Neuromorphic Engineering I

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Material courtesy of Elisabetta Chicca
Bielefeld University, Germany
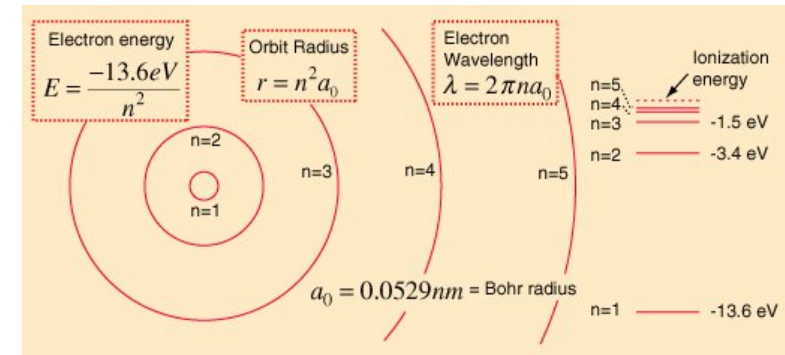
Institute of Neuroinformatics
University of Zurich and ETH Zurich

Zurich, 2013/2014 Fall Semester

Isolated atoms
Hydrogen Energy Levels

Electron in atoms have quantized energy levels

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/hyde.html

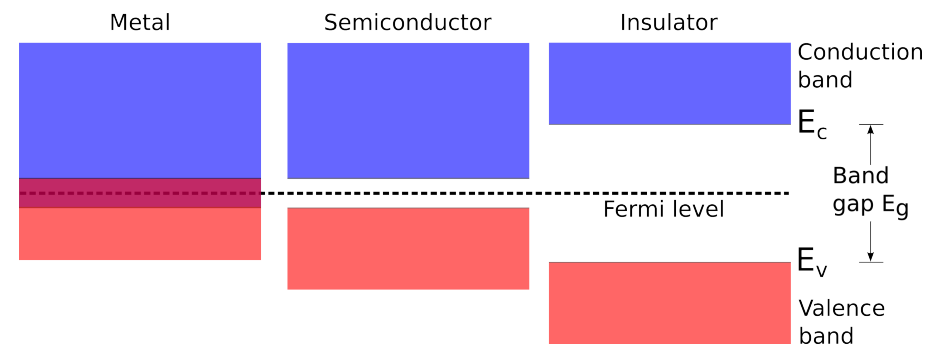
Solids
Energy bands

In a solid the forces of attraction and repulsion between atoms affect the electron energy levels.

An energetic balance is found at the proper inter-atomic spacing for the crystal.

The electronic energy levels form continuous bands of energies.

Metals, semiconductors and insulators
Energy bands

Metals, semiconductors and insulators
Energy bands

At 0 K the electrons occupy the lowest energy states available, every state in the valence band is filled and the conduction band is empty.

Current flow in solids: an applied electric field can generate current flow only if electrons can move into new energy states. There must be empty states available to the electrons.

Metals, semiconductors and insulators
Energy bands

Insulators: at 0 K the valence band is completely filled and the conductance band is completely empty. No charge transport can take place in either bands.

Semiconductors: at 0 K semiconductors are equivalent to insulators. The size of the band gap E_g is much smaller in semiconductors (e.g. 1.1 eV for silicon (Si), 5 eV for diamond (C)).

Metals: the energy bands overlap, thus electrons and empty energy states are intermixed. At 0 K an applied electric field can generate current flow (electrons can move freely).

Metals, semiconductors and insulators
### Energy bands

Room temperature
In semiconductors (materials with small band gap E_g) the number of electrons available for conduction is significantly increased by thermal energy. This number is negligible in insulators (materials with big band gap E_g).
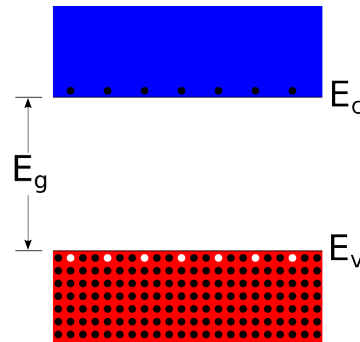
Electrons and holes

In semiconductors, at room temperature, thermal energy allows electrons in the valence band to be excited across the band gap to the conduction band.
These electrons leave unoccupied states in the valence band (called *holes*).
This process generates the so called Electron-Hole Pairs (EHPs).

Electrons and holes (free charges)
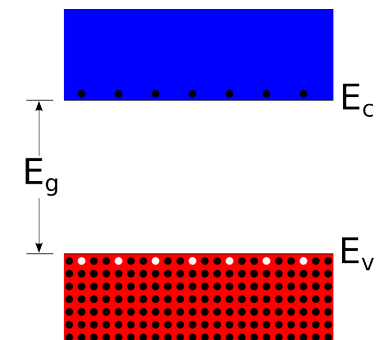
Electrons in the conduction band are surrounded by a large number of unoccupied energy states (e.g.: pure Si at room temperature: 10^{10} EHP/cm^3, 10^{22} atoms/cm^3). These unoccupied energy states allow the few electrons in the conductance band to move freely. Similarly, holes in the valence band can move and generate a current flow of positive charges.

E_c

E_g

E_v

Intrinsic semiconductors

Intrinsic semiconductor: perfect crystal of semiconductor material with no impurities or defects.
At 0 K there are no free electron-hole pairs, therefore no charge carriers are available.
Si valence electrons are constrained in covalent bonds[1] of the crystal lattice. The energy required to break a covalent bond is the band gap energy E_g.
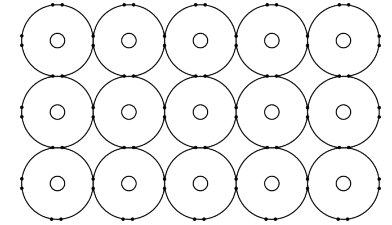
[1] Chemical bond formed by the sharing of one or more electrons, especially pairs of electrons, between atoms.

Intrinsic semiconductors

At room temperature, thermal energy can break covalent bonds and generate electron-hole pairs.
The free electron and the hole are spread out over several lattice spacing (not as depicted in the figure), as predicted by quantum mechanics.
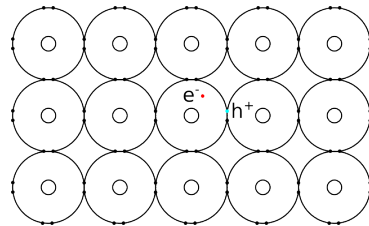Electrons concentration in the conductance band
$n = $ *electrons per cm*^{3}
Holes concentration in the valence band
$p = $ *holes per cm*^{3}
Intrinsic carrier concentration
$n_i = n = p$

e^- h^+

Extrinsic semiconductors

Extrinsic or *doped* materials are created by purposely introducing impurities into the crystal. Impurities can generate extrinsic negative or positive charge carriers.

Donor impurities: atoms with more valence electrons than intrinsic atoms. The extra electrons require very little thermal energy to be excited to the conductance band.

Acceptor impurities: atoms with less valence electrons than intrinsic atoms. The missing electrons are recruited from the valence band of intrinsic atoms, creating holes in the intrinsic semiconductor.

Extrinsic semiconductors

n-type materials

Donor atoms for Silicon and Germanium (4 electrons) are Phosphorus and Arsenic (5 electrons).

Extrinsic semiconductors

p-type materials
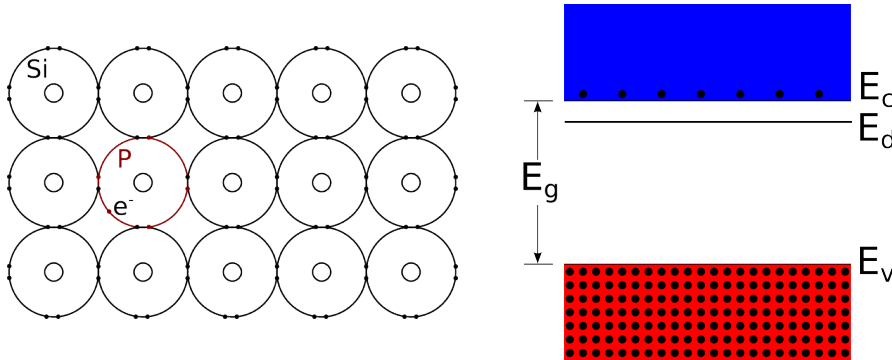
Acceptor atoms for Silicon and Germanium (4 electrons) are Boron and Aluminium (3 electrons).

Distribution of electrons

Fermi-Dirac distribution function
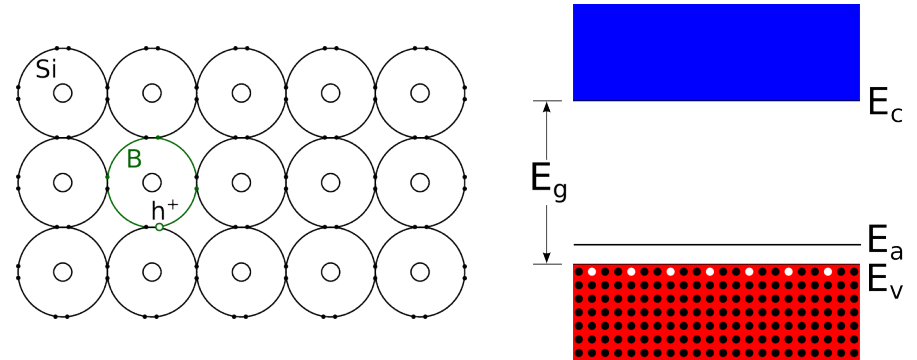
The distribution of electrons at thermal equilibrium is

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$f(E)$ is the probability that an energy state with energy E is occupied by an electron at absolute temperature T. k is the Boltzmann's constant ($k = 8.62 \times 10^{-5} eV/K$) and E_F is the Fermi level. Independently of the absolute temperature, the probability of occupancy of an energy state at the Fermi level is $1/2$.

Concentration of electrons

The electrons concentration in the conduction band is given by the integral, over the range of energies accessible by electrons, of the density of states at energy E, weighted by the probability $f(E)$ for an electron to have this given energy:

$$n_0 = \int_{E_C}^{\infty} f(E)N(E)dE$$

$N(E)dE$ (cm^{-3}) is the density of states in the energy range dE.

The density of states in the conduction band can be derived using the three dimensional Schrödinger equation for a free electron, with the addition of a set of periodic boundary conditions representing the crystal lattice (see Streetman 1995, appendix IV). The subscript 0 in n_0 is used to indicate equilibrium conditions.

Given that $f(E)$ tends to zero for large energies, $f(E)N(E)$ decreases rapidly above E_C.

Concentration of electrons
Derivation

$$f(E) \approx e^{(E_F - E)/kT} \; \text{ for } (E - E_F) > 3kT, \quad N_C(E) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\sqrt{E - E_C}$$

where m_c is the effective mass of the electron in the crystal and $\hbar = h/2\pi$ is the normalized Plank constant ($h = 6.626 \cdot 10^{-34} \; Js$).

$$n_0 = \int_{E_C}^{\infty} N_C(E) f(E) dE = \int_{E_C}^{\infty} N_C(E) \cdot e^{\frac{E_F - E}{kT}} dE = \int_{E_C}^{\infty} N_C(E) \cdot e^{-\frac{E - E_C}{kT}} e^{-\frac{E_C - E_F}{kT}} dE$$

$$\boxed{n_0 = N_C \cdot e^{-\frac{E_C - E_F}{kT}}}$$

where $N_C = \int_{E_C}^{\infty} N_C(E) \cdot e^{-\frac{E - E_C}{kT}} dE$ is the *effective densities of states*.

Concentration of holes
Derivation

$$1 - f(E) \approx e^{-(E_F - E)/kT} \; \text{for } (E_F - E) > 3kT, \quad N_V(E) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\sqrt{E_V - E}$$

where m_v is the effective mass of the hole in the crystal and $\hbar = h/2\pi$ is the normalized Plank constant ($h = 6.626 \cdot 10^{-34} \; Js$).

$$p_0 = \int_{-\infty}^{E_V} N_V(E)(1 - F(E)) dE = \int_{-\infty}^{E_V} N_V(E) \cdot e^{-\frac{E_F - E}{kT}} e^{\frac{E_V - E_V}{kT}}$$

$$\boxed{p_0 = N_V \cdot e^{\frac{E_V - E_F}{kT}}}$$

where $N_V = \int_{-\infty}^{E_V} N_V(E) \cdot e^{\frac{E - E_V}{kT}} dE$ is the *effective densities of states*.

The product $n_0 p_0$

For a given material and temperature, the product of n_0 and p_0 is a constant:

$$n_0 p_0 = N_C \cdot e^{-\frac{E_C - E_F}{kT}} N_V \cdot e^{\frac{E_V - E_F}{kT}} = N_C N_V e^{-\frac{E_C - E_V}{kT}} = N_C N_V e^{-\frac{E_g}{kT}}$$

Intrinsic carriers are created in pairs (by thermal energy), so their concentrations $n_i = N_C e^{-(E_C - E_i)/kT}$ and $p_i = N_V e^{(E_V - E_i)/kT}$ (where E_i is the intrinsic Fermi level) are equal. Given that $n_i p_i = N_C N_V e^{-E_g/kT}$, we can write the intrinsic concentration as

$$n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}}$$

which leads to

$$\boxed{n_0 p_0 = n_i^2, \; n_0 = n_i \cdot e^{\frac{E_F - E_i}{kT}}, \; p_0 = n_i \cdot e^{\frac{E_i - E_F}{kT}}}$$
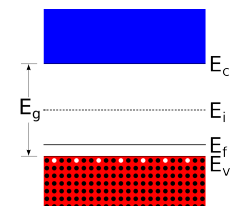
Extrinsic semiconductors
Fermi level and carrier concentrations

For $N_d \gg n_i$ and $N_a = 0$, $n_0 \simeq N_d$ and $p_0 \simeq \frac{p_i^2}{N_d}$
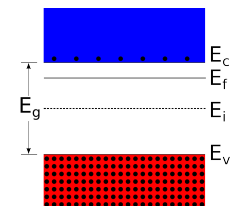
For $N_a \gg p_i$ and $N_d = 0$, $p_0 \simeq N_a$ and $n_0 \simeq \frac{n_i^2}{N_a}$

Example: $N_a > N_d > 0$, this is a *p*-type material (more holes than electrons, Fermi level in the lower part of the band gap). All free electrons created by donors recombine with holes created by acceptors. The resultant holes concentration is therefore $N_a - N_d$. This process is called *compensation*.

Space charge neutrality: $p_0 + N_d^+ = n_0 + N_a^-$

$p_0 = n_0 + (N_a^- - N_d^+) \simeq N_a - N_d$, if the material is *p*-doped ($N_a > N_d$) and all impurities are ionized ($N_a^- = N_a$ and $N_d^+ = N_d$).

E_g · · · · · · E_c

· · · · · · E_i

E_f

E_v
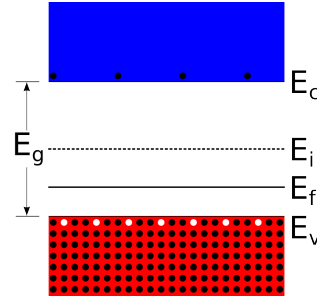
When an electric field is applied to a semiconductor a *drift* of carriers is generated, thus producing a *current flow*. Two important quantities are needed to calculate the current flow: carrier concentrations and mobilities.

Free charges in semiconductors are subject to collisions with the lattice and the impurities. These processes are temperature dependent (thermal motion of the lattice atoms and carriers' velocity) and affect carrier mobilities.

An electric field $\mathscr{E}_x$ applied in the *x*-direction produces a force $-q\mathscr{E}_x$ on each electron. This force produces, on average a net motion of electrons in the *x*-direction, affecting the total *x*-component momentum p_x as follows:

$$-nq\mathscr{E}_x = \frac{dp_x}{dt}$$

At steady state, the acceleration provided by the electric field is balanced by decelerations due to collisions. The differential change in momentum due to collisions is

$$dp_x = -p_x \frac{dt}{\bar{t}} \rightarrow \frac{dp_x}{dt} = -\frac{p_x}{\bar{t}}$$

where $\bar{t}$ is the *mean free time* (mean time between two collisions).

At steady state the changes of momentum due to the electric field and collisions must balance:

$$-nq\mathscr{E}_x - \frac{p_x}{\bar{t}} = 0$$

We can therefore express the average momentum per electron $< p_x >$ as

$$< p_x > = \frac{p_x}{n} = -q\bar{t}\mathscr{E}_x$$

We can therefore write the electrons average net velocity $< v_x >$ produced by the electric field $\mathscr{E}_x$ as

$$< v_x > = \frac{< p_x >}{m_n^*} = -\frac{q\bar{t}}{m_n^*}\mathscr{E}_x$$

where m_n^* is the electron effective mass (which is a property of the material band structure).

The electron current density produced by the electric field $\mathscr{E}_x$ is given by the electron charge $-q$ multiplied by the number of electron crossing a unit area per unit time $n < v_x >$

$$J_x = \frac{nq^2\bar{t}}{m_n^*}\mathscr{E}_x$$

Drift of carriers in electric field

Ohm's law states that the current density is directly proportional to the electric field:

$$J_x = \sigma_n \mathscr{E}_x, \text{ where } \sigma_n \equiv \frac{nq^2\bar{t}}{m_n^*} \text{ is the conductivity}$$

The electron *mobility* is defined as

$$\mu_n \equiv \frac{q\bar{t}}{m_n^*}$$

therefore, the conductivity can be rewritten as $\sigma_n = qn\mu_n$.

The current density can be rewritten as

$$\boxed{J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x}$$

Diffusion of carriers

Diffusion of carriers is generated by the presence of a carrier concentration gradient (as for gas molecules). The gradient in carrier concentration generates a net motion from regions with high concentration to regions with low concentration. Let us consider an arbitrary function $n(x)$ describing the electron distribution in the x-direction.

The electron flux density (rate of electron flow per unit area) from x_1 to $x_2 = x_1 + 2\bar{l}$ is

$$\Phi_n(x_0) = \frac{\bar{l}}{2\bar{t}}(n_1 - n_2)$$

where $\bar{l}$ is the mean free path between collisions, $\bar{t}$ is the mean free time between collisions, $n_1 = n(x_1)$ and $n_2 = n(x_2)$ are the electron concentrations at x_1 and x_2 respectively.

Diffusion of carriers

Given that $\bar{l}$ is a small distance, we can write

$$n_1 - n_2 = \frac{n(x) - n(x + \Delta x)}{\Delta x}\bar{l}$$

with $x = x_1 + \bar{l}$ and $\Delta x = \bar{l}$. In the limit of small free path we can rewrite the electron flux density as follows:

$$\Phi_n(x_0) = \frac{\bar{l}^2}{2\bar{t}} \lim_{\Delta x \to 0} \frac{n(x) - n(x + \Delta x)}{\Delta x} = -\frac{\bar{l}^2}{2\bar{t}}\frac{dn(x)}{dx} = -D_n\frac{dn(x)}{dx}$$

where $D_n = \bar{l}^2/2\bar{t}$ is the electron diffusion coefficient.

Current density

$$\boxed{\begin{aligned} J_n^{diff} &= +qD_n\frac{dn(x)}{dx} \\ J_p^{diff} &= -qD_p\frac{dp(x)}{dx} \end{aligned}}$$

where J_n^{diff} and J_p^{diff} are the diffusion currents crossing a unit area due to electrons and holes respectively.

Both electrons and holes move in the direction of decreasing carrier concentration. Because of their opposite charge ($-q$ for electrons, $+q$ for holes) the currents they generate are in opposite directions (given the same concentration gradient).

Total current density

The total current density is the sum of the electron and holes current densities.

$$J(x) = J_n(x) + J_p(x)$$

In presence of an electric field, each of these current densities have a drift and a diffusion component:

$$J_n(x) = q\mu_n n(x)\mathscr{E}_x + qD_n\frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)\mathscr{E}_x - qD_p\frac{dp(x)}{dx}$$

Diffusion and recombination

Recombination of charge carriers should be considered when dealing with diffusion processes. Recombination can alter the carrier distribution, thus affecting diffusion. Let us consider the number of carriers entering a volume $\Delta x A$ (where Δx is a differential length of a semiconductor sample with area A in the plan orthogonal to x) per unit time:

$$\left.\frac{\partial p}{\partial t}\right|_{x\rightarrow x+\Delta x} = \frac{1}{q}\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

where $J_p(x)$ is the hole current density and $\delta p/\tau_p$ is the recombination rate. For $\Delta x \rightarrow 0$, we can use the derivative form to write the *continuity equation* for holes and electrons

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

Diffusion equation

For negligible drift current, we can replace the current density by the diffusion current ($J_p = -qD_p\frac{\partial \delta p}{\partial x}$, $J_n = qD_n\frac{\partial \delta n}{\partial x}$) and obtain the *diffusion equation* for holes and electrons:

$$\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta n}{\partial t} = D_n\frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

Diffusion equation
Steady state

When a steady state distribution of excess carriers is maintained, all time derivatives in the diffusion equation are zero. The diffusion equation for the steady state condition becomes:

$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p\tau_p} \equiv \frac{\delta p}{L_p^2}$$

$$\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2}$$

where $L \equiv \sqrt{D\tau}$ is the diffusion length.

The origins of the p-n junction

Russel Ohl and colleagues

Bell Telephone Laboratories, March 1940.

Before and during World War II ultrashort-wave radio communication research was pushed by the need for radar applications. George Southworth unsuccessfully tried to detect ultrashort radio waves with specially designed vacuum tubes and copper oxide rectifiers. He then bought a few old "cat's whisker" crystal detectors from a tiny second-hand radio shop and got promising results. Southworth asked Ohl to make a comprehensive study of crystal detectors to determine the best material. Ohl had already worked extensively with crystal detectors, trying to remove odd behaviors by reducing impurities. Ohl tested a rod of silicon and he wrote in his lab notebook that "near one end of the rod there is a change in the crystal structure indicated by a crack." He also noticed that the amount of current that could flow through the crystal was strongly affected by light. The crack was the point of contact between highly purified silicon and impurities rich silicon (resulting from slowly cooling the ingot from which the rod was cut). The two types of silicon were called p-type (producing a positive voltage under light exposure) and n-type (producing a negative voltage under light exposure). The barrier between the two types of silicon was named **p-n junction**.

The origins of the p-n junction
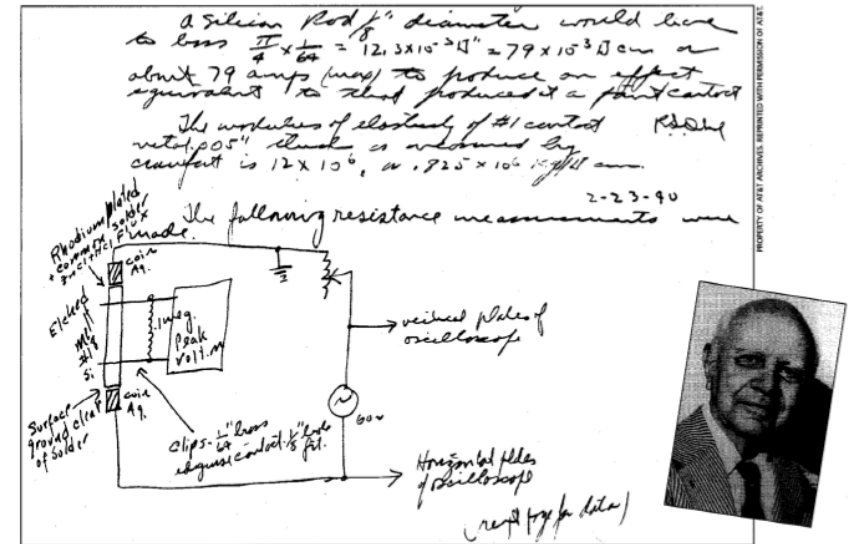
Source: Riordan and Hoddeson, *The origins of the p-n junction*, IEEE Spectrum, 1997

Invariance of the Fermi level at equilibrium

Let us consider two different materials (e.g.: a metal and a semiconductor, two differently doped semiconductors, etc.) in contact with each other, such that charge carriers can move from one to the other. At thermal equilibrium the net current and the net transfer of energy have to be null $\rightarrow$ the net transfer of electrons between the two material has to be null.

The rate of transfer of electrons from one material to the other is proportional to the number of filled state in the first material and the number of empty states in the second material:

$$1 \rightarrow 2 \quad \propto N_1(E)f_1(E) \cdot N_2(E)[1 - f_2(E)]$$
$$2 \rightarrow 1 \quad \propto N_2(E)f_2(E) \cdot N_1(E)[1 - f_1(E)]$$

where $N_1(E)$ and $N_2(E)$ are the density of states for material 1 and 2 respectively, $f_1(E)$ and $f_2(E)$ are the Fermi distributions for material 1 and 2 respectively.

Invariance of the Fermi level at equilibrium

At equilibrium

$$N_1(E)f_1(E) \cdot N_2(E)[1 - f_2(E)] = N_2(E)f_2(E) \cdot N_1(E)[1 - f_1(E)]$$

which leads to

$$N_1 N_2 f_1 - N_1 N_2 f_1 f_2 = N_1 N_2 f_2 - N_1 N_2 f_1 f_2 \rightarrow f_1(E) = f_2(E)$$

$$\frac{1}{1 + e^{(E - E_{F1})/kT}} = \frac{1}{1 + e^{(E - E_{F2})/kT}} \rightarrow E_{F1} = E_{F2}$$

At equilibrium the Fermi level throughout materials in intimate contact is constant:

$$\frac{dE_F}{dx} = 0 \quad \forall x$$

The contact potential

Let us imagine to put together two pieces of semiconductor: an *n*-type and a *p*-type semiconductor. Once the two pieces are joint, concentration gradients of electrons and holes are created. Because of that holes diffuse to the *n*-type material and electrons diffuse to the *p*-type material.

Holes diffusing away from the *p*-type material create uncompensated acceptor ions N_a^- and electrons diffusing away from the *n*-type material create uncompensated donor ions N_d^+. These charges generate an electric field directed from the *n* to the *p*-type material. This field gives rise to a drift current from the *n*-type to the *p*-type material, opposite to the diffusion current.

The contact potential

V_0 = contact potential
qV_0 = bands separation
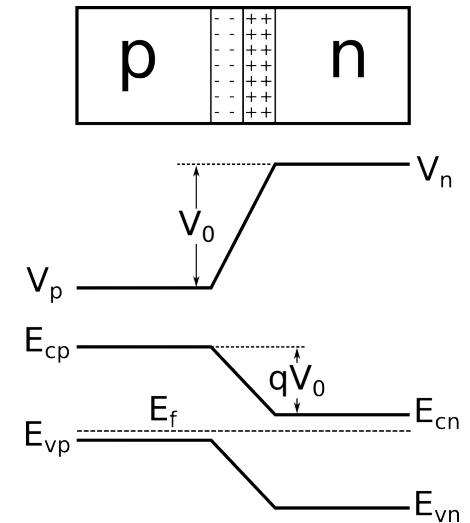E_F = Fermi level

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Equilibrium

At equilibrium the following conditions have to be met:

- No net current can flow: the drift currents exactly compensate the diffusion currents.
- No accumulation of carrier can occur: for each type of carrier, the drift current exactly compensate the diffusion current

$$J_n(diff) - J_n(drift) = 0$$
$$J_p(diff) - J_p(drift) = 0$$

Electron current

Assuming that the *x* direction is from *p* to *n*, we can write the electron current as follows:

$$J_n(x) = q \left[D_n \frac{dn(x)}{dx} + \mu_n n(x) \mathscr{E}(x) \right] = 0$$

Rearranging terms, we can write:

$$\frac{\mu_n}{D_n} \mathscr{E}(x) = -\frac{1}{n(x)} \frac{dn(x)}{dx}$$

Using the Einstein relation ($D/\mu = kT/q$) and writing the electric field as a function of the potential gradient ($\mathscr{E} = -dV(x)/dx$), we obtain:

$$\frac{q}{kT} \frac{dV(x)}{dx} = \frac{1}{n(x)} \frac{dn(x)}{dx}$$

We can solve this equation by integrating it over the appropriate limits:

$$\frac{q}{kT} \int_{V_p}^{V_n} dV = \int_{n_p}^{n_n} \frac{dn}{n}$$

This integration gives:

$$\frac{q}{kT}(V_n - V_p) = \ln n_n - \ln n_p = \ln \frac{n_n}{n_p}$$

Given that the contact potential V_0 is the difference $V_p - V_n$, we can write it as a function of the equilibrium hole concentration on the two sides of the junction:

$$V_0 = \frac{kT}{q} \ln \frac{n_n}{n_p} \rightarrow \frac{n_n}{n_p} = e^{qV_0/kT}$$

Given that $p_p n_p = p_n n_n = n_i^2$, we can write:

$$\boxed{\frac{n_n}{n_p} = \frac{p_p}{p_n} = e^{\frac{qV_0}{kT}}}$$

Assuming that the x direction is from p to n, we can write the hole current as follows:

$$J_p(x) = q \left[-D_p \frac{dp(x)}{dx} + \mu_p p(x) \mathscr{E}(x) \right] = 0$$

Rearranging terms, we can write:

$$\frac{\mu_p}{D_p} \mathscr{E}(x) = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

Using the Einstein relation ($D/\mu = kT/q$) and writing the electric field as a function of the potential gradient ($\mathscr{E} = -dV(x)/dx$), we obtain:

$$-\frac{q}{kT} \frac{dV(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

We can solve this equation by integrating it over the appropriate limits:

$$-\frac{q}{kT} \int_{V_p}^{V_n} dV = \int_{p_p}^{p_n} \frac{dp}{p}$$

This integration gives:

$$-\frac{q}{kT}(V_n - V_p) = \ln p_n - \ln p_p = \ln \frac{p_n}{p_p}$$

Given that the contact potential V_0 is the difference $V_n - V_p$, we can write it as a function of the equilibrium hole concentration on the two sides of the junction:

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Given that $p_p n_p = p_n n_n = n_i^2$, we can write:

$$\boxed{\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{\frac{qV_0}{kT}}}$$

If we assume that in each side of the junction the majority carrier concentration is equal to the impurity concentration, we can express the contact potential as follows:

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \tag{1}$$

where N_a is the acceptor concentration in the p-type material, N_d is the donor concentration in the n-type material and n_i is the intrinsic electron concentration.

When an external voltage is applied to the p-n junction, its current flow strongly depend on the sign of the voltage:

- *Forward bias*: the *p* region as a positive external voltage bias relative to the *n* region. Current can freely flow from the *p* to the *n* region.
- *Reverse bias*: the *p* region as a negative external voltage bias relative to the *n* region. No current can flow in the p-n junction.

p-n junctions can also be used as photocells (see p-n junction history), light emitters (LEDs), voltage variable capacitor. Junctions can be used as "building blocks" for constructing TRANSISTORS.
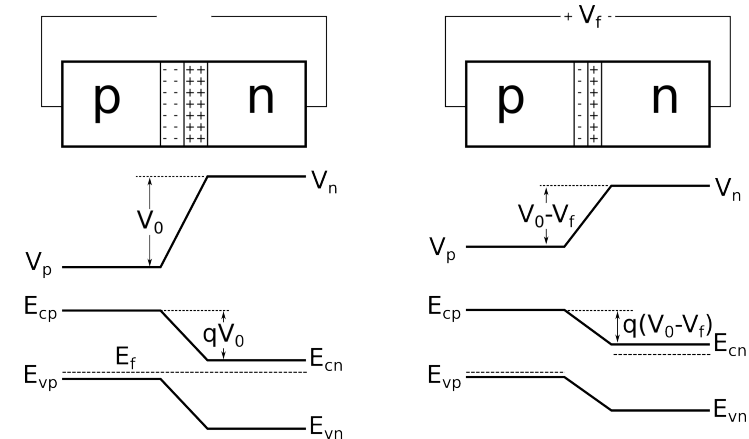
We can assume that the applied voltage V_f appears entirely across the depletion region. The forward bias raises the potential on the *p*-type material relative to the *n*-type material. The electric field decreases (applied field opposite to built-in field), therefore decreasing the width of the depletion region. Given the lowered energy barrier majority carriers can easily diffuse from one side to the other (holes from *p* to *n* and electrons from *n* to *p*).

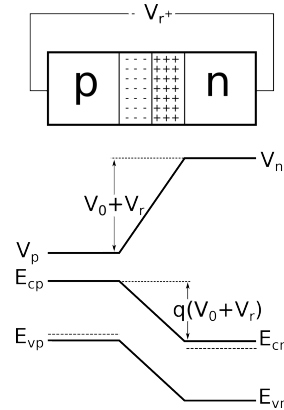The diffusion current can be large for forward bias.

The drift current is independent of the energy barrier.
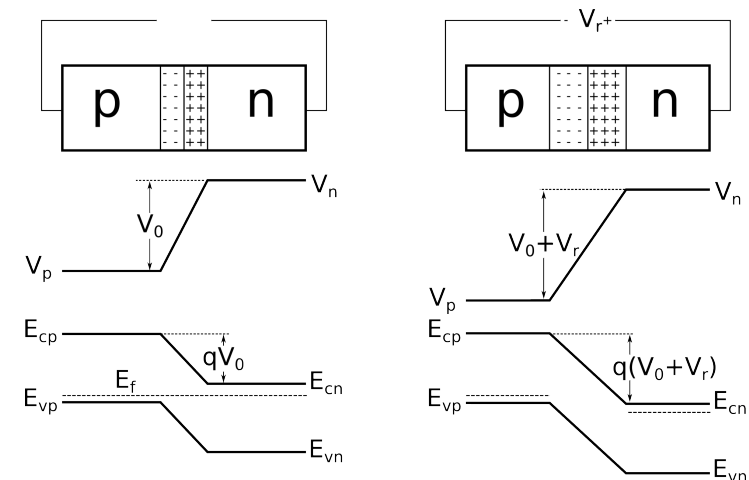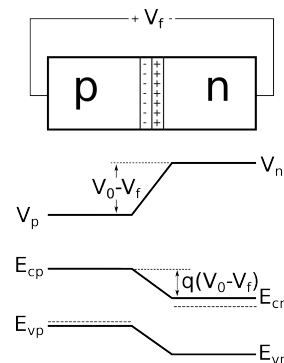
Reverse bias

The reverse bias lowers the potential on the *p*-type material relative to the *n*-type material. The electric field increases (applied field in the same direction as the built-in field), therefore increasing the width of the depletion region. Given the increased energy barrier virtually no majority carriers can diffuse from one side to the other.

The diffusion current is negligible for reverse bias.

The drift current is independent of the energy barrier.

Diode equation
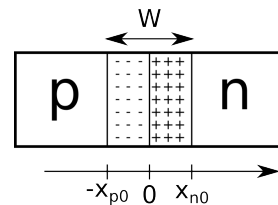### Equilibrium ratio of electron concentration

Diffusion of majority carriers across the depletion region varies the minority carrier concentrations in the opposite side. In presence of an external voltage bias the equilibrium ratio of hole concentration ($n_n/n_p = e^{qV_0/kT}$) becomes

$$\frac{n(x_{n0})}{n(-x_{p0})} = e^{q\frac{V_0 - V}{kT}}$$

where the external voltage bias *V* is positive for forward bias and negative for reverse bias.

Neglecting changes in majority carrier concentrations, we can write:

$$\frac{n_n}{n(-x_{p0})} = \frac{n_n}{n_p}e^{\frac{-qV}{kT}} \rightarrow \frac{n(-x_{p0})}{n_p} = e^{\frac{qV}{kT}}$$

Diode equation
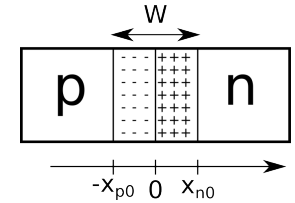### Equilibrium ratio of hole concentration

Diffusion of majority carriers across the depletion region varies the minority carrier concentrations in the opposite side. In presence of an external voltage bias the equilibrium ratio of hole concentration ($p_p/p_n = e^{qV_0/kT}$) becomes

$$\frac{p(-x_{p0})}{p(x_{n0})} = e^{q\frac{V_0 - V}{kT}}$$

where the external voltage bias *V* is positive for forward bias and negative for reverse bias.

Neglecting changes in majority carrier concentrations, we can write:

$$\frac{p_p}{p(x_{n0})} = \frac{p_p}{p_n}e^{\frac{-qV}{kT}} \rightarrow \frac{p(x_{n0})}{p_n} = e^{\frac{qV}{kT}}$$

Diode equation
Minority carrier distribution

The excess of minority carrier concentration at the edges of the depletion region can be written as follows:

$$\Delta n_p = n(-x_{p0}) - n_p = n_p e^{\frac{qV}{kT}} - n_p = n_p(e^{\frac{qV}{kT}} - 1)$$
$$\Delta p_n = p(x_{n0}) - p_n = p_n e^{\frac{qV}{kT}} - p_n = p_n(e^{\frac{qV}{kT}} - 1)$$

As minority carriers diffuse into the material (away from the depletion region) they recombine with majority carriers. The resulting excess of minority carrier distribution can be derived as a solution of the steady state diffusion equation (see slide 32):

$$\delta p(x_n) = \Delta p_n e^{-\frac{x_n}{L_p}} = p_n(e^{\frac{qV}{kt}} - 1)e^{-\frac{x_n}{L_p}}$$
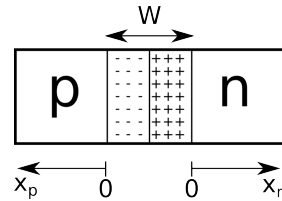$$\delta n(x_p) = \Delta n_p e^{-\frac{x_p}{L_n}} = n_p(e^{\frac{qV}{kt}} - 1)e^{-\frac{x_p}{L_n}}$$

where x_p is the distance from the edge of depletion region in the *p*-type material ($-x_{p0}$) and x_n is the distance from the edge of depletion region in the *n*-type material (x_{n0}).

Diode equation
Current
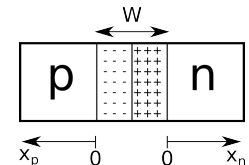
The hole and electron diffusion currents can be easily calculated:

$$I_p(x_n) = -qAD_p\frac{d\delta p(x_n)}{dx_n} = qA\frac{D_p}{L_p}\Delta p_n e^{-\frac{x_n}{L_p}}$$

$$I_n(x_p) = qAD_n\frac{d\delta p(x_p)}{dx_p} = -qA\frac{D_n}{L_n}\Delta n_p e^{-\frac{x_p}{L_n}}$$

The total diffusion current injected into the material at the junction is obtained for $x_p = 0$ and $x_n = 0$ respectively in the p and n diffusion currents:

$$I_p(x_n = 0) = qA\frac{D_p}{L_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{\frac{qV}{kT}} - 1)$$

$$I_n(x_p = 0) = -qA\frac{D_n}{L_n}\Delta n_p = -qA\frac{D_n}{L_n}n_p(e^{\frac{qV}{kT}} - 1)$$

where I_n is opposite to the x_p direction.

Diode equation
Current

Taking the x-direction as reference, we can write the total current as follows:

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{\frac{qV}{kT}} - 1\right) = I_0\left(e^{\frac{qV}{kT}} - 1\right)$$

For reverse bias $V_r > 3kT/q$ the exponential term is negligible compared to the unity term and the diode equation becomes $I = -I_0$.

Next week

Metal-Oxide-Semiconductor Transistor

Momentum

The momentum p of a particle is the product of its mass m and velocity v:

$$p = mv$$

A force F applied to a particle for a time interval Δt produces a change in the particle's momentum given by:

$$\Delta p = F\Delta t$$

This equation can be expressed in a differential form (Newton's second law) stating that the rate of change in momentum is equal to the force applied to the particle:

$$F = \frac{dp}{dt}.$$

Diffusion equation $\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{L_p^2}$

$$ay'' + by' + cy = 0$$

If $b^2 - 4ac > 0$ then the general solution of the differential equation is:

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x} \text{ with } r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We have $a = 1$, $b = 0$ and $c = -1/L^2$ so the general solution is:

$$y = c_1 e^{x/L} + c_2 e^{-x/L}$$

c_1 and c_2 can be determined by imposing boundary conditions.