Chapter 1

Introduction to Simple Epidemic Model

1.1 Simple SIR (without births and deaths)

Program 2.1 is a simple SIR model (page 19 of the book). These are the equations and the code of the model:

Equations

$$\frac{dS}{dt} = -\beta * S * I \tag{1.1}$$

$$\frac{dI}{dt} = \beta * S * I - \gamma * I \tag{1.2}$$

$$\frac{dR}{dt} = \gamma * I \tag{1.3}$$

Pharo code

```
|solver system dt beta gamma values stepper diag colors maxTime|
dt := 1.0.
beta := 1.4247.
gamma := 0.14286.
maxTime := 70.0.
system := ExplicitSystem block: [ :x :t| |c|
    c := Array new: 3.
    c at: 1 put: (beta negated) * (x at: 1) * (x at: 2).
    c at: 2 put: (beta * (x at: 1) * (x at: 2)) - (gamma * (x at: 2)).
```

```
    c at: 3 put: gamma * (x at: 2).
    c
    ].
stepper := RungeKuttaStepper onSystem: system.
solver := (ExplicitSolver new) stepper: stepper; system: system; dt: dt.
state := { 1-1e-6. 1e-6. 0}.
values := (0.0 to: maxTime by: dt) collect: [ :t| |state| state := stepper doStep: state
                                                time: t stepSize: dt ].
diag := OrderedCollection new.
colors := Array with: Color blue with: Color red with: Color green.
1 to: 3 do: [ :i|
   diag add:
      ((GETLineDiagram new)
         models: (1 to: maxTime+1 by: 1);
         y: [ :x| (values at: x) at: i ];
         color: (colors at: i))
    ].
builder := (GETDiagramBuilder new).
builder compositeDiagram
   xAxisLabel: 'Time in days';
   yAxisLabel: 'Number of Individuals';
   regularAxis;
   diagrams: diag.
builder open.
```

1.2 SIR model with births and deaths

Equations

$$\frac{dS}{dt} = \mu - \beta * S * I - \mu * S \tag{1.4}$$

$$\frac{dI}{dt} = \beta * S * I - \gamma * I - \mu * I \tag{1.5}$$

$$\frac{dR}{dt} = \gamma * I - \mu * R \tag{1.6}$$

Pharo code

```
|solver system dt beta gamma values stepper diag mu colors maxTime|
dt := 1.0.
mu := 1/(70*365.0).
beta := 520/365.0.
gamma := 1/7.0.
maxTime := 60*365.
```

```
system := ExplicitSystem block: [ :x :t| |c|
    c := Array new: 3.
    c at: 1 put: mu − (beta ∗ (x at: 1) ∗ (x at: 2)) − (mu ∗ (x at:1)).
    c at: 2 put: (beta ∗ (x at: 1) ∗ (x at: 2)) − (gamma ∗ (x at: 2)) − (mu ∗ (x at:2)).
    c at: 3 put: (gamma ∗ (x at: 2)) − (mu ∗ (x at: 2)).
    c
    ].

stepper := RungeKuttaStepper onSystem: system.
solver := (ExplicitSolver new) stepper: stepper; system: system; dt: dt.
state := { 0.1. 1e−4. 1−0.1−1e−4}.
values := (0.0 to: maxTime by: dt) collect: [ :t| |state| state := stepper doStep: state
                                                time: t stepSize: dt ].
diag := OrderedCollection new.
colors := Array with: Color blue with: Color red with: Color green.
1 to: 3 do: [ :i|
   diag add:
      ((GETLineDiagram new)
         models: (1 to: maxTime+1 by: 1);
         y: [ :x| (values at: x) at: i ];
         color: (colors at: i))
    ].
builder := (GETDiagramBuilder new).
builder compositeDiagram
   xAxisLabel: 'Time in days';
   yAxisLabel: 'Number of Individuals';
   regularAxis;
   diagrams: diag.
builder open.
```

1.3 SIR model with disease induced mortality and density dependent transmission

Equations

$$\frac{dX}{dt} = \nu - \beta * X * Y - \mu * X \tag{1.7}$$

$$\frac{dY}{dt} = \beta * X * Y - \frac{\gamma + \mu}{1 - \rho} * Y \tag{1.8}$$

$$\frac{dZ}{dt} = \gamma * Y - \mu * Z \tag{1.9}$$

1.4 SIR model, disease induced mortality and frequency dependent transmission

Equations

$$\frac{dX}{dt} = \nu - \beta * X * Y/N - \mu * X \tag{1.10}$$

$$\frac{dY}{dt} = \beta * X * Y/N - \frac{\gamma + \mu}{1 - \rho} * Y \tag{1.11}$$

$$\frac{dZ}{dt} = \gamma * Y - \mu * Z \tag{1.12}$$

Pharo code

1.5 SIS model without births or deaths

Equations

$$\frac{dS}{dt} = \gamma * I - \beta * S * I \tag{1.13}$$

$$\frac{dI}{dt} = \beta * S * I - \gamma * I \tag{1.14}$$

1.6 SEIR model with births and deaths

1.7 SIR with a carrier state