Gaits

Contact-aware Control, Lecture 10

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Bipedal walking and running can be broken into phases:

- *double-support* - two feet touch the ground;
- *single-support* - one foot touches the ground;
- *flight phase* - no foot touches the ground.

Running is defined as gate that includes a flight phase; walking doesn't.

Simple model of walking includes single-support phase only (when one foot touches the ground, the other leaves it at the same time).

A single-support phase walking is characterized by step length, time allocated for a single step, velocity of the center of mass, position of the next support (place where the foot lands). These values are not independent.

Double-support walking is additionally characterized by the duration of the double-support phase.

We can identify two popular approaches to generating gates:

- Raibert-style approach: we decide on the desired center of mass velocity and compute step length (position of the next foothold).
- ZMP-style approach: we decide sequence of footholds and compute the center of mass trajectory.

Let us consider an inverted pendulum:

$$J\ddot{\varphi} + lmg\cos(\varphi) = 0. \tag{1}$$

Kinetic energy T and potential energy P of the system are defined as:

$$T = \frac{J\dot{\varphi}^2}{2}, \qquad P = \cos(\varphi)lmg. \tag{2}$$

Assuming no dissipation of energy and no actuation, the total energy of the system is conserved:

$$\frac{J\dot{\varphi}^2}{2} + \cos(\varphi)lmg = \text{const} \tag{3}$$

Let us model a single support walking phase as inverted pendulum moving from the time t_1 when the angle $\varphi(t_1) = \alpha$ to the time t_2 when the angle $\varphi(t_2) = \beta$.

Potential energy at the beginning of the single-support phase the potential energy $P_1 = \cos(\alpha)lmg$ and angular velocity $\omega_1 = \dot{\varphi}(t_1)$, and the end of the single-support phase it is $P_2 = \cos(\beta)lmg$ and $\omega_2 = \dot{\varphi}(t_2)$.

Total energy is conserved:

$$\frac{J\omega_1^2}{2} + \cos(\alpha)lmg = \frac{J\omega_2^2}{2} + \cos(\beta)lmg \tag{4}$$

$$\cos(\beta) - \cos(\alpha) = \frac{J}{2lmg}(\omega_1^2 - \omega_2^2) \tag{5}$$

Inverted pendulum walking, 2

Thus we have the following relation between ω_1 and ω_2:

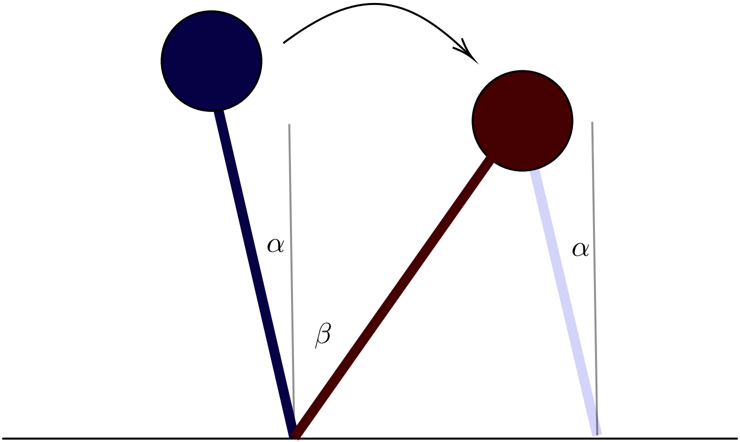
$$\omega_1^2 - \omega_2^2 = \frac{2lmg}{J}(\cos(\beta) - \cos(\alpha)) \tag{6}$$

where $|\alpha| < \frac{\pi}{2}$ and $|\beta| < \frac{\pi}{2}$.

If $|\beta| = |\alpha|$, the r.h.s. of the equation becomes 0, hence $\omega_1 = \omega_2$.

If $|\beta| > |\alpha|$, the r.h.s. of the equation is negative, hence $\omega_1 < \omega_2$.

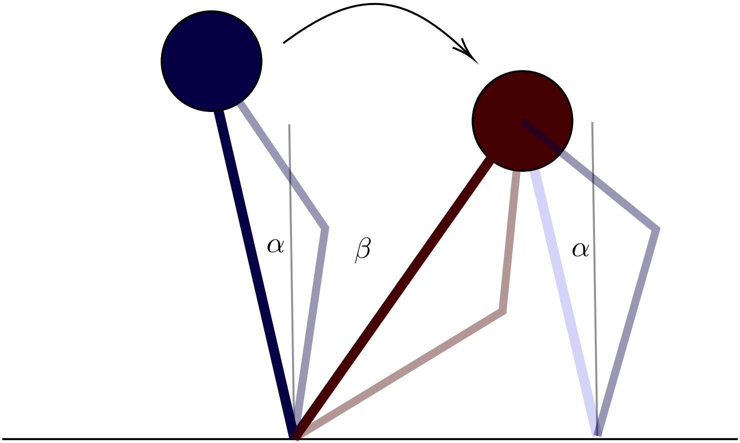
If $|\beta| < |\alpha|$, the r.h.s. of the equation is positive, hence $\omega_1 > \omega_2$.


α
β
α

The pendulum model indicates that, if we want to increase kinetic energy, we need to decrease the angle $|\alpha|$ by absolute value. If we fix β, this corresponds to a shorter step length.

If we want to decrease the kinetic energy, we need to increase the angle $|\alpha|$ by absolute value. If we fix β and $\alpha < 0$ (the pendulum is in the II quadrant at the beginning of the support phase), this corresponds to a longer step length.

Inverted pendulum walking - illustration 2



This model visually corresponds to how humans run:

Figure 1: Credit: runnersworld.com

Since the total energy of the system of the pendulum remains, if the kinetic energy of the end of the single-support phase increased, it means the potential energy decreased by the same value. It implies that the center of mass lowered its position at the end of the single-support phase compared with its start. Given a constant α, it would imply a "shorter leg" during the next single-support phase.

We can solve this problem by introducing a loaded spring into the robot leg. The deformation of the spring is related to the change of the leg length. Stiffness of the spring is a hyper-parameter.

This idea is quite natural: during walking with positive acceleration, people step on a leg partially bent at the knee (effective length between the foot at the hip is lower) which then rapidly straightens.

By introducing a flight phase, we solve the problem of spring "pulling" on the mass.

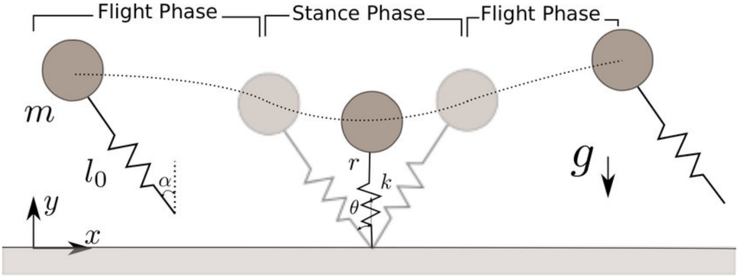


Figure 2: Credit: Dai, H. and Tedrake, R., 2012. Optimizing robust limit cycles for legged locomotion on unknown terrain

Assuming forward motion (no change in orientation), we can propose the following formula for computing the next step location:

$$p_k = r_{\mathrm{CoM}} + r_s + v\Delta t + k(v - v_{\mathrm{des}}) \tag{7}$$

where p_k the next step position, r_{CoM} - center of mass (CoM) position, r_s - vector from CoM to the robot shoulder, Δt - step time, v - CoM velocity, v_{des} - desired CoM velocity and k - gain coefficient.

- If $v_{\text{des}} > v$, the component $k(v - v_{\text{des}})$ is negative, hence the step becomes **shorter**.
- If $v_{\text{des}} < v$, the component $k(v - v_{\text{des}})$ is positive, hence the step becomes **longer**.
- If $v_{\text{des}} = v$, the component $k(v - v_{\text{des}})$ is equal to zero, hence the step remains the same.

This corresponds to what we had earlier: fixing β shorter step corresponds to a smaller α, hence more kinetic energy at the end of the single-support phase, and vice-versa.

Read more

- M. H. Raibert, H. B. Brown, and M. Chepponis, “Experiments in Balance with a 3D One-Legged Hopping Machine,” The International Journal of Robotics Research, vol. 3, no. 2, pp. 75–92, 1984.

Lecture slides are available via Github, links are on Moodle

You can help improve these slides at:
github.com/SergeiSa/Contact-Aware-Control-Fall-2023