

P8160 - Bayesian modeling of hurricane trajectories

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Contents

Background and Objectives	2
Methods	2
Bayesian Model	2
MCMC Algorithm	2
MCMC Results	4
Results	10
Seasonal Analysis	10
Prediction	10
Discussion	14
Group Contributions	14
Appendix	15

Background and Objectives

As hurricanes affect people, the ability to forecast hurricanes is essential for minimizing the risks in suffered areas. Hereby, a hierarchical Bayesian strategy for modeling North Atlantic hurricane counts since 1950 is illustrated. Model integration would be expected to achieve through a Markov chain Monte Carlo algorithm. Contingent on the availability of values for the covariates, the models can be used to explore the seasonality among track data and make predictive inferences on hurricanes.

The data given has 703 observations and following features are recorded for each hurricanes in the North Atlantic:

- ID: ID of the hurricanes
- Season: In which the hurricane occurred
- Month: In which the hurricane occurred
- Nature: Nature of the hurricane (ET-Extra tropical;DS-Disturbance;NR-Not rated;SS-Sub tropical; TS-Tropical storm)
- time: dates and time of the record
- Latitude and Longitude: The location of a hurricane check point
- Wind.kt: Maximum wind speed (in Knot) at each check point

Methods

Baysian Model

MCMC Algorithm

Given the fact that we cannot directly sample from the joint posterior distribution calculated above. We choose to use Gibbs sampling. Gibbs sampling is a sequential sampler that samples from the conditional distribution of each parameter so that the final collection of samples forms a Markov chain whose stationary distribution is the joint distribution that we are interested in.

In our case, conditional posteriors are more manageable than the joint posterior. Conditional posteriors for each parameter are calculated as the following:

!!! ADD CONDITIONAL POSTERIOR

Still, we could not find a regular distribution that has the density function that matches with the conditional posterior of σ^2 . Therefore, we incorporated Metropolis Hasting Random Walk into Gibbs sampling. Metropolis Hasting Random Walk finds approximated values of σ^2 by accepting and rejecting proposed value with an acceptance probability that helps maintain the detailed balance condition. In our algorithm, we work with log-scaled posterior values instead of the original-scaled posterior because $\frac{1}{\sigma^N}$ dominates the entire posterior value where N is the total number of observations in the data set.

In our investigation of hurricane wind speed, we used Gibbs sampling and Metropolis Hasting Random Walk as the following:

Algorithm 1 Gibbs Sampling

```

1: function GIBBS(starting_values, V,S,X,D,Y, nitr, $\nu$ ,H, hurricane_idx)
2:   create posts results lists to collect results for B,  $\mu$ , A,  $\gamma$ ,  $\sigma^2$ 
3:    $B^1 \leftarrow f(B|start_{\mu}, start_{\sigma^2}, start_A, start_{\gamma}, Y)$ 
4:    $\mu^1 \leftarrow f(\mu|B^1, start_{\sigma^2}, start_A, start_{\gamma}, Y)$ 
5:    $A^1 \leftarrow f(A|B^1, \mu^1, start_{\sigma^2}, start_{\gamma}, Y)$ 
6:    $\gamma^1 \leftarrow f(\gamma|B^1, \mu^1, start_{\sigma^2}, A^1, Y)$ 
7:    $\sigma^2\_chain \leftarrow metropolis\_hasting\_sigma\_squared(start_{\sigma^2}, iter = 500, a = 0.8, Y, D, B^1, X, \gamma^1)$ 
8:    $(\sigma^2)^1 \leftarrow mean(\sigma^2\_chain[401:500])$ 
9:   for i in 2:nitr do
10:     $B^i \leftarrow f(B|\mu^{i-1}, (\sigma^2)^{i-1}, A^{i-1}, \gamma^{i-1}, Y)$ 
11:     $\mu^i \leftarrow f(\mu|B^i, (\sigma^2)^{i-1}, A^{i-1}, \gamma^{i-1}, Y)$ 
12:     $A^i \leftarrow f(A|B^i, \mu^i, (\sigma^2)^{i-1}, \gamma^{i-1}, Y)$ 
13:     $\gamma^i \leftarrow f(\gamma|B^i, \mu^i, (\sigma^2)^{i-1}, A^i, Y)$ 
14:     $\sigma^2\_chain \leftarrow metropolis\_hasting\_sigma\_squared((\sigma^2)^{i-1}, iter = 500, a = 0.8, Y, D, B^i, X, \gamma^i)$ 
15:     $(\sigma^2)^i \leftarrow mean(\sigma^2\_chain[401:500])$ 
16:   end for
17:   return posts results lists that collect results for B,  $\mu$ , A,  $\gamma$ ,  $\sigma^2$ 
18: end function

```

Figure 1: Gibbs Sampling

Algorithm 2 Metropolis Hasting sigma squared

```

1: function METROPOLIS HASTING SIGMA SQUARED(start_sigma_squared, nitr, a, B, X,D,Y, H,
hurricane_idx)
2:   Initialize chain
3:   chain[1]  $\leftarrow$  start_sigma_squared
4:   for i in 2:itr do
5:     curr  $\leftarrow$  chain[i - 1]
6:     prop  $\leftarrow$  a sample from Unif(curr-a, curr+a)
7:     prop_posterior  $\leftarrow$  sigma_squared_log_posterior(prop,...)
8:     curr_posterior  $\leftarrow$  sigma_squared_log_posterior(curr,...)
9:     posterior_ratio  $\leftarrow$  exp(prop_posterior - curr_posterior)
10:     $\alpha \leftarrow min(posterior\_ratio, 1)$  ▷ acceptance probability
11:    u  $\leftarrow$  runif(1)
12:    if u <  $\alpha$  then
13:      chain[i]  $\leftarrow$  prop ▷ accept proposal
14:    else
15:      chain[i]  $\leftarrow$  curr ▷ reject proposal
16:    end if
17:   end for
18:   return chain
19: end function

```

Figure 2: Metropolis Hasting Random Walk

We decide starting values of each parameters by fitting a linear mixed effects model that is the same as the proposed model in a frequentist way and then extract its corresponding coefficients. Starting values for γ are the fixed effects coefficients in the linear mixed effects model.

Table 1: Starting Values for Gibbs Sampling

Terms	Starting Value
γ	Fixed effect coefficients in the fitted Linear Mixed Effects Model
β_{i_1, \dots, i_4}	Random effect coefficients in the fitted Linear Mixed Effects Model
σ^2	Residual Variance in the fitted Linear Mixed Effects Model
β_0	Sum of random effect and fixed effect intercepts in the fitted Linear Mixed Effects Model
μ	Mean of the β_{i_1, \dots, i_n} for each predictor
$A = \Sigma^{-1}$	Mean of the Wishart distribution with $\nu = 5$ and scale matrix with 0.7 on the main diagnoal and 0.2 on off-diagnoal

With the stating values above, we assume that the true σ^2 will be around 27 and tried a few values for window size a . Finally, with $a = 0.8$, σ^2 samples converge pretty quickly and the acceptance rate is around 46% with 500 iterations. Therefore, we set $a = 0.8$ in each iteration of Gibbs Sampling and burn the first 400 samples of σ^2 .

Although our algorithm is not the most efficient algorithm, it has two main advantages: 1) It is simple to compute and does not rely on the calculation of gradient of log densities as Hamiltonian Monte Carlo; 2) By incorporating the Random Walk, we have the flexibility to control the range of sampling area when the conditional distribution of parameter of interest is difficult to simulate from and when our initial prior is not informative .

MCMC Results

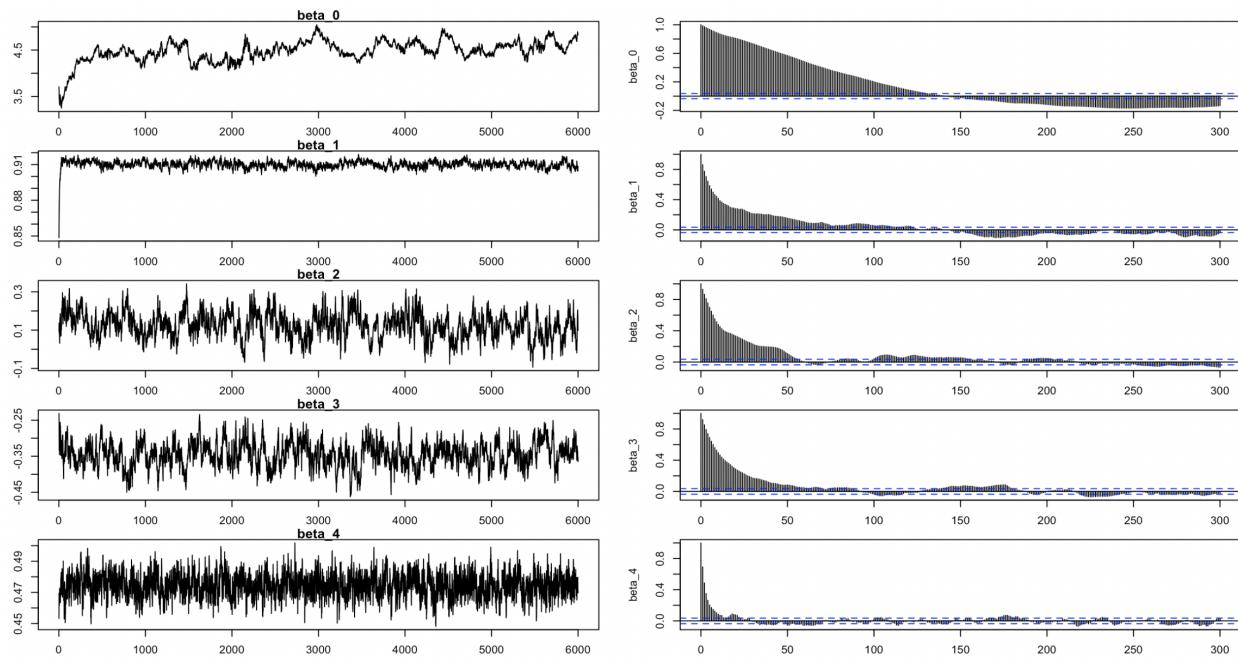
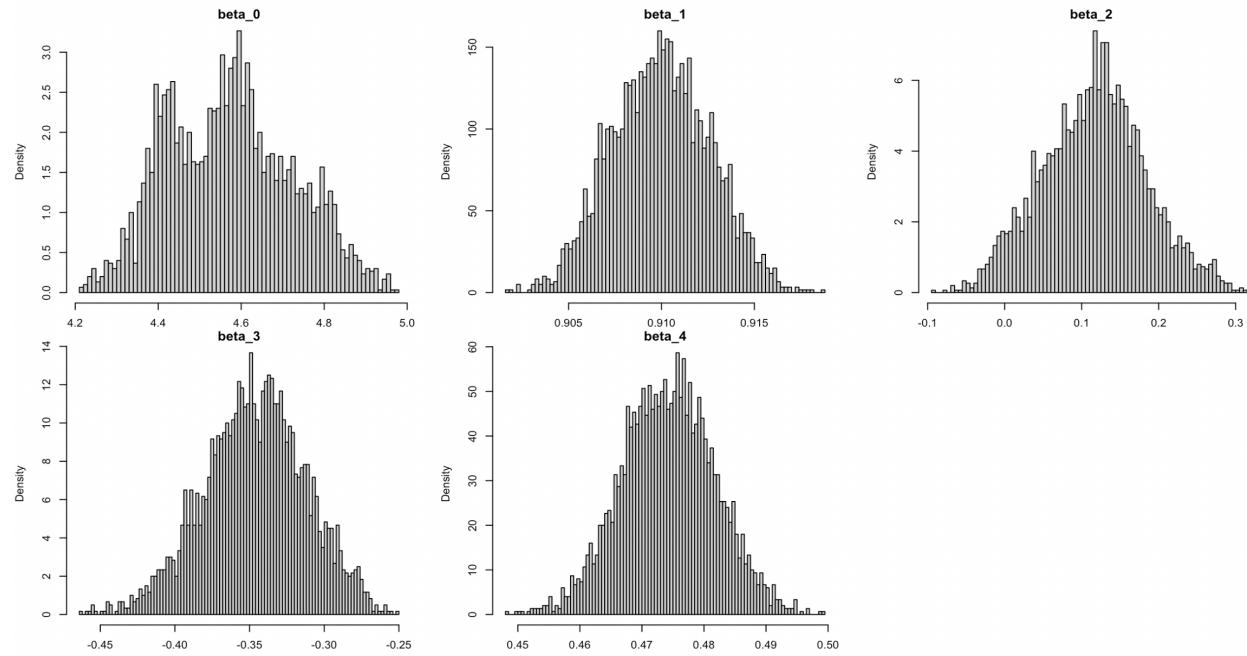
We ran our algorithm with these starting values for 6000 iterations and decided to burn the first 3000 samples for each parameters for analysis in the rest of this project. We use diagnostic plots to evaluate the quality of these samples. We created trace plots for all 6000 samples for each parameter, and autocorrelation plots and histogram plots for the second half of the samples after burn-in.

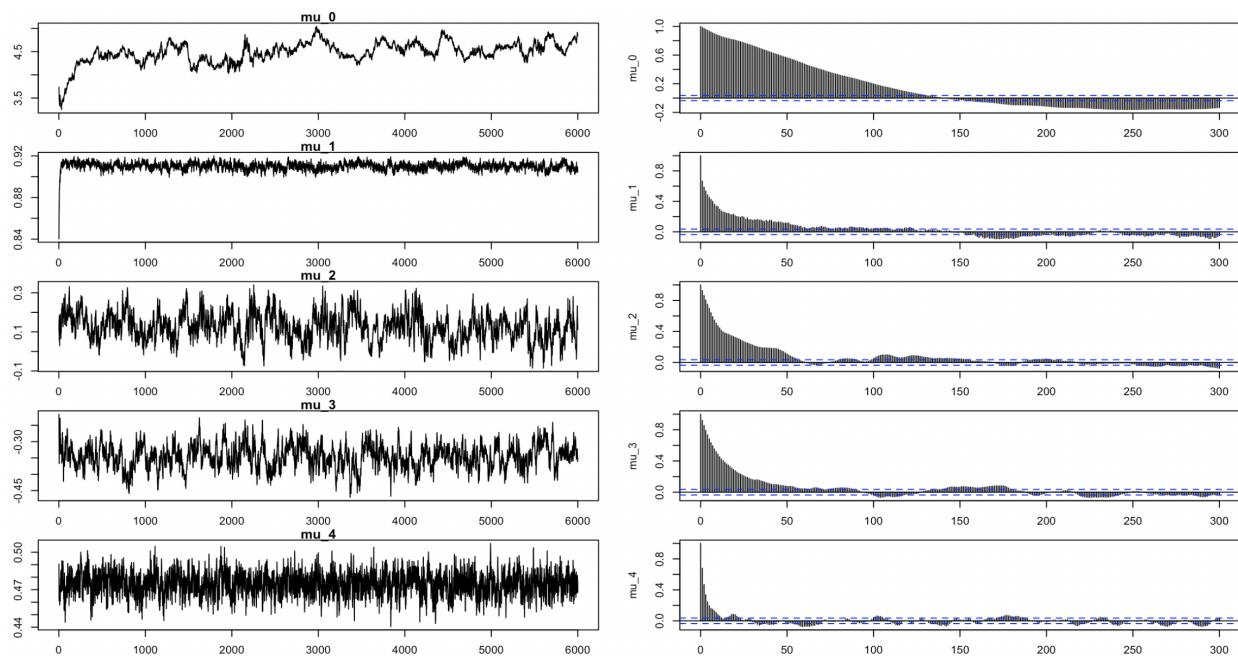
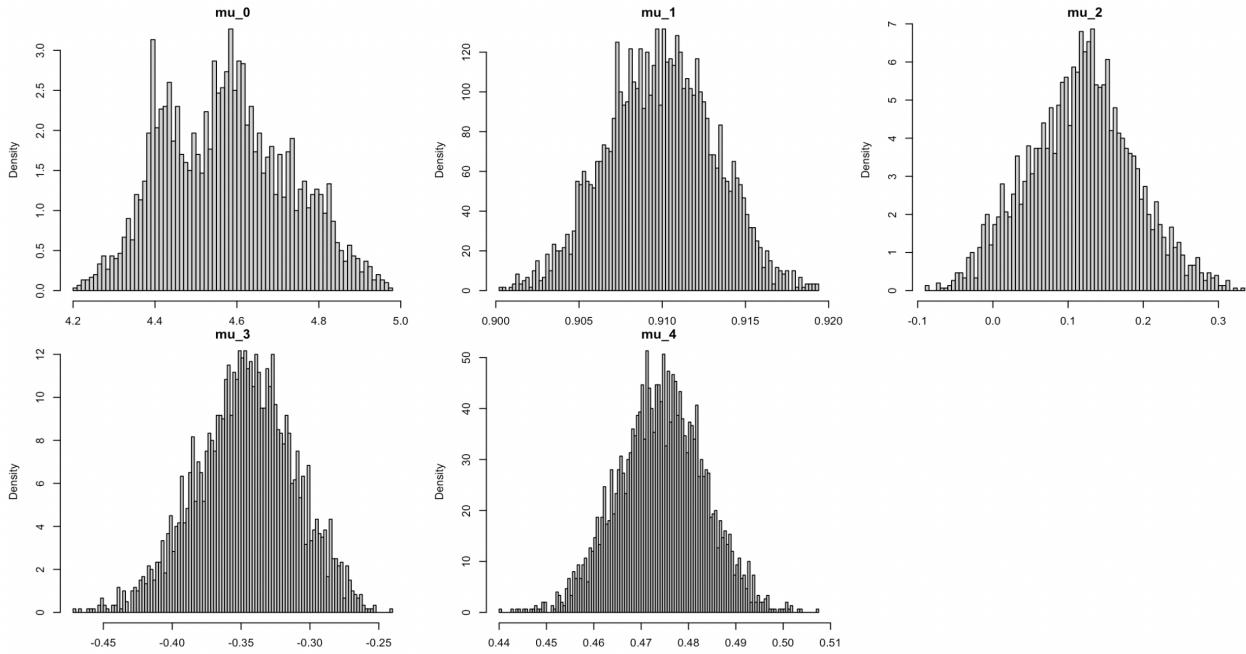
For a good set of samples, we expect to see the chain in trace plot stablize after a short amount of iterations. After the burn-in, we expect to see small or no autocorrelation and somewhat normal histograms.

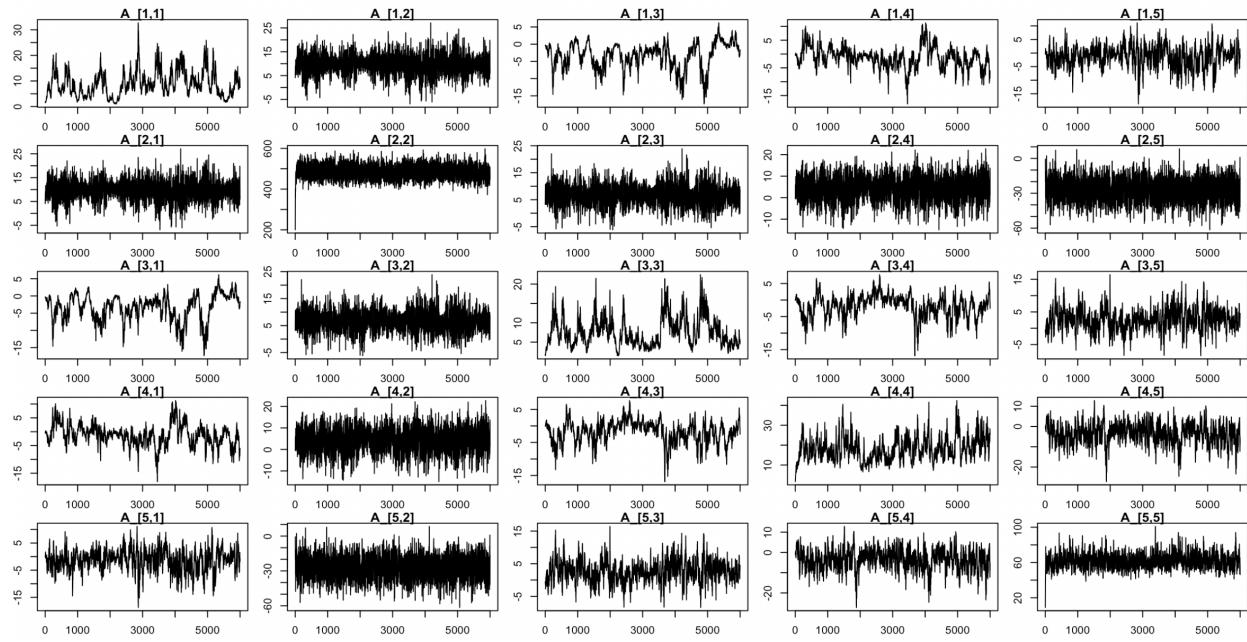
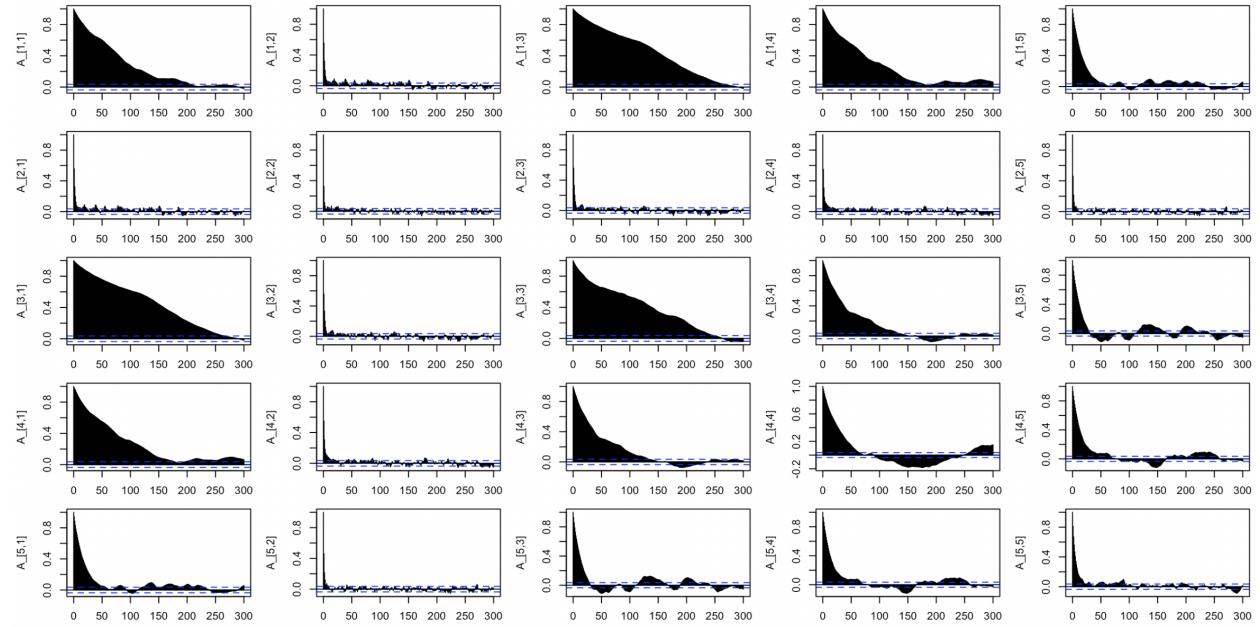
Overall, MCMC samples for the fixed effect parameters γ converge pretty well and their histograms have a nice bell shpae curve. There is very small autocorellation except γ_{10} which is the coefficient for the Season variable.

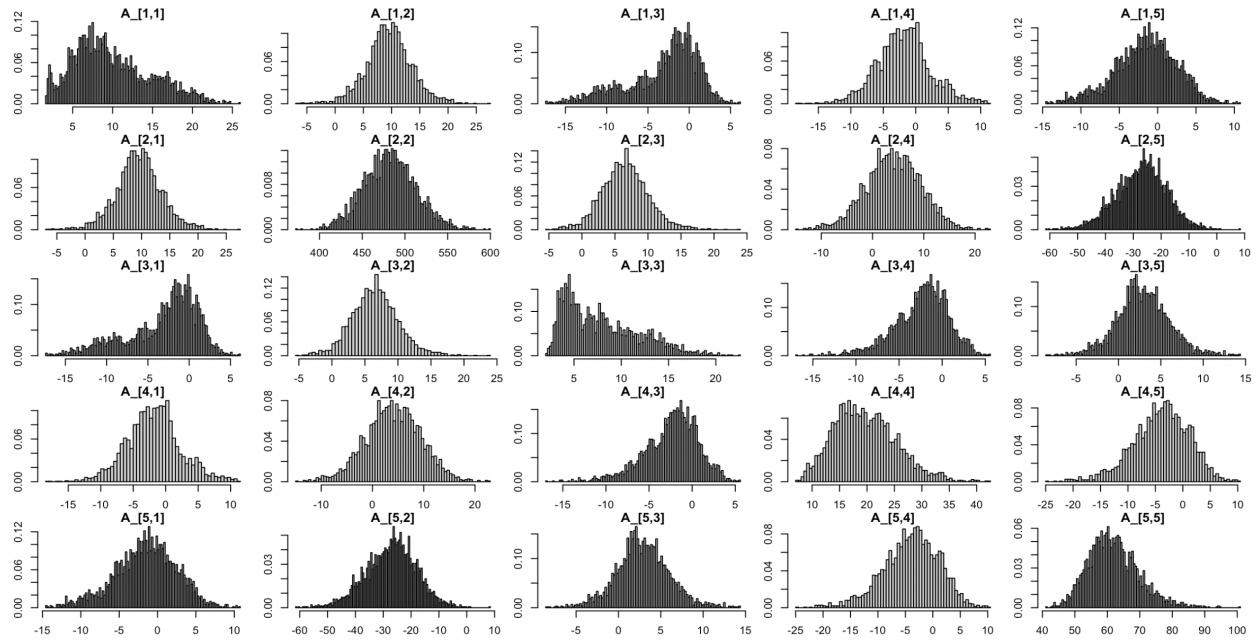
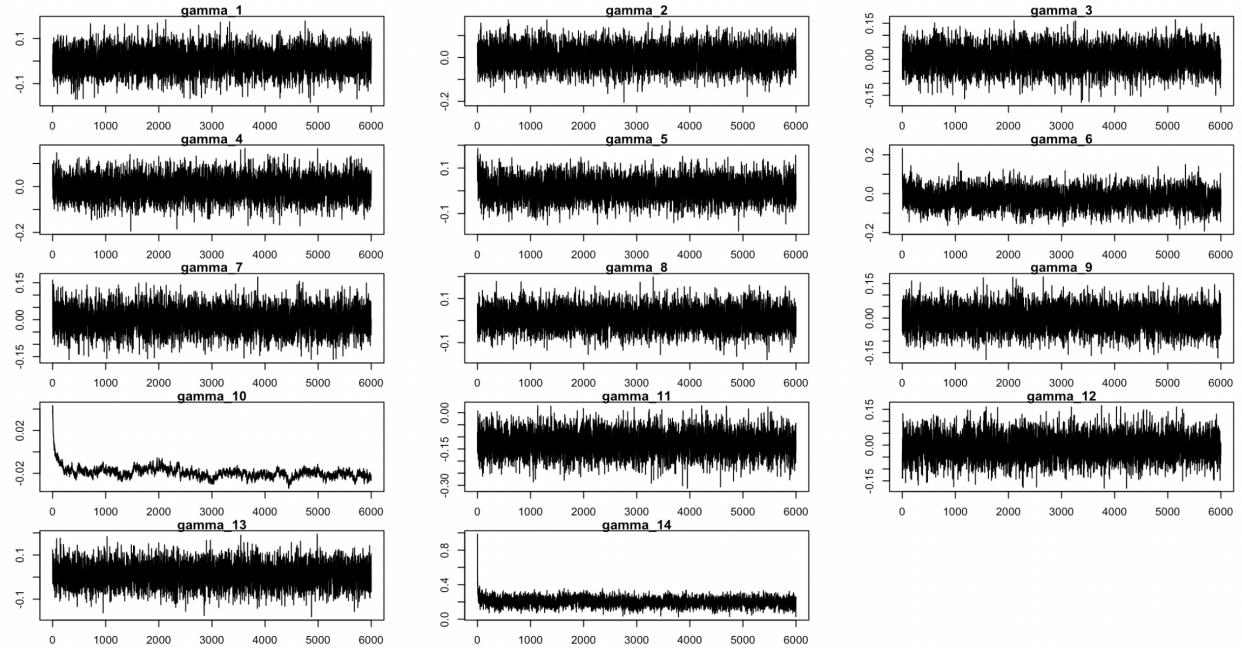
On the other hand, MCMC samples for the random effects parameters have more oscilliations in the trace plots even though most of them converge. For β_{i_0, \dots, i_4} and μ , post burn-in samples have somewhat normal histograms and their autocorrelations decrease to a tolerable range of values within 50 lags. Samples for the inverse of the covariance matrix for β_{i_0, \dots, i_4} are tricky to converge. By the design of covariance matrix, if the samples of a variance term have too much oscilliations, the osciliations will propagate to other terms in the covairiance matrix as well. But still, a good number of terms in the inverse of the covariance matrix have converging samples that have small autocorrelation.

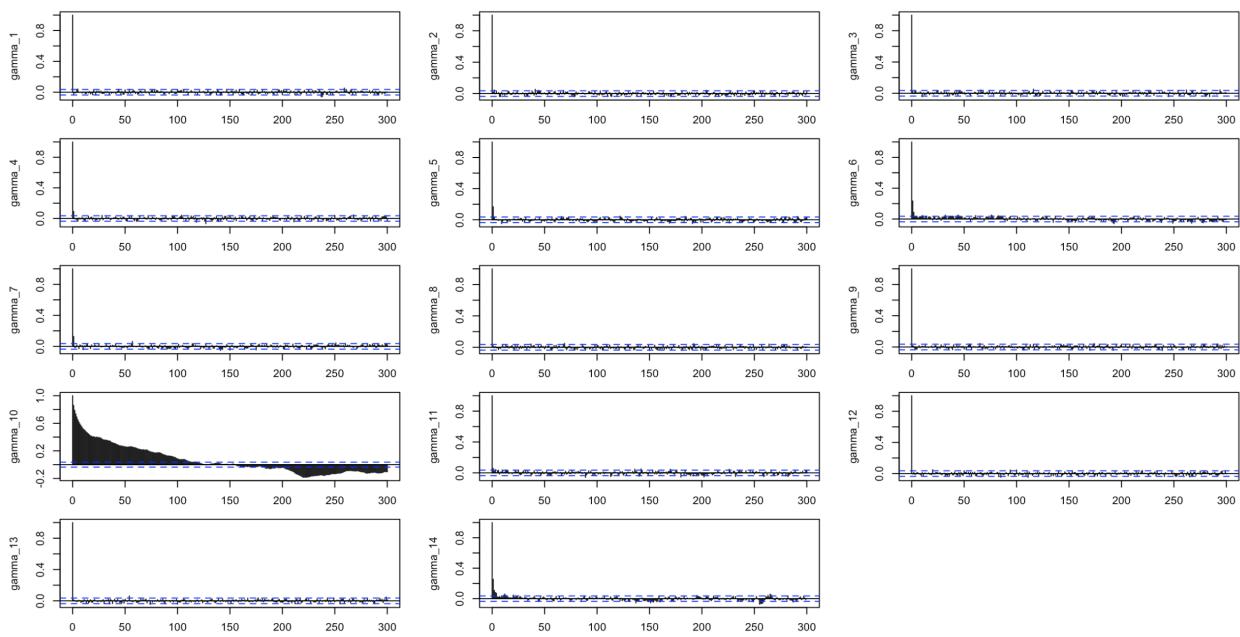
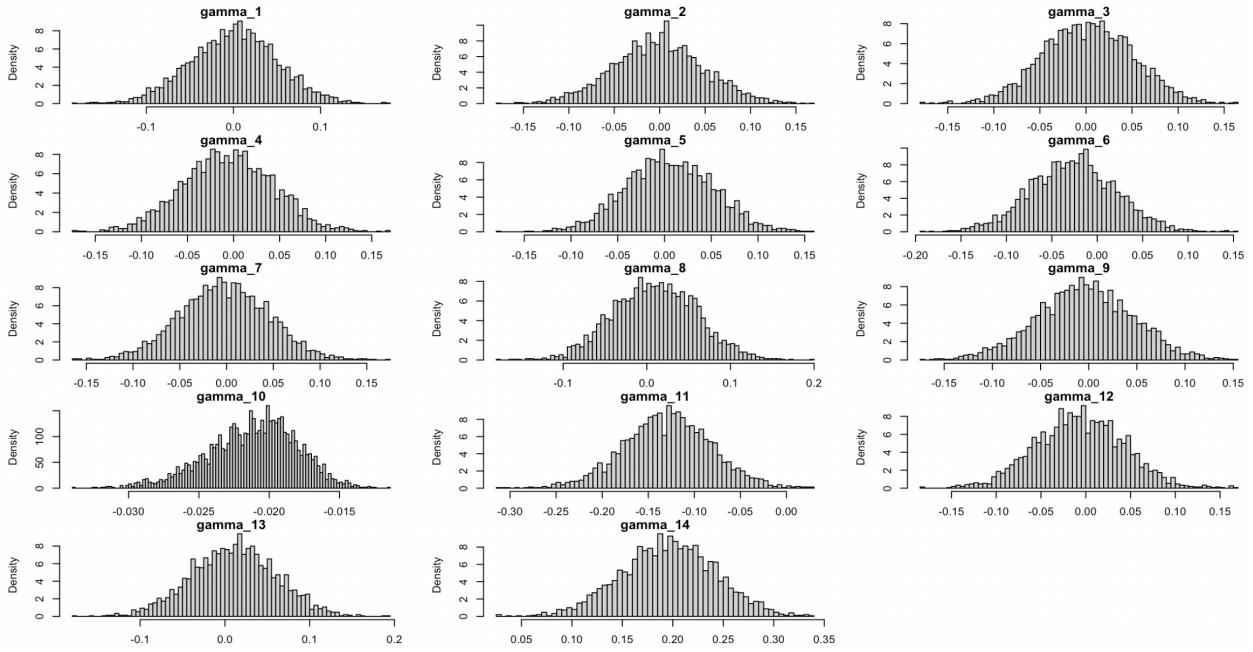
Finally, for σ^2 , our random walk is doing a good job. Samples do converge and have small correlation.

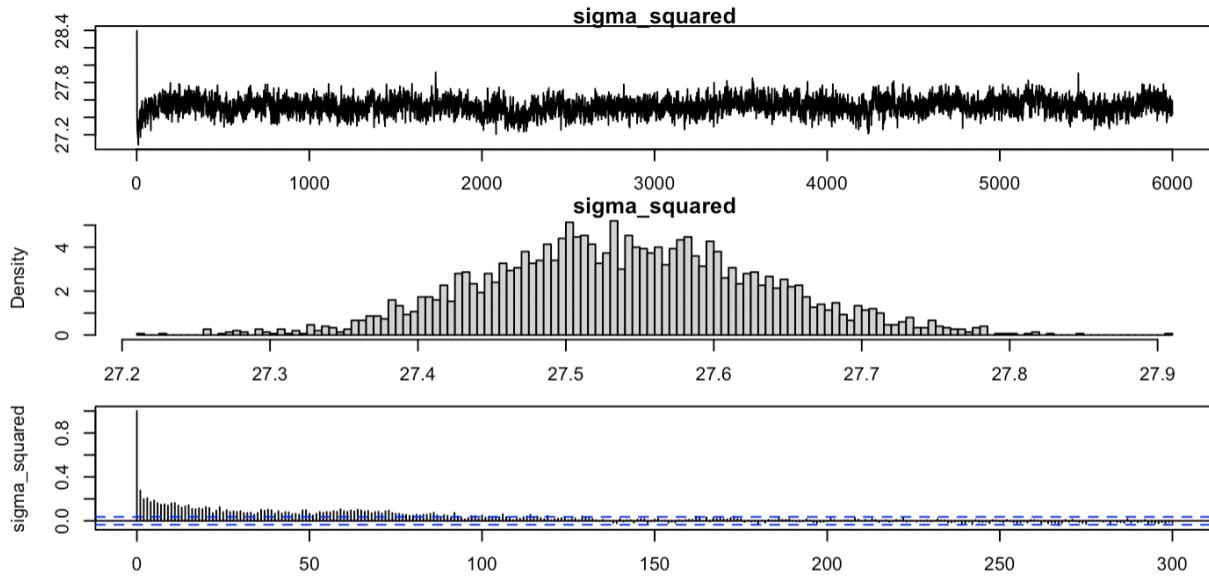
Figure 3: Trace plots and ACF plots for β Figure 4: Histograms for β

Figure 5: Trace plots and ACF plots for μ Figure 6: Histograms for μ

Figure 7: Trace plots for $A = \Sigma^{-1}$ Figure 8: ACF plots for $A = \Sigma^{-1}$

Figure 9: Histograms for $A = \Sigma^{-1}$ Figure 10: Trace plots for γ

Figure 11: ACF plots for γ Figure 12: Histograms for γ

Figure 13: Histograms for σ^2

Results

Seasonal Analysis

Prediction

Accurately predicting hurricane wind speeds is crucial in hurricane forecasting. To achieve this, we calculated the posterior sample means of each parameters by averaging the last three thousand values in each MCMC chain, and used these means to predict wind speeds at each time point. The estimated results of parameters are shown below.

	gamma_estimate
April	0.001
May	-0.001
June	0.002
July	-0.006
August	0.005
September	-0.026
October	-0.003
November	0.010
December	-0.004
Season	-0.021
ET	-0.129
NR	-0.007
SS	0.010
TS	0.196

Figure 14: Fixed Effects Gamma Estimate for Each Covariate

	ABLE.1950	BAKER.1950	CHARLIE.1950	DOG.1950	EASY.1950
intercept	4.751	4.704	4.497	4.372	4.516
Wind_prev	0.895	0.849	0.894	0.941	0.936
Lat_change	0.444	0.320	0.160	-0.167	0.078
Long_change	-0.490	-0.300	-0.352	-0.511	-0.384
Wind_change	0.456	0.354	0.470	0.487	0.546

Figure 15: Subset of Random Effects Beta Estimated for Each Hurricane

With these parameter estimates, we made wind speed prediction for hurricane Ginger 1971. As shown in the figure below, overall our predictions are very close to observed wind speed for Ginger 1971 but there exists a short amount of lag in our prediction. It is also worth noting that after fitting the first wind speed peak, the model seems to keep predicting sharp peaks even though the observed data has a small plateau after continuous build up in wind speed.

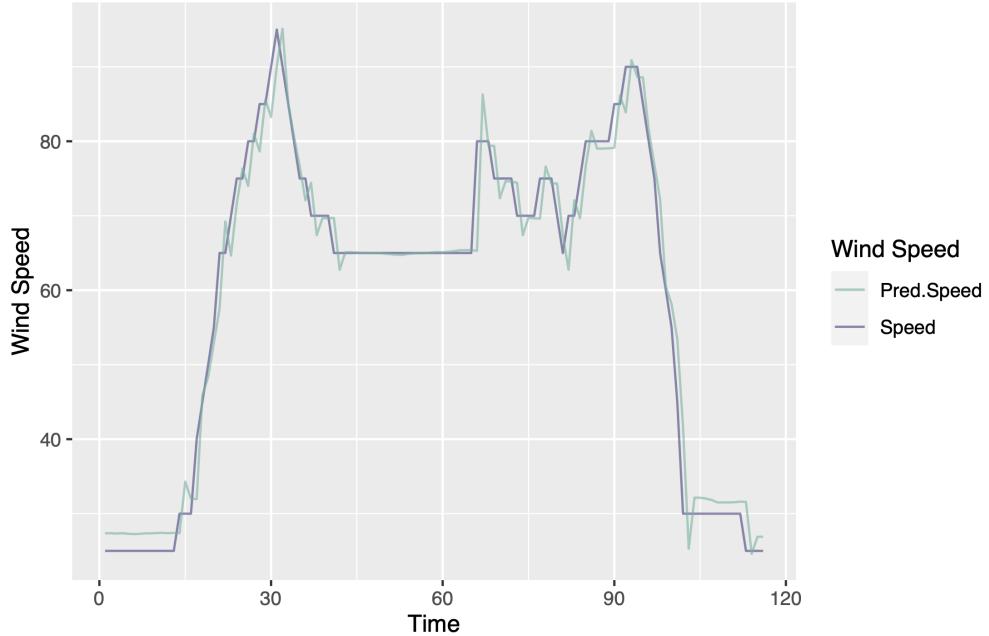


Figure 16: Observed Wind Speed vs. Predicted Wind Speed for Ginger 1971

To assess the overall performance of our Bayesian model across all hurricanes, we used the Root Mean Squared Error (RMSE) to compare the predicted wind speeds and the observed wind speeds. The table below shows the top 10 hurricanes with the least RMSE. The first hurricane has only six observations but still our model achieves a small RMSE of 1.041. This result indicates that our model can make accurate predictions even for hurricanes with fewer observations.

Hurricane ID	Number of Observation	Prediction RMSE
SIXTEEN.2008	6	1.041
FABIAN.1991	9	1.259
FOUR:UNNAMED.1988	5	1.373
TEN.2005	19	1.422
LORENZO.2001	14	1.475
FIFTEEN:UNNAMED.1988	4	1.845
FABIAN.1997	15	1.939
ANDREW.1986	14	1.943
TEN.2011	7	1.984
CINDY.1987	20	1.997

Figure 17: Prediction Standard Error of Wind Speed in Ascending Order

Next, we further analyzed the model’s performance on hurricanes with different number of observations. We broke down all observations into two groups: 1) a group of hurricanes with more than 50 observations, labeled ‘more observations’ ; 2) another group of hurricanes with fewer than 50 observations, labeled ‘fewer observations’. The average RMSE for both categories is around 4, suggesting that our model’s accuracy is not significantly affected by the amount of observations available. It is true that the performance on hurricanes with more observations is more stable and has a smaller variability. This finding is reassuring and indicates that our model is effective in predicting wind speeds for hurricanes with varying amounts of data.

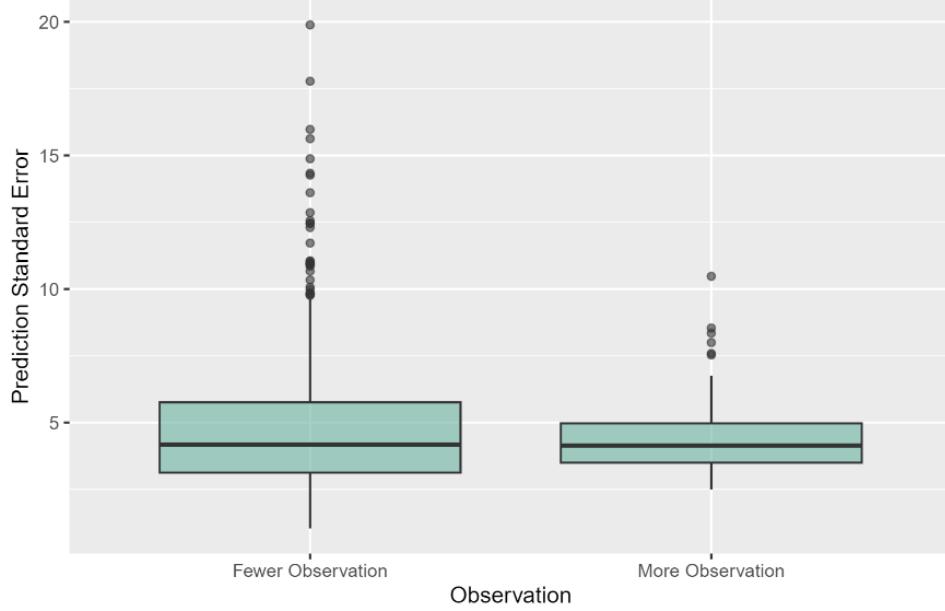


Figure 18: Prediction Standard Error of Wind Speed by Number of Observation

Additional plots are presented below to evaluate the model’s performance. The figure displaying the average prediction by month shows that the model tends to overestimate in January and underestimate in April by a tiny amount. Overall, our model performs pretty well across different months. When we evaluate our predictions by year, we also see consistent performance averaging across different hurricane types and months.

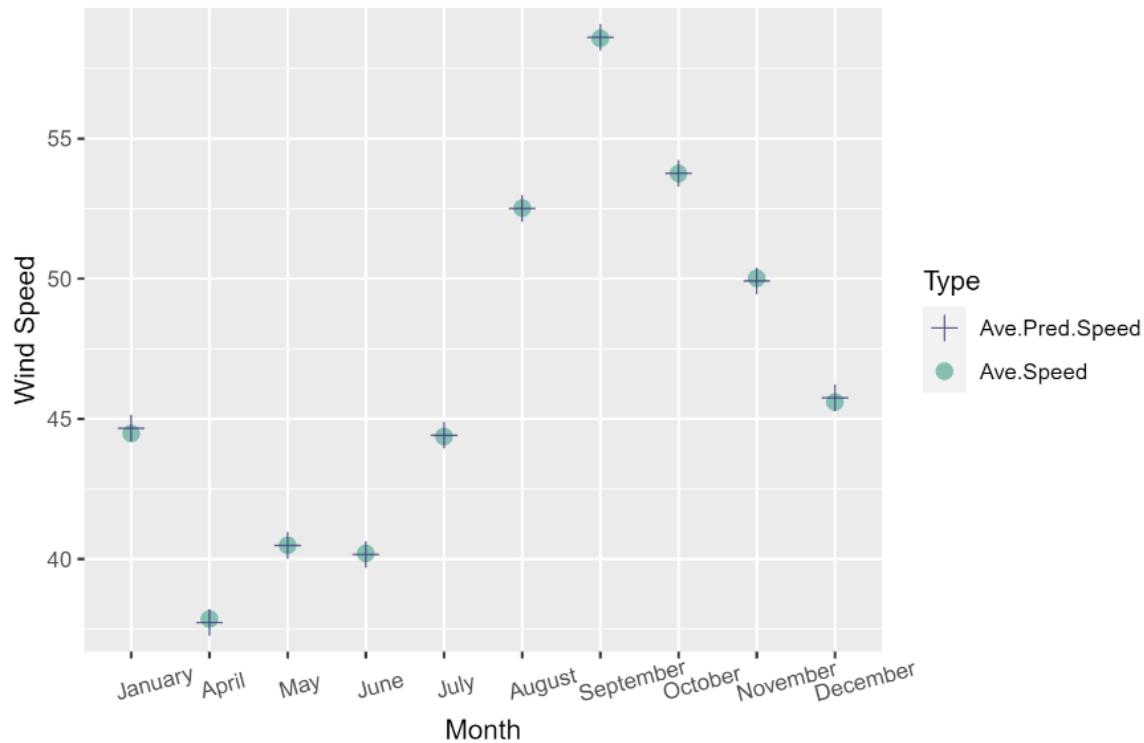


Figure 19: Observed Wind Speed vs. Predicted Wind Speed by Month

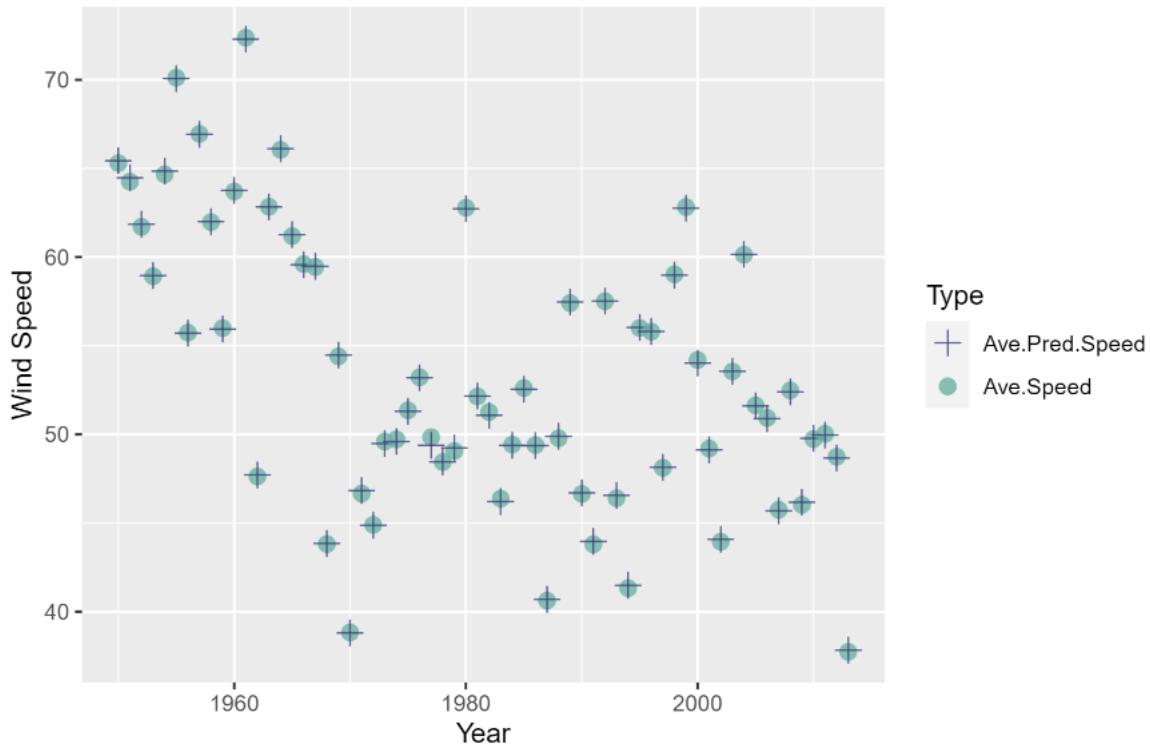


Figure 20: Observed Wind Speed vs. Predicted Wind Speed by Year

In conclusion, our analysis using RMSE and additional graphs demonstrates that our Bayesian model is capable of making accurate predictions for hurricane wind speeds. These results suggest that our model is effective in predicting wind speeds for hurricanes with varying amounts of data, and performs consistently across different months, years, and types of hurricanes. These findings have significant implications in the field of hurricane prediction and can inform future research in this area.

Discussion

In summary, our application of the MCMC algorithm to the hurricane Bayesian model yields valuable insights into the prediction of hurricane wind speeds. Our analysis revealed significant seasonal differences in wind speeds and did not find any evidence to support the claim of increasing wind speeds over the years. These findings emphasize the importance of accounting for seasonal differences when modeling hurricane wind speeds. Although our final model did not achieve perfect convergence for every parameter, we were still able to produce reliable predictions. The use of MCMC has allowed us to overcome the challenges posed by the complicated distribution and high-dimensional space of hurricane wind speed data, demonstrating the usefulness of this algorithm in Bayesian analysis.

Gibbs Sampling is a powerful tool in Bayesian statistics that is particularly useful for handling complicated distributions in high-dimensional spaces. Its main advantages include its flexibility to incorporate Random Walk and its ability to explore and sample the model parameter space efficiently. This is particularly helpful when direct sampling is practically impossible in high-dimensional spaces.

Despite its many benefits, our sampler has some drawbacks that need to be addressed. One major issue is the assumption of prior distribution, which can introduce bias and affect the estimation of the posterior distribution. To find more informative priors, we can look for external sources on hurricane forecasting and create exploratory plots for some random samples of the data set. Additionally, the method cannot guarantee convergence due to the correlated samples, which can lead to errors in the estimation. To address this issue, our method can be improved by upgrading to a more developed sampler such as Hamiltonian Monte Carlo, increasing number of samples, or trying different parameterizations of the model. These solutions can lead to more accurate and efficient estimation of the posterior distribution.

In the end, we must also note that the data set only includes hurricane observations from 1950 up to 2013. Our results are not generalizable to predict wind speed of hurricanes beyond this time range especially due to global warming in recent years.

Group Contributions

Appendix