Task

Given a system:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ n & -2 & -10/n & -2 \\ -5 & -n/10 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} x \end{cases}$$

where n is your number in your group list (ask your TA to give you your number if you don't have one).

1. Find its transfer function representation ($y(s)/u(s) = W(s)$).
2. Propose an ODE representation of the system.
3. Propose a controller (control law $u = -Kx$) that makes the system stable. Do it via pole placement and as an LQR. For LQR show the cost function you chose.
4. Show stability of the closed-loop system via eigenvalue analysis.
5. Find stability margins by analysing Bode diagram for the system.
6. Simulate closed-loop system.
7. Modify the control law in such a way that the state of the system converges to $x_0 = \begin{bmatrix} 2 + 0.1n \\ n - 5 \\ 0 \\ 0 \end{bmatrix}$. Show resulting control law. Simulate the system and demostrate convergence via graphs of state dynamics and error dynamics.
8. Discretize the system with $\Delta t = 0.01$. Write equations of the discrete dinamics.
9. Propose a control law for the discrete system via pole-placement and LQR (show cost function for the LQR).
10. Show eigenvalue analisys of the slosed-loop dynamics of the discrete system (with the proposed discrete control law. Demonstrate stability.
11. Simulate the discrete system. Show graphs.