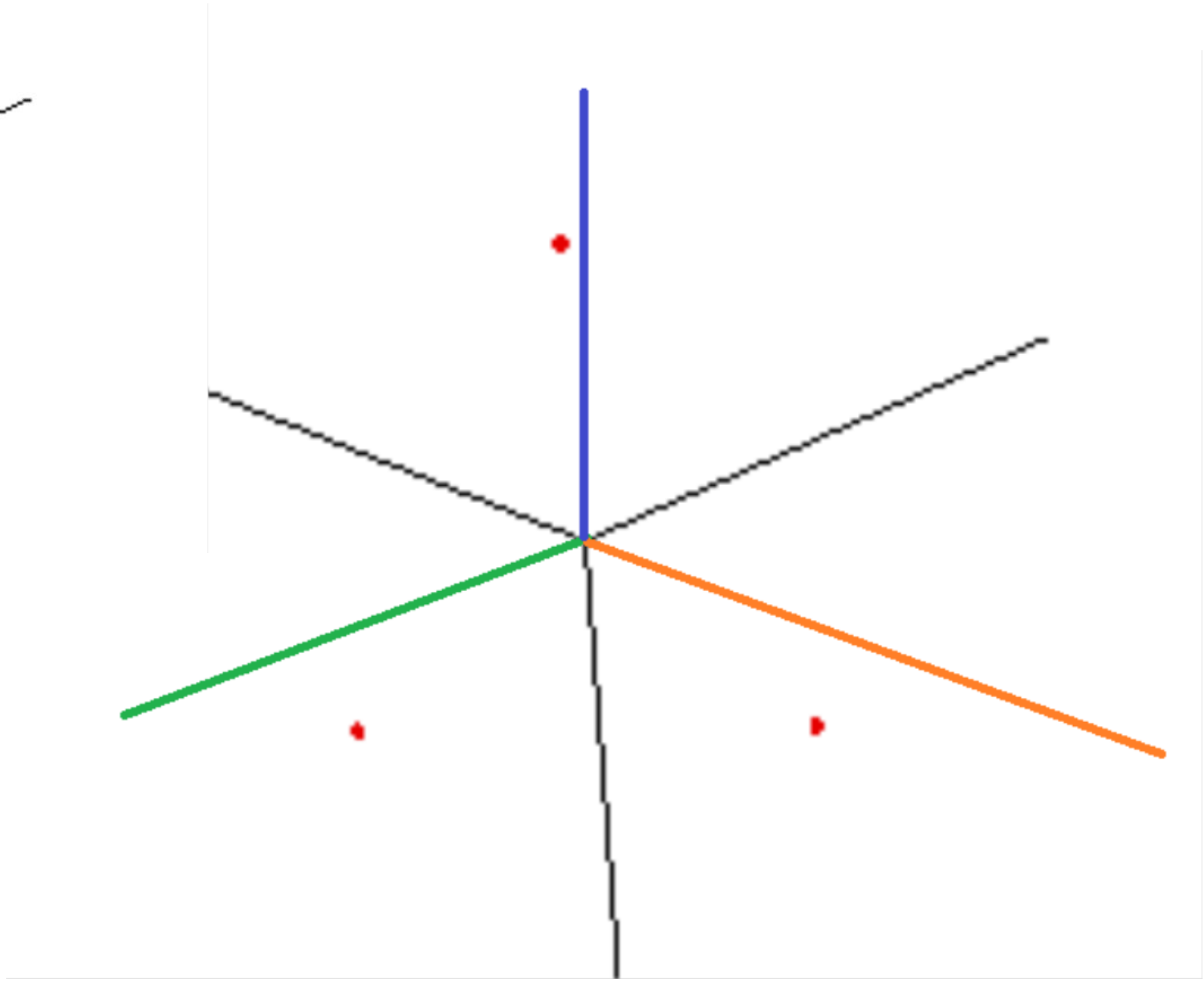


Closest Voronoi diagram of three points

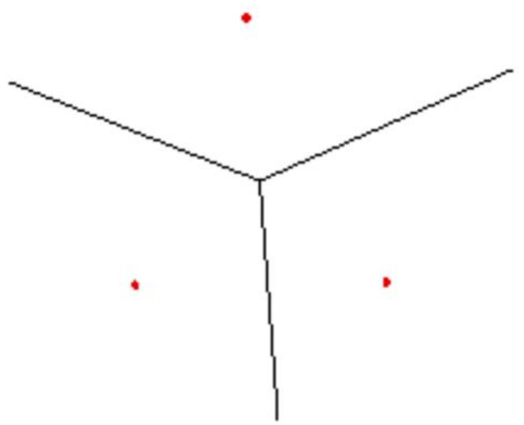


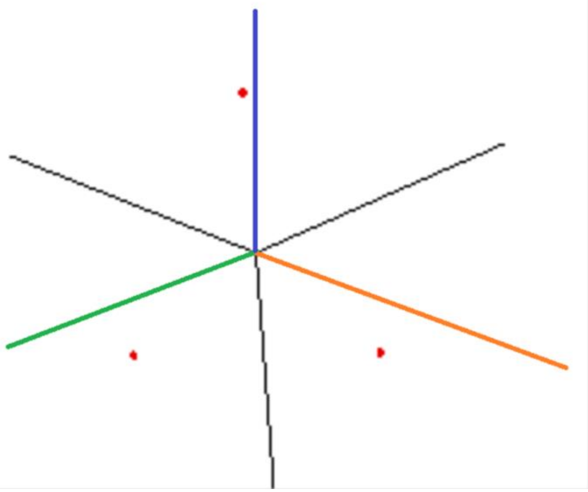
Farthest Voronoi diagram of three points

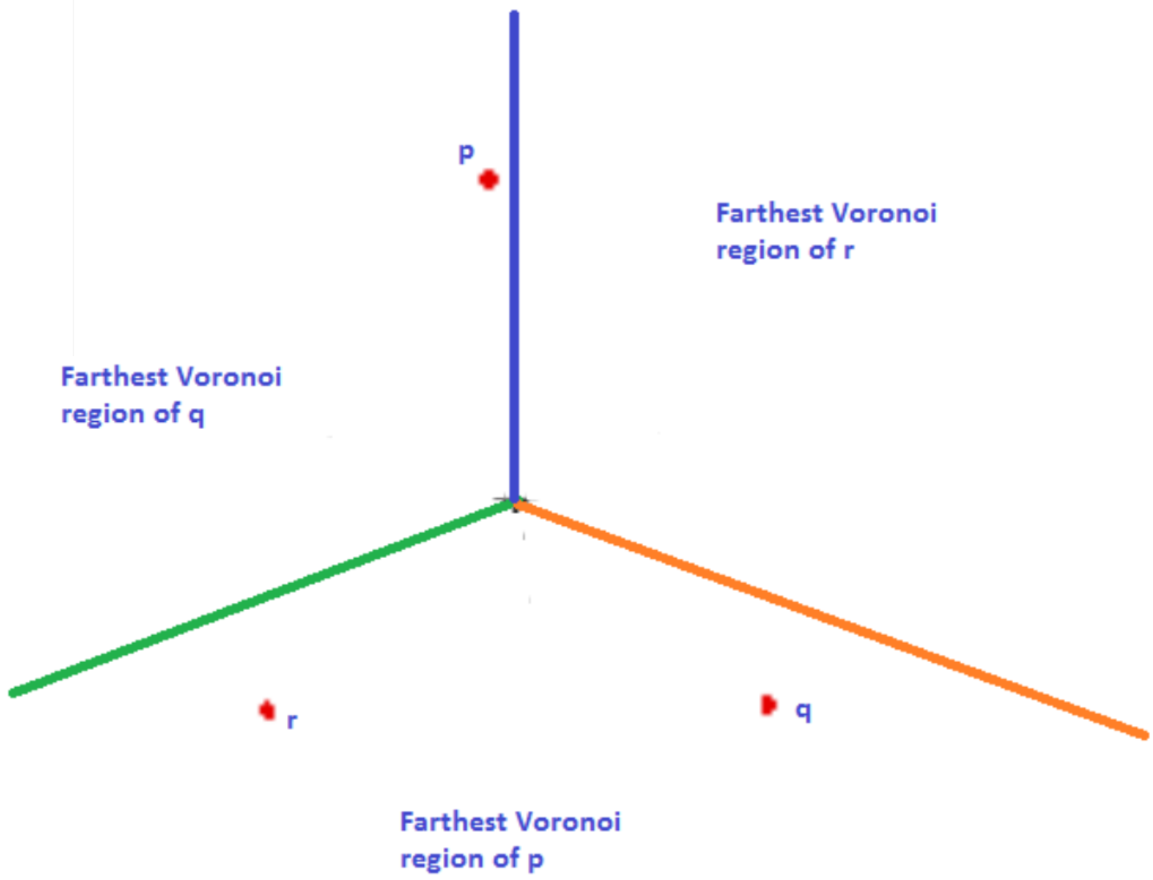




Farthest Voronoi diagram of three points







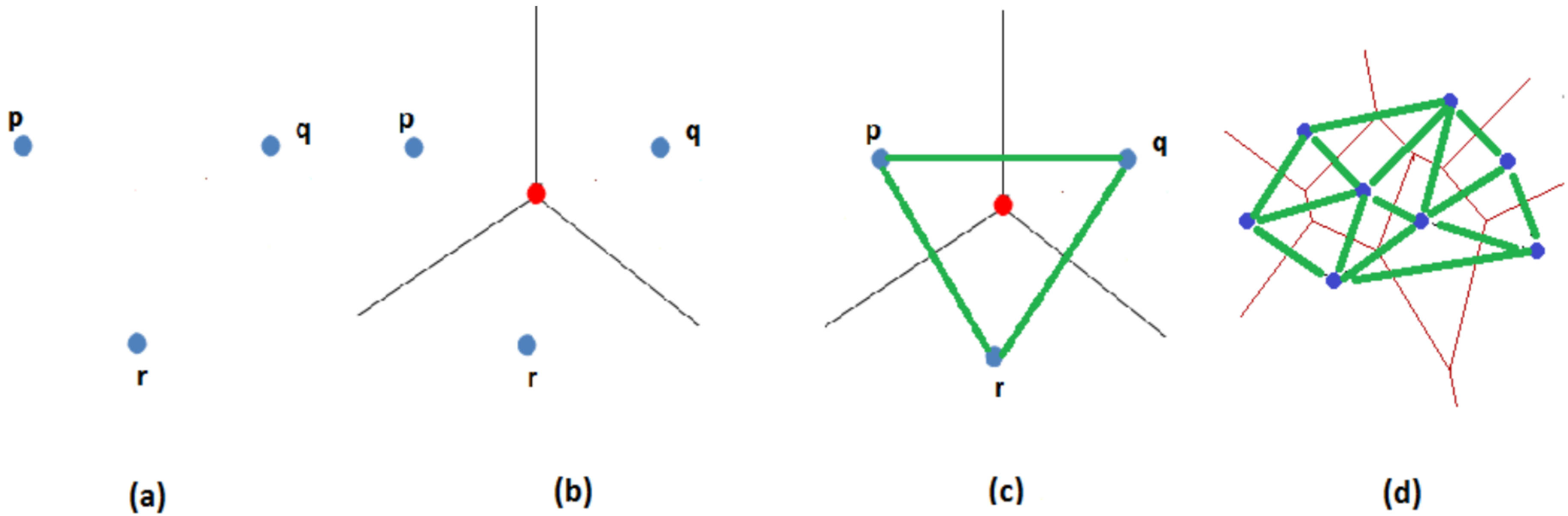
Exercise

- Draw Farthest Voronoi diagram of 4 points
  - 4 points arranged in different ways
- Draw Farthest Voronoi diagram of 5 points
  - 5 points arranged in different ways

Delaunay Triangulation (DT)

Delaunay Triangulation (DT)

- Straight line dual of a Voronoi diagram
- Example : 3 points, its VD, its DT

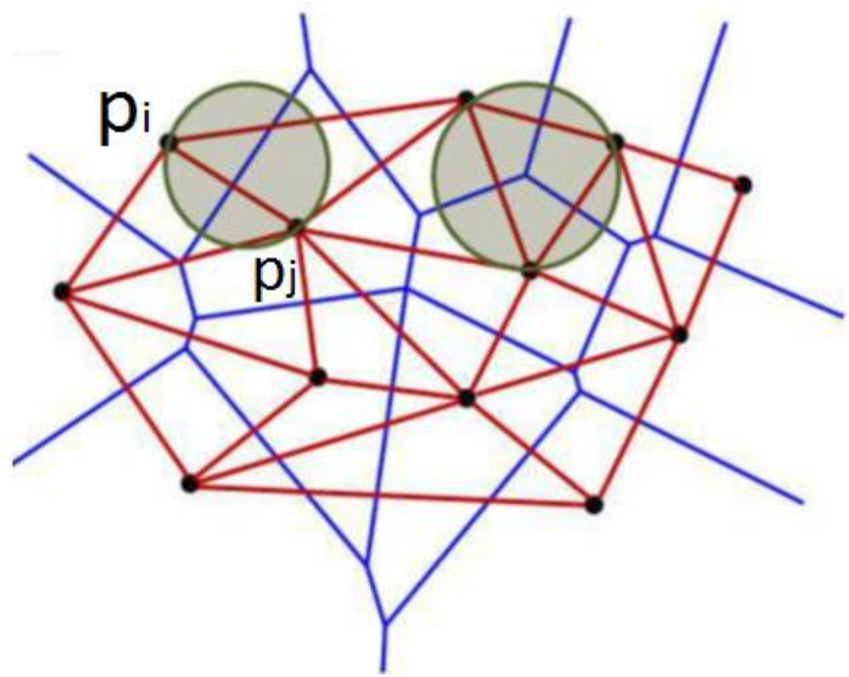


Delaunay Triangulation (DT)

- Named after Boris Delaunay
- **Boris Nikolaevich Delaunay** or **Delone** (Russian Mathematician)
- He invented Delaunay Triangulation in 1934
- Delone sets is his another contribution : Applied widely in coding theory and approximation algorithms

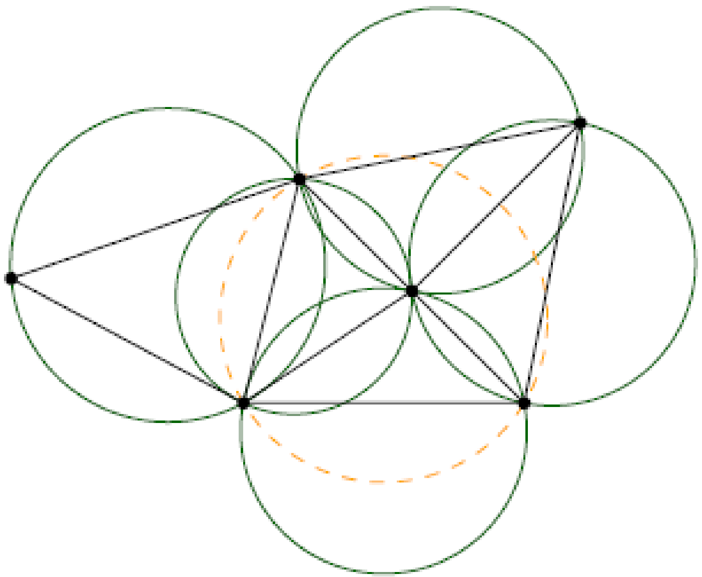
Properties : Delaunay triangulation

- **Empty circle property**: Two points p_i and p_j are connected by an edge in the Delaunay triangulation, if and only if there is an empty circle passing through p_i and p_j .
- **Circumcircle property:** The circumcircle of any triangle in the Delaunay triangulation is empty (contains no other points of P).



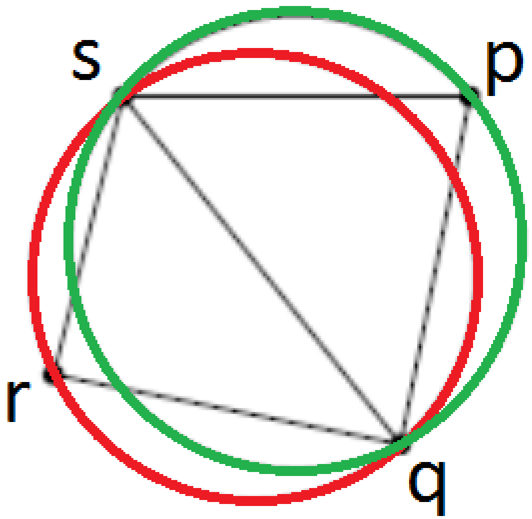
Delaunay triangulation

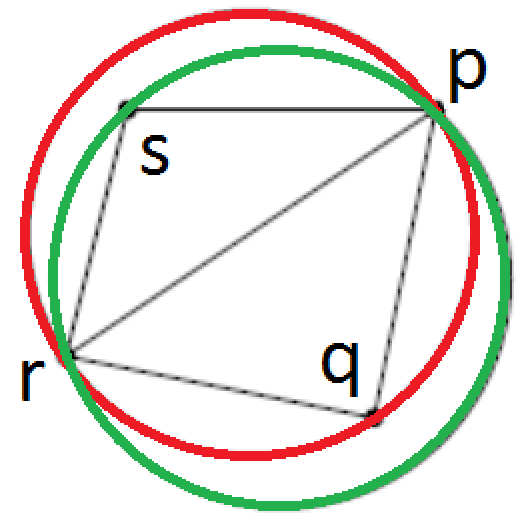
- A triangulation of a finite point set P is called a Delaunay triangulation, if the circumcircle of every triangle (Δ) is empty, that is, there is no point from P in the interior of any Δ.



Delaunay triangulation

- Consider four points in convex position ie. if all the points are vertices of their convex hull
- How many triangulations are possible?

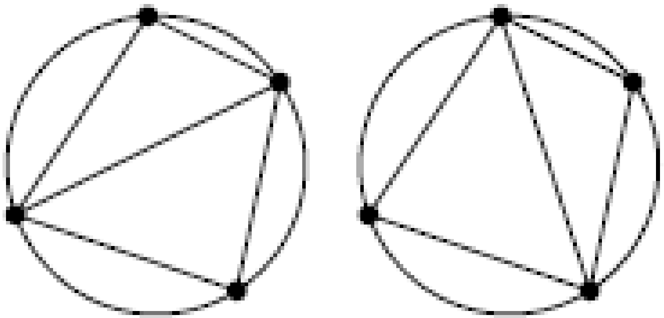




- One is Delaunay triangulation, the other one is not

Delaunay triangulation

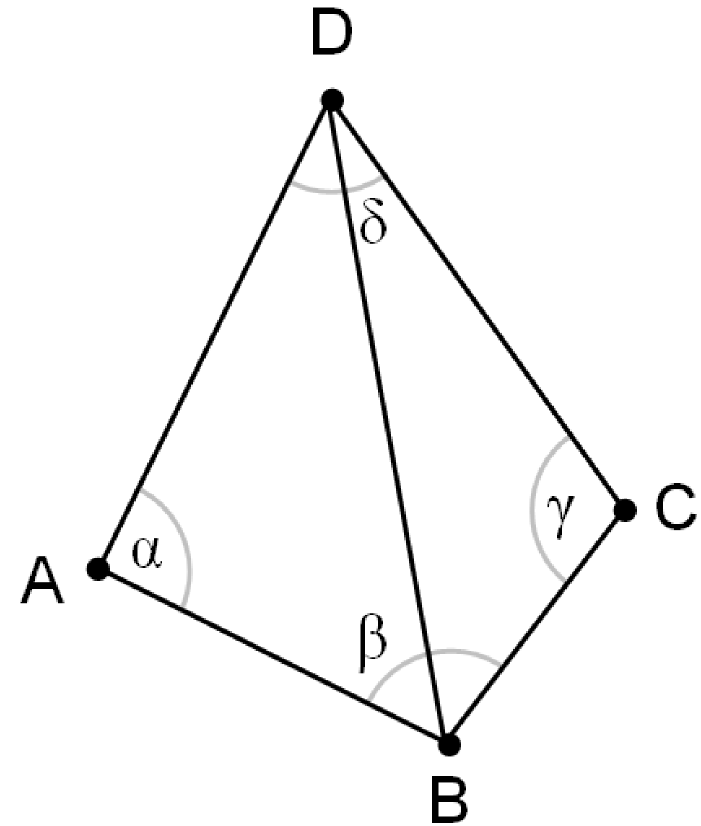
- Two Delaunay triangulations are also possible



- In this case, all the four points are on a common circle – or cocircular.
- Given a set P of four points that are in convex position, but not cocircular, then P has exactly one Delaunay triangulation. Check this out with an example.

More about circumcircle property

- Consider two triangles ABD and BCD with the common edge BD
- If the sum of the angles α and γ is greater than 180°, the triangulation does not meet the Circumcircle property of DT



Another property of DT

- DT maximizes the minimum angle or it avoids "skinny" triangles
- Compared to any other triangulation of the points, the smallest angle in the Delaunay triangulation is at least as large as the smallest angle in any other triangulation
- Now we will focus on Angle optimal triangulation and use that to define Delaunay triangulation

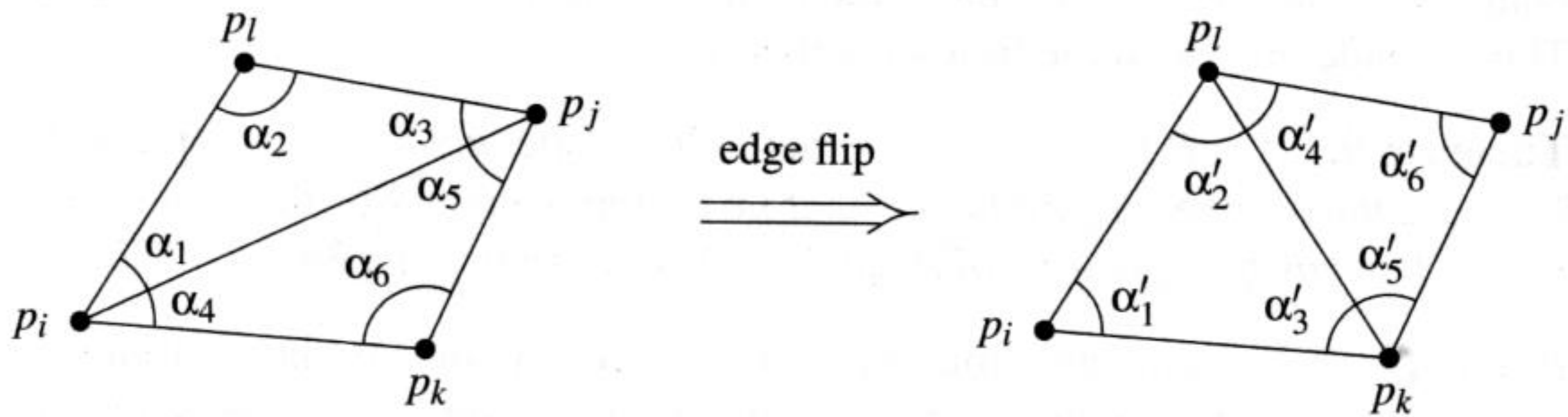
Angle Optimal Triangulations

- Create *angle vector* of the sorted angles of triangulation *T*, $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_m) = A(T)$, with α_1 being the smallest angle.
- Let $(\alpha'_1, \alpha'_2, \alpha'_3, \ldots \alpha'_m) = A(T')$
- $A(T)$ is larger than $A(T')$ iff there exists an i

  such that $\alpha_j = \alpha'_j$ for all $j < i$ and $\alpha_i > \alpha'_i$
- Best triangulation is a triangulation that is *angle optimal,* i.e. has the largest angle vector.

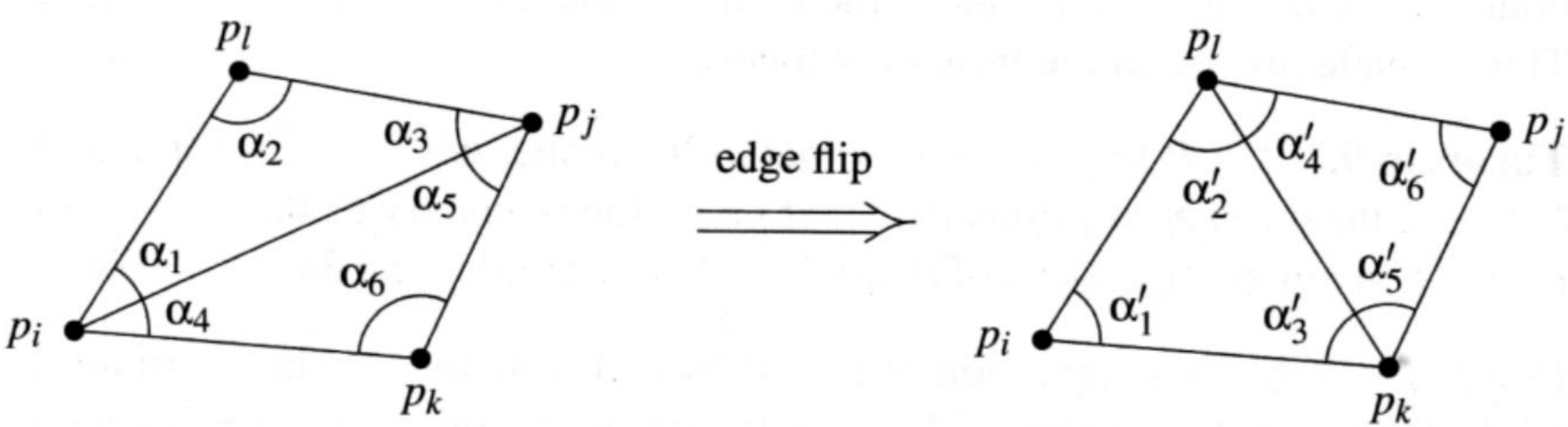
Angle Optimal Triangulations

- Consider two adjacent triangles of a Triangulation:
- If the two triangles form a convex quadrilateral, we could have an alternate triangulation by performing an *edge flip* on their shared edge



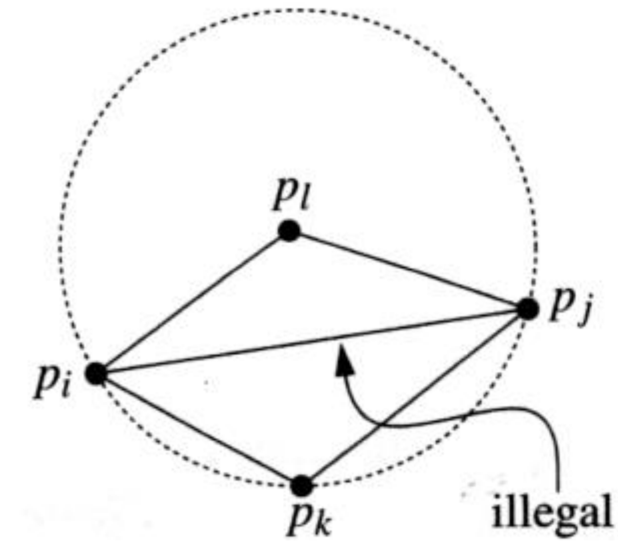
Illegal edges

- Edge *e* is illegal if: $$\min_{1\leqslant i\leqslant 6} \alpha_i < \min_{1\leqslant i\leqslant 6} \alpha'_i.$$
- Only difference between *T* containing *e* and *T*' with *e* flipped are the six angles of the quadrilateral.
- If triangulation *T* contains an illegal edge e, we can make A(*T*) larger by flipping e.
- In this case, *T* is an *illegal triangulation*.



Illegal edges

- If p_i, p_j, p_k, p_l form a convex quadrilateral and do not lie on a common circle, exactly one of p_ip_j and p_kp_l is an illegal edge.



- The edge p_ip_j is illegal iff p_l lies inside the circle

Illegal Triangulation

- If triangulation *T* contains an illegal edge e, we can make A(*T*) larger by flipping e.
- In this case, *T* is an *illegal triangulation*.

Computing Legal Triangulations

- Compute a triangulation of input points *P*.
- Flip illegal edges of this triangulation until all edges are legal.
- Algorithm terminates because there is a finite number of triangulations.

References

- J. O Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998
- *Farthest Voronoi App ?*
- *https://www.coursera.org/lecture/geometric-algorithms/angle-optimal-triangulations-9z9k6*
- Notes by John Augustine, IIT Madras
- https://dccg.upc.edu/people/vera/wp-content/uploads/2013/06/GeoC-Voronoi-algorithms.pdf by Professor Vera Sacristan

THANK YOU