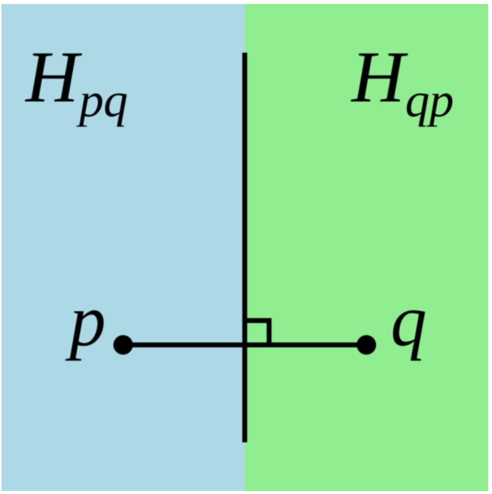
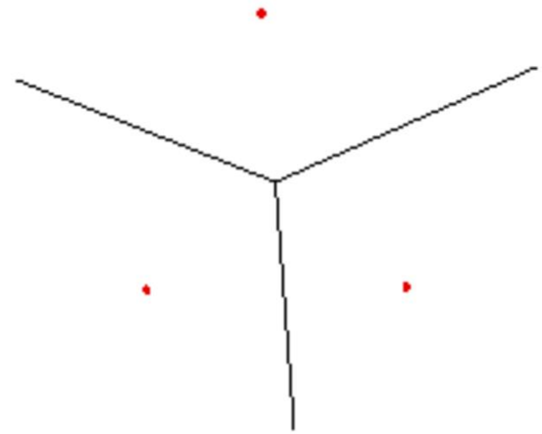
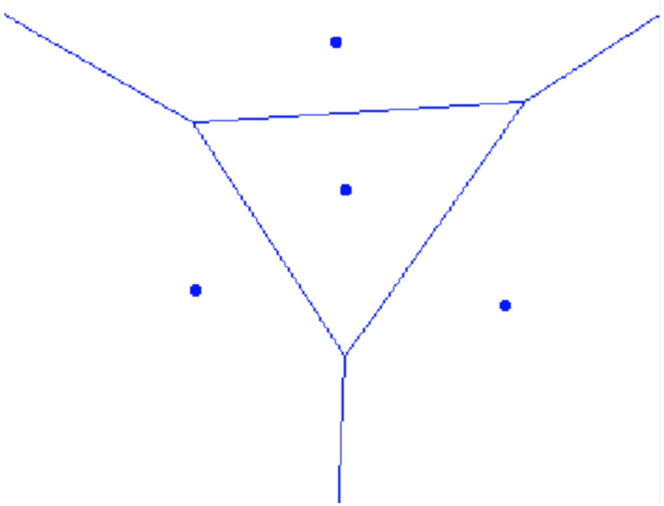
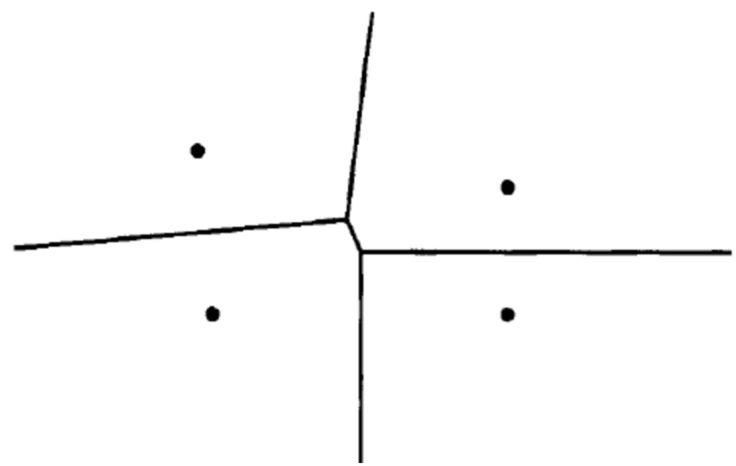
What do we observe ?





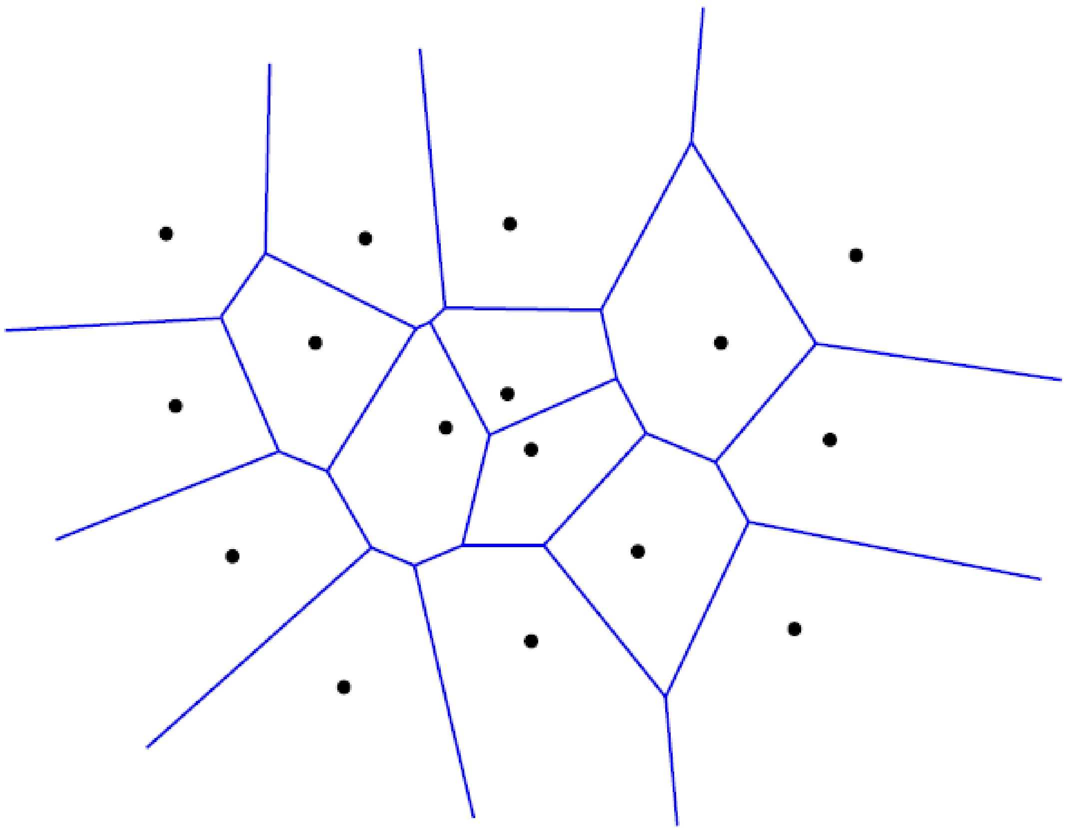




VD Properties

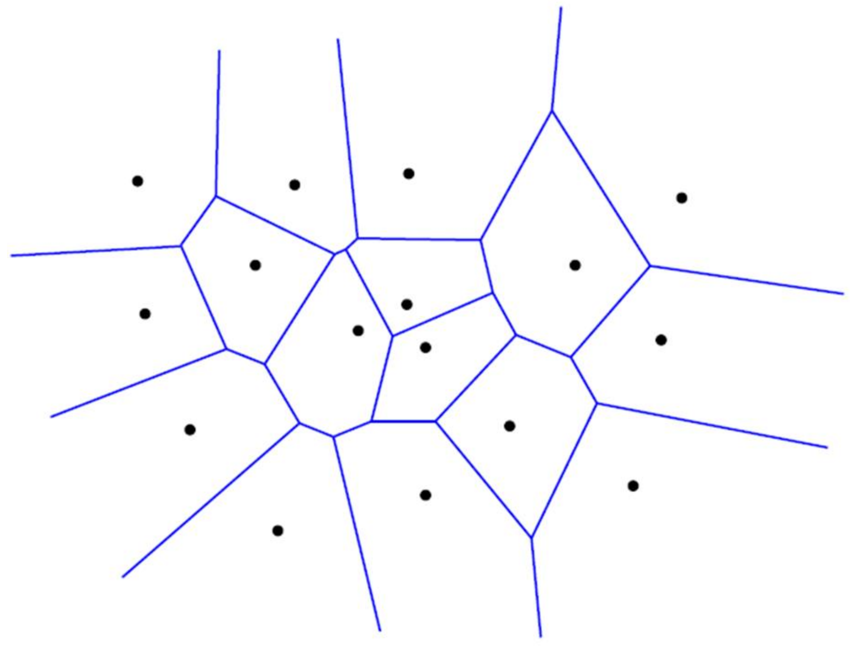
Theorem

- Every vertex of the Voronoi diagram is the common intersection of exactly three edges of the diagram



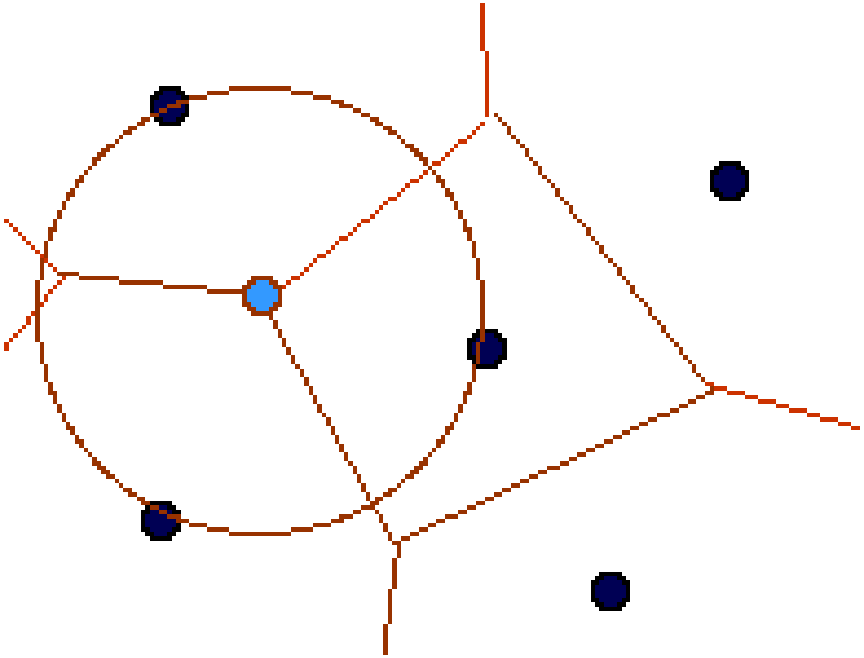
Observation

- Voronoi vertices are centers of circles defined by the three points
- Voronoi diagram is a regular graph of degree three



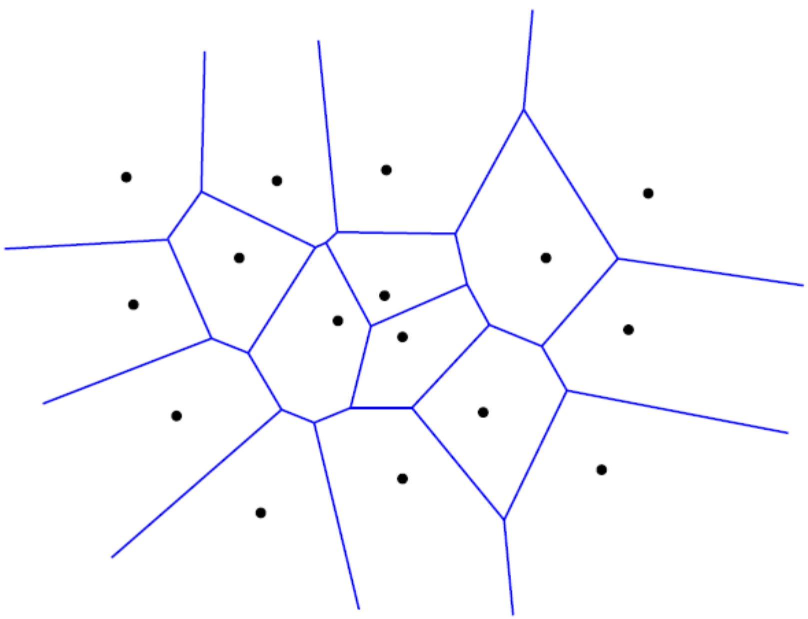
Theorem

- For every vertex v of the Voronoi diagram of S, the circle C(v) contains no other sites of S



Theorem

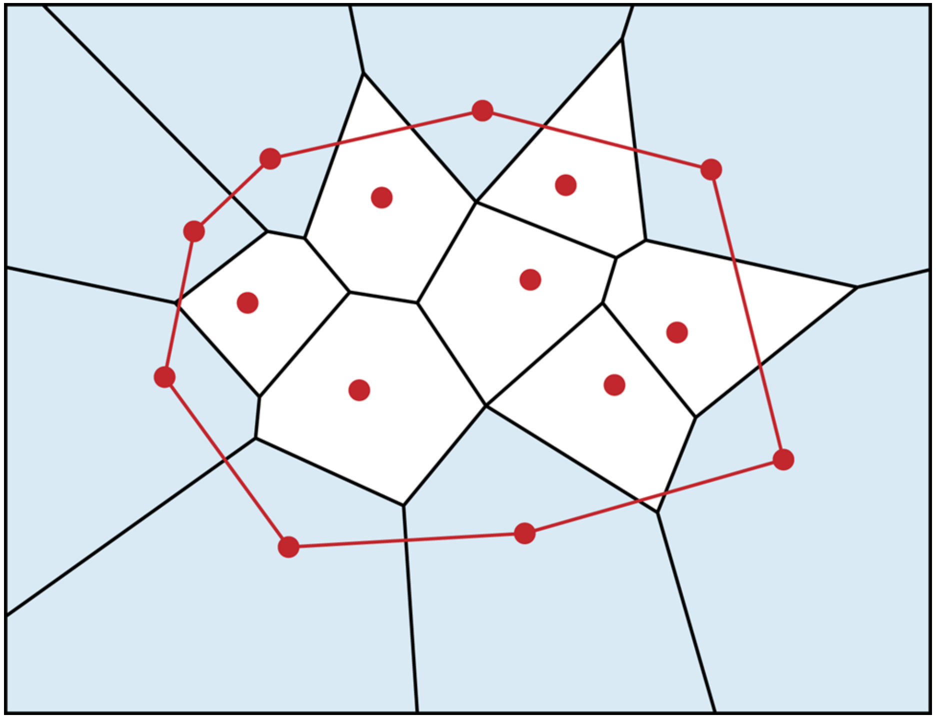
- Every nearest neighbor of p_i in S defines an edge of the Voronoi polygon V(i)



Other related theorems

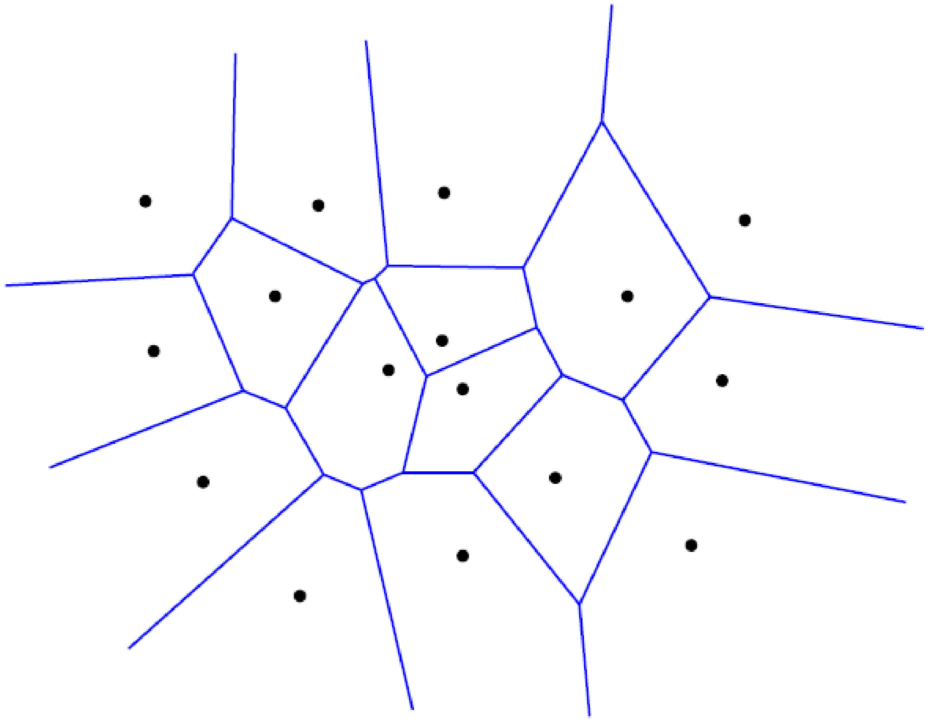
Theorem

- Polygon V(i) in a VD of S is unbounded if and only if p_i is a point on the boundary of the convex hull of the set S



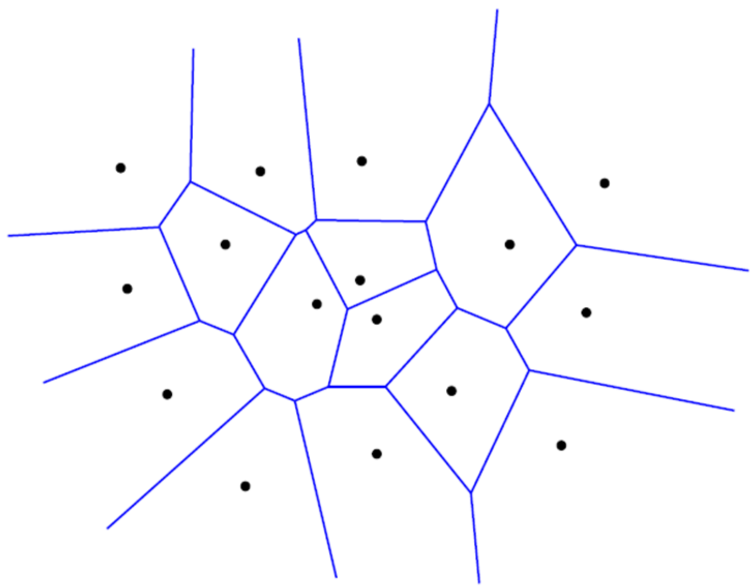
Theorem

- The straight line dual of the Voronoi diagram is a triangulation of S



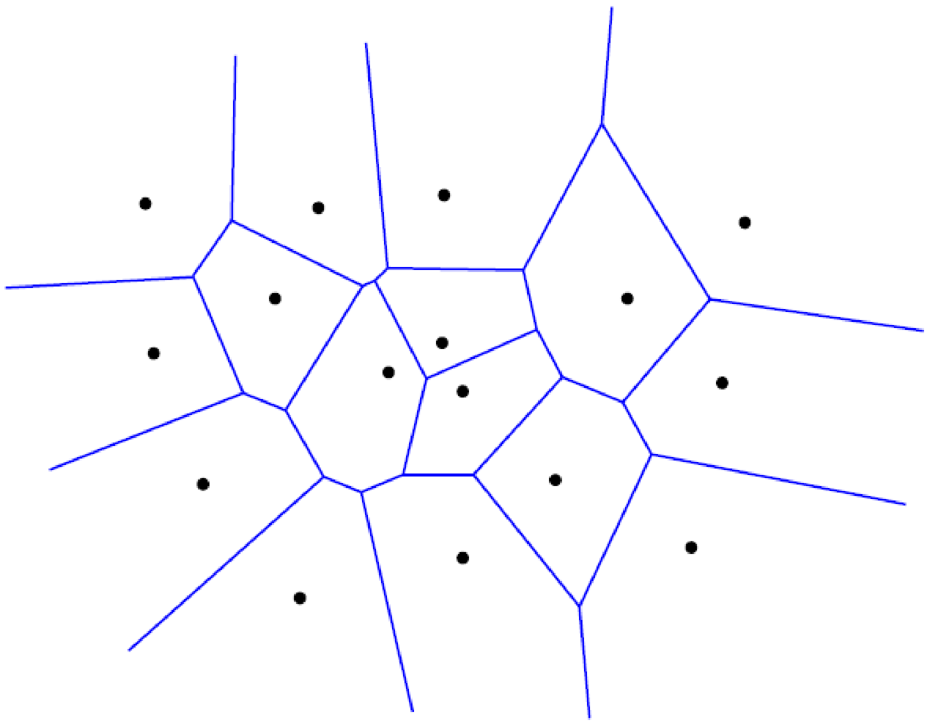
Straight line dual of a Voronoi diagram

- Graph embedded in the plane
- Obtained by adding a straight line segment between each pair of points of S whose Voronoi polygons share an edge

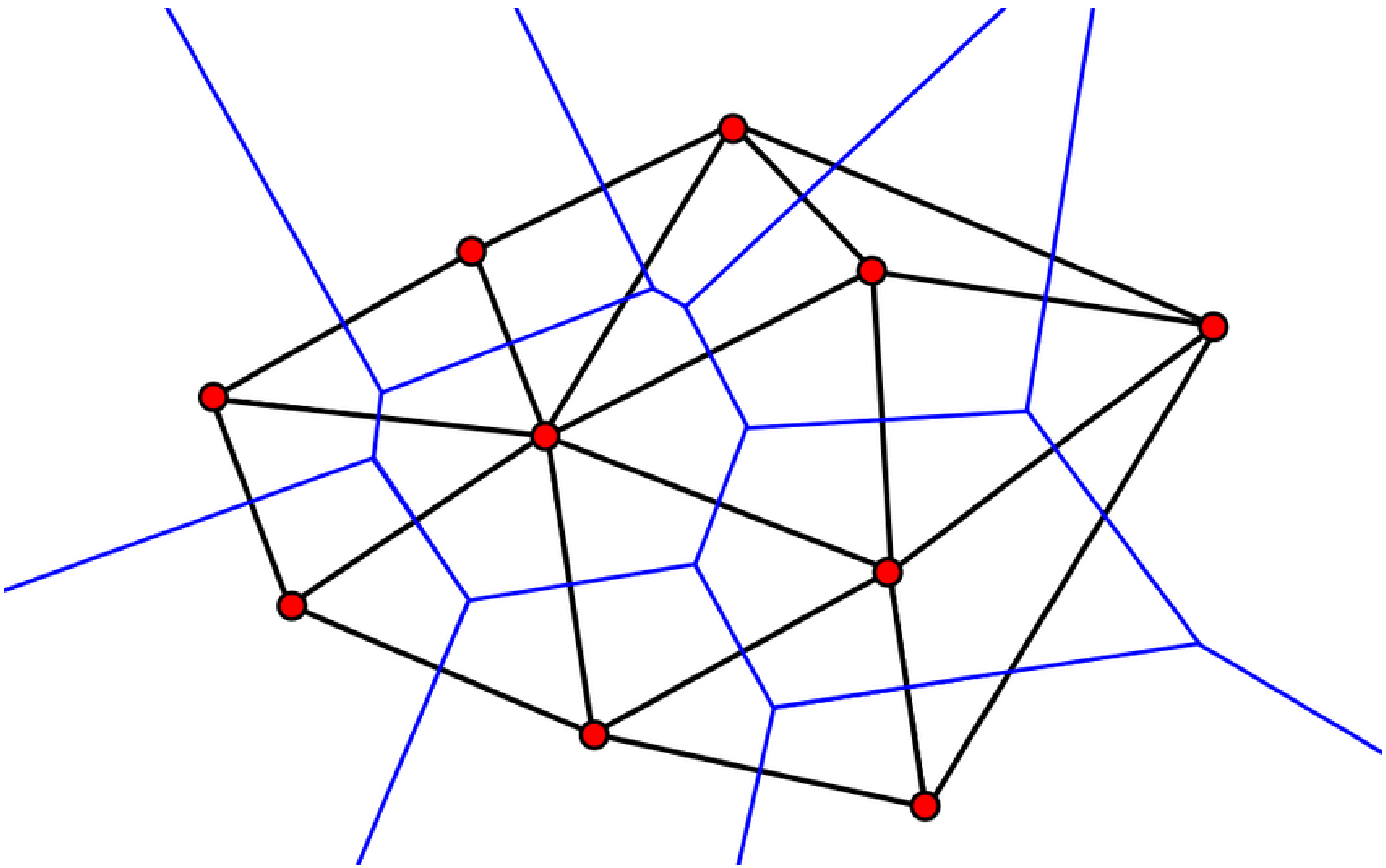


The straight line dual of the VD

- As stated by the theorem: The straight line dual of the Voronoi diagram is a triangulation of S
- What is a straight line dual of a Voronoi diagram?

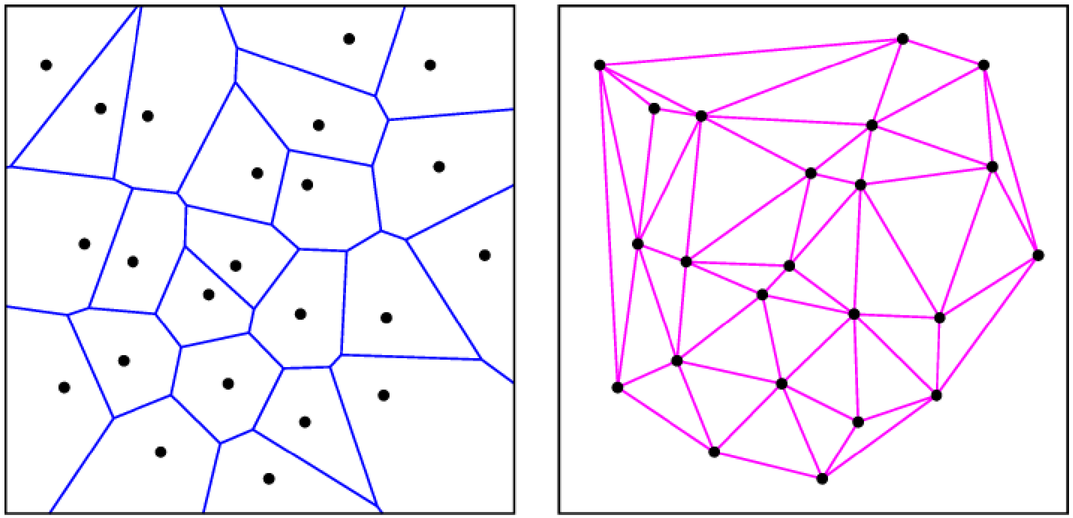


Example: Delaunay triangulation (straight line dual of the VD)embedded on its VD



Straight line dual of a Voronoi diagram

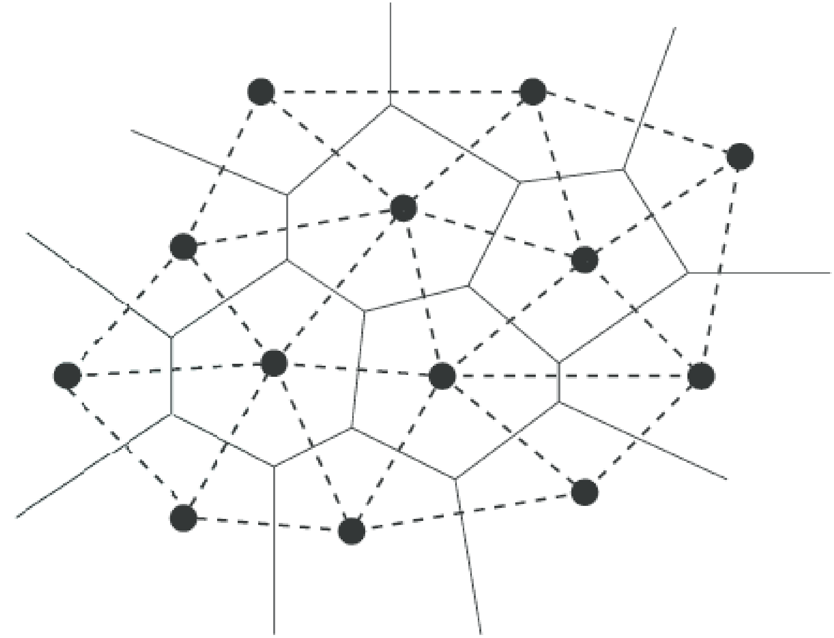
- Graph embedded in the plane
- Obtained by adding a straight line segment between each pair of points of S whose Voronoi polygons share an edge



Summary: VD Properties

- Thm1: Every vertex of the Voronoi diagram is the common intersection of exactly three edges of the diagram
- Thm2:For every vertex v of the Voronoi diagram of S, the circle C(v) contains no other point of S
- Thm3:Every nearest neighbor of p_i in S defines an edge of the Voronoi polygon V(i)
- Thm4:Polygon V(i) is unbounded if and only if p_i is a point on the boundary of the convex hull of the set S
- Thm5:The straight line dual of the Voronoi diagram is a triangulation of S

Delaunay Properties



D1. $\mathcal{D}(P)$ is the straight-line dual of $\mathcal{V}(P)$. This is by definition.

D2. $\mathcal{D}(P)$ is a triangulation if no four points of P are cocircular: Every face is a triangle. This is Delaunay's theorem. The faces of $\mathcal{D}(P)$ are called *Delaunay triangles*.

D3. Each face (triangle) of $\mathcal{D}(P)$ corresponds to a vertex of $\mathcal{V}(P)$.

D4. Each edge of $\mathcal{D}(P)$ corresponds to an edge of $\mathcal{V}(P)$.

D5. Each node of $\mathcal{D}(P)$ corresponds to a region of $\mathcal{V}(P)$.

D6. The boundary of $\mathcal{D}(P)$ is the convex hull of the sites.

D7. The interior of each (triangle) face of $\mathcal{D}(P)$ contains no sites. (Compare **V5**.)

Theorem

The straight line dual of Voronoi diagram(S) for a set S of N>=3 points in general position (no three points from S are collinear and no four points from S are co circular) is a triangulation : the unique Delaunay triangulation of S.

Algorithms for Voronoi Construction

References

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THANK YOU