① $X_1, \dots, X_n$ – выборка $\xi \sim Pois(\theta)$, $P(x_i = k) = \frac{\theta^k}{k!} e^{-\theta}$, $k \in \mathbb{N}_0$.

Ⓐ $\theta = M\xi = \eta_1(\theta) = \bar{x} \Rightarrow \hat{\theta} = \eta_1^{-1}(\bar{x}) = \bar{x}$

$\eta_1 = id \Rightarrow \eta_1^{-1} = id$

Ⓑ $\ln \mathcal{L} = \sum_{i=1}^{n} \ln\left(\frac{\theta^{x_i}}{x_i!} e^{-\theta}\right) = -n\theta + \ln\theta \cdot \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} (x_i)!$

$\Rightarrow (\ln \mathcal{L})' = -n + \frac{1}{\theta} \sum_{i=1}^{n} x_i \Rightarrow \hat{\theta} = \bar{x}$

Ⓒ Несмещенность:

$M\hat{\theta} = M\left(\frac{1}{n}\sum_{i=1}^{n} \xi_i\right) = \frac{1}{n}\sum_{i=1}^{n} M\xi = M\xi = \theta.$

Состоятельность:

$\hat{\theta} \xrightarrow[n\to\infty]{P} \theta$?

$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} x_i \Rightarrow \frac{1}{n}\sum_{i=1}^{n} \xi = \bar{\xi} \xrightarrow[n\to\infty]{P} M\xi = \theta.$

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$\Rightarrow \hat{\theta} \xrightarrow[n\to\infty]{P} \theta.$

② $x_1, \dots, x_n$ из $\xi \sim N(\mu, \sigma^2)$; $p_\xi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\mathcal{L} = \prod_{i=1}^{n} p_\xi(x_i) = \left(\frac{1}{\sqrt{2\pi}}\right)^n \cdot \frac{1}{\sigma^n} \cdot \exp\left(-\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}\right)$

$\ln \mathcal{L} = -n \ln\sqrt{2\pi} - n\ln\sigma - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$

$\frac{\partial \ln \mathcal{L}}{\partial \mu} = 0 = +\sum_{i=1}^{n} \frac{x_i - \mu}{\sigma^2} \Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0 \Rightarrow \mu = \bar{x}$

$\frac{\partial \ln \mathcal{L}}{\partial \sigma} = 0 = -\frac{n}{\sigma} + \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^3} \Rightarrow \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^2$

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$

$\Rightarrow$ ОМП: $\boxed{\hat{\theta} = \left(\bar{x} ; \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)}$