The Guggenheim Museum in Bilbao (Image courtesy: BBC.com)

What is the minimum number of guards to safeguard the whole gallery?

What is the minimum number of guards?

Placing minimum number of guards

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Placing minimum number of guards

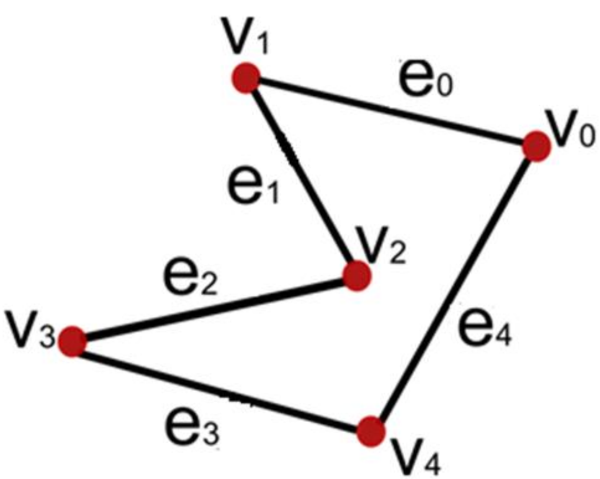
Art Gallery Problem [Victor Klee, 1973]

- Input : Art Gallery
- Output : Minimum number of guards that can safe-guard or cover the interior walls of the gallery

- The Guggenheim Museum in Bilbao: hard to supervise (Image courtesy: BBC.com)

Input representation

- How do we represent the input / art gallery in geometric terms?
- Polygon


v1
e0
v0
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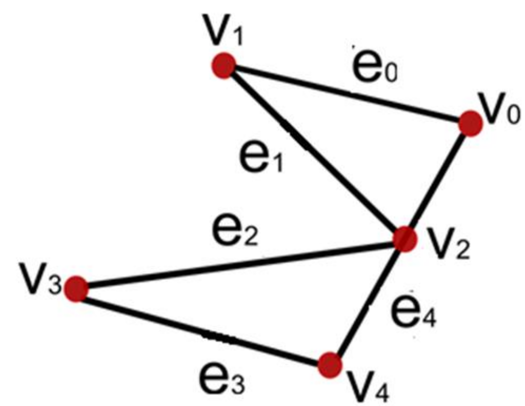
v1
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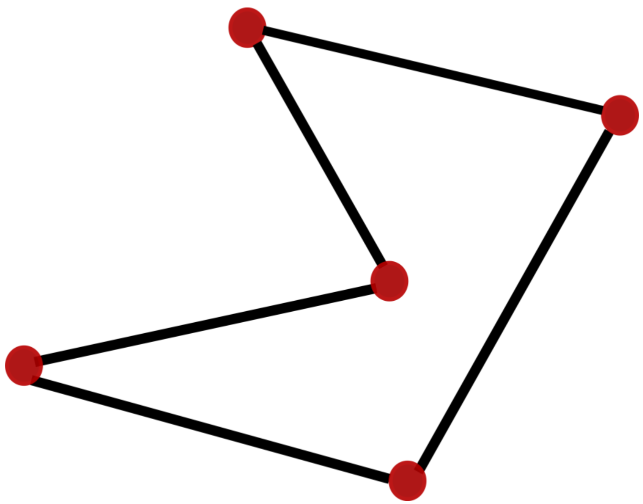
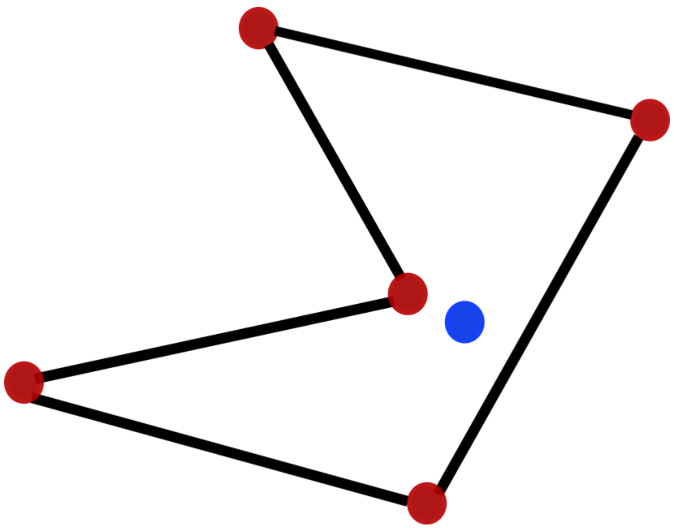
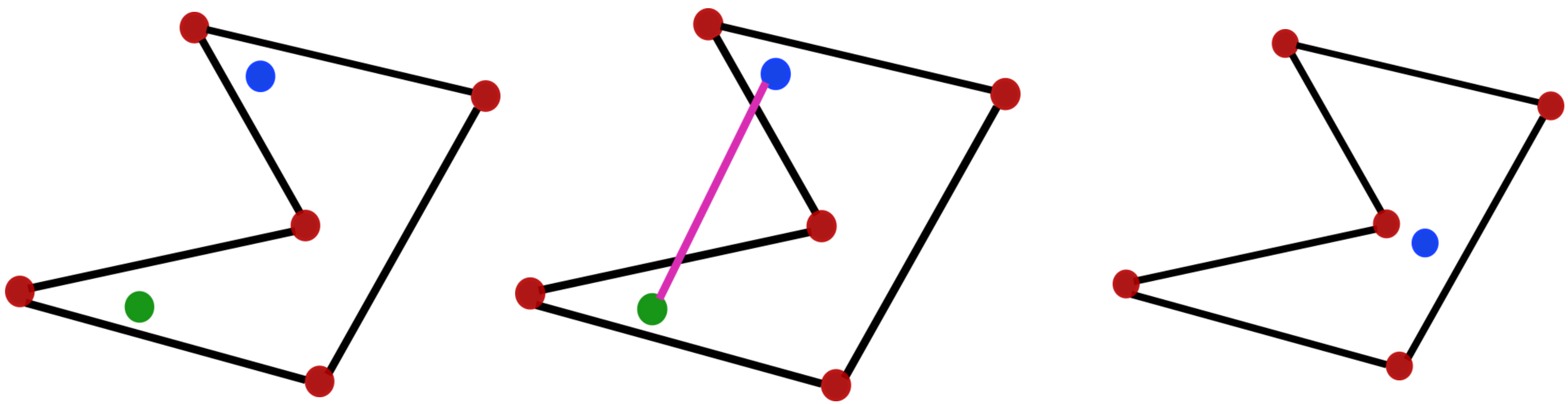
Placing Guards

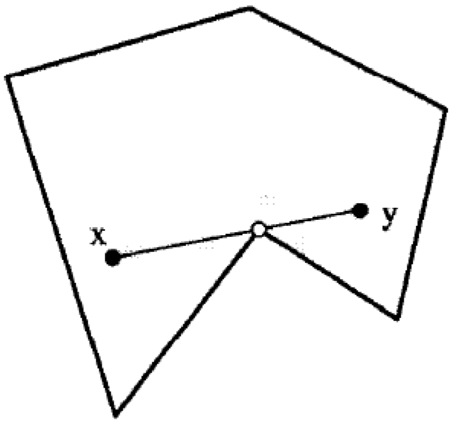




Visibility



Visibility



- The vertex is the Grazing contact of line xy
- Definition of visibility which we already know: Point *x* is visible to point *y* iff the closed segment *xy* is nowhere exterior to the polygon
- According to the above definition: *x* is visible to *y*, even though there is a grazing contact between them

Definition of clear visibility

- A vertex can block vision in the case of clear visibility

x has *clear visibility* to y if $xy \subseteq P$ and $xy \cap \partial P \subseteq \{x, y\}$.

Visibility : Covering a polygon

- A set of guards cover a polygon if every point in the polygon is visible to some guard
- What we have to do is: Given a simple Polygon *P* with *n* vertices, compute the minimum number of guards which cover *P*

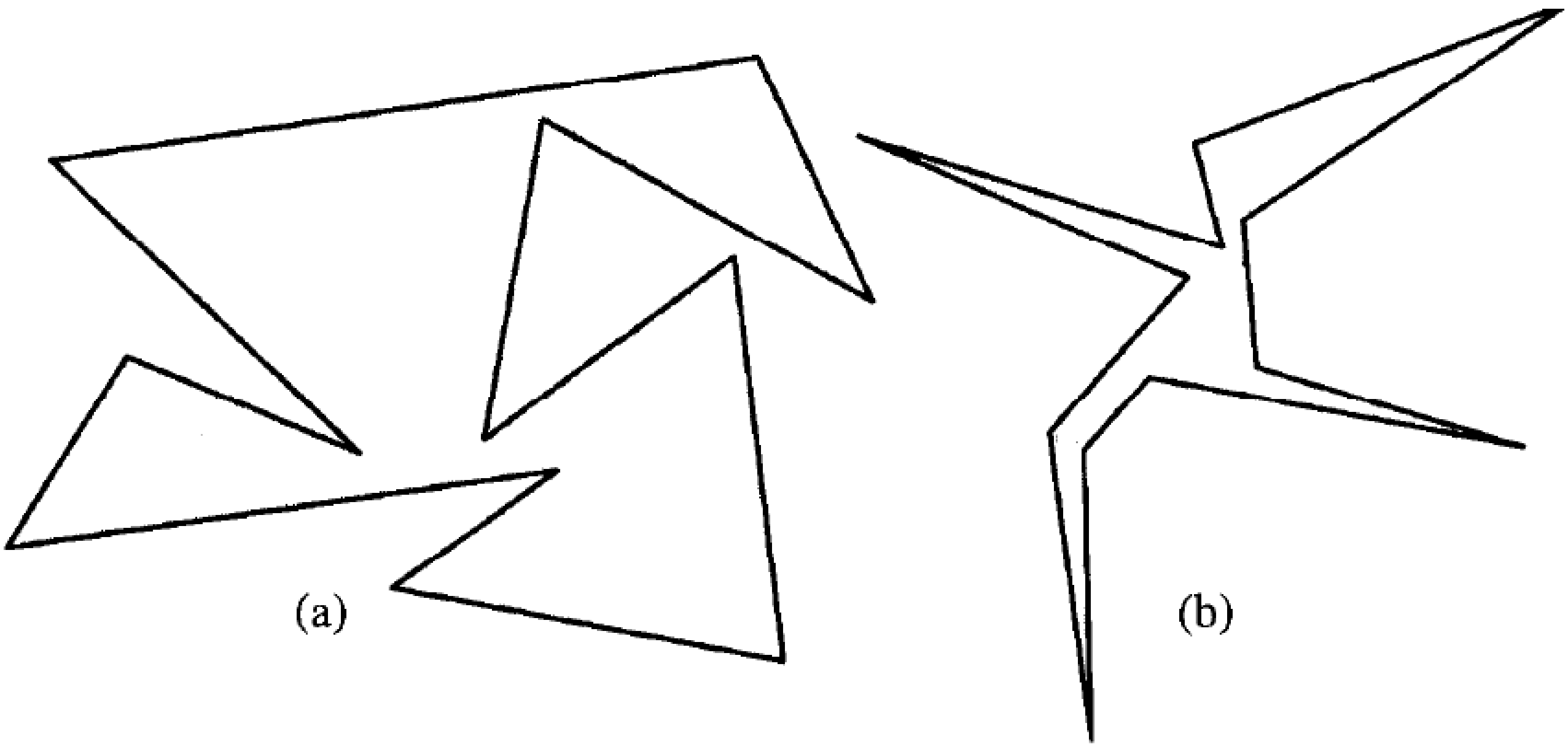
How many guards are needed to cover the Polygon?

- For any given fixed polygon P with *n* vertices, there is some minimum number of guards for complete coverage
- Suppose we obtained *g* as the number of guards for P
- Is *g* the most that is ever needed for all possible polygons of *n* vertices ?

Exercise

- Draw two different polygons P_1 and P_2 with *n* vertices

- Show that P_1 and P_2 require different number of guards



(a) (b)

Example

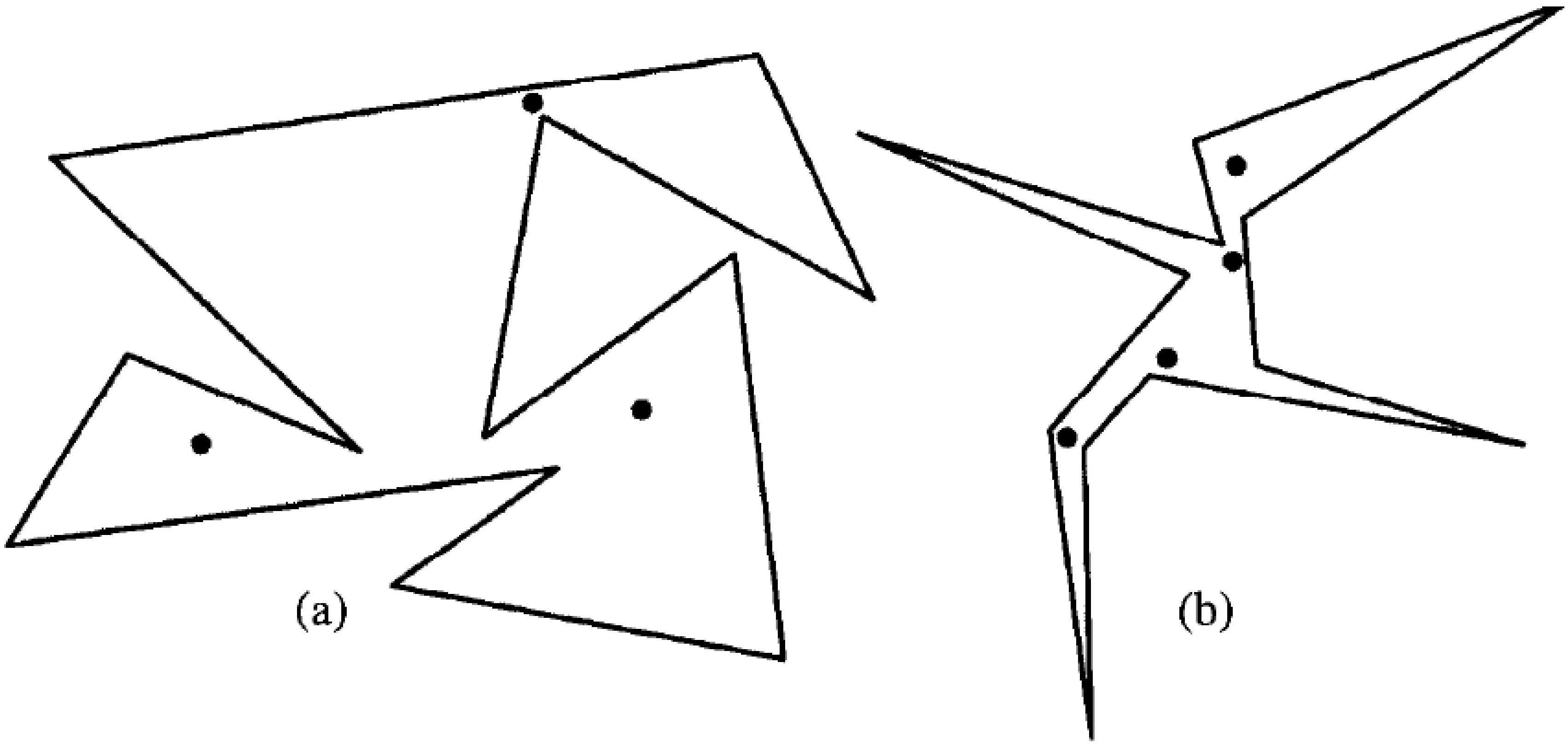


FIGURE 1.3 Two polygons of $n = 12$ vertices: (a) requires 3 guards; (b) requires 4 guards.

Now we know

- For any given fixed polygon P with *n* vertices, there is some minimum number of guards for complete coverage
- Suppose we obtained *g* as the number of guards for P
- Is *g* the most that is ever needed for all possible polygons of *n* vertices ?
- Not necessarily

Our objective

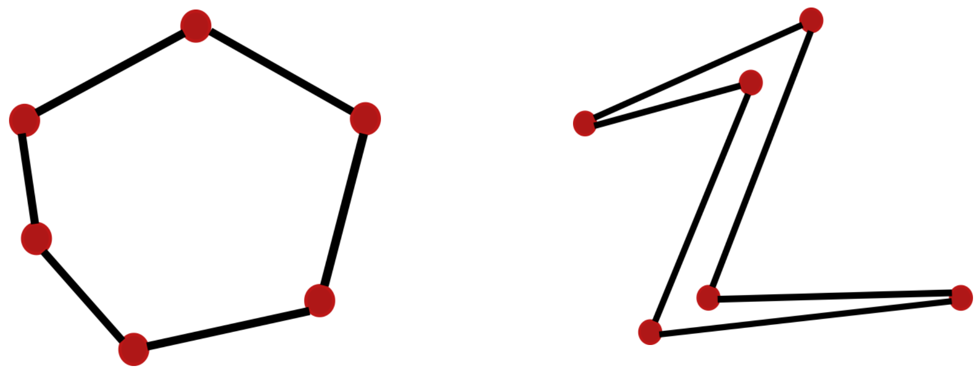
- Find out the largest number of guards that any polygon of *n* vertices needs, where *n* is an integer known to us
- Victor Klee posed this question formally as: **Express as a function of *n*, the smallest number of guards that suffice to cover any polygon of *n* vertices**
- This number of guards is said to be sufficient and necessary to cover P
  - Necessary because at least that many guards are needed for some polygons
  - Sufficient because that many always suffice for any polygon

Formal definition of our objective

- $g(\mathrm{P})$: smallest number of guards needed to cover polygon P
- $g(\mathrm{P}) = \min_S |\{S \text{ covers P}\}|$
  - S is a set of points and $|S|$ is the cardinality of S
- Let P_n be a polygon of n vertices
- $G(n)$ is the maximum of $g(\mathrm{P}_n)$ over all polygons of n vertices
  - $G(n) = \max_{P_n} g(\mathrm{P}_n)$
- Our objective is to find out what is $G(n)$

Our objective in simple words

- There can be different polygons with *n* vertices



- Find the minimum # of guards for each *P*
- $g(P) = \min_S \{S \text{ covers } P\}$
- Get the maximum # among those
- $G(n) = \max_{P_n} g(P_n)$
- **Max over Min Formulation**

Empirical exploration to compute G(n)

- For a triangle, $G(3)$ = ?
- For a quadrilateral, $G(4)$ = ?
- For a pentagon, $G(5)$ = ?
- For a hexagon, G(6) = ?
- For a heptagon, G(7) = ?
- For an octagon, G(8) = ?
- For a nonagon (enneagon), G(9)=?
- For a decagon, G(10) = ?
- For a hendecagon (undecagon or endecagon), G(11) = ?
- For a dodecagon, G(12) = ?

Empirical exploration to compute G(n)

- For a triangle, $G(3) = 1$
- For a quadrilateral, $G(4) = 1$
- For a pentagon, $G(5) = 1$
- For a hexagon, G(6) = 2
- For a heptagon, G(7) = 2
- For a octagon, G(8) = 2
- For a nonagon (enneagon), G(9)=3
- For a decagon, G(10) = 3
- For a hendecagon (undecagon or endecagon), G(11) = 3
- For a dodecagon, G(12) = 4

Reference

- J. O Rourke, *Computational Geometry in C*, 2/e, Cambridge University Press, 1998

Thank you