CS2006D Discrete Structures

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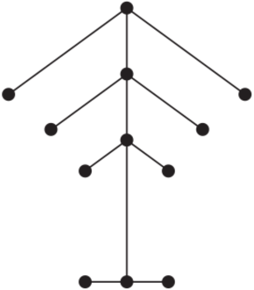
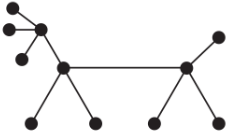
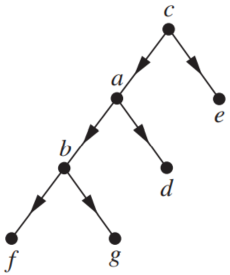
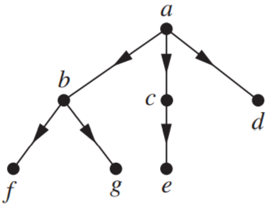
- A connected graph that contains no simple circuits is called a tree
- Any tree must be a simple graph
- An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices
- The graphs containing no simple circuits that are not necessarily connected are called forests
- Each connected component of a forest is a tree





- A rooted tree is a tree in which one vertex has been designated as the root and every edge is directed away from the root
- We can change an unrooted tree into a rooted tree by choosing any vertex as the root
- Different choices of the root produce different rooted trees
- We usually draw a rooted tree with its root at the top of the graph

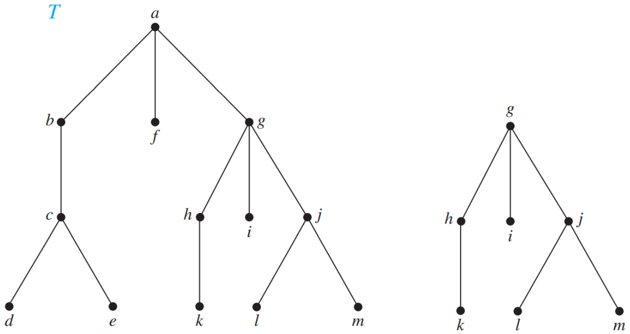




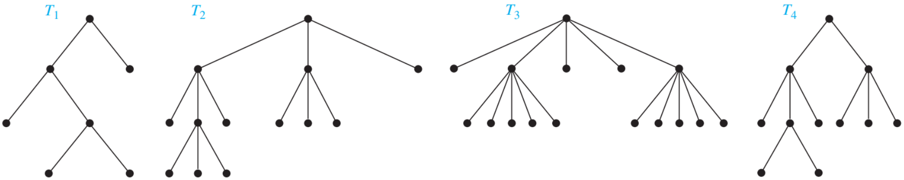
- ⋆ If v is a vertex in T other than the root, the parent of v is the unique vertex u such that there is a directed edge from u to v
- ⋆ When u is the parent of v, v is called a child of u
- ⋆ Vertices with the same parent are called siblings
- ⋆ The ancestors of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root
- ⋆ The descendants of a vertex v are those vertices that have v as an ancestor

Rooted Trees

- A vertex of a rooted tree is called a leaf if it has no children
- Vertices that have children are called internal vertices
- The **root** is an internal vertex unless it is the only vertex in the graph, in which case it is a leaf
- If a is a vertex in a tree, the subtree with a as its root is the **subgraph** of the tree consisting of a and its **descendants** and all edges incident to these descendants

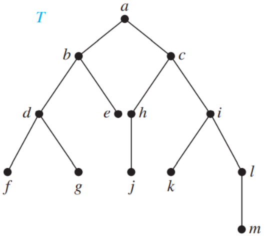


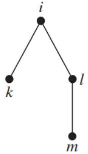
- A rooted tree is called an m-ary tree if every internal vertex has no more than m children
- The tree is called a full m-ary tree if every internal vertex has exactly m children
- An m-ary tree with $m = 2$ is called a binary tree
- Which among the following are full m-ary trees?



Ordered Rooted Tree

- An ordered rooted tree is a rooted tree where the children of each internal vertex are ordered
- Ordered rooted trees are drawn so that the children of each internal vertex are shown in order from left to right
- In an ordered binary tree (usually called just a binary tree), if an internal vertex has two children, the first child is called the left child and the second child is called the right child
- The tree rooted at the left child of a vertex is called the left subtree of this vertex, and the tree rooted at the right child of a vertex is called the right subtree of the vertex





Properties of Trees

- Consider a simple graph G. G is a tree if and only if G satisfies any two of the following properties
  1. G is connected
  2. G is acyclic
  3. G has n vertices has $n - 1$ edges
- A full m-ary tree with i internal vertices contains $n = mi + 1$ vertices
- A full m-ary tree with
  (i) n vertices has $i = (n - 1)/m$ internal vertices and $l = [(m - 1)n + 1]/m$ leaves
  (ii) i internal vertices has $n = mi + 1$ vertices and $l = (m - 1)i + 1$ leaves
  (iii) l leaves has $n = (ml - 1)/(m - 1)$ vertices and $i = (l - 1)/(m - 1)$ internal vertices

Q Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?