Shortest Path Planning

Computational Intelligence, Lecture ?

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Spring 2023

Content

If we want to plan a path on a 2D map, we can represent obstacle-free space regions as a nodes, and possible transitions between the obstacle-free space regions as graph edges.

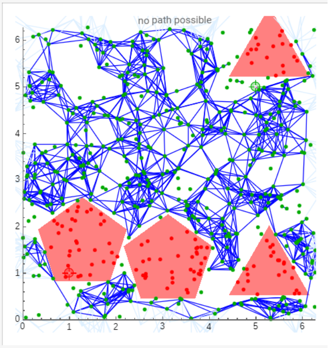

Figure 1: Path planning as graph search; Credit: https://demonstrations.wolfram.com/ProbabilisticRoadmapMethod/

Consider a directed graph (each edge has a direction assigned to it):

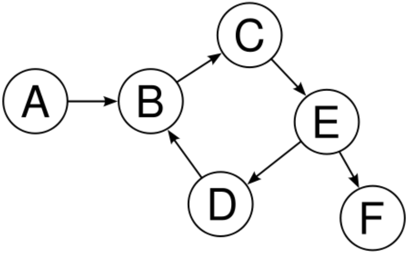

Figure 2: Directed graph; Credit: https://github.com/HQarroum/directed-graph

How can we find a shortest path from a start node to a finish node on it?

Shortest path (1)

We assign index variable x_i to i-th edge; each index variable is positive $x_i \geq 0$.

If $x_i = 1$ the edge is part of the path. We assume that otherwise $x_i = 0$ (which will be enforced by the other constraints). Adding a cost d_i associated with each edge (e.g. Euclidean distance) we get a linear cost $l(\mathbf{x})$:

$$l(\mathbf{x}) = \mathbf{x}^\top \mathbf{d} \tag{1}$$

Shortest path (2)

Since each edge connects one node (e.g. node u) to another (e.g. node v), we can label all index variables x with superscripts, denoting nodes that they connect - $x^{u,v}$.

Our goal will be to count how many path segments enter and leave each node. For any normal node the number will be equal:

$$-\sum_{\forall i} x^{i,v} + \sum_{\forall j} x^{v,j} = 0 \tag{2}$$

Shortest path (3)

We know that for the starting node s, there will only be one path segment leaving it:

$$-\sum_{\forall i} x^{i,s} + \sum_{\forall j} x^{s,j} = 1 \tag{3}$$

For the finishing node f we have only one path segment entering it:

$$-\sum_{\forall i} x^{i,f} + \sum_{\forall j} x^{f,j} = -1 \tag{4}$$

Together the problem becomes:

$$
\begin{aligned}
&\underset{\mathbf{x}}{\text{minimize}} \quad \mathbf{x}^\top \mathbf{d}, \\
&\text{subject to} \quad \begin{cases}
-\sum_{\forall i} x^{i,v} + \sum_{\forall j} x^{v,j} = 0, \quad \forall v \\
-\sum_{\forall i} x^{i,s} + \sum_{\forall j} x^{s,j} = 1, \\
-\sum_{\forall i} x^{i,f} + \sum_{\forall j} x^{f,j} = -1, \\
\mathbf{x} \geq 0.
\end{cases}
\end{aligned}
\tag{5}
$$

And with that, the problem can be solved as an LP.

Lecture slides are available via Github, links are on Moodle

You can help improve these slides at:
github.com/SergeiSa/Computational-Intelligence-Slides-Spring-2023