Introduction. Electrodynamics

Mechatronics, Lecture 1

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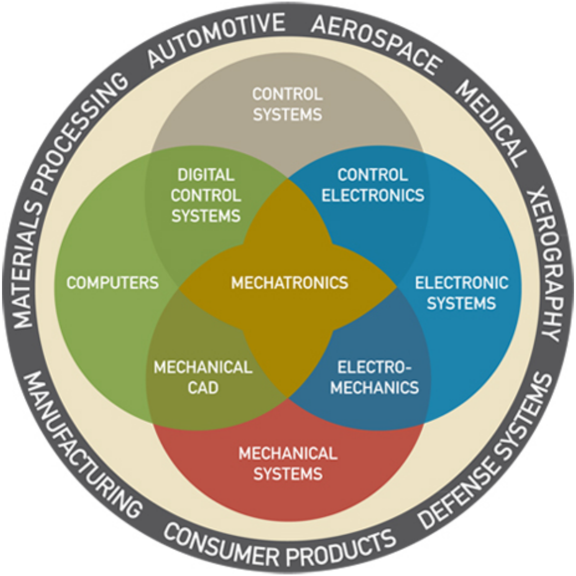


Figure 1: An vision of mechatronics

- Originally Mechatronics was thought of as combination of *Mechanics* and *Electronics*, hence the name.
- The key idea was that one can make a better product if its mechanical and electrical design are done simultaneously.
- Now we combine not only mechanical and electrical design, but also sensors and code.

Nothing is absolute, and neither is Mechatronics. It is beneficial to design highly optimized mechatronic modules (especially in terms of performance), and it is beneficial to produce at high-volume simple components (especially in terms of economics).

One can see that a successful biped Atlas by Boston Dynamics is full of custom highly optimized mechatronic modules, while highly successful quadruped A1 is full of off-the-shelf components.

For us the course will be an opportunity to study some of the key instances of mechatronic products - motors. We will focus on various aspects of mechanical, electrical, control, sensing and programming aspects of working with motors, and especially on unity of the approaches across these differing disciplines.

Content

- Ohm's law
- Kirchhoff's laws
- RL, RC, RLC circuits
- Lorentz force
- Electric power
- Joule heating

Ohm's law, 1

Ohm's law can be expressed in the following way:

$$V = IR \tag{1}$$

where V is voltage, I is current and R is resistance. We can think of this law in the following terms:

The voltage across a conductor is equal to the current flowing through it, times the conductor's resistance.

Ohm's law, 2

Ohm's law is equivalently expressed as:

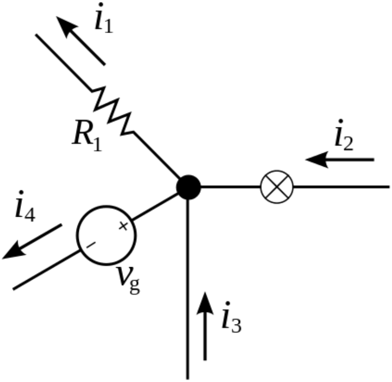
$$I = V/R \tag{2}$$

To compute current flowing through a conductor, we divide the voltage across the conductor by its resistance.

Kirchhoff's current law is:

$$\sum I_j = 0 \tag{3}$$

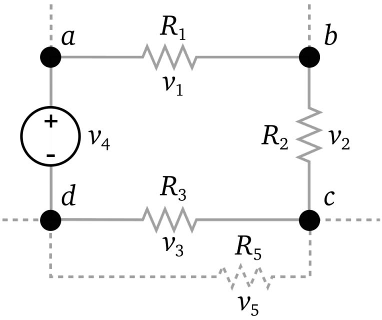
where I_j are currents flowing into a node (junction) and flowing out of a node.



Kirchhoff's voltage law is:

$$\sum V_j = 0 \tag{4}$$

where V_j are voltage across elements in a closed loop.

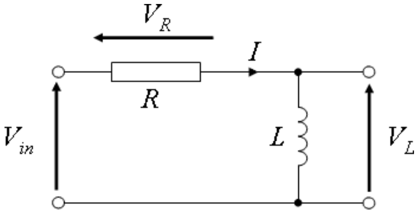


Kirchhoff's laws can be formulated as:

- *The algebraic sum of currents in a network of conductors meeting at a point is zero.*
- *The directed sum of the potential differences (voltages) around any closed loop is zero.*

RL circuit, 1

RL circuit (resistor-inductor circuit) in the simplest case contains a power source, a resistor and an inductor.



We can model it as a first order differential equation (ODE):

$$L\frac{dI}{dt} + IR = V \tag{5}$$

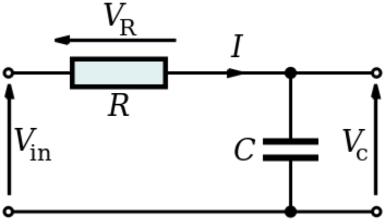
where L is inductance, I is current in the circuit, R is the resistance of the resistor and V is the voltage of source (input / battery, etc).

RL circuit, 2

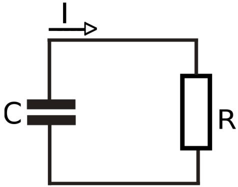
Differential equation $L\frac{dI}{dt} + IR = V$ can be viewed as follows:

- The equation represents Kirchhoff's voltage law written as a differential equation.
- Element IR is the voltage across the resistor; element $L\frac{dI}{dt}$ is the voltage across the inductor.
- Being a differential equation, it acts as a *filter*.

RC circuit (resistor-capacitor circuit) in the simplest case contains a power source, a resistor and a capacitor.



(a) Voltage input



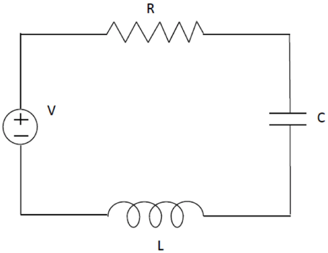
(b) Short circuit

Short circuit model can be described as:

$$C\frac{dV}{dt} + \frac{V}{R} = 0 \tag{6}$$

where C is the capacitance, V is the voltage across the capacitor [1].

RLC circuit (resistor-inductor-capacitor circuit):



We can model it as:

$$L\frac{dI}{dt} + IR + V(0) + \frac{1}{C}\int_0^t I(\tau)d\tau = V(t) \tag{7}$$

Straight wire with a current flowing through it, when placed in magnetic field, experiences force acting on it:

$$\mathbf{f} = I\mathbf{w} \times \mathbf{b} \tag{8}$$

where $\mathbf{f}$ is the force, I is the current, $\mathbf{w}$ is a vector whose length equals to the length of the wire and direction is equal to the direction of the current; $\mathbf{b}$ is the magnetic field.

Electric power is a *the rate of doing work*. In care of resistor, the formula of computing electric power is:

$$W = RI^2 \tag{9}$$

where R is the resistance of a conductor and I is the current flowing through the conductor.

Alternatively, it can be computed as:

$$W = VI \tag{10}$$

where V is voltage.

We can compute the *power of heating* generated by an electrical conductor:

$$P = RI^2 \tag{11}$$

where R is the resistance of a conductor and I is the current flowing through the conductor.

Note that the heating appears to be exactly equivalent to a power computed for a resistor. We can think of it as ”electrical power of a resistor is completely converted into heat”.

Read more

- Stanford. Chapter 7 Impedance and Bode Plots

[1] The derivation of this equation can be summed up as "The current through the resistor must be equal in magnitude (but opposite in sign) to the time derivative of the accumulated charge on the capacitor".

Lecture slides are available via Github, links are on Moodle

You can help improve these slides at:
github.com/SergeiSa/Mechatronics-2023