Exercise for MA-INF 2201 Computer Vision WS22/23
24.11.2022
Submission on 01.12.2022

1. A function is *submodular* when it satisfies the equation:

$$P(\beta,\gamma) + P(\alpha,\delta) - P(\beta,\delta) - P(\alpha,\gamma) \geq 0$$

for all $\alpha, \beta, \gamma, \delta$ such that $\beta > \alpha$ and $\delta > \gamma$. Show that:

1.1. the *Quadratic Function* $P(\omega_m, \omega_n) = c(\omega_m - \omega_n)^2$ is *submodular*, ***(2 points)***

1.2. the *Potts model* $P(\omega_m, \omega_n) = c(1 - \delta(\omega_m - \omega_n))$ with is not *submodular*, by providing a counter-example to the above criterion. ***(2 points)***

2. Provide a graph structure using the *alpha expansion* method that encodes the initial state of 6 nodes (a,b,c,d,e,f) with initial states $\beta\beta\gamma\alpha\alpha\gamma$ for the case where the label α is expanded. ***(4 points)***

3. Denoise the binary image using a Markov random field. For cut, use max-flow/min-cut algorithm. Read a noisy binary image *noise.png*

3.1. Create a graph for the image using all the pixels as nodes. Each pixel (node) are connected to the "source node" and the "sink node" with directed edges as well as the directed edges between its left, top, right and bottom neighboring pixel. ***(2 points)***

3.2. The Unaries are defined by

$$\begin{aligned} P(x_n|w_n = 0) &= Bern_{x_n}[\rho] \\ P(x_n|w_n = 1) &= Bern_{x_n}[1-\rho] \\ U_n(w_n) &= -log(P(x_n|w_n)) \end{aligned}$$

where, $\rho = 0.7$. Use the following three combinations of pairwise values and display all their denoised outputs. ***(4 points)***

i. $P(w_m = 0, w_n = 0) = P(w_m = 1, w_n = 1) = 0.005$
and $P(w_m = 0, w_n = 1) = P(w_m = 1, w_n = 0) = 0.2$

ii. $P(w_m = 0, w_n = 0) = P(w_m = 1, w_n = 1) = 0.005$
and $P(w_m = 0, w_n = 1) = P(w_m = 1, w_n = 0) = 0.35$

iii. $P(w_m = 0, w_n = 0) = P(w_m = 1, w_n = 1) = 0.005$
and $P(w_m = 0, w_n = 1) = P(w_m = 1, w_n = 0) = 0.55$

Note: For min-cut/max-flow algorithm install "PyMaxflow"

4. Extend the algorithm in question 3 for a grayscale image *noise2.png* using Alpha Expansion Algorithm. There are only three labels $[l_1, l_2, l_3]$ where l_1=1, l_2=2 and l_3=3 corresponding to gray values of (0,128,255) respectively. Unary costs are defined as:

$$\begin{aligned} P(x_n = l_i|w_n = l_i) &= \rho_{l_1} \\ P(x_n = l_j|w_n = l_i) &= \frac{(1-\rho_{l_1})}{2} \; \forall i \neq j \end{aligned}$$

where, $\rho_{l_i} = 0.05$ Define the pairwise cost using Potts Model:

$$P(\omega_m, \omega_n) = (1 - \delta(\omega_m - \omega_n))$$

(6 points)