5LIK0: Embedded Signal Processing Systems, Assignment 1

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Important Notes:

- This assignment assumes you are familiar with the screen-cast lectures up to and including Lecture 3.C and the Lecture Notes up to and including Section 4, the Singular Value Decomposition.
- This assignment may be executed together with **one** other student, or individually (if you do it individually you have to do all the work alone).
- The results have to be submitted electronically on the 5LIK0 Canvas page in the form of a single zip archive with five MATLAB scripts. Make sure that the submission indicates both student IDs as described below.
- Pay close attention to the instructions in the individual exercises. In particular, **use exactly the script names and the variable names as described in the problems**. The evaluation of your scripts is automated and any deviation could therefore lead to a failing result! You may use the provided script templates.
- Any form of collaboration on the assignments with other persons than your partner is not allowed.
- If there are any questions about the assignment, they can be asked on Teams or during the lab sessions.
- Use the files `data1.mat`, `data2.mat`, `data4.mat`, `data5.mat`, as provided on Canvas with this assignment.
- Your solution scripts **may not** rely on the *Symbolic Math Toolbox* of MATLAB. It will not be available on the system that evaluates your scripts. You should **not** use, for instance, the `solve` command. If in doubt, consult `https://mathworks.com/help/symbolic/referencelist.html`.
- The automatic evaluation will leave sufficient room for rounding errors due to MATLAB calculations.
- There are 5 problems. You can gain 100 points in total. The grade is determined by dividing the total number of points gained by 10.

The deadline for submission of Assignment 1 is Friday May 31, 23.59h. It should be submitted on Canvas as a single zip file named `assignment1_<id1>_<id2>.zip`, where <id1> and <id2> are your respective student ID numbers. The zip archive should include only the MATLAB script solutions to the problems below.

Problem 1 (25 POINTS)

Write a single script, called `assignment1_1.m`, in which you load the variables from `data1.mat` (with the command `load("data1.mat")`), and do the following. Your script should end having defined the following variables: `a1`, `a2`, `V`, `x`, `U` and `e`, in the MATLAB workspace.

(a) (*5 points*) Determine two orthonormal vectors in the kernel of the matrix in variable `A` and assign the results to variables `a1` and `a2`.

(b) (*5 points*) Determine a minimal basis (i.e., with the least number of vectors) for the column span of the matrix `B`. Store the basis vectors in a matrix in variable `V` which has the basis vectors as it columns.

(c) (*5 points*) Determine a vector $\mathbf{x}$ such that $||\mathbf{x}|| = 2$ and $\mathbf{C}^{\dagger}\mathbf{C}\mathbf{x} = \mathbf{x}$, where $\mathbf{C}$ is the matrix in variable `C`. Store the vector $\mathbf{x}$ in the variable `x`.

(d) (*5 points*) Determine an isometry $\mathbf{U}$ with the *maximum* number of columns such that $\mathbf{U}^H\mathbf{D} = \mathbf{0}$, where $\mathbf{D}$ is the matrix in variable `D`. Store the matrix $\mathbf{U}$ in the variable `U`.

(e) (*5 points*) Let $[\mathbf{e}_1 \ \ \mathbf{e}_2 \ \ \mathbf{e}_3]$ be the matrix represented by variable `E`. Determine a vector $\mathbf{e}_4 \neq \mathbf{0}$, such that the set $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ of vectors is *dependent*. Store the vector $\mathbf{e}_4$ in variable `e`.

Problem 2 (15 POINTS)

Write a single script, called `assignment1_2.m`, in which you load the variables from `data2.mat`, and do the following. Your script should end having defined the following variables: `v`, `B`, and `C`, in the MATLAB workspace.

(a) (*5 points*) Given the matrix $\mathbf{A}$ in variable `A`, find a vector $\mathbf{v}$ such that $||\mathbf{v}|| \leq 1$ and $||\mathbf{A}\mathbf{v}|| > 5$. Store the vector $\mathbf{v}$ in the variable `v`.

(b) (*5 points*) Determine a matrix $\mathbf{B} \in \mathbb{C}^{3\times 3}$ such that there exists a vector $\mathbf{x}_1$ such that $\frac{||\mathbf{A}\mathbf{x}_1||}{||\mathbf{x}_1||} = 1$ and a vector $\mathbf{x}_2$ such that $\frac{||\mathbf{A}\mathbf{x}_2||}{||\mathbf{x}_2||} = 2$. Store the matrix $\mathbf{B}$ in the variable `B`.

(c) (*5 points*) Determine a matrix $\mathbf{C}$, such that the condition number $CN(\mathbf{C}) = 4$, the induced Euclidean norm $||\mathbf{C}|| = 3$, and the rank of $\mathbf{C}$, $r = 2$. Store the matrix $\mathbf{C}$ in the variable `C`.

Problem 3 (25 POINTS)

Write a single script, called `assignment1_3.m`, in which you do the following. Your script should end having defined the following variables: `U`, `V`, `A`, `x`, and `b`, in the MATLAB workspace.

(a) (*5 points*) Compute a *unitary* matrix in $\mathbb{R}^{2\times 2}$ such that the first row of the matrix is equal to $[0.9602 \ \ -0.2794]$. Store the matrix in the variable `V`.

(b) (*5 points*) Determine an arbitrary 3 by 3 unitary matrix in $\mathbb{R}^{3\times 3}$ (it does not have to re *random*, it can be any fixed matrix) **that does not include** 0 **as any of its elements** and store the matrix in the variable `U`.

(c) (*5 points*) Compute the matrix that has the singular values $\sigma_1 = 3$ and $\sigma_2 = 2$, that has the columns of `U` (from part b) as its *left* singular vectors and has the columns of `V` (from part a) as its *right* singular vectors. Store the resulting matrix $\mathbf{A}$ in the variable `A`.

(d) (*5 points*)
Determine a vector $\mathbf{x}$ such that $\frac{||\mathbf{A}\mathbf{x}||}{||\mathbf{x}||} = \frac{\sqrt{13}}{\sqrt{2}}$ and store it in the variable `x`.

(e) (*5 points*) Determine a vector $\mathbf{b}$ such that the least squares solution $\mathbf{y}$ of the equation $\mathbf{A}\mathbf{y} = \mathbf{b}$ is such that $||\mathbf{A}\mathbf{y} - \mathbf{b}||^2 = 1$ and store the result in the variable `b`.

Problem 4 (15 POINTS)

Write a single script, called `assignment1_4.m`, in which you load the variables from `data4.mat`, and do the following. Your script should end having defined the following variables: `a`, `b`, and `c` in the MATLAB workspace.

Given the provided vectors `x`, `y`, and `z`, each containing 50 samples x_k, y_k and z_k, for $1 \leq k \leq 50$, i.e., $x_1 =$ `x(1)`, $z_2 =$ `z(2)`, etcetera. Assume that the data samples can be approximately modelled with the equation

$$z_k = a \cdot \cos\left(x_k \cdot \frac{\pi}{180}\right) + b \cdot \sin\left(y_k \cdot \frac{\pi}{180}\right) + c$$

Find the optimal parameters a, b, and c that minimize the following error E of the approximation.

$$E = \frac{1}{50}\sum_{k=1}^{50} |z_k - a \cdot \cos\left(x_k \cdot \frac{\pi}{180}\right) + b \cdot \sin\left(y_k \cdot \frac{\pi}{180}\right) + c|^2$$

Assign the value of the optimal parameters a, b, and c, to the variables `a`, `b`, and `c`, respectively.

Problem 5 (20 POINTS)

Write a single script, called `assignment1_5.m`, in which you load the variables from `data5.mat`, and do the following. Your script should end having defined the following variables: `W1`, `E1`, `I1`, `W2`, `E2`, `A2`, and `I2` in the MATLAB workspace.

(a) (*10 points*) Assume we have a beam forming setup with $M = 5$ antenna elements, and $d = 3$ sources. Assume that we know the channel matrix $\mathbf{A}_1$ from direction finding methods and assume that our optimization goal is *model matching* using the Zero Forcing receiver. The provided channel matrix $\mathbf{A}_1$ is loaded in variable `A1` and the matrix $\mathbf{X}_1$ of received antenna array samples in variable `X1`. Compute the optimal beam forming matrix $\mathbf{W}_1$ (such that $\mathbf{Y} = \mathbf{W}_1^H \mathbf{X}_1$ gives the optimal outputs according to the model matching objective) and store it in the variable `W1`.

Compute the average output error E_1 for the signal samples in the provided matrix $\mathbf{S}_1$ in variable `S1`, i.e.,

$$E_1 = \frac{1}{N} \sum_{k=1}^{N} ||\mathbf{W}_1^H \mathbf{x}[k] - \mathbf{s}[k]||^2,$$

where N is the number of samples in the provided data set, $\mathbf{x}[k]$ is column k of matrix $\mathbf{X}_1$ and $\mathbf{s}[k]$ is column k of matrix $\mathbf{S}_1$. Store the result E_1 in the variable `E1`.

Compute the *interference* of this beam forming matrix, which we define as follows.

$$I_1 = ||\mathbf{W}_1^H \mathbf{A}_1 - \mathbf{I}||_F^2$$

Store the computed interference I_1 in the variable `I1`.

(b) (*10 points*) Assume that we have a similar setup with $M = 3$ and $d = 3$. Assume that in this case we know a set of training symbols in matrix $\mathbf{S}_2$, in variable `S2`, and that our optimization goal is to minimize the *output error*, i.e., to find a beam forming matrix $\mathbf{W}_2$ that minimizes $||\mathbf{W}_2^H \mathbf{X}_2 - \mathbf{S}_2||_F^2$ for the provided data samples in $\mathbf{X}_2$, in variable `X2`. Compute the optimal beam forming matrix $\mathbf{W}_2$ and store the result in variable `W2`. Then, compute the average output error E_2 for the signal samples in the provided matrix $\mathbf{S}_2$, i.e.,

$$E_2 = \frac{1}{N} \sum_{k=1}^{N} ||\mathbf{W}_2^H \mathbf{x}[k] - \mathbf{s}[k]||^2,$$

where N is the number of samples in the provided data set, $\mathbf{x}[k]$ is column k of matrix $\mathbf{X}_2$ and $\mathbf{s}[k]$ is column k of matrix $\mathbf{S}_2$. Store the result E_2 in the variable `E2`.

Compute an estimate $\hat{\mathbf{A}}_2$ of the channel matrix as the least squares solution to the equation

$$\mathbf{X}_2 = \hat{\mathbf{A}}_2 \mathbf{S}_2$$

Store the estimate of the channel in variable `A2`. Then compute the amount of interference, I_2, caused by $\mathbf{W}_2$ on channel $\hat{\mathbf{A}}_2$, defined as:

$$I_2 = ||\mathbf{W}_2^H \hat{\mathbf{A}}_2 - \mathbf{I}||_F^2$$

and store the result in variable `I2`.