CS 217 – Algorithm Design and Analysis

Shanghai Jiaotong University, Fall 2019

Handed out on Monday, 2021-03-22
First submission and questions due on Thusday, 2021-04-01
You will receive feedback from the TA.
Final submission due on Monday, 2021-04-05

3 Minimum Spanning Trees

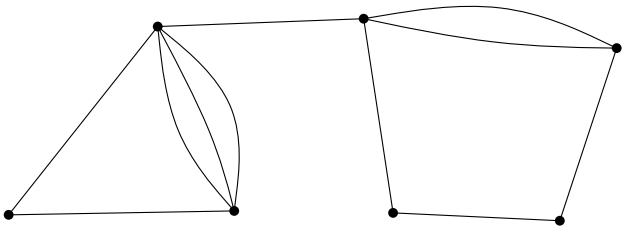
Throughout this assignment, let G be a weighted graph, i.e., $G = (V, E, w)$ with $w : E \to \mathbb{R}^+$. For $c \in \mathbb{R}$ and a weighted graph $G = (V, E, w)$, let $G_c := (V, \{e \in E \mid w(e) \leq c\})$. That is, G_c is the subgraph of G consisting of all edges of weight at most c.

Exercise 1. Let T be a minimum spanning tree of G, and let $c \in \mathbb{R}$. Show that T_c and G_c have exactly the same connected components. (That is, two vertices $u, v \in V$ are connected in T_c if and only if they are connected in G_c). You are encouraged to draw pictures to illustrate your proof!

Exercise 2. For a weighted graph G, let $m_c(G) := |\{e \in E(G) \mid w(e) \leq c\}|$, i.e., the number of edges of weight at most c (so G_c has $m_c(G)$ edges). Let T, T' be two minimum spanning trees of G. Show that $m_c(T) = m_c(T')$.

Exercise 3. Suppose G is connected, and no two edges of G have the same weight. Show that G has exactly one minimum spanning tree!

A *multigraph* is a graph that can have multiple edges, called "parallel edges". Without defining it formally, we illustrate it:



A multigraph.

All other definitions, like connected components and spanning trees are the same as for normal (simple) graphs. However, when two spanning trees use different parallel edges, we consider them different:

The same multigraph with two different spanning trees.

Exercise 4. How many spanning trees does the above multigraph on 7 vertices have? Justify your answer!

Exercise 5. Suppose you have a polynomial-time algorithm that, given a multigraph H, computes the number of spanning trees of H. Using this algorithm as a subroutine, design a polynomial-time algorithm that, given a weighted graph G, computes the number of minimum spanning trees of G.