CV

Sheet 01

Task 3

Definition of convolution for countinuous functions:

$$x(t) * h(t) = \int x(\tau)\, h(t-\tau)\, d\tau, \quad t \in \mathbb{R}$$

Associativity of convolutions:

$$x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$$

Proof:

$$x(t) * (h_1(t) * h_2(t))$$

$$= x(t) \times \int h_2(\tau) h_1(t-\tau)\, d\tau$$

$$= \int x(\psi) \int h_2(\tau) h_1(t-\psi-\tau)\, d\tau\, d\psi$$

$$= \int h_2(\tau) \int x(\psi) h_1(t-\tau-\psi)\, d\psi\, d\tau$$

$$= \int h_2(\tau)(x * h_1)(t-\tau)\, d\tau = ((x * h_1) * h_2)(t)$$

$$= (x(t) * h_1(t)) * h_2(t)$$

□