

$$1. U = x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26$$

$$U'_x = 3x^2 + 3y^2 - 39 \quad U'_y = 6xy - 36 \quad U'_z = 2z + 2$$

$$U''_{xy} = 6y \quad U''_{yx} = 6y \quad U''_{zy} = 0 \quad U''_{zx} = 0 = U''_{xz} = U''_{yz}$$

$$~~U''_{xx} = 6x~~ \quad U''_{xx} = 6x \quad U''_{yy} = 6x \quad U''_{zz} = 2$$

$$2. U = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$$

$$U'_x = -\frac{256}{x^2} + \frac{2x}{y} \quad U'_y = -\frac{x^2}{y^2} + \frac{2y}{z} \quad U'_z = -\frac{y^2}{z^2} + 2z$$

$$U''_{xy} = -\frac{2x}{y^2} \quad U''_{yx} = -\frac{2x}{y^2} \quad U''_{zx} = 0 \quad U''_{zy} = -\frac{2y}{z^2}$$

$$U''_{yz} = -\frac{2y}{z^2} \quad U''_{xx} = \left(\frac{256}{x^3} + \frac{2}{y} \right) \cdot 2, \quad U''_{yy} = \left(\frac{x^2}{y^3} - \frac{2}{z} \right) \cdot 2$$

$$U''_{zz} = 2 \left(\frac{y^2}{z^3} + 1 \right)$$

$$3. u = x^2 + y^2 + z^2, \vec{c}(-9, 8, -12), M = (8; -12; 9)$$

$$u'_x = 2x, u'_y = 2y, u'_z = 2z$$

$$\text{grad } u = (16, -24, 18), |\vec{c}| = \sqrt{(-9)^2 + 8^2 + (-12)^2} = 17$$

$$\vec{c}_0 = \frac{\vec{c}}{|\vec{c}|} = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17}\right)$$

$$u'_{\vec{c}} = \left(-\frac{144}{17}\right) + \left(-\frac{192}{17}\right) + \left(-\frac{216}{17}\right) = \cancel{32} - 32 + \frac{8}{17}$$

$$4. u = e^{x^2 + y^2 + z^2}, \vec{d} = (4, -13, -16), L(-16; 4; -13)$$

$$u'_x = 2xe^{x^2 + y^2 + z^2}, u'_y = 2ye^{x^2 + y^2 + z^2}, u'_z = 2ze^{x^2 + y^2 + z^2}$$

$$|\vec{d}| = \sqrt{4^2 + (-13)^2 + (-16)^2} = 21, \vec{d}_0 = \left(\frac{4}{21}, \frac{-13}{21}, \frac{-16}{21}\right)$$

$$\text{grad } u = (-32e^{441}, 8e^{441}, -26e^{441})$$

$$u'_{\vec{d}} = e^{441} \cdot \left(\frac{-32 \cdot 4 - 13 \cdot 8 - 16 \cdot 26}{21}\right) = \cancel{-304}$$

$$= -e^{441} \cdot \frac{648}{21}$$

$$6. \quad U = x^2y + \frac{1}{3}y^3 + 2x^2 + 3y^2 - 1$$

$$\begin{cases} U'_x = 2xy + 4x = 0 \\ U'_y = x^2 + y^2 + 6y = 0 \end{cases} \Rightarrow y = -2$$

$$\Rightarrow x^2 + 4 - 12 = 0 \Rightarrow x_1 = -2\sqrt{2} \quad x_2 = 2\sqrt{2}$$

$$(-2\sqrt{2}, -2) \quad (2\sqrt{2}, -2)$$

$$U''_{xx} = 2y + 4 \quad U''_{xy} = 2x \quad U''_{yx} = 2x \quad U''_{yy} = 2y + 6$$

$$\begin{pmatrix} 0 & -4\sqrt{2} \\ -4\sqrt{2} & 2 \end{pmatrix} \quad \Delta_1 = 0 \rightarrow \text{ne pordoroe} \\ \Delta_2 = 0 - 32 = -32$$

$$\begin{pmatrix} 0 & 4\sqrt{2} \\ 4\sqrt{2} & 2 \end{pmatrix} \quad \Delta_1 = 0 \rightarrow \text{ne pordoroe}$$

$$y = \frac{1}{1 + e^{-ax+b}} \left(y - \frac{1}{1 + e^{-ax+b}} \right)^2$$

$$U'_a = - \frac{2x \left(y - \frac{1}{1 + e^{-ax+b}} \right) \cdot e^{-ax+b}}{(e^{-ax+b} + 1)^2}$$

$$U'_b = \frac{2 \left(y - \frac{1}{1 + e^{-ax+b}} \right) \cdot e^{-ax+b}}{(e^{-ax+b} + 1)^2}$$