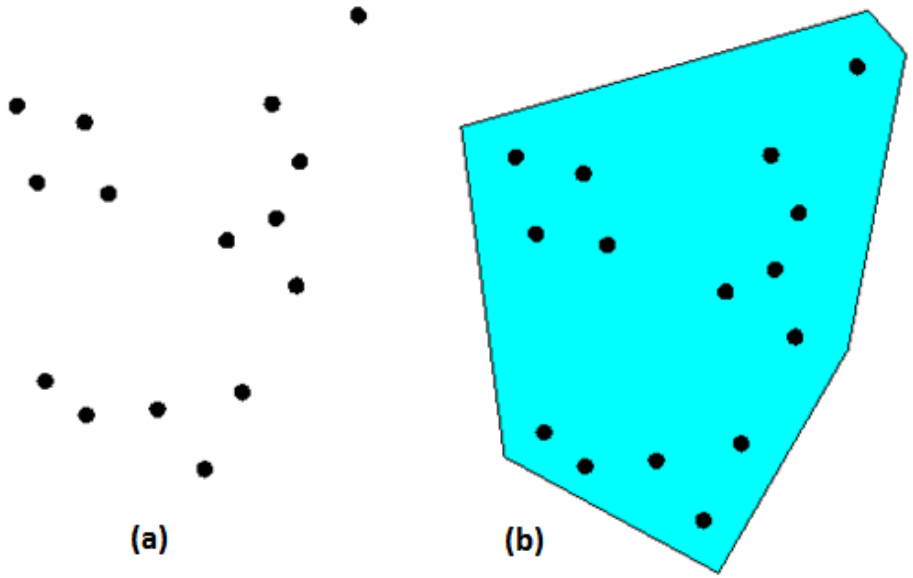
# Application of Delaunay Triangulation

- One of the important area where Delaunay Triangulation is used:

- **RECONSTRUCTION / CONTOURING / BOUNDARY DETECTION**

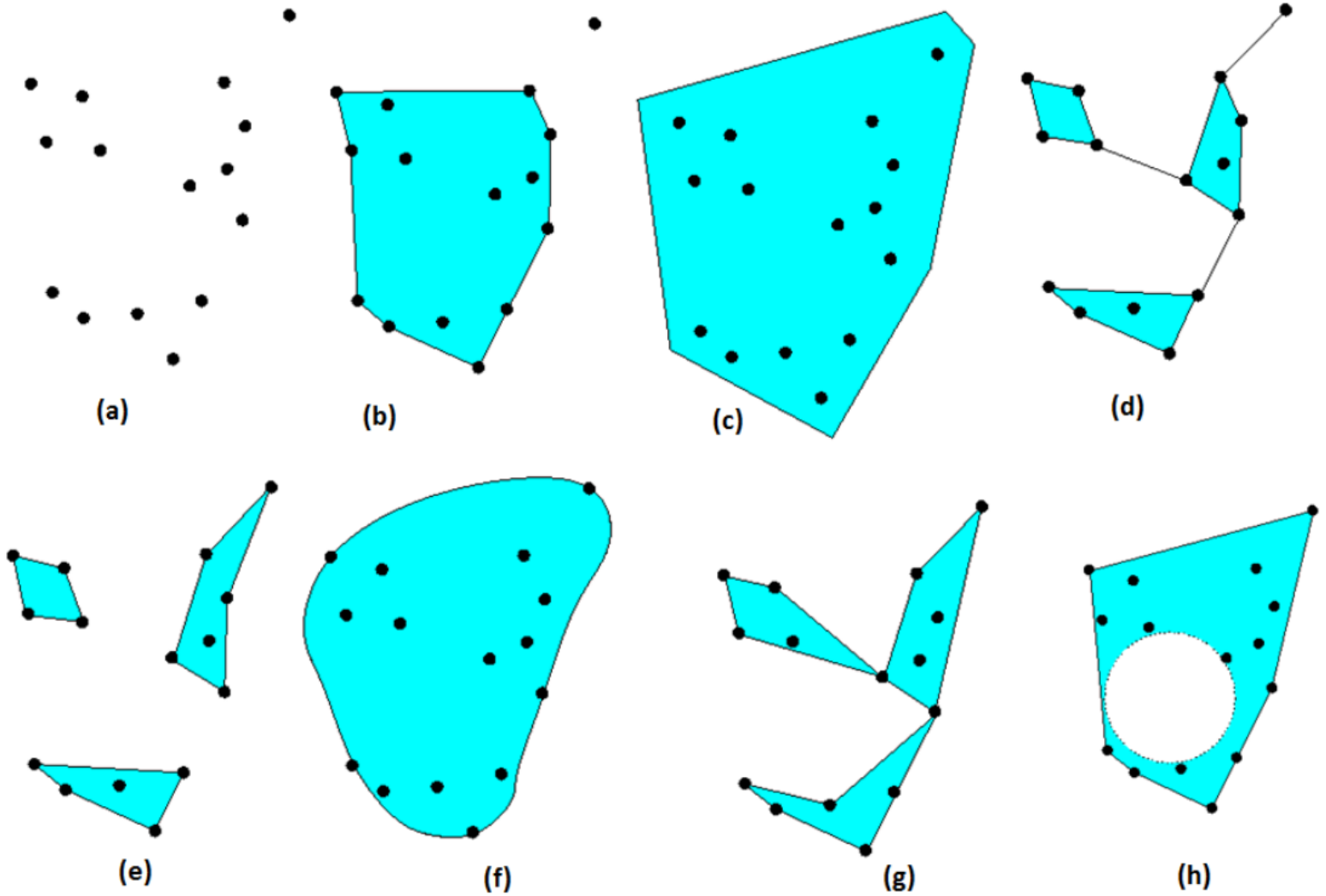
# Reconstruction problem

- To construct a region that characterizes the shape of a given finite set of points (P) in the plane
- Alternate Definition: Computation of an approximation of the unknown shape induced by a given point set



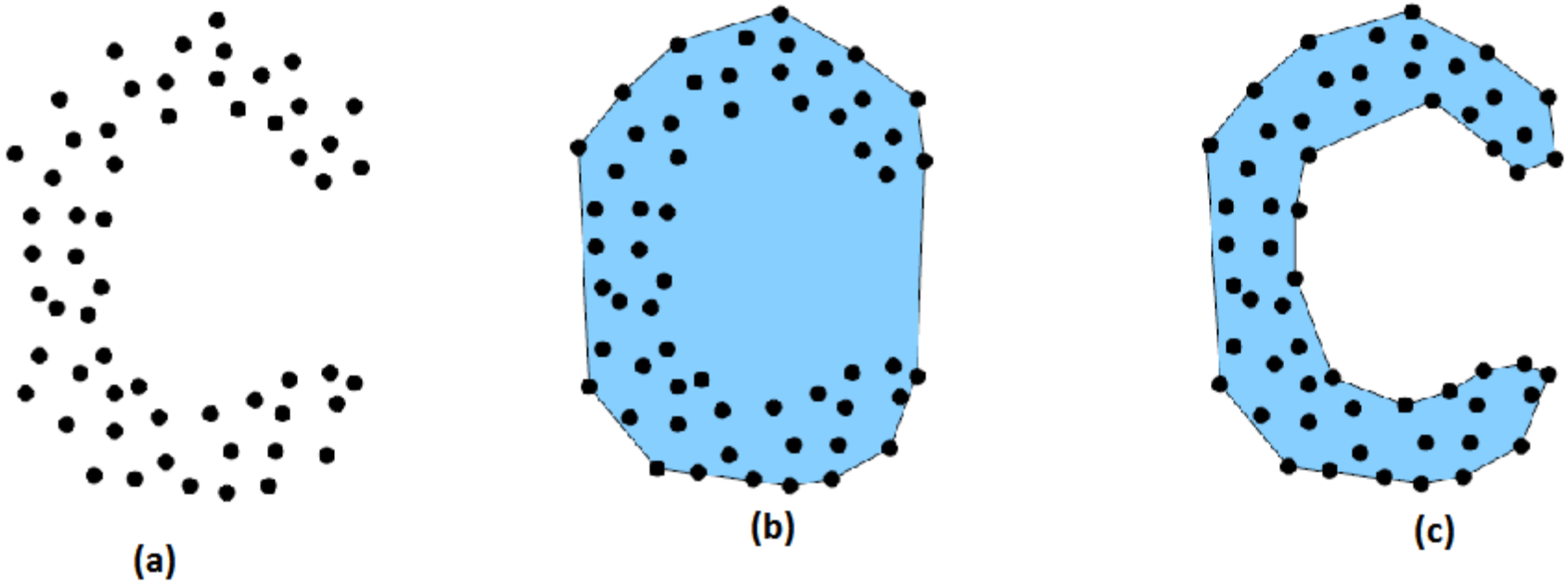

- Is this the only region occupied by P?

# Different regions for the same point set




- Which region characterizes P?
- Is convex hull the solution?

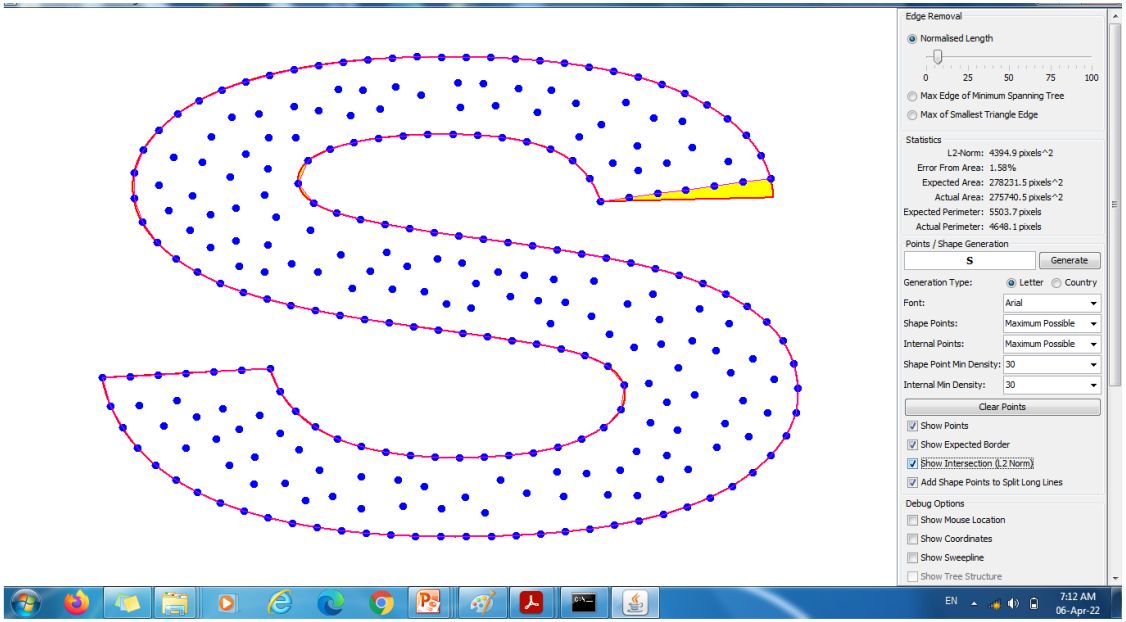
# Convex hull

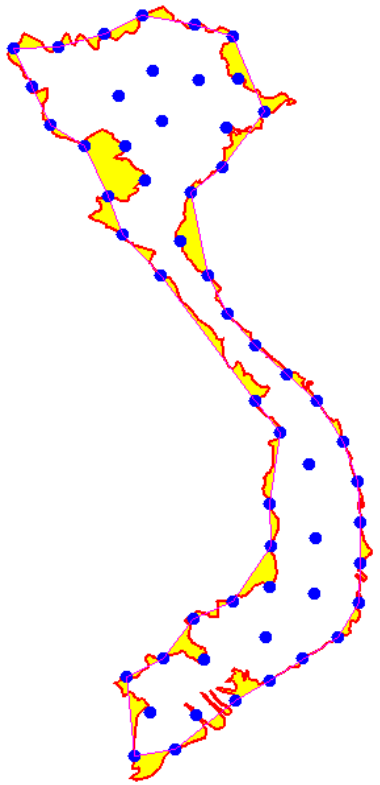



- Convex hull does not capture the shape characterized by P

# χ-shape (Characteristic shape)

- Algorithm to construct Characteristic shape is proposed by Matt Duckham, Lars Kulik, Michael F. Worboys, and Antony Galton
- Efficient generation of simple polygons for characterizing the shape of a set of points in the plane, Pattern Recognition, 41(10):3224- 3236, 2008.

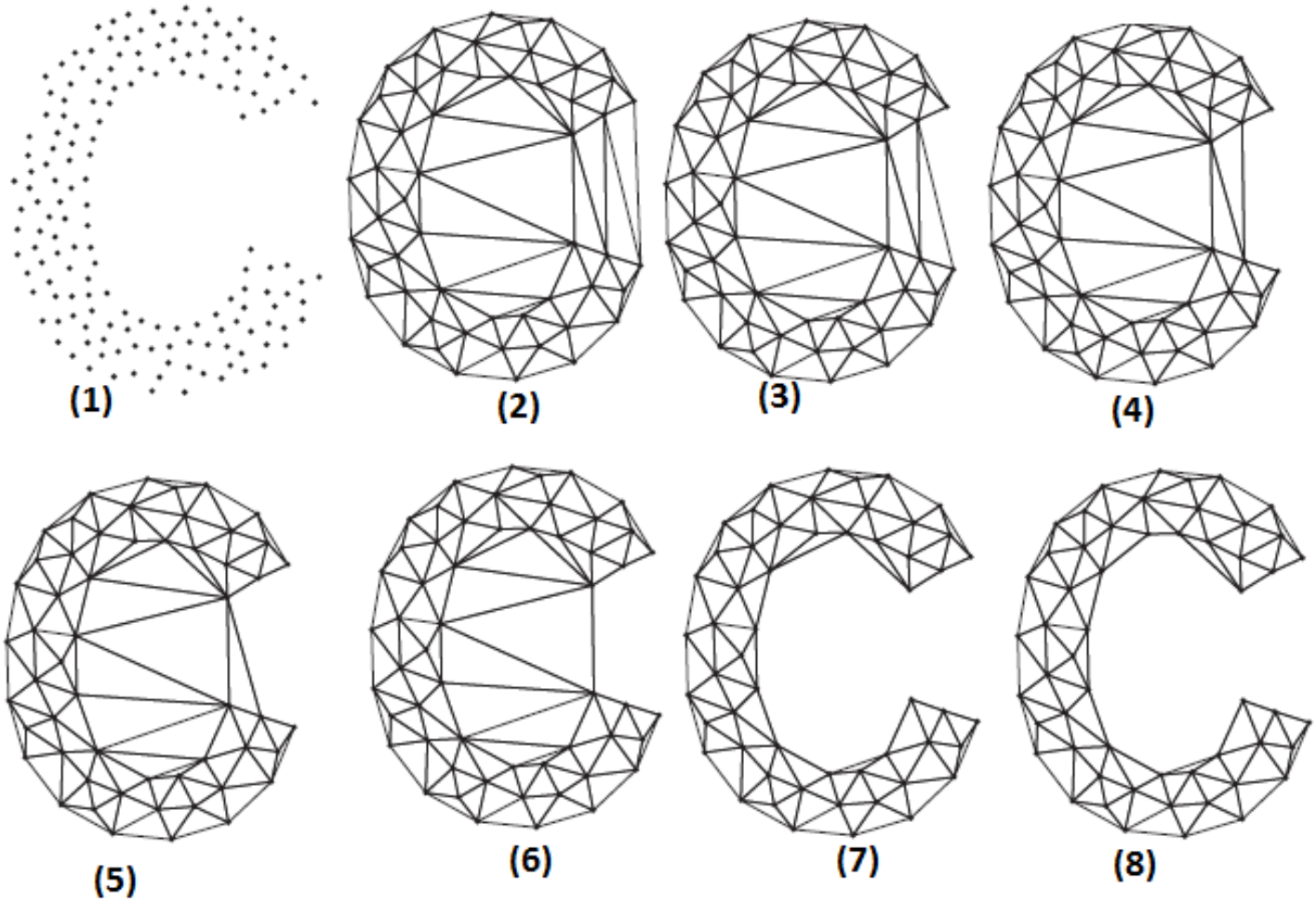


# χ-shape (Characteristic shape) Algorithm

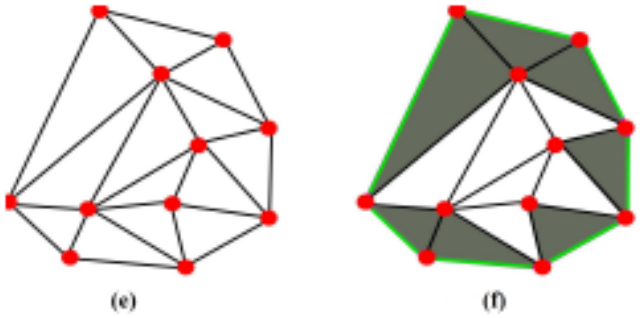
- Input : Set of points P
- Output : χ-shape
- It is a simple polygon
- It contains all the points of P ; and
- It bounds an area contained within and possibly equal to the convex hull of the points of P.
- Algorithm is based on Delaunay Triangulation of a given set of points

# Illustration of the algorithm



# χ-shape Algorithm

- The algorithm is based on “shaving” (removing) exterior edges from a Delaunay triangulation based on:
- Length of the edges --- Constraint 1
- Regularity constraint --- Constraint 2
- Exterior triangles and exterior edges

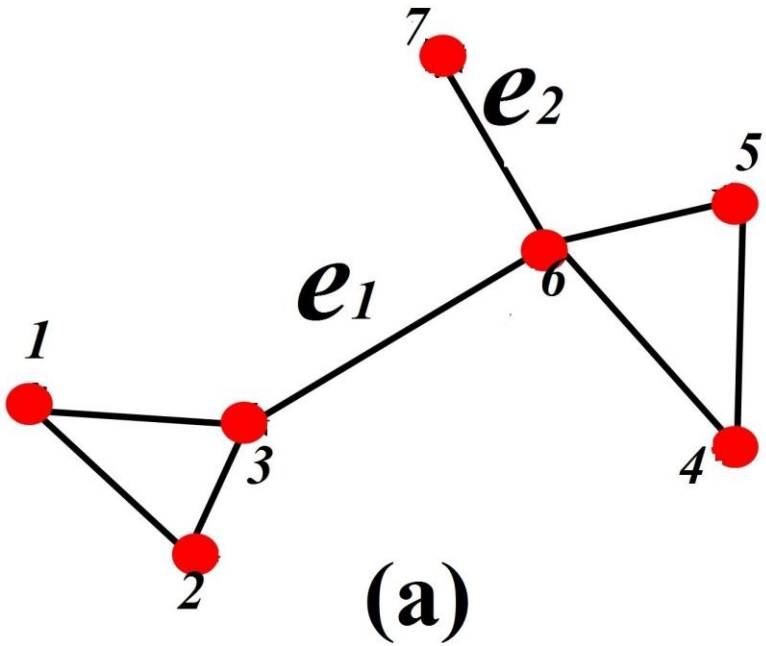


# Constraint 1 :Length

- An edge is removed if the length of the edge is greater than a user given parameter
- Otherwise it is retained in the Delaunay Triangulation itself
- This gives a flexibility for the user to try out different parameters and see which output is better

# Constraint 2 :Regularity

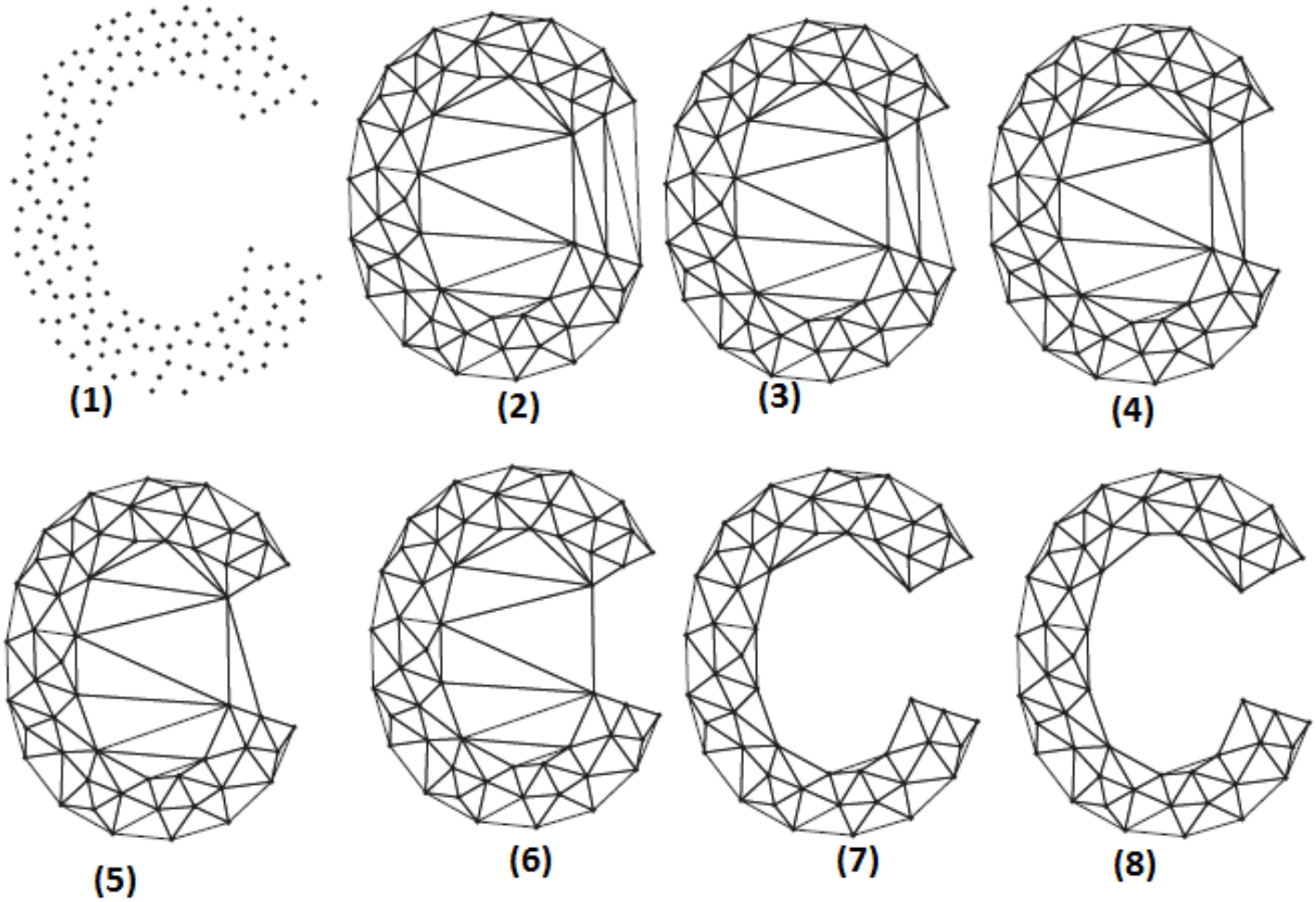
- An edge is removed, if after its removal, the resultant graph satisfies a regularity constraint
- If the resultant graph(after removal of an edge) does not have cut vertex, bridge or dangling edge, then it satisfies regularity constraint



# χ-shape Algorithm

(1) Generate the Delaunay triangulation of the set of input points $P$;
(2) Remove the longest exterior edge from the triangulation such that:
    (a) the edge to be removed is longer than the length parameter $l$; and
    (b) the exterior edges of the resulting triangulation form the boundary of a simple polygon;
(3) Repeat 2. as long as there are more edges to be removed
(4) Return the polygon formed by the exterior edges of the triangulation

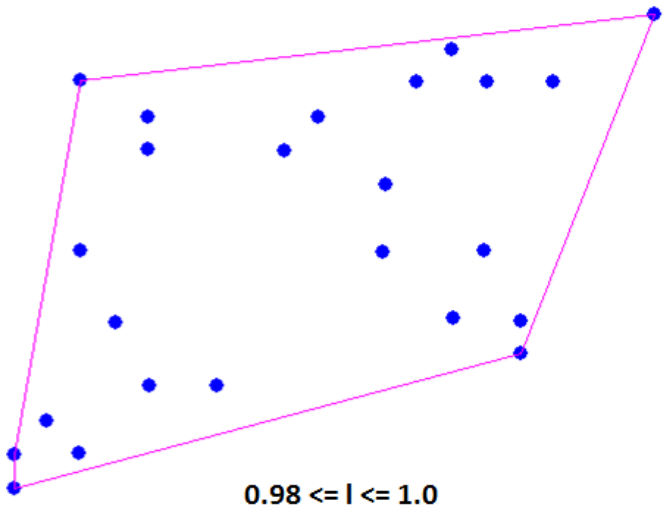
# Illustration of the algorithm

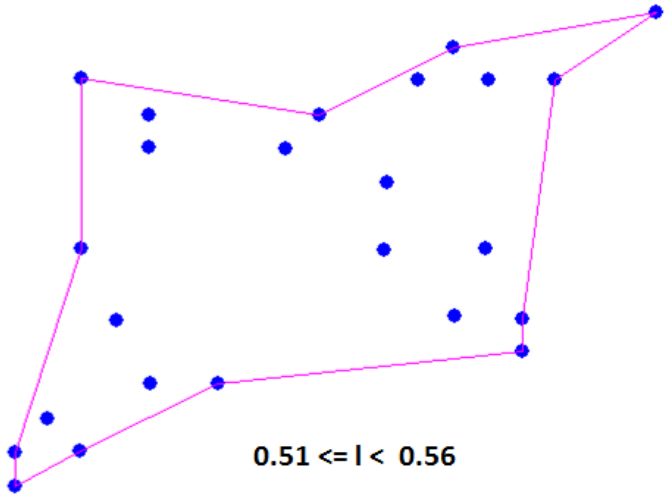


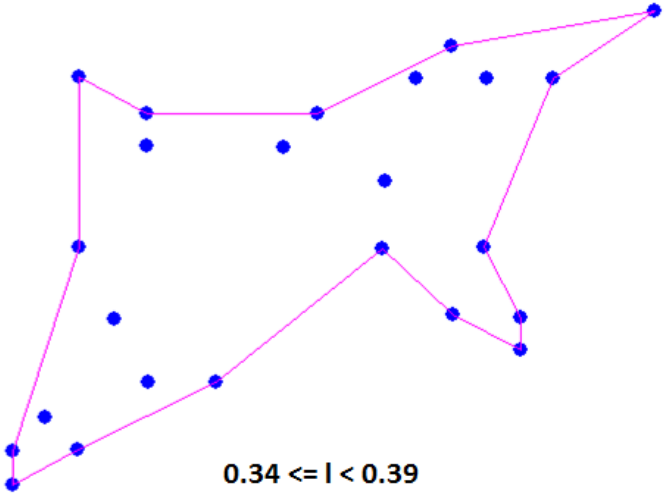
# Parameter – length (l)

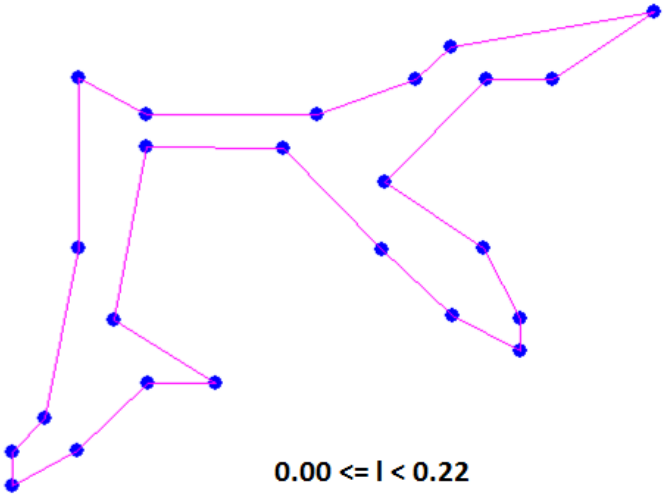
- Global parameter – value given by the user
- Gives flexibility for the user to visually choose the reconstructed output

# Different values of length - illustration

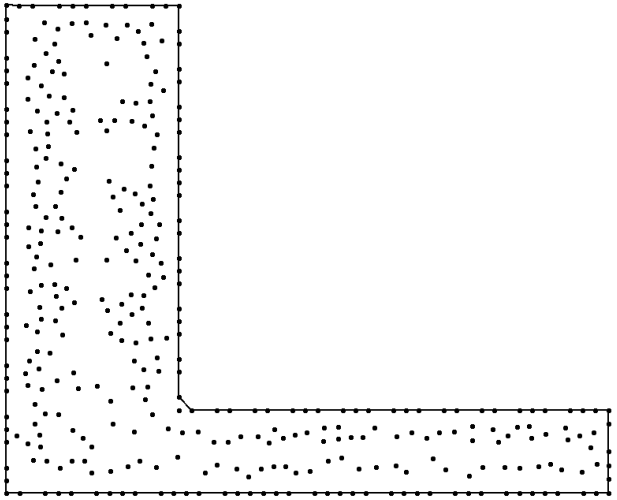


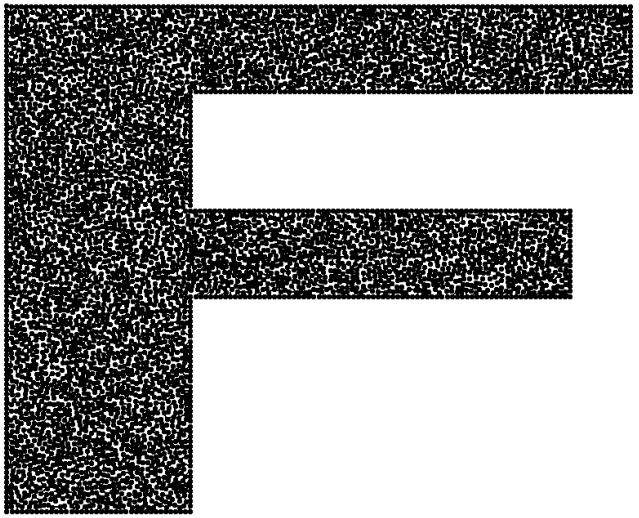



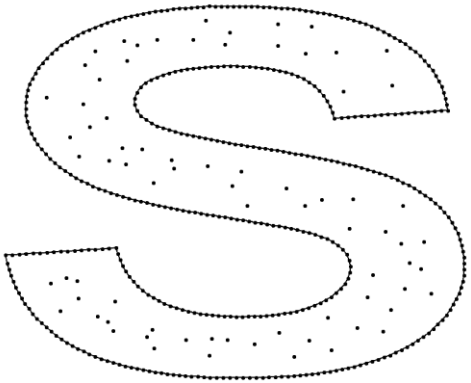



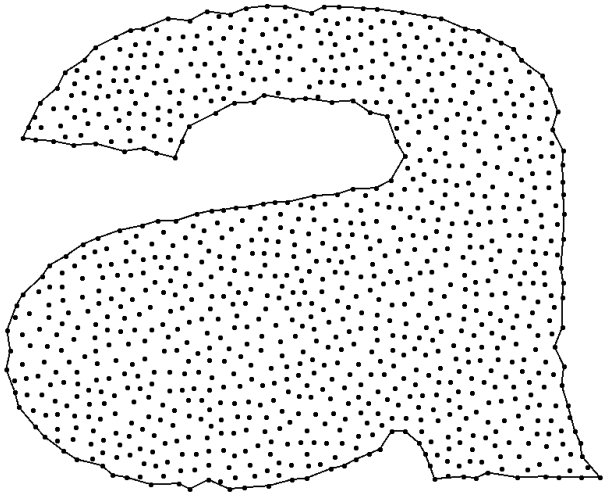



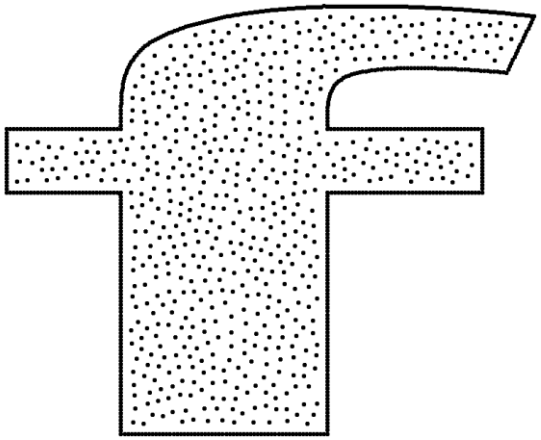
# Few results of χ-shape Algorithm



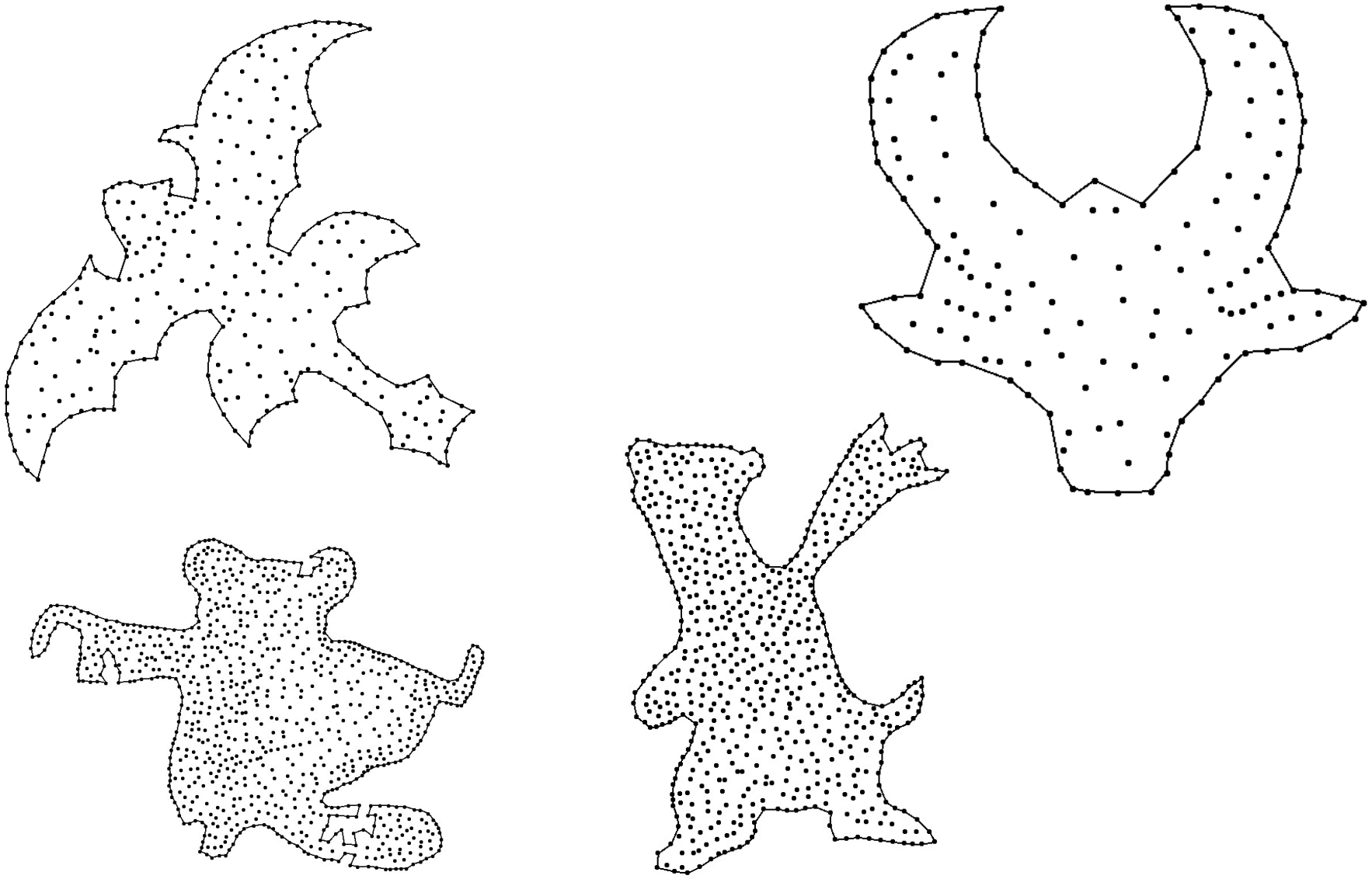








# Few results of χ-shape Algorithm



# Another Reconstruction Algorithm

## α-shape

# α-shape

- H. Edelsbrunner, D. G. Kirkpatrick, R. Seidel, On the shape of a set of points in the plane, IEEE Transactions on Information Theory 29 (4) (1983) 551–558.
- α-shape algorithm : The software is available in Demo Folder of CGAL – 2D Alpha Shape

# α-shape

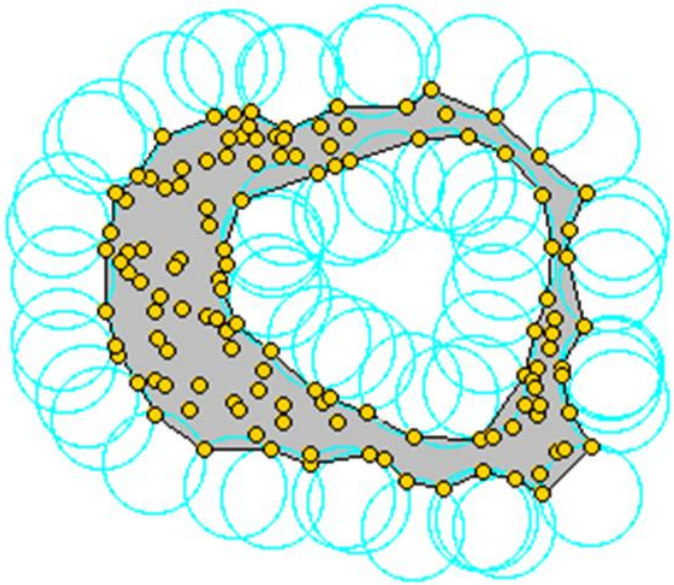
- α-shape : One of the earliest attempt to characterize a set of points is by Edelsbrunner et al.
- H. Edelsbrunner, D. G. Kirkpatrick, R. Seidel, On the shape of a set of points in the plane, IEEE Transactions on Information Theory 29 (4) (1983) 551–558.
- When it was proposed in 1983, it was not meant for reconstruction purposes
- Later its 3D counter part was designed for reconstruction
- EDELSBRUNNER H., MUCKE E. P.: Three-dimensional alpha shapes. ACM Trans. Graph. 13, 1 (Jan. 1994), 43–72.

# α-shape

- Input : A set of points
- Output: α-shape
- A shape which characterizes the set of points
- Not necessarily a polygon
- It can be disconnected components

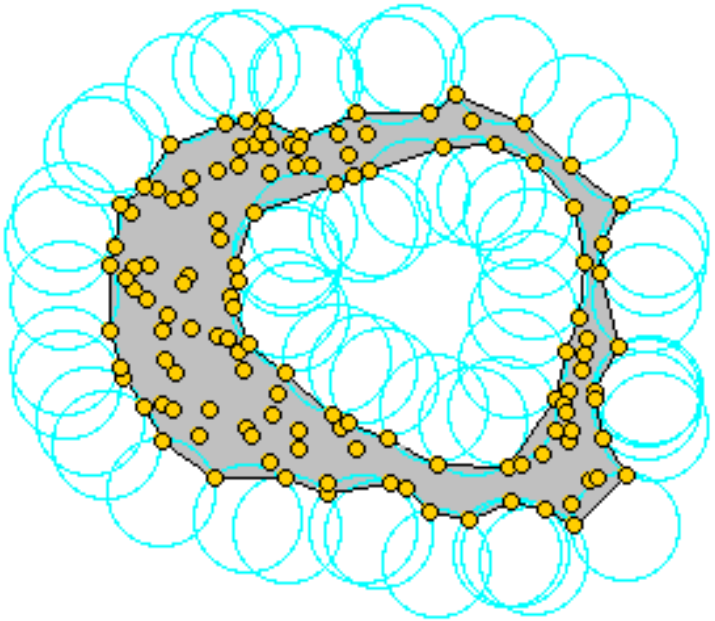
# α-shape : Intuition

- Imagine a huge mass of ice-cream making up the space $R^3$ and containing the points as "hard" chocolate pieces.
- Using one of these sphere-formed ice-cream spoons, carve out all parts of the ice-cream block, without bumping into chocolate pieces, thereby even carving out holes in the inside (e.g. parts not reachable by simply moving the spoon from the outside).

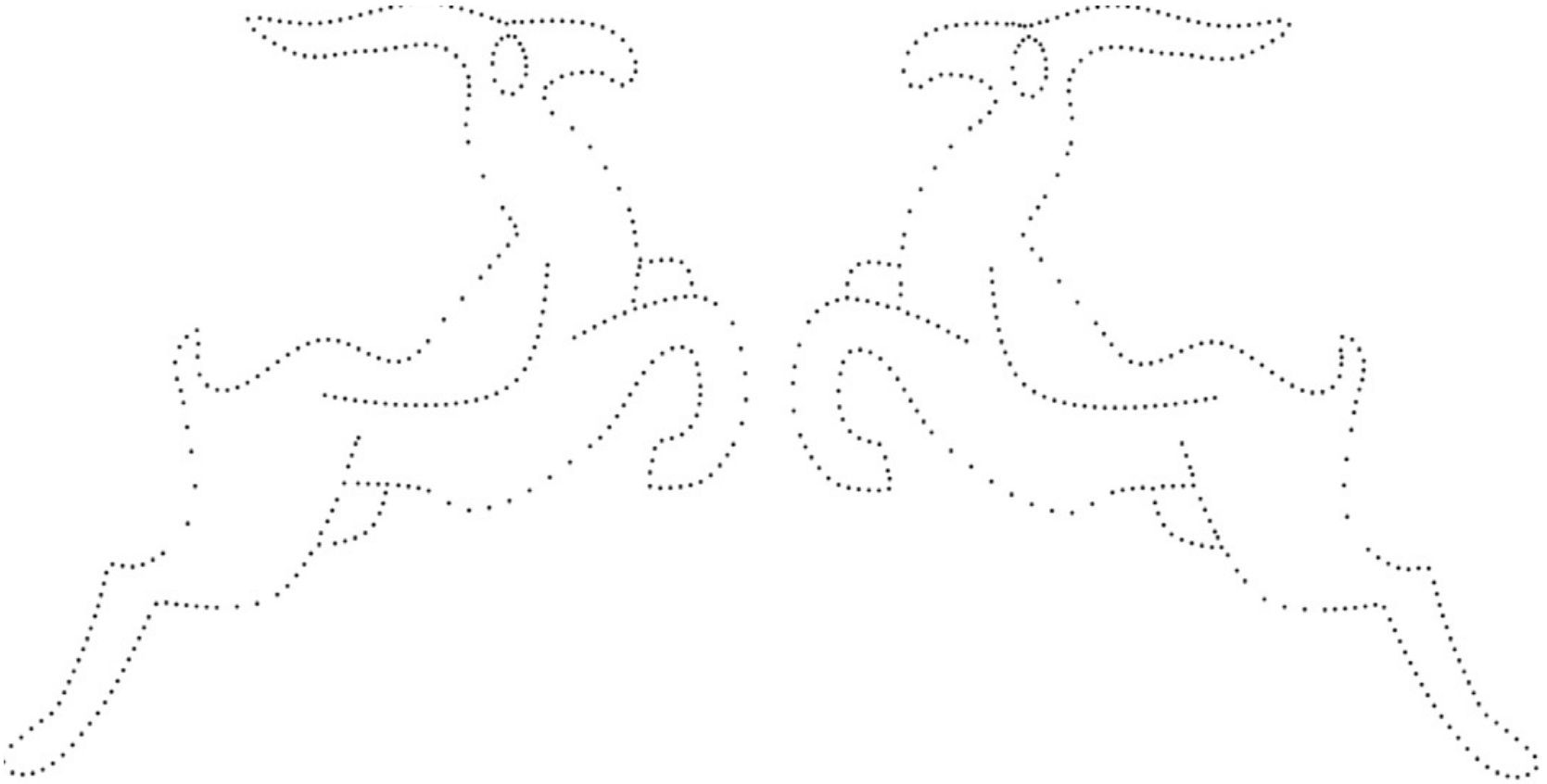


# α-shape

- We will eventually end up with a (not necessarily convex) object bounded by caps, arcs and points. If we now straighten all "round" faces to triangles and line segments, we have an intuitive description of what is called the *α*-shape of *S*. The drawing above provides an example of this process in 2D (where our ice-cream spoon is simply a circle).

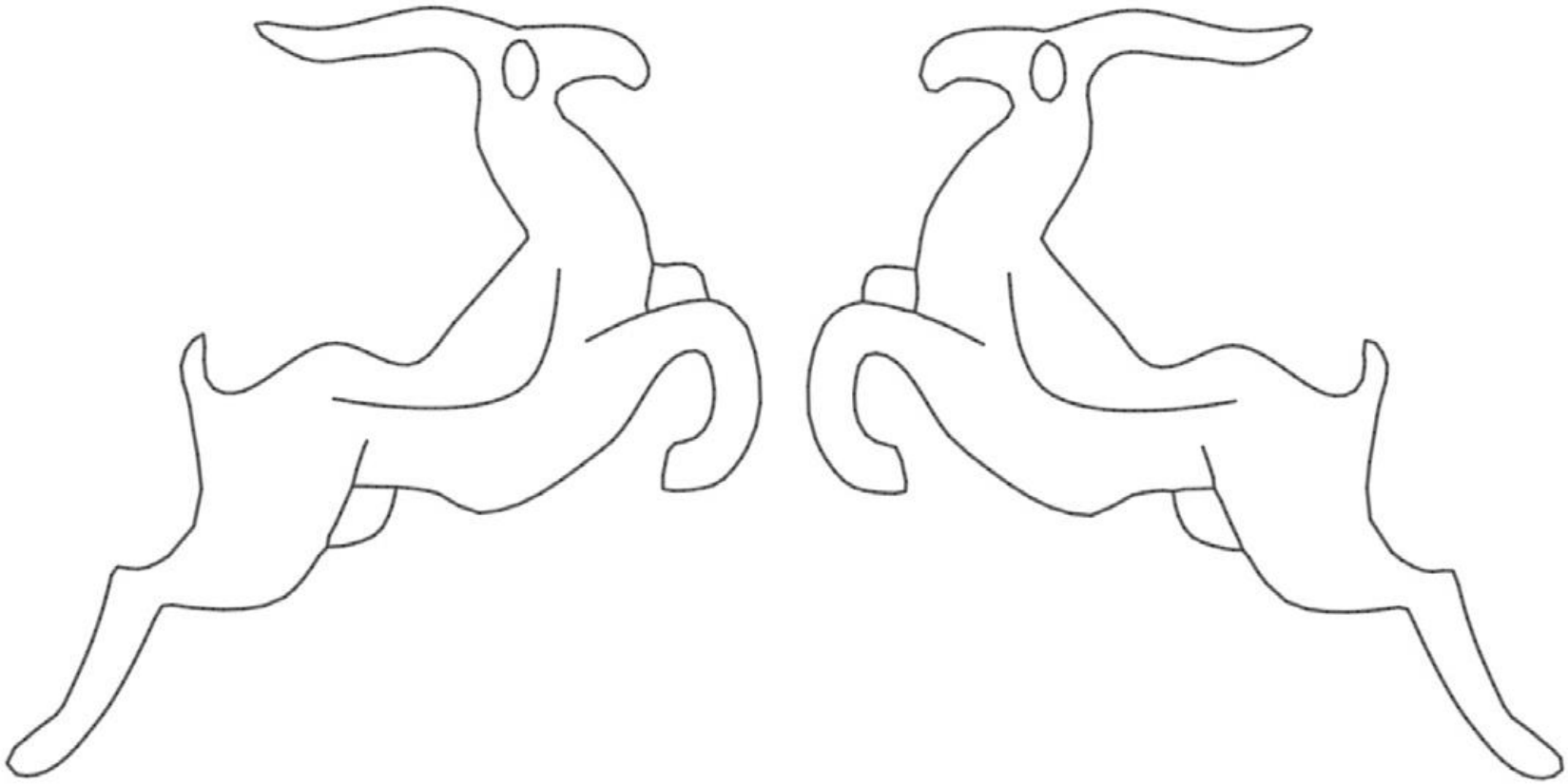


# An input

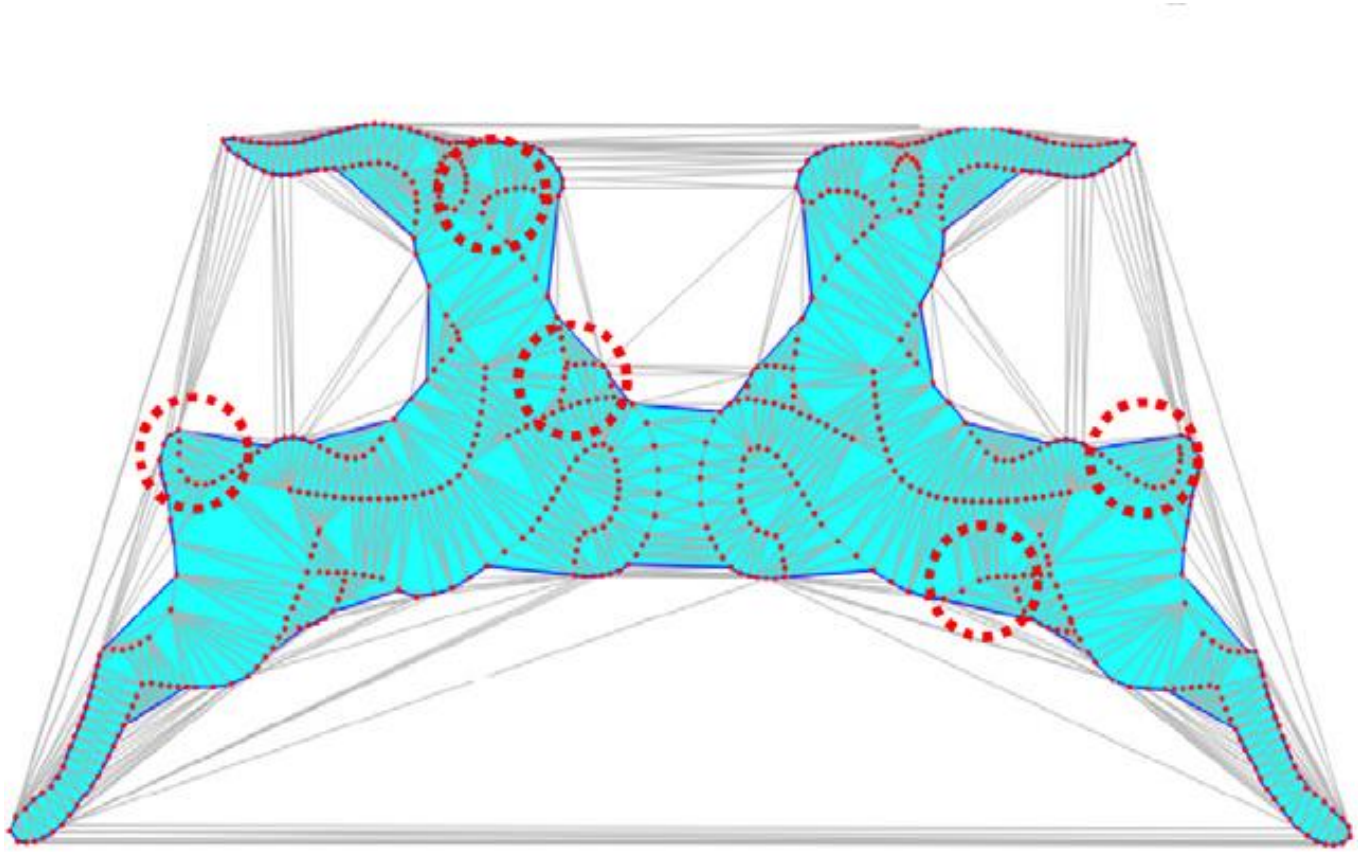


(a) Input

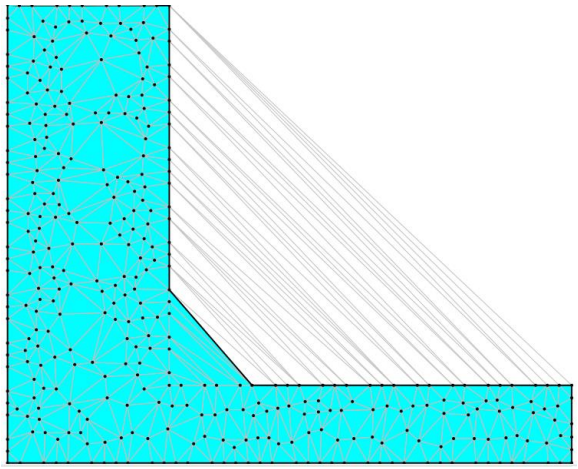
# Expected output

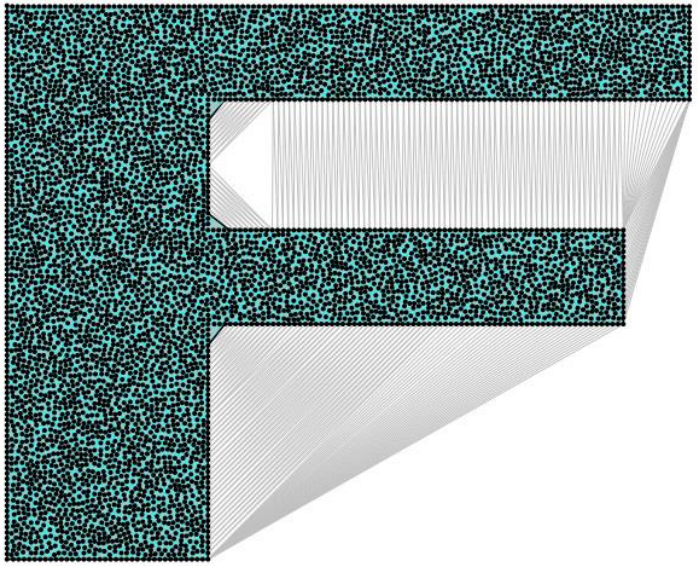


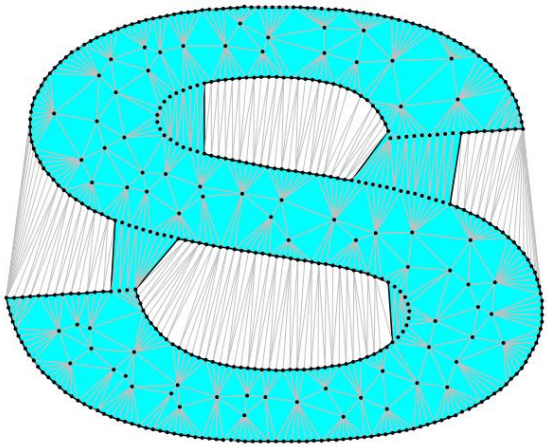
# The best Alpha shape obtained

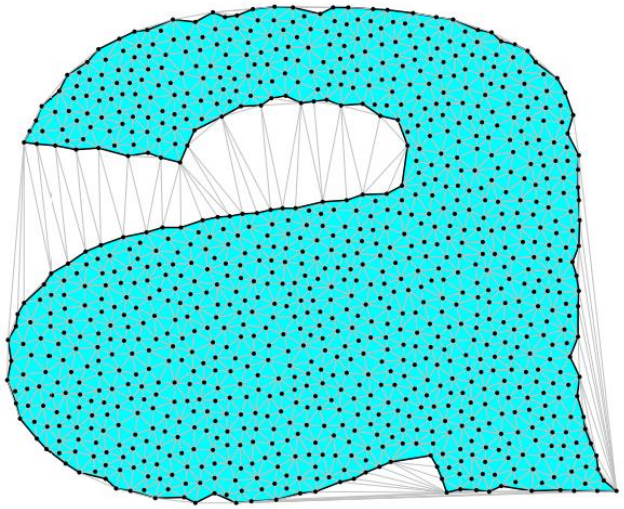


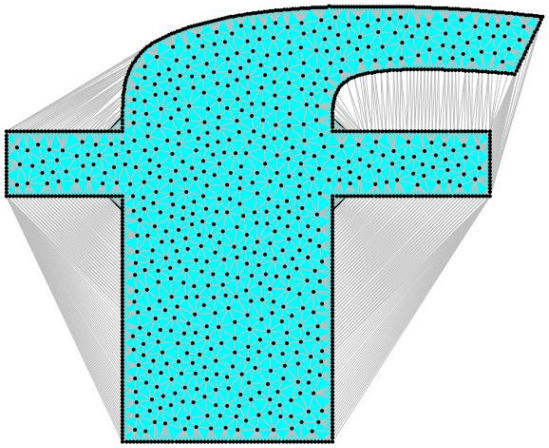
# Some results of α-shape algorithm





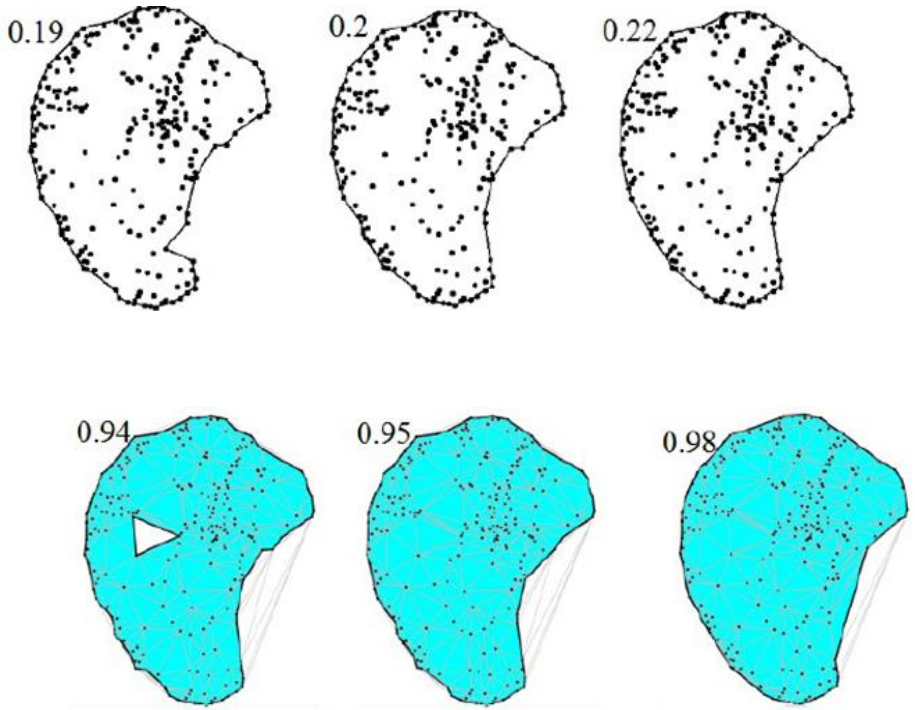






# Parametric algorithms

- Both the algorithms are parametric algorithms
- χ -shape
- α-shape



# Parameter tuning-Pros & Cons

- **Pros**- flexibility to choose the best shape
- **Cons** :
- Human involvement->errors
- Requires prior knowledge on the shape
- Consumes user time
- Labor intensive and expensive

# Project for MVR

Figure 5: Input image converted to jpeg

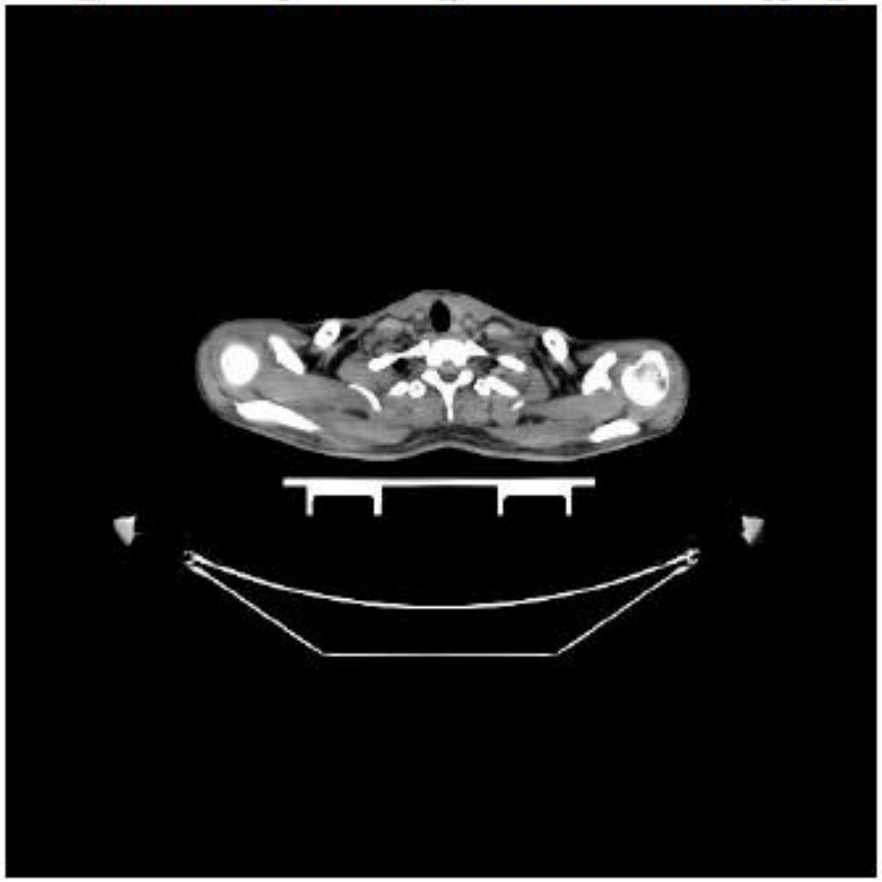


Figure 6: Selecting tumour area

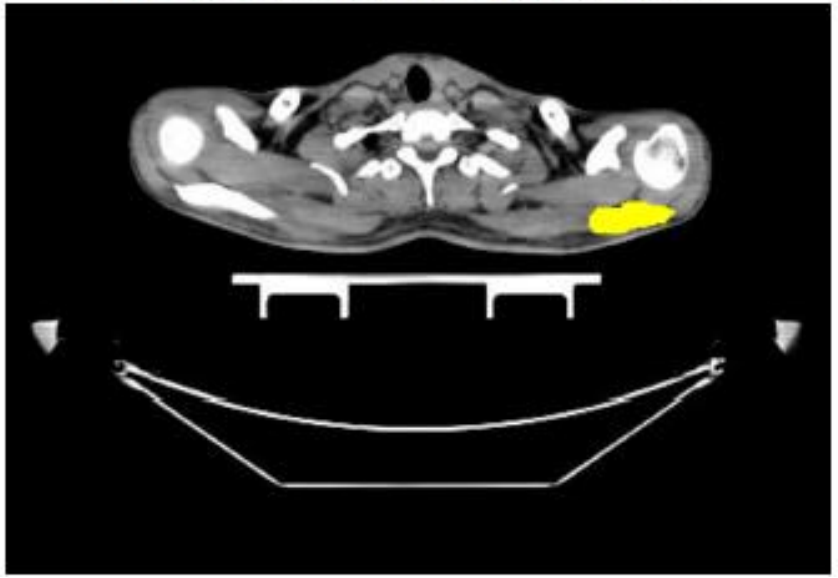
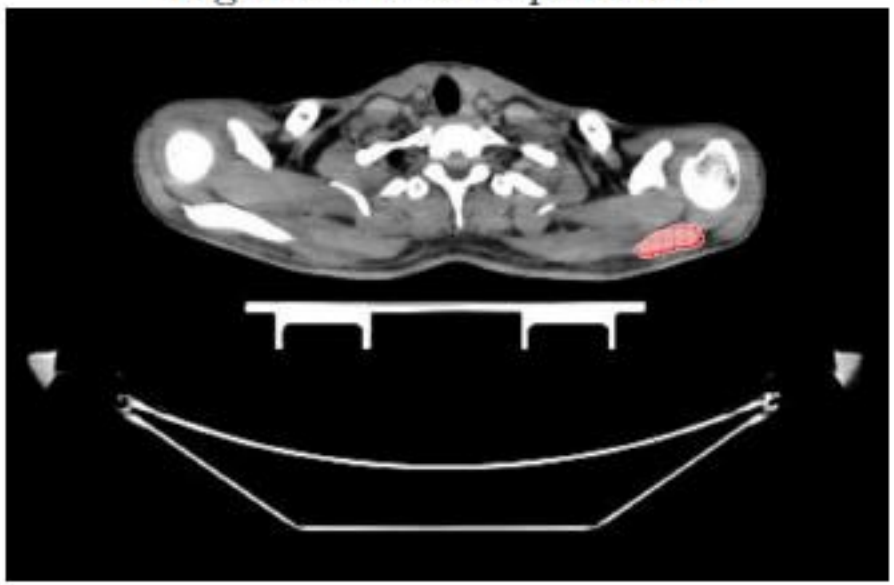
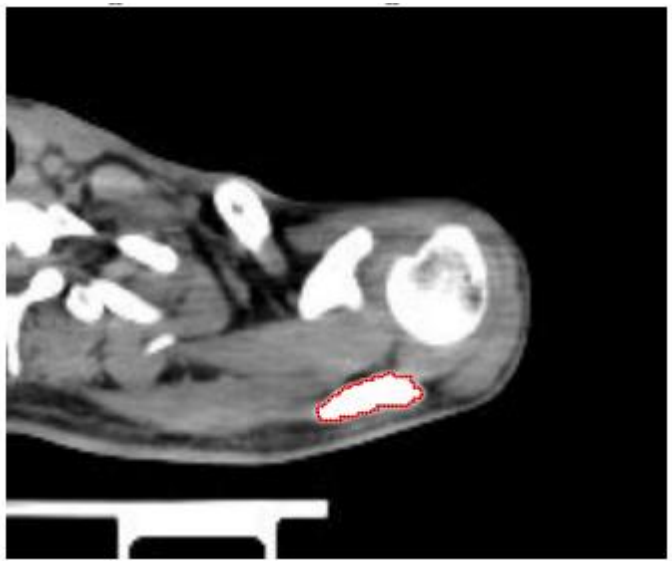


Figure 7: Create point set



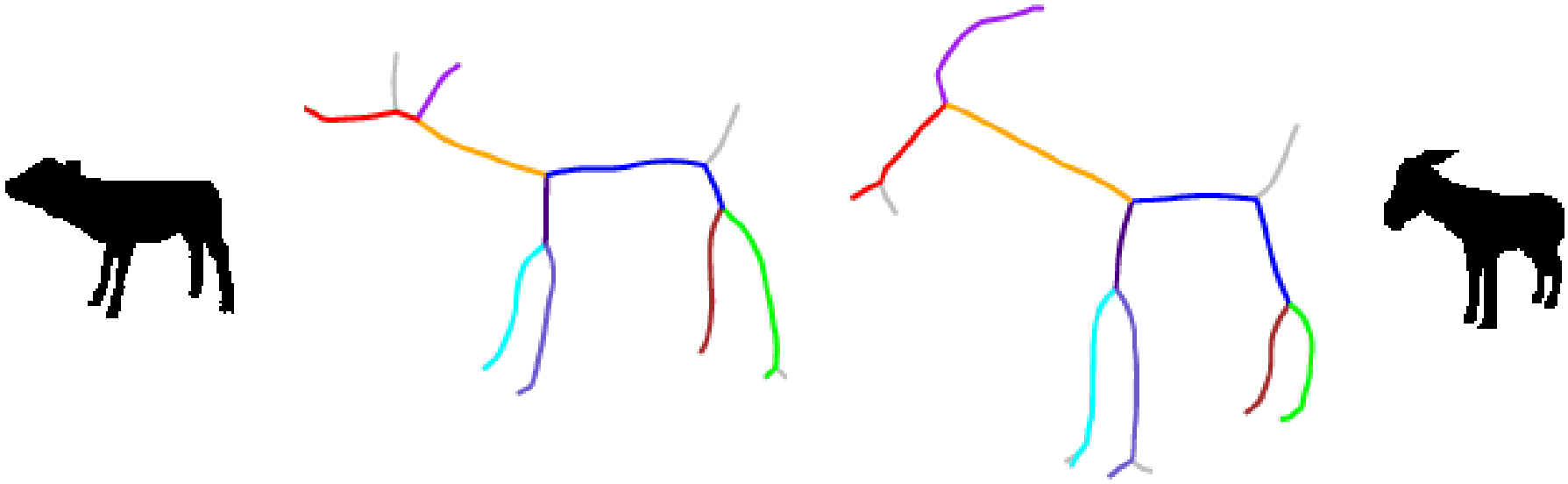


# Medial Axis

# Medial Axis

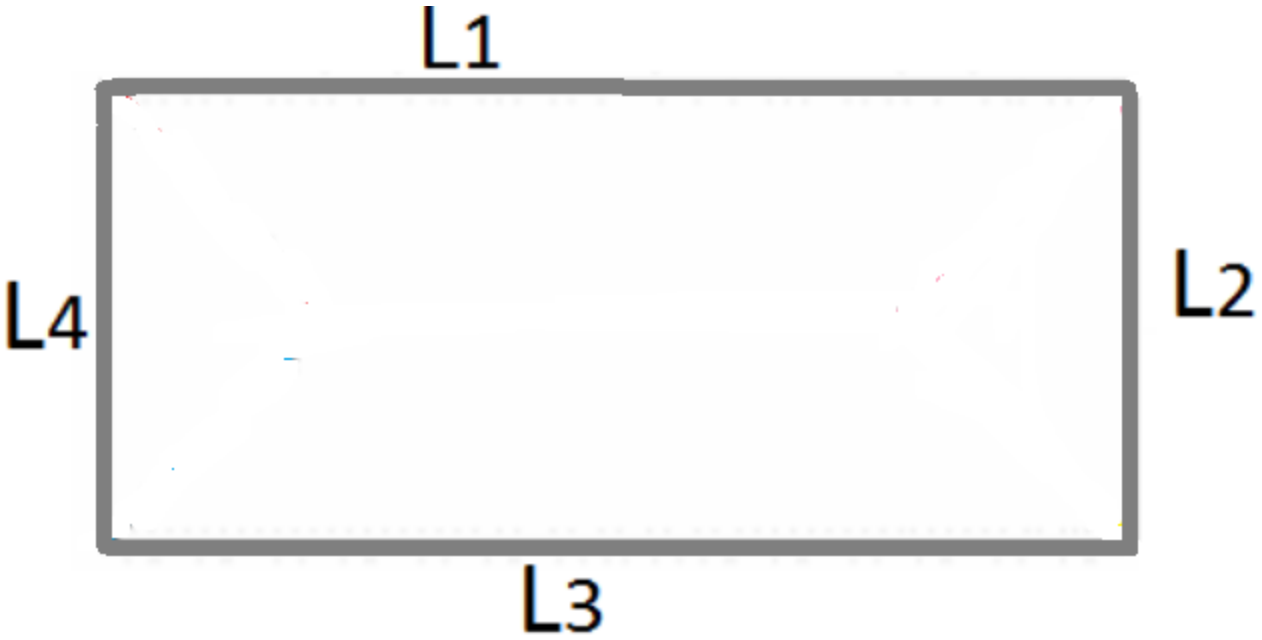
- Medial axis of a shape is a shape descriptor of that shape

# Other examples



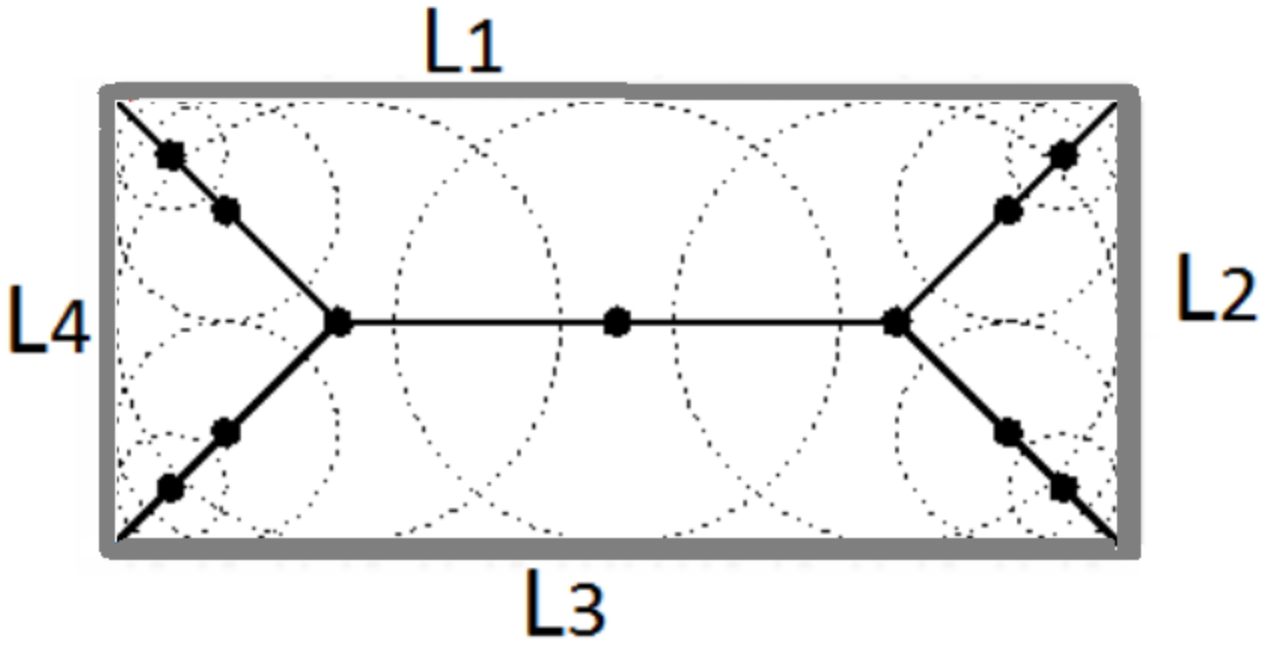
# Medial Axis

- Medial axis was first introduced by Blum in 1967 to describe biological shape



# Medial Axis

- It can be viewed as the locus of the centre of a maximal disk as it rolls inside an object

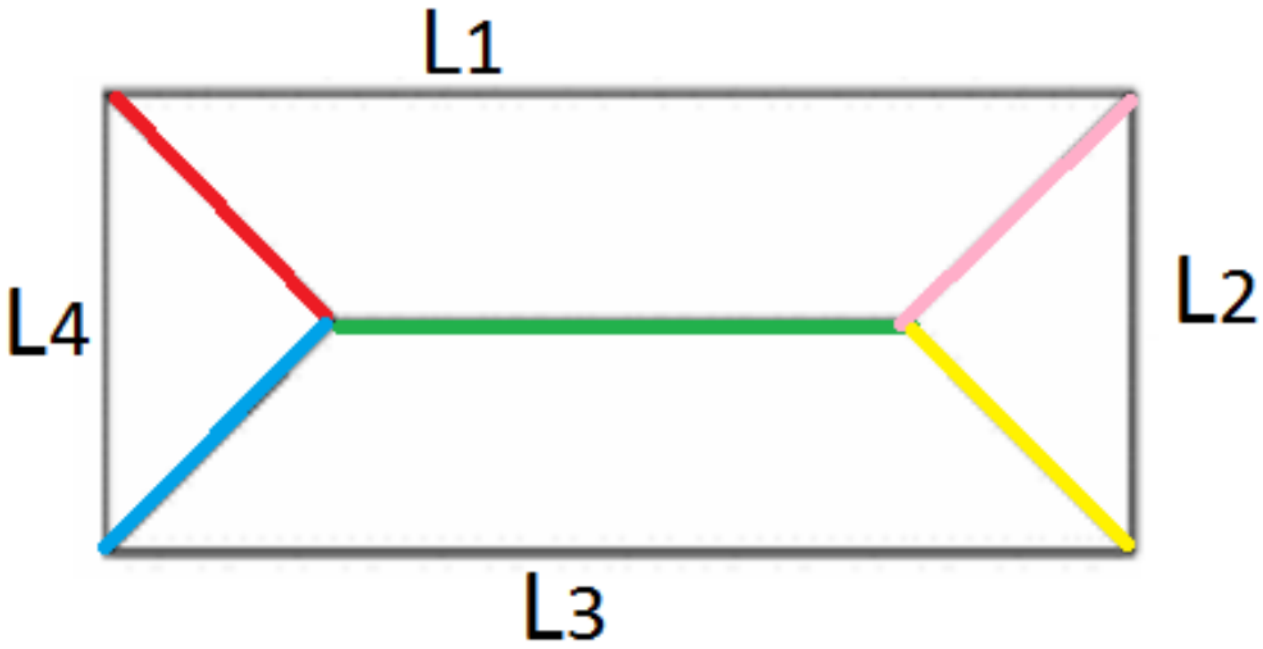


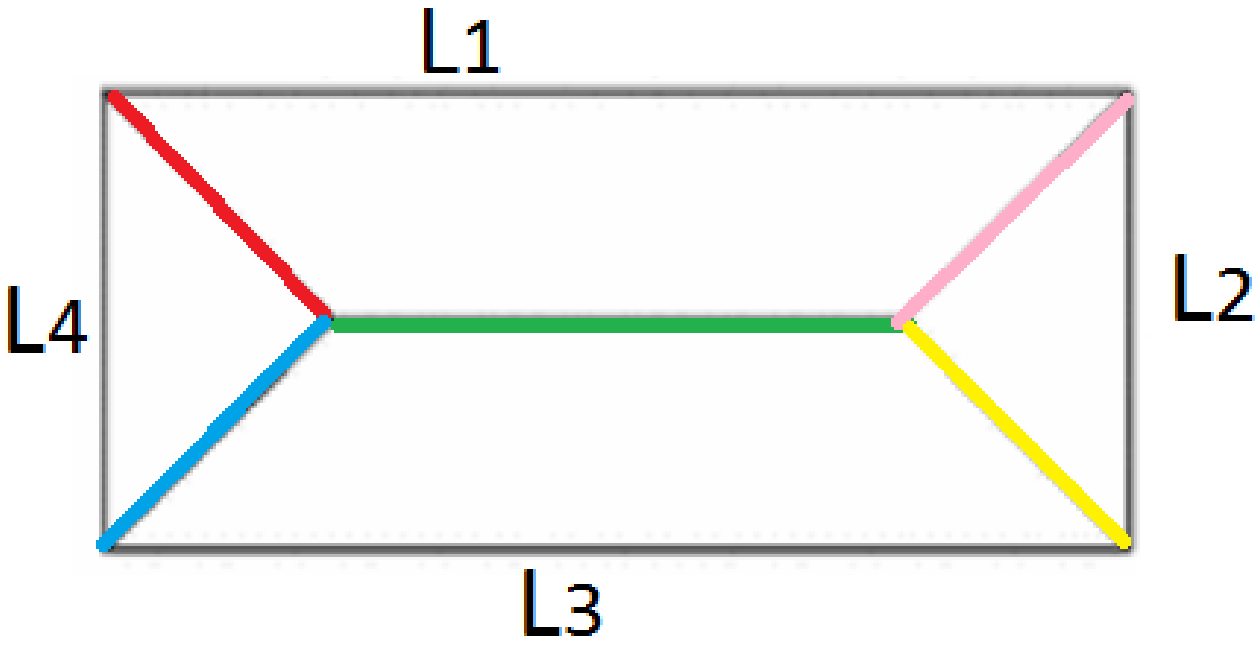

- A closed disc (or ball) is said to be maximal in a subset D of the 2-D space if it is contained in D but is not a proper subset of any other disc (or ball) contained in D.

# Medial Axis Transform (MAT)

- The medial axis together with the associated radius function of the maximally inscribed discs is called the **medial axis transform** (**MAT**).
- The medial axis transform is a complete shape descriptor
- It can be used to reconstruct the shape of the original domain.

# How to draw Medial Axis?

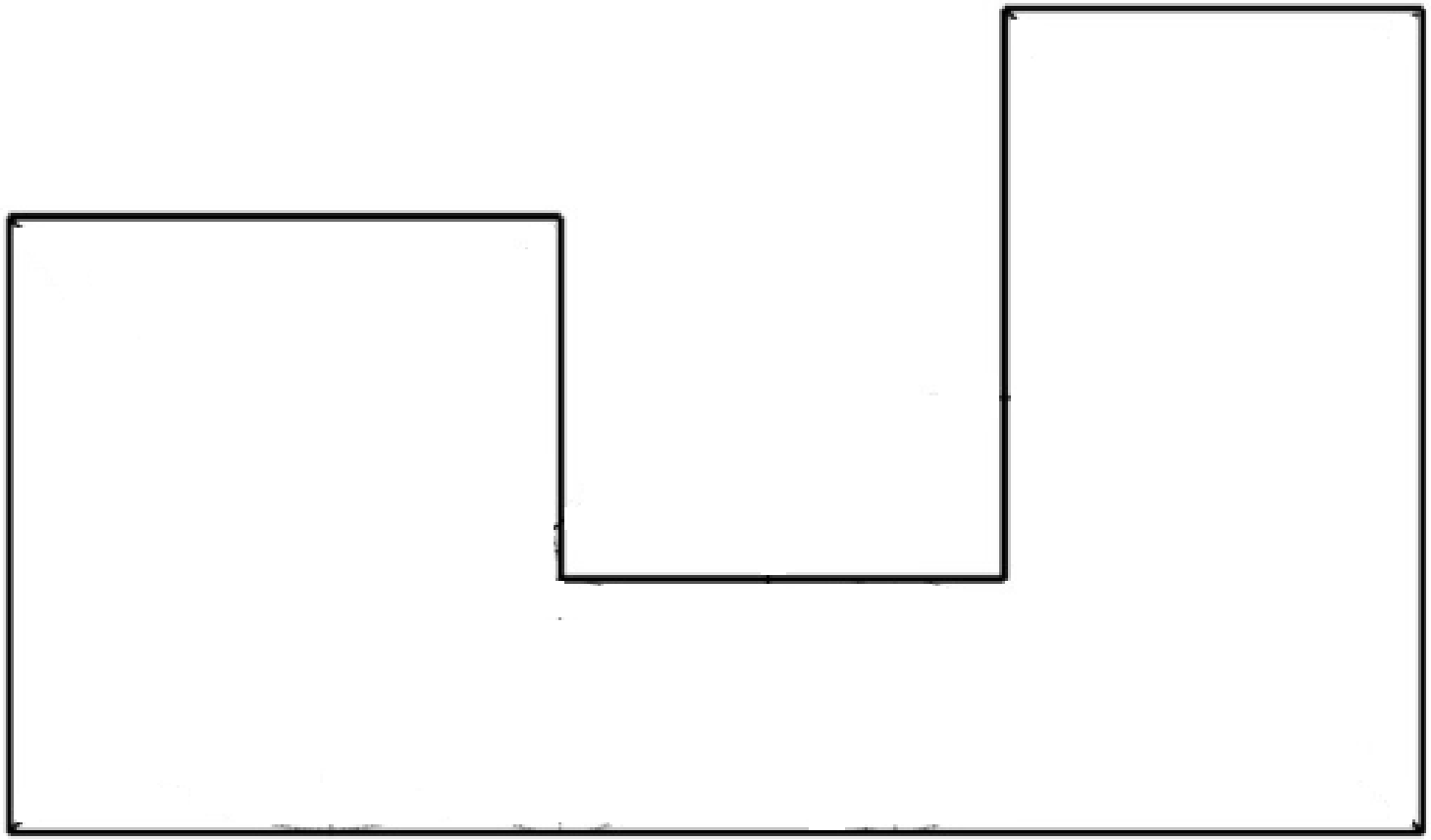




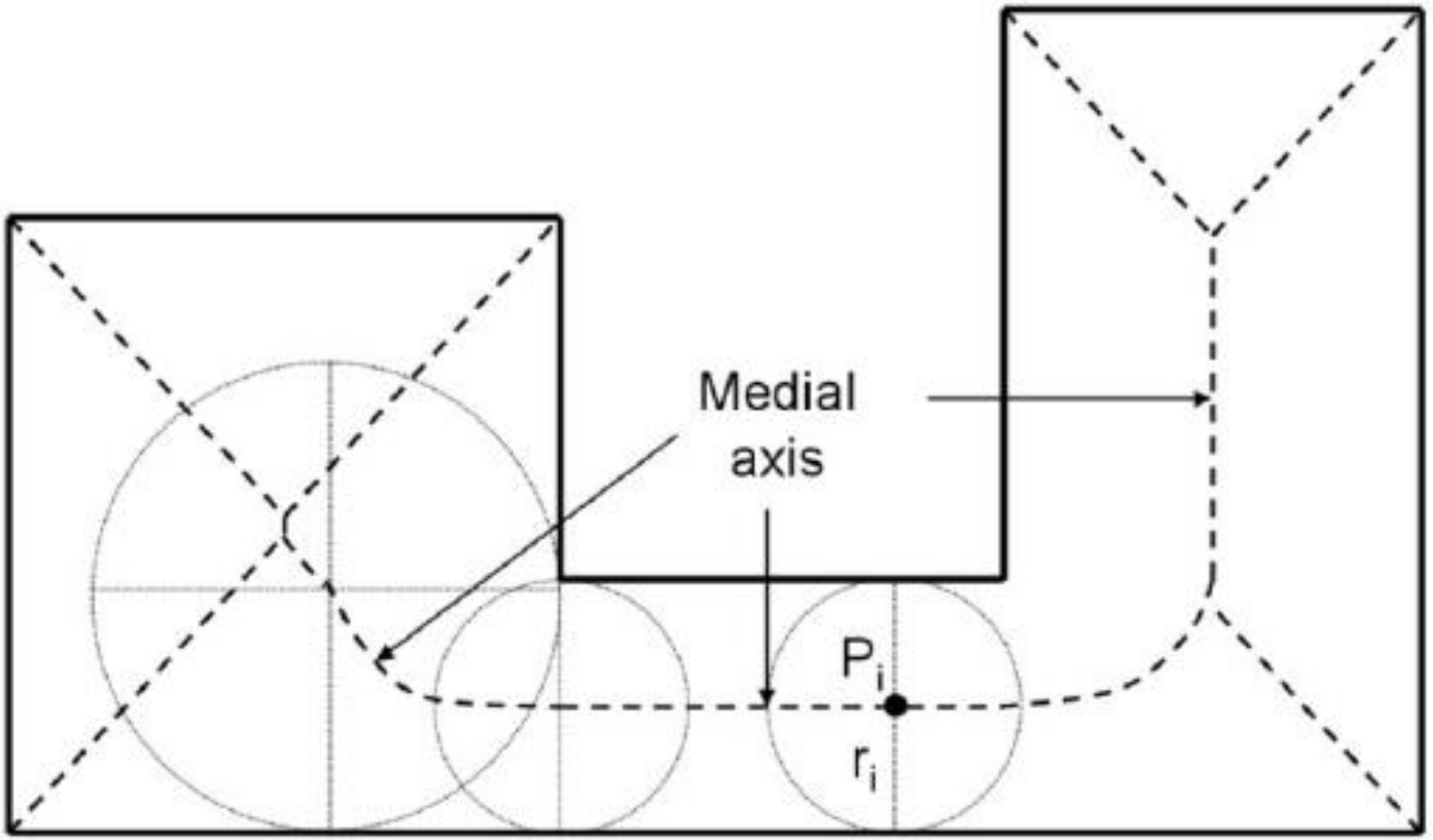

- Red colored line : angular bisector of L1 and L4
- Blue colored line : angular bisector of L3 and L4
- Pink colored line : angular bisector of L1 and L2
- Yellow colored line : angular bisector of L2 and L3
- Green colored line : Bisector of L1 and L3

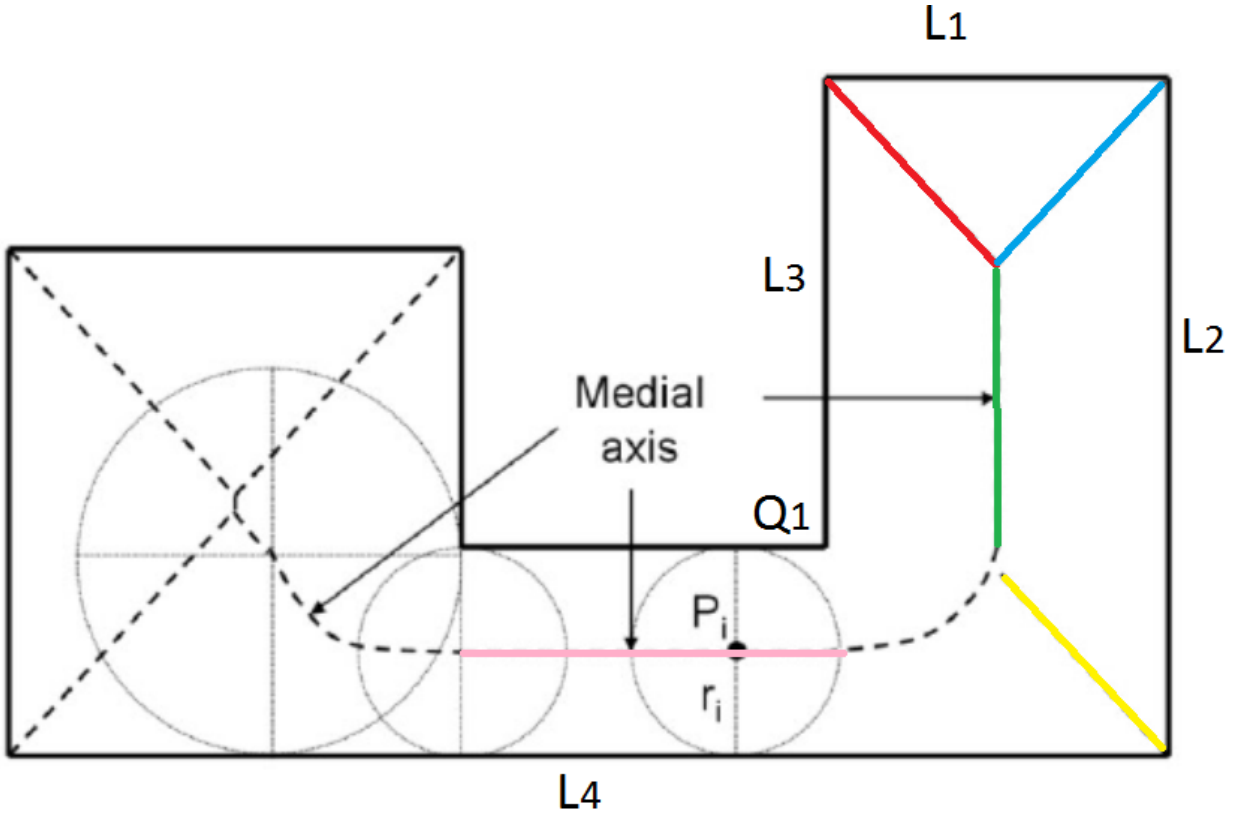
# Exercise

- Draw Medial axis  of the following object

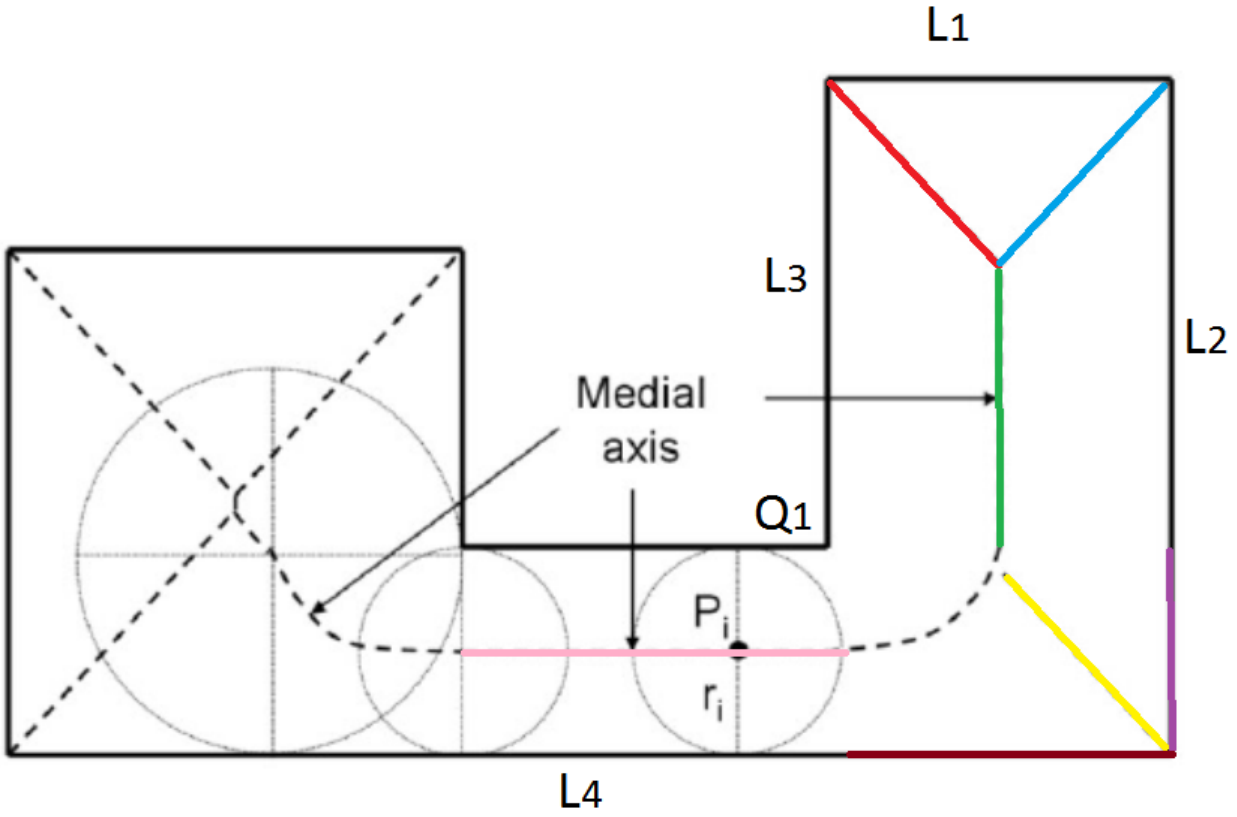


# Medial Axis





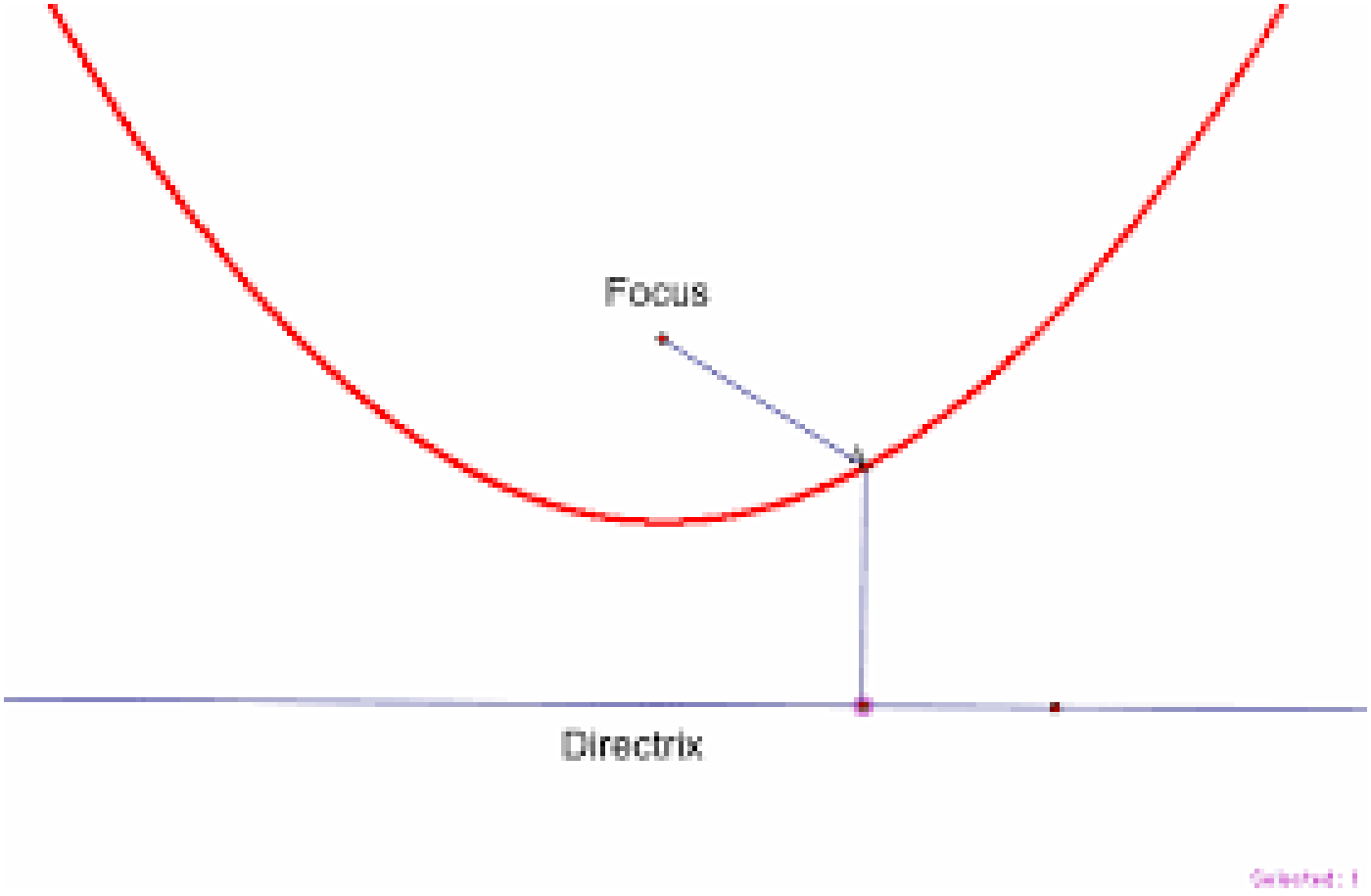

- Red colored line : angular bisector of L1 and L3
- Blue colored line : angular bisector of L1 and L2
- Green colored line : bisector of L2 and L3
- Yellow colored line : angular bisector of L2 and L4
- Pink colored line : Bisector of L4 and the line parallel to that
- Dotted line between pink colored line and green colored line ?



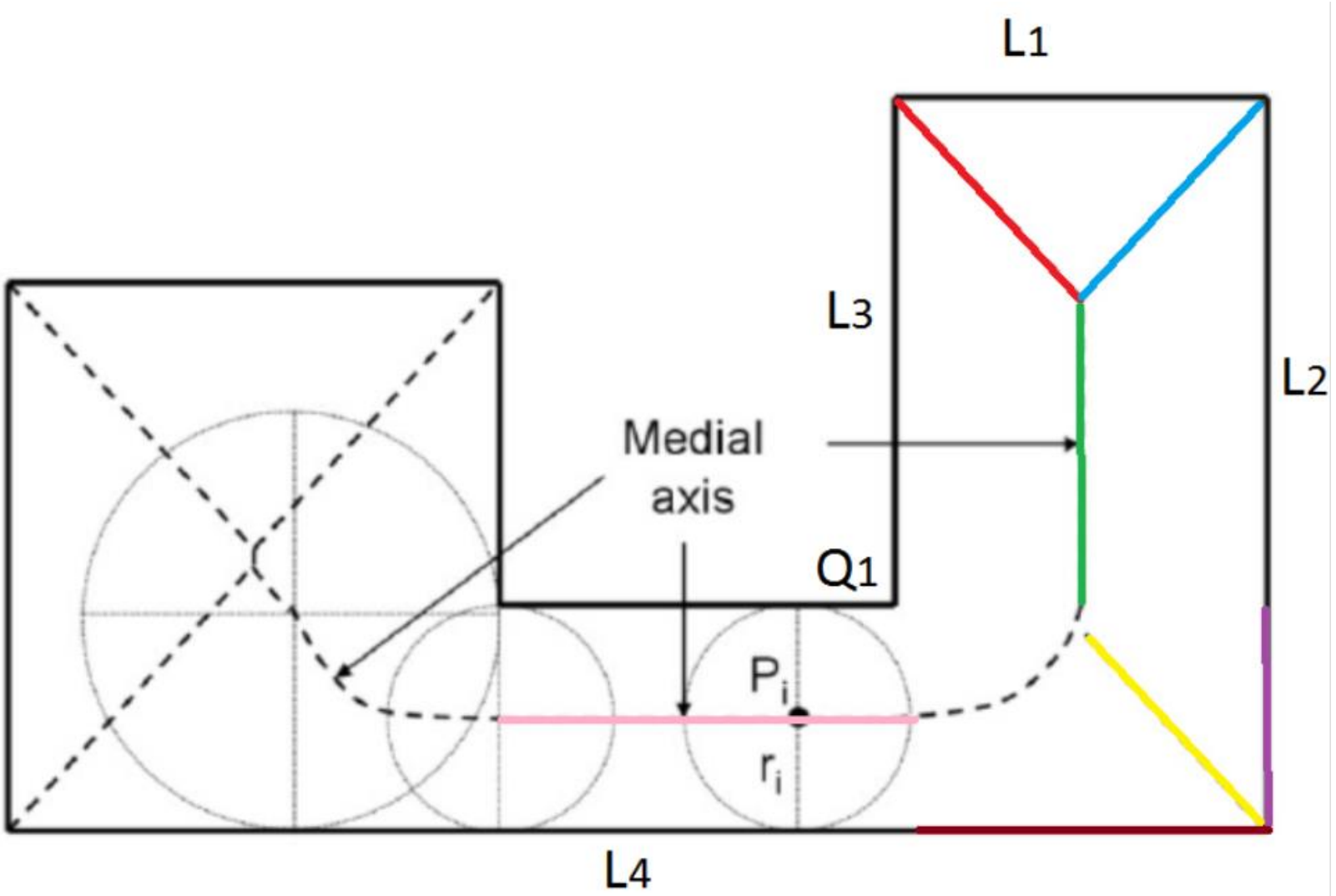

- Dotted line between pink colored line and green colored line : Bisector between Q1 and part of L2 (purple colored) , Bisector between Q1 and part of L4 (brown colored)
- What is the bisector between a point and a line?

# Parabola

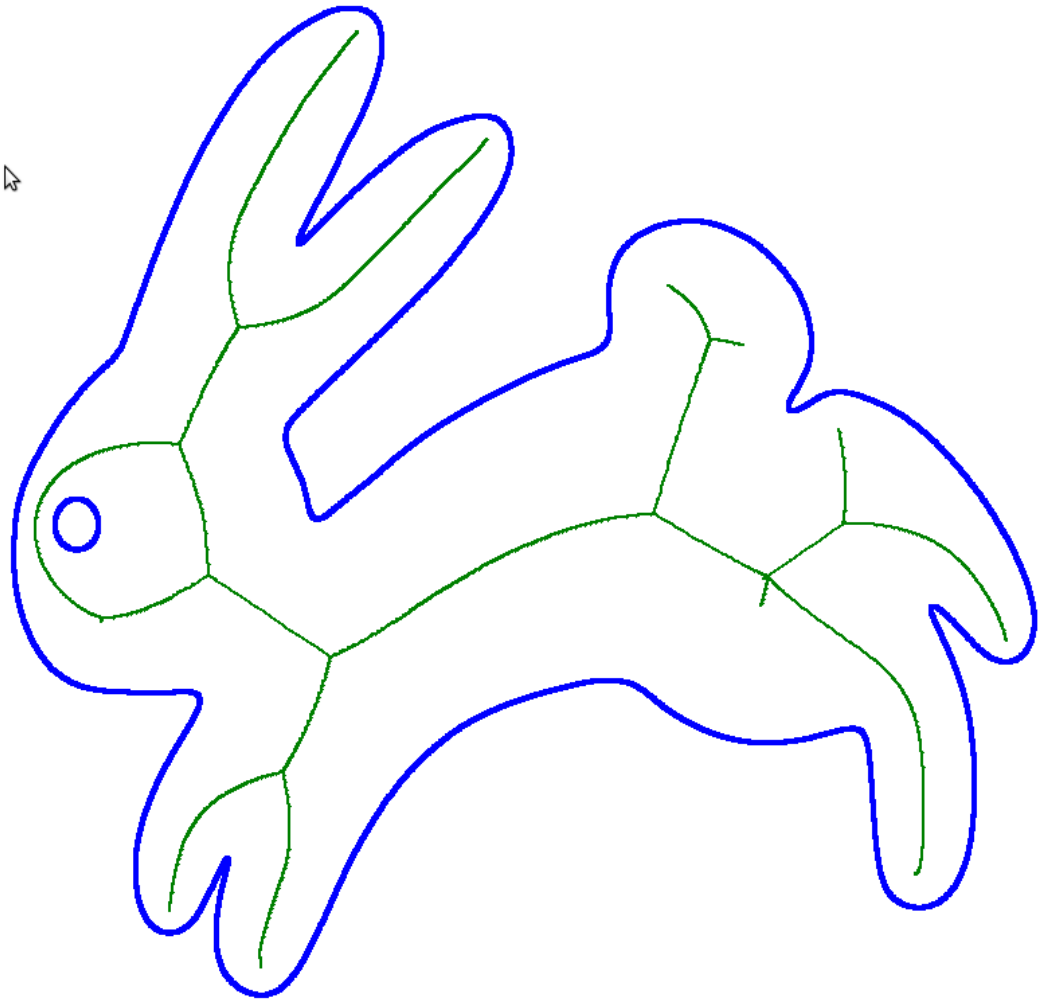
- A **parabola** is the set of all **points** in a plane that are **equidistant** between a fixed **point** (focus) and a **line** (directrix)



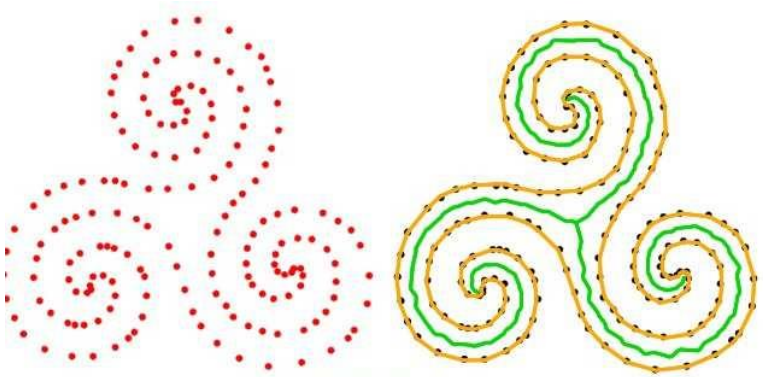
# Two parts of parabola: Q1 with part of L2 & Q1 with part of L4



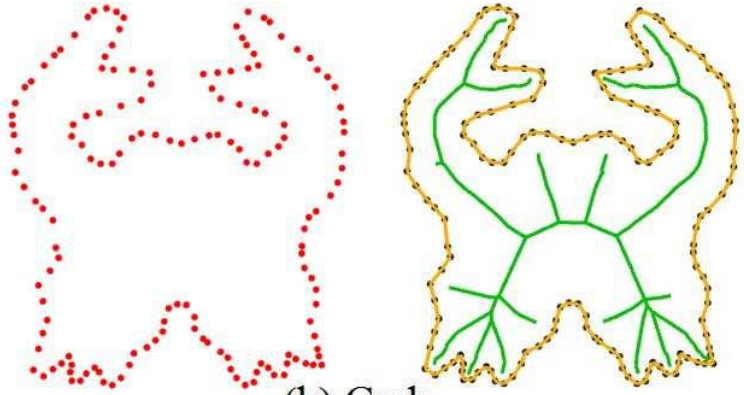
# Another example of Medial Axis



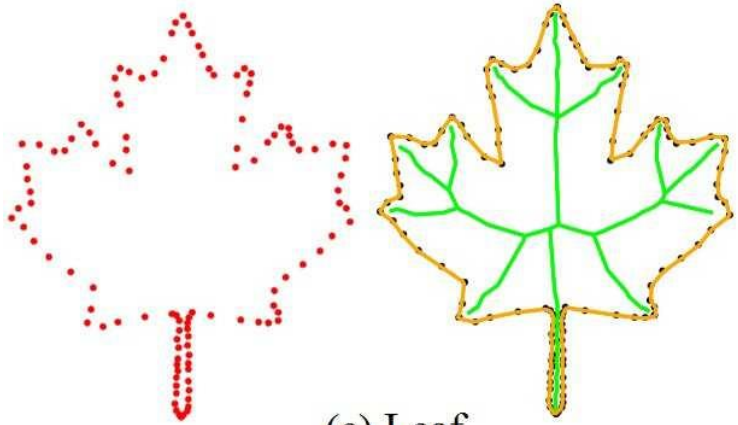
# Examples



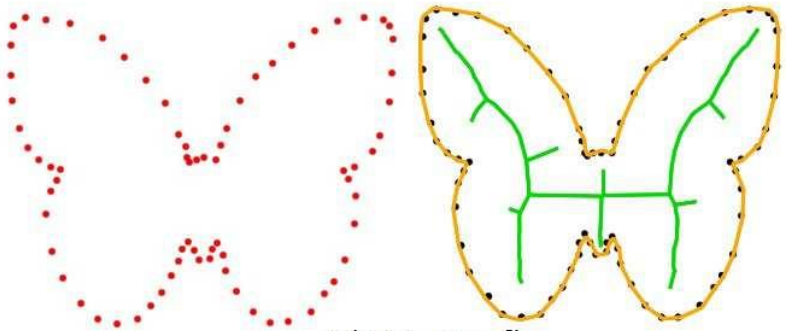
(a) Spiral



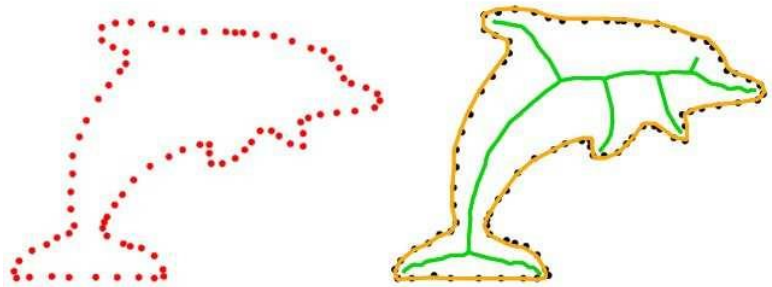
(b) Crab



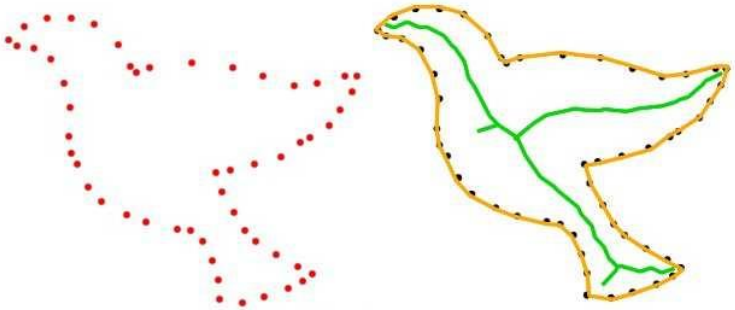
(c) Leaf



(d) Butterfly



(e) Shark



(f) Dove

# Exercise

- Draw Medial axis for different figures

# References for reconstruction


- Matt Duckham, Lars Kulik, Michael F. Worboys, and Antony Galton, *Efficient generation of simple polygons for characterizing the shape of a set of points in the plane*, Pattern Recognition, 41(10):3224-3236, 2008.
- χ-shape algorithm : Link for the software http://ambientspatial.net/ddo/?p=143
- H. Edelsbrunner, D. G. Kirkpatrick, R. Seidel, On the shape of a set of points in the plane, IEEE Transactions on Information Theory 29 (4) (1983) 551–558.
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- https://en.wikipedia.org/wiki/Medial_axis
- **Constructing medial axis transform of planar domains with curved boundaries,** M.Ramanathan and B.Gurumoorthy, Computer Aided Design, Volume 35, Issue 7, June 2003, Pages 619-632

I enjoyed taking classes for u ☺
& learned a lot from/due to u

Thank you

# Best wishes