DC motor - properties

Mechatronics, Lecture 5

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Content

Dynamics of the motor with payload is:

$$\begin{cases} L\dot{I} + RI + C_w\omega = u \\ J\dot{\omega} + F\omega = C_\tau I - \tau_p \end{cases} \tag{1}$$

where τ_p is the payload torque.

Let us consider stead-state $\omega = \omega^* = \text{const}$ and $I = \text{const}$:

$$\begin{cases} RI + C_w\omega^* = u \\ F\omega^* = C_\tau I - \tau_p \end{cases} \tag{2}$$

Knowing the desired angular velocity ω^* and payload torque τ_p can solve the steady-state equations for current and voltage:

$$I = \frac{F}{C_\tau}\omega^* + \frac{1}{C_\tau}\tau_p \tag{3}$$

$$u = \frac{FR + C_w C_\tau}{C_\tau}\omega^* + \frac{R}{C_\tau}\tau_p \tag{4}$$

We can also consider a steady state equations with $\omega = 0$, $\varphi = \text{const}$ and $I = \text{const}$. In this case, the current and voltage becomes:

$$I = \frac{1}{C_\tau}\tau_p \tag{5}$$

$$u = \frac{R}{C_\tau}\tau_p \tag{6}$$

We can compute electrical power P_e consumed by the DC motor:

$$P_e = Iu \tag{7}$$

The mechanical power P_p associated with the payload:

$$P_p = \omega \tau_p \tag{8}$$

DC motor, efficiency, 1

Multiplying steady-state equations by I and ω we get:

$$\begin{cases} RI^2 + C_w I\omega = Iu \\ F\omega^2 = C_\tau I\omega - \tau_p\omega \end{cases} \tag{9}$$

Since $P_e = Iu$ and $P_p = \omega\tau_p$ we get:

$$\begin{cases} P_e = RI^2 + C_w I\omega \\ P_p = C_\tau I\omega - F\omega^2 \end{cases} \tag{10}$$

This allows us to compute *efficiency* P_p/P_e:

$$\frac{P_p}{P_e} = \frac{C_\tau I\omega - F\omega^2}{C_w I\omega + RI^2} \tag{11}$$

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Let us note that efficiency $\frac{P_p}{P_e}$ approaching 1 would mean that the entire power input is transformed to mechanical power output.

We note that $W_h = RI^2$ is the power of heating generated by the resistor (motor winding). If we assume F is small enough for $F\omega^2$ to be negligible ($F\omega^2 << |C_\tau I \omega|$), we get a simpler expression:

$$\frac{P_p}{P_e} = \frac{C_\tau I \omega}{C_w I \omega + W_h} \tag{12}$$

It intuitively understandable that power of heating directly impedes the transformation of electrical power into mechanical.

If we assume W_h is small enough ($W_h << |C_w I \omega|$), we get a simpler expression:

$$\frac{P_p}{P_e} = \frac{C_\tau I \omega}{C_w I \omega} = \frac{C_\tau}{C_w} \tag{13}$$

There is a common assumption that torque constant is equal to the back-EMF constant: $C_\tau \sim C_w$. This gives us $\frac{P_p}{P_e} \sim 1$.

We can view the ratio $\frac{C_\tau}{C_w}$ as an approximate efficiency measure, excluding the effects of viscous friction and heating power.

Let us consider the power equations once more:

$$\begin{cases} P_e = RI^2 + C_w I\omega \\ P_p = C_\tau I\omega - F\omega^2 \end{cases} \tag{14}$$

Assuming that the shaft remains still ($\omega = 0$) we get:

$$\begin{cases} P_e = RI^2 = W_h \\ P_p = 0 \end{cases} \tag{15}$$

This means that while mechanical power is zero, electrical power is not. This equation illustrates that the entire power input in this regime is being converted into heat.

Reducer effect

Let us note that reducer does not influence electrical power. We can show that ideal reducer does not affect the mechanical power either.

Let the output angular velocity be $\omega_p = \frac{1}{N}\omega$ and the output torque be $\tau_p = N\tau$. Then mechanical power becomes:

$$P_p = \frac{1}{N}\omega N\tau = \omega\tau \tag{16}$$

Meaning that ideal reducer has no influence on mechanical power. Actual physical reducers do influence output power, making it lower. The information on the efficiency of a reducer is usually available. Note that the power loss in the reducer can be modeled by taking into account viscous friction associated with the gearbox.

Read more

- Verstraten, T., Mathijssen, G., Furnemont, R., Vanderborght, B. and Lefeber, D., 2015. Modeling and design of geared DC motors for energy efficiency: Comparison between theory and experiments. Mechatronics, 30, pp.198-213.

Lecture slides are available via Github, links are on Moodle

You can help improve these slides at:
github.com/SergeiSa/Mechatronics-2023