Modeling Basic Aspects of Cyber-Physical Systems, Part II

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Motivation

- Educating designers
- Developing expressive, efficient, and robust modeling and simulation tool

Vision

Design better simulation tool to support virtual testing and analysis of **cyber-physical systems** at both **industrial** and **academic** levels.

Problem

Today to model and analyze a cyber physical system, one need to connect several domains, this brings forward the challenge of adding features to support **innovative design process**.

Key Idea

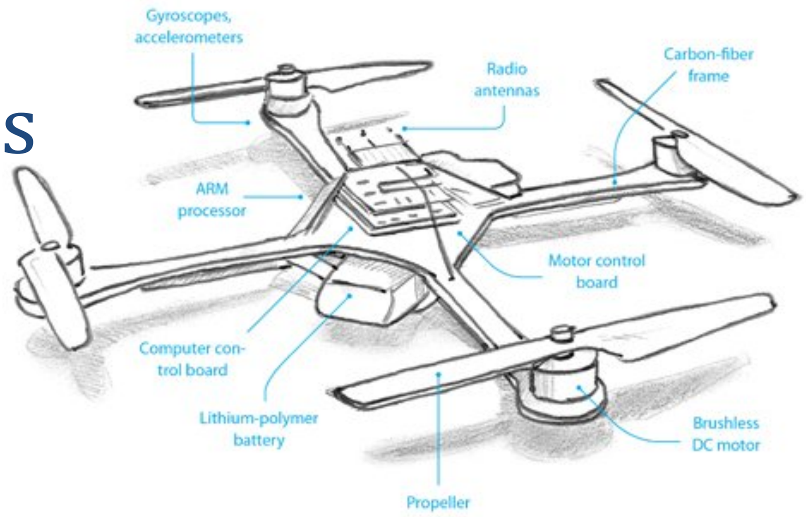
We are working on a tool called **Acumen**. Small language for hybrid systems modeling. We use Acumen to model and analyze two sophisticated standard **robotics** examples and add features to support **design innovation process**.

Newtonian and Lagrangian Formulations

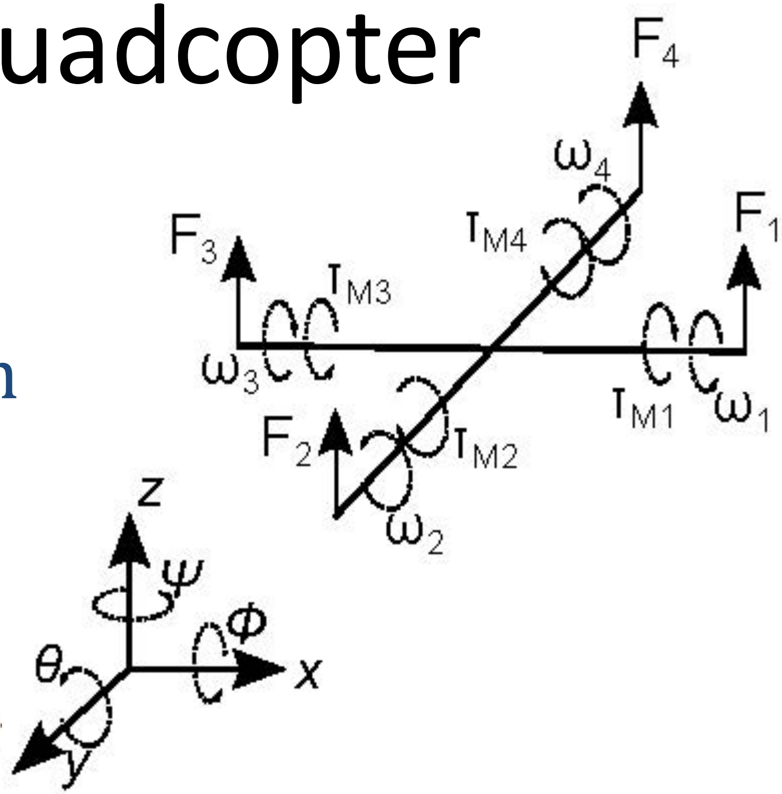
- Why Newtonian?
  - Non-conservative forces
  - Vector base
  - Solve constraints equations
- Why Lagrangian?
  - Direct specification
  - Calculate potential and kinetic energies
  - Compact without constraints

Case Study 1- Quadcopter

- Complex mechatronic system
- 6-degree of freedom
- Underactuation
- Cover different domains



Case Study 1- Quadcopter



- Mathematical Model
  - Newton-Euler formulation
  - Vectors

$$\sum F = m\bar{a} = G + RT$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \dot{\phi}C_\phi T_\theta + \dot{\theta}\frac{S_\phi}{C_\theta^2} & -\dot{\phi}S_\phi C_\theta + \dot{\theta}\frac{C_\phi}{C_\theta^2} \end{bmatrix}$$

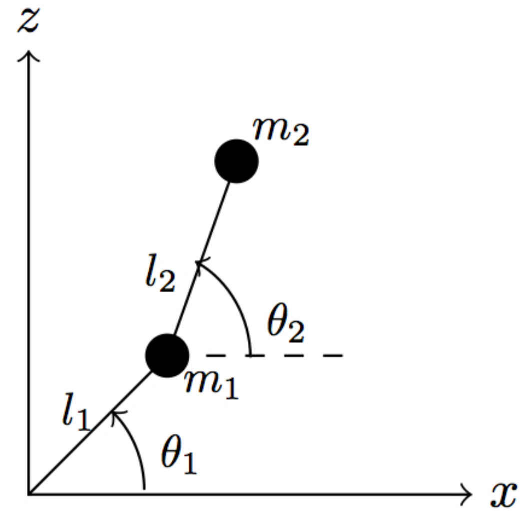
$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m} \begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix}$$

$$+ \nu + W_\eta^{-1} \dot{\nu}$$

Case Study 1- Quadcopter

- Acumen Model

Case Study 2a- Double Pendulum



- Mathematical Model
  - Lagrangian formulation
  - Generalized Coordinates

$$\forall i \in \{1 \ldots |q|)\}, \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q$$

- description of components

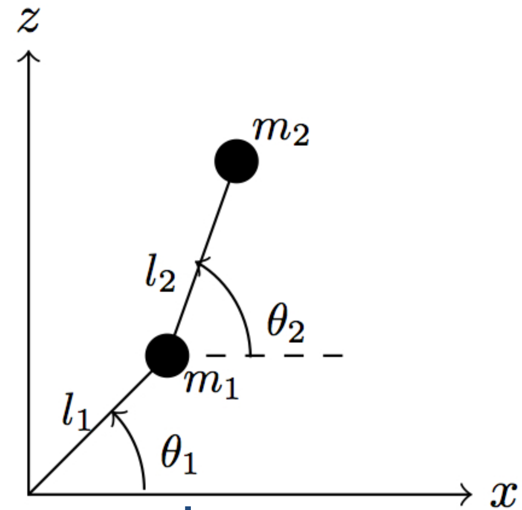
$$q = (\theta_1, \theta_2)$$

- define total kinetic and potential energy

$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$V = m_1 g z_1 + m_2 g z_2$$

Case Study 2a- Double Pendulum



- identify non-conservative forces

$$Q = 0$$

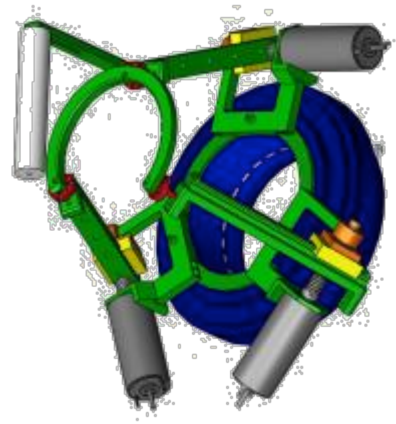
- substitution in Euler-Lagrangian equation

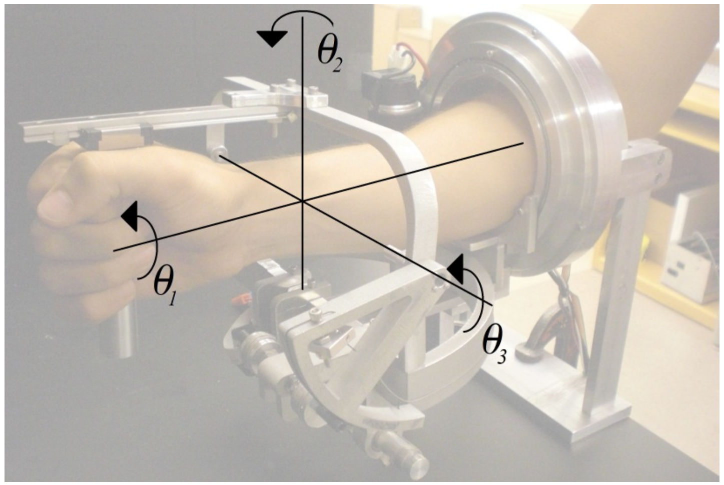
$$\ddot{\theta}_1 = m_2 l_2(\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + (m_1 + m_2) g \cos(\theta_1))/(-l_1(m_1 + m_2))$$

$$\ddot{\theta}_2 = m_2 l_1(\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + m_2 g \cos(\theta_2))/ - l_1 m_2$$

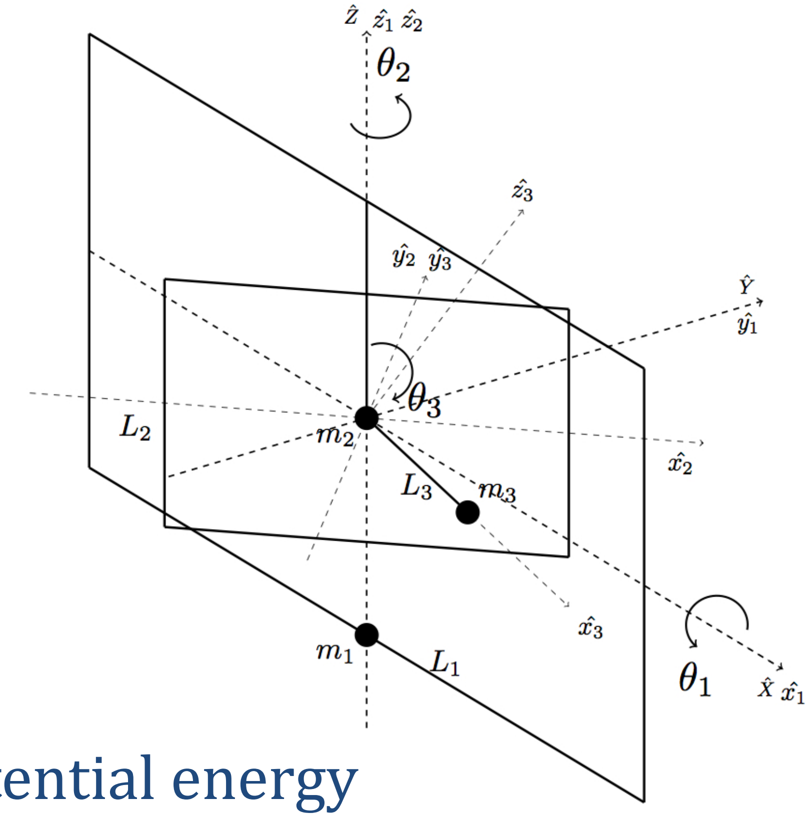
Case Study 2- RICEWRIST

- Rehabliation
- 3-degree of freedom
- Gibral
- Cover different domains





Case Study 2- RICEWRIST



- Mathematical Model
  - Lagrangian formulation
  - Generalized Coordinates

$$\forall i \in \{1 \ldots |q|)\}, \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q$$

- description of components

$$q = (\theta_1, \theta_2, \theta_3)$$

- define total kinetic and potential energy

$$V = m_1 g h_1 + m_2 g h_2 + m_3 g h_3$$
$$= -m_1 g l_2 \cos(\theta_1) + m_3 g l_3 \sin(\theta_1)\sin(\theta_3)$$

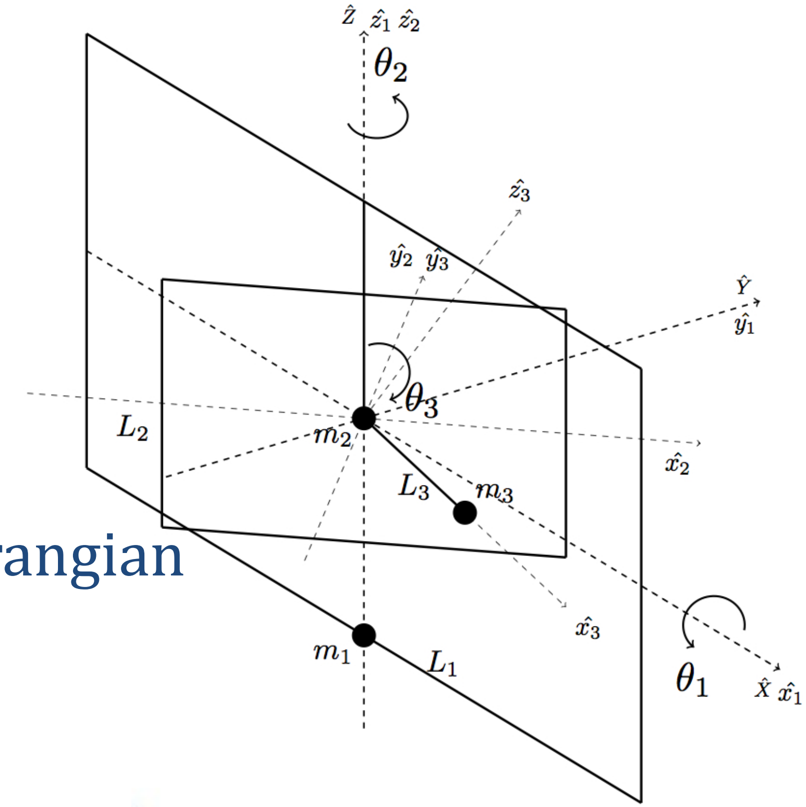
Case Study 2- RICEWRIST

- identify non-conservative forces

$$Q = 0$$

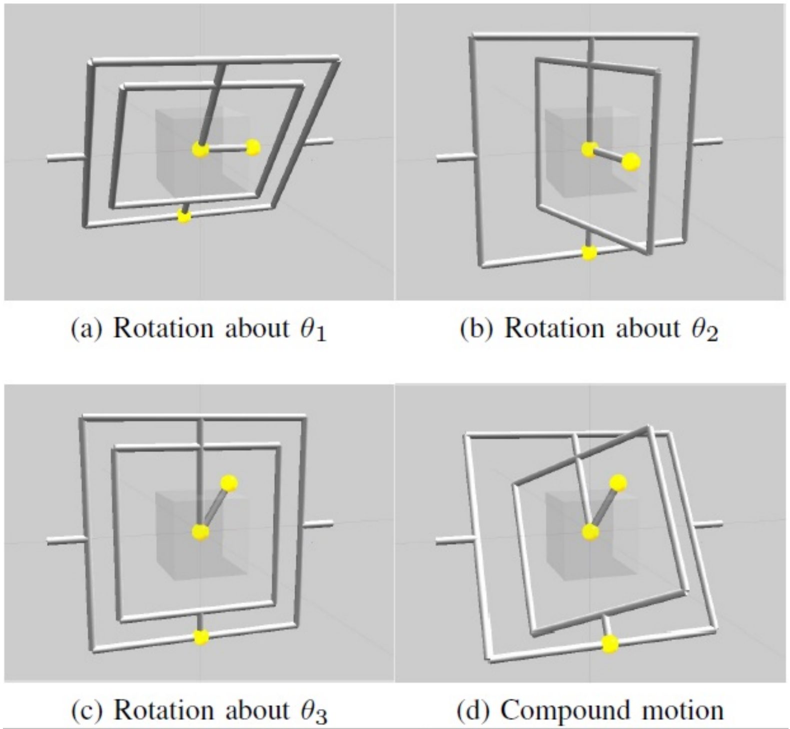
- substitution in Euler-Lagrangian equation

$$\frac{1}{2}\frac{d}{dt}\left(I_2\frac{\partial\omega_2\cdot\omega_2}{\partial\dot{\theta}_2}+I_3\frac{\partial\omega_3\cdot\omega_3}{\partial\dot{\theta}_2}\right)\Bigg)+$$
$$-\frac{1}{2}\left(I_2\frac{\partial\omega_2\cdot\omega_2}{\partial\theta_2}+I_3\frac{\partial\omega_3\cdot\omega_3}{\partial\theta_2}\right)=0$$



Case Study 1- RICEWRIST

- Acumen Model



(a) Rotation about θ_1 (b) Rotation about θ_2

(c) Rotation about θ_3 (d) Compound motion

Summary

- Untyped core formulation.
- Effectively modeling.
- Newton-Euler formulation.
- Lagrangian formulation.
- Partial static derivatives inclusion.

Thank you for your attention!

Questions

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