

# Lecture 35: VAE Applications

Instructor: Sergei V. Kalinin

# VAE applications for real problems

- We often deal with complex data sets containing the information on physics of objects we seek to understand
- This can be spectral data sets (EELS in STEM, CITS in STM, complex spectroscopies in PFM) or single, multimodal, or hyperspectral images
- Often, we seek to reduce dimensionality and explore similarities in these data sets.

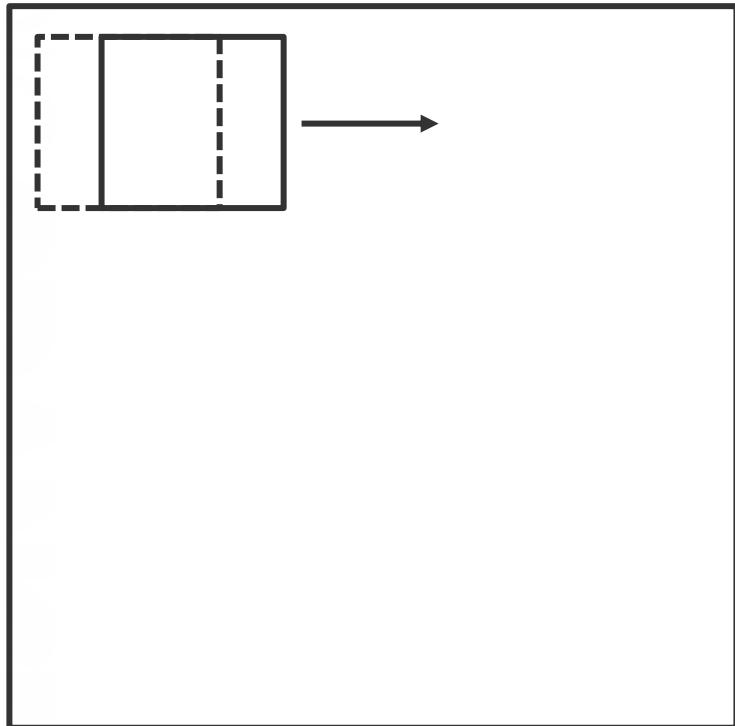
## Two things matter: descriptors and ML method

- In analysis of EELS or CITS data, very often our descriptor is the spectrum at each pixel. Typical analysis will be either linear or non-linear dimensionality reduction or clustering:
  - Linear dimensionality reduction: PCA, NMF, BLU
  - Clustering: k-means, GMM
  - Manifold learning: ISO, UMAP, tSNE, DBSCAN
  - Neural nets: SOFM, AEs, VAEs
- Typical result will be the components (representing behavior), and loading maps representing spatial variability of these behaviors. By construct, components will not depend on the relative spatial positions of pixel.

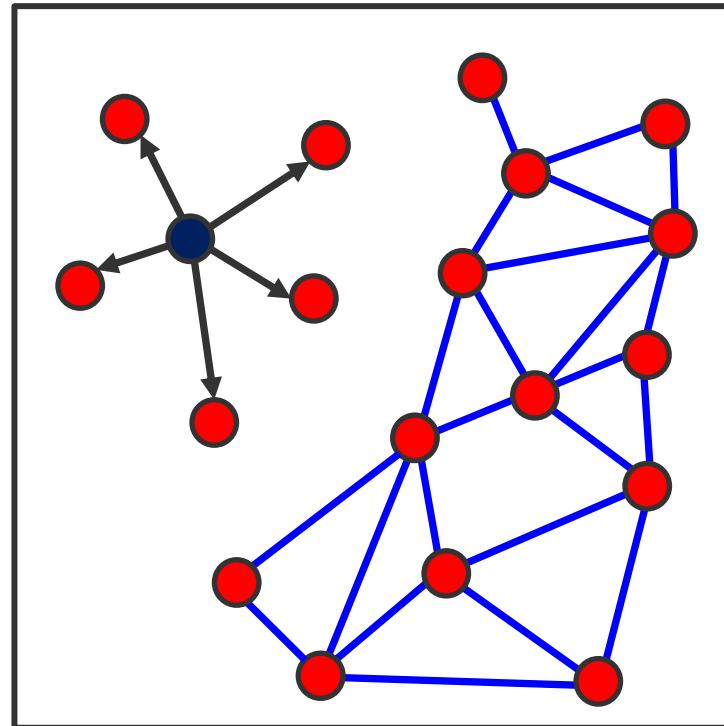
# Describing the building blocks

- The classical physical descriptions (symmetry, etc) can be defined locally only in Bayesian sense
- We can argue that local descriptors are simple, if not necessarily known
- And the rules that guide their emergence are also simple, if not known

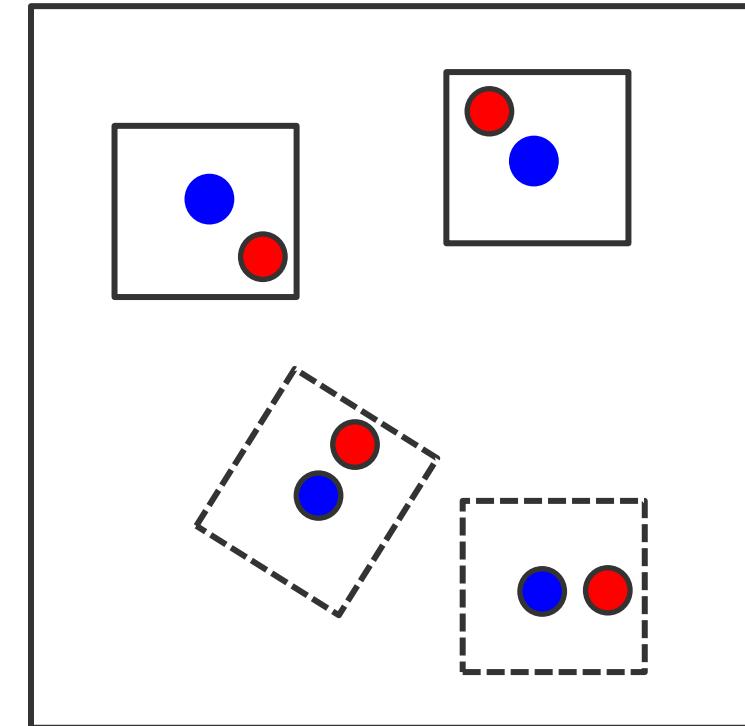
**Continuous translational symmetry**



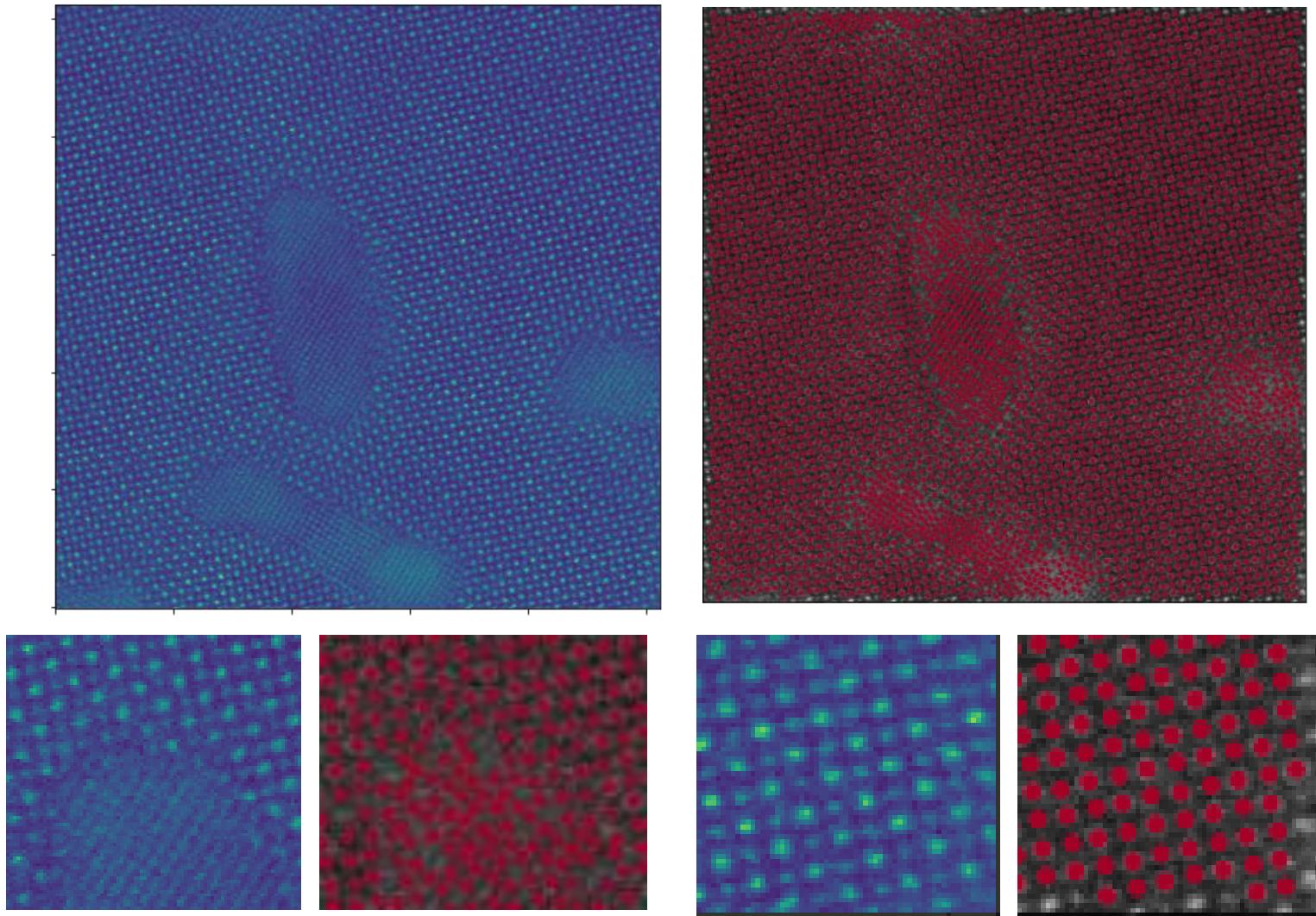
**Atom based descriptions**



**Localized sub-images**



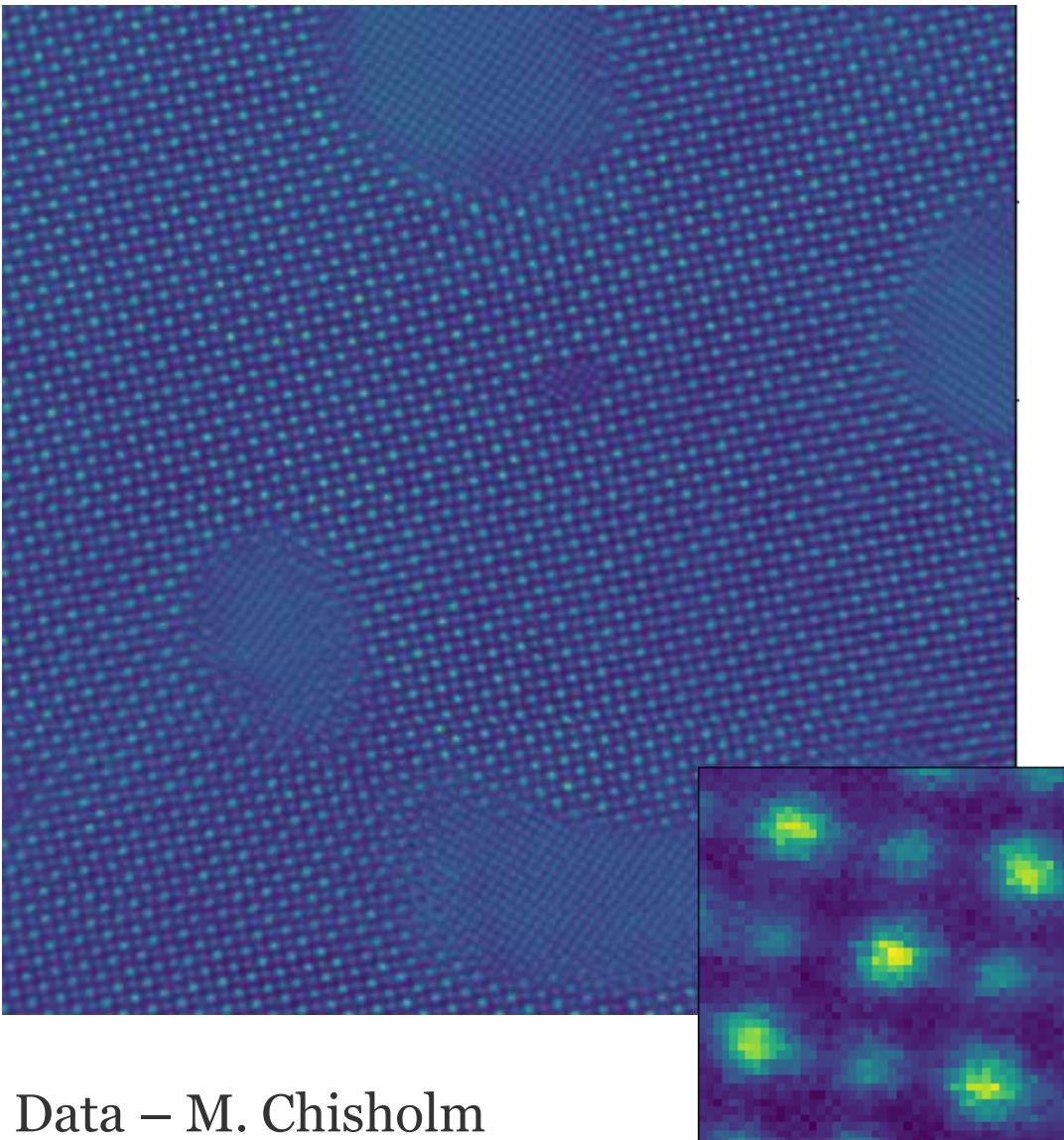
# Let's put it all together!



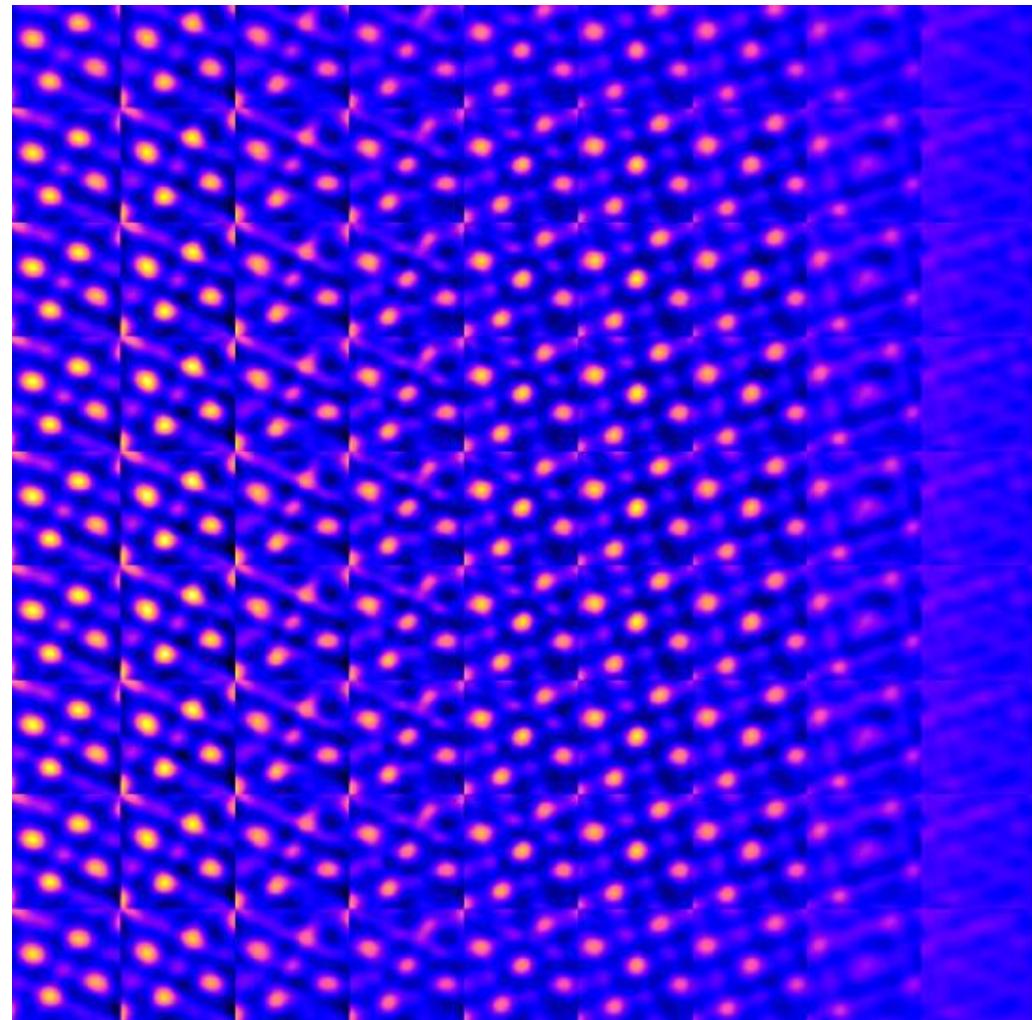
**Step 1:** Find all atoms (or all that you can) – use maximum finders, blob-log, or DCNNs

**Step 2:** Create descriptors – patches centered on atoms. Keep track on what part of image (or stack) it has came from

# Analysis of the NiO-LSMO

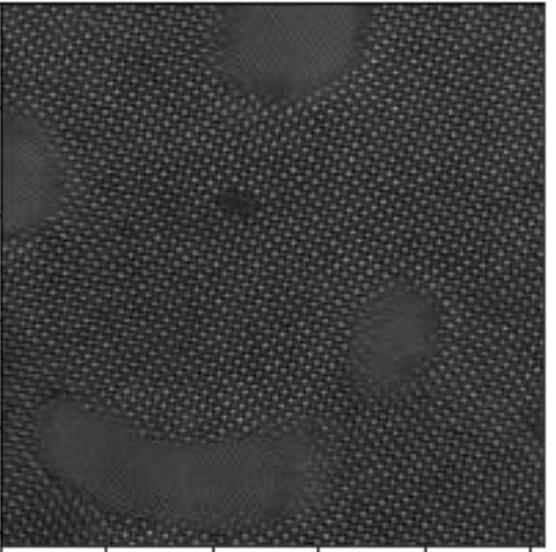


Data – M. Chisholm

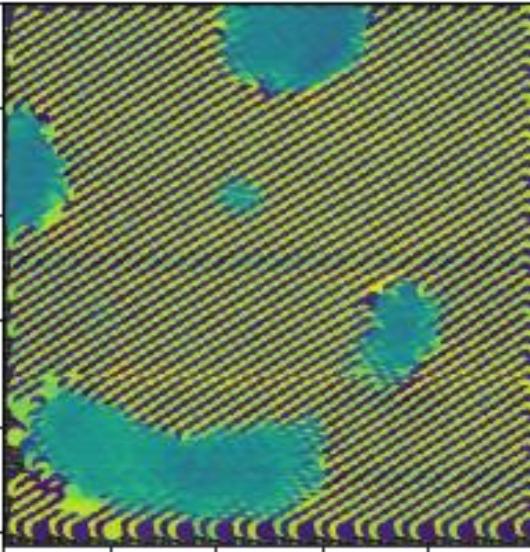


# Let's look at latent space

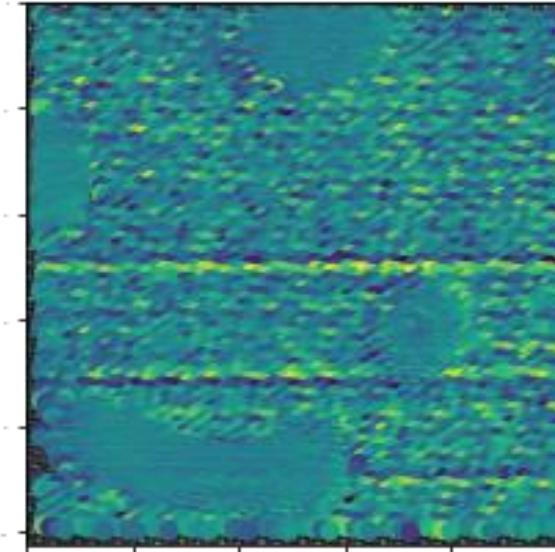
Image



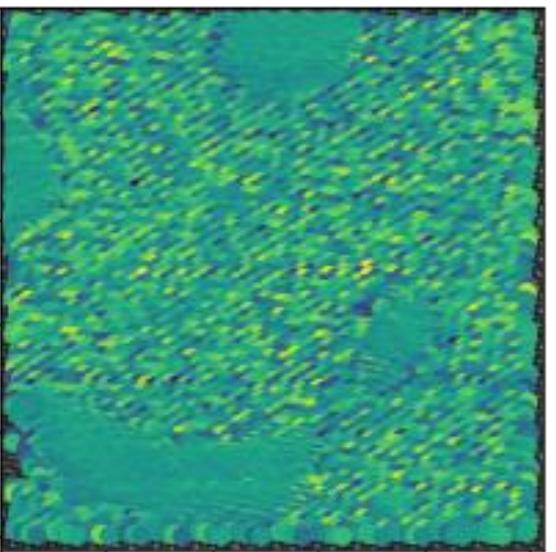
Angle



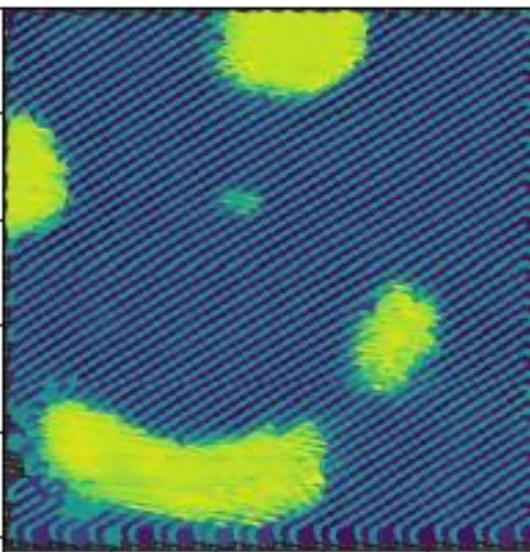
X Offset



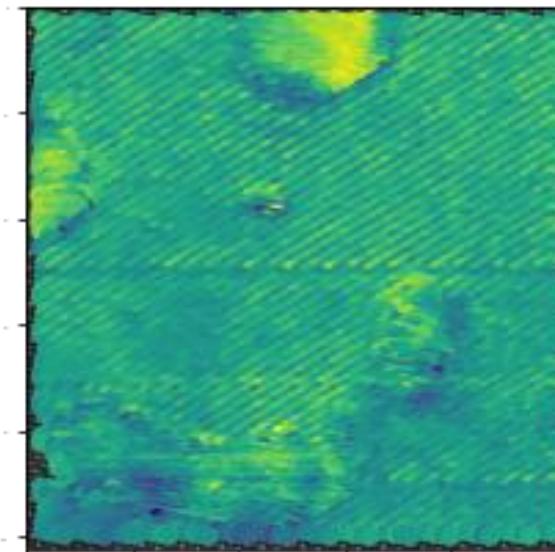
Y Offset



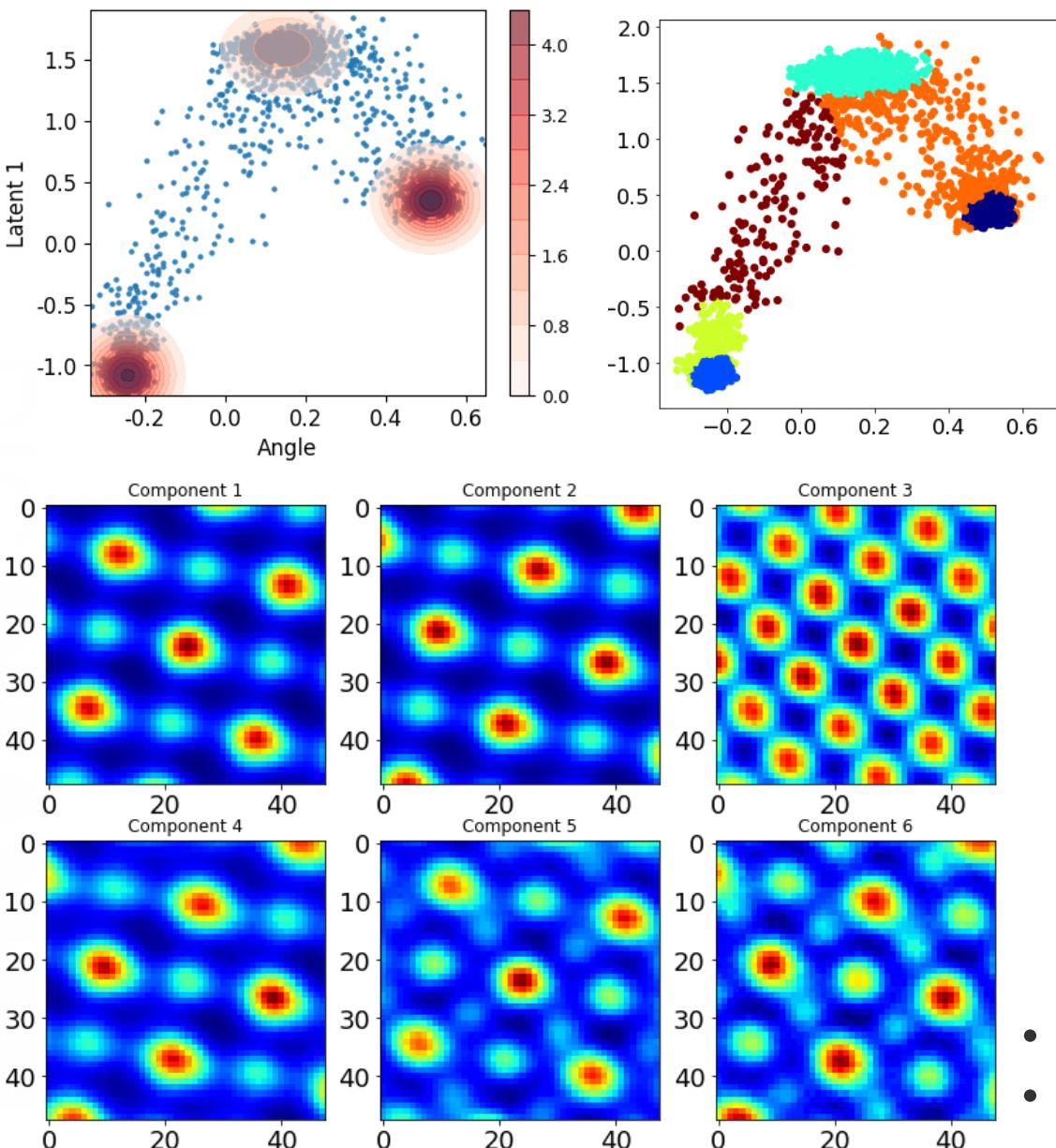
Latent 1



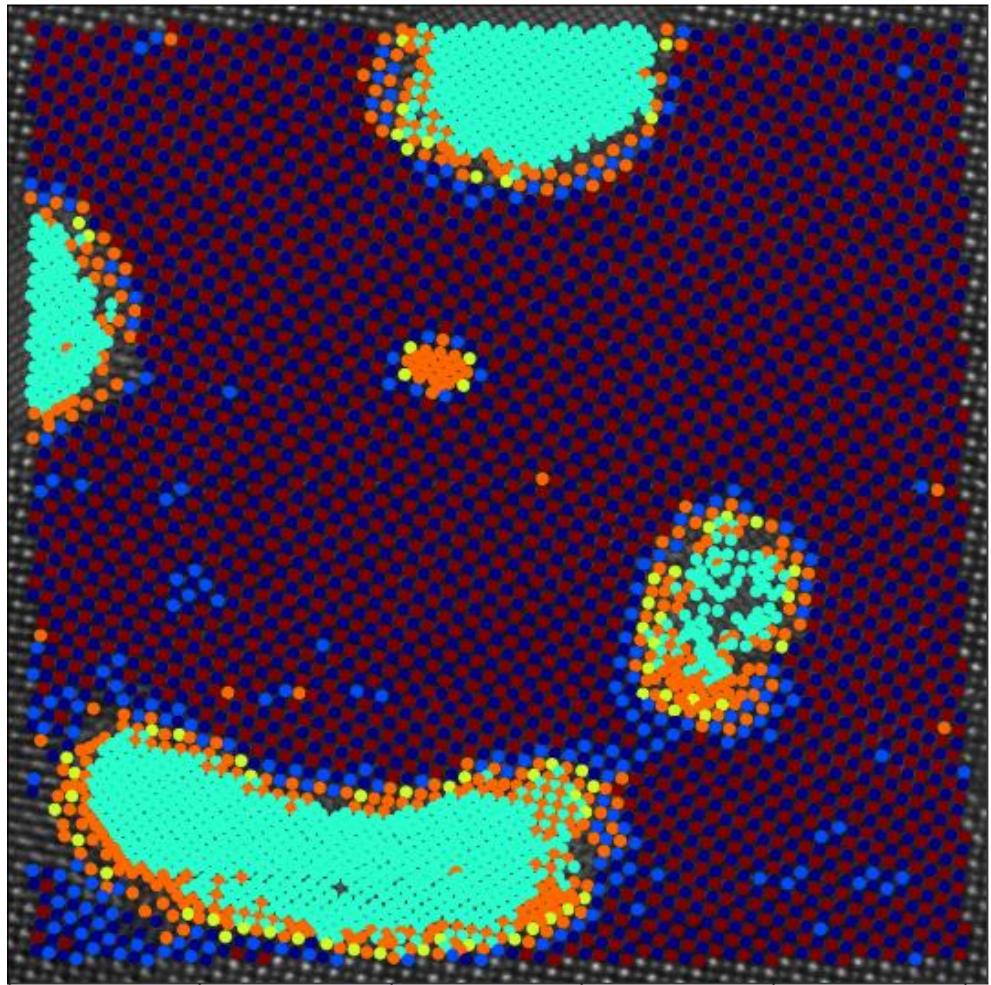
Latent 2



# Exploring latent distributions

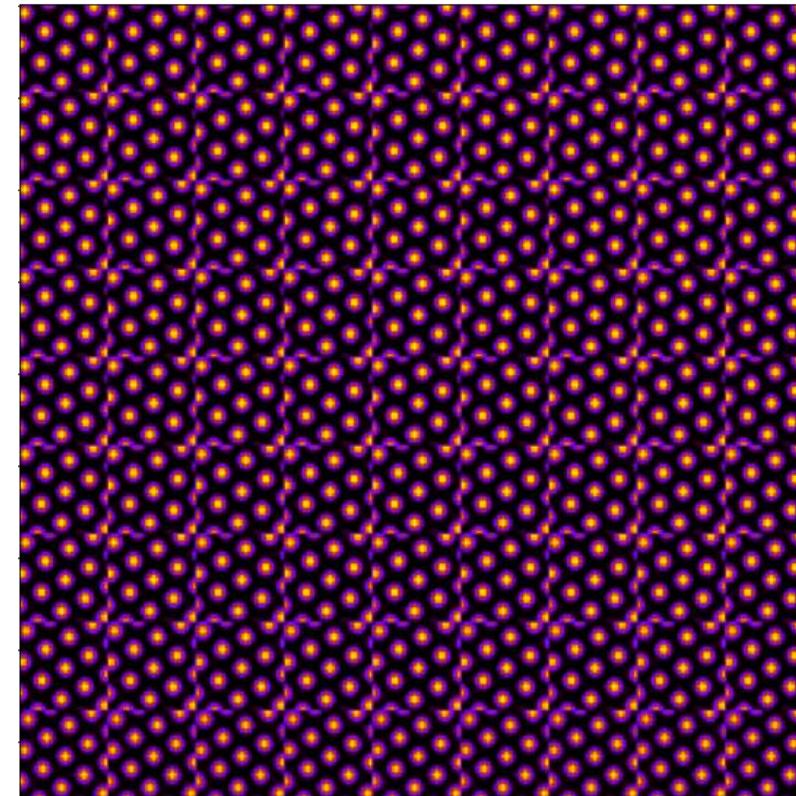
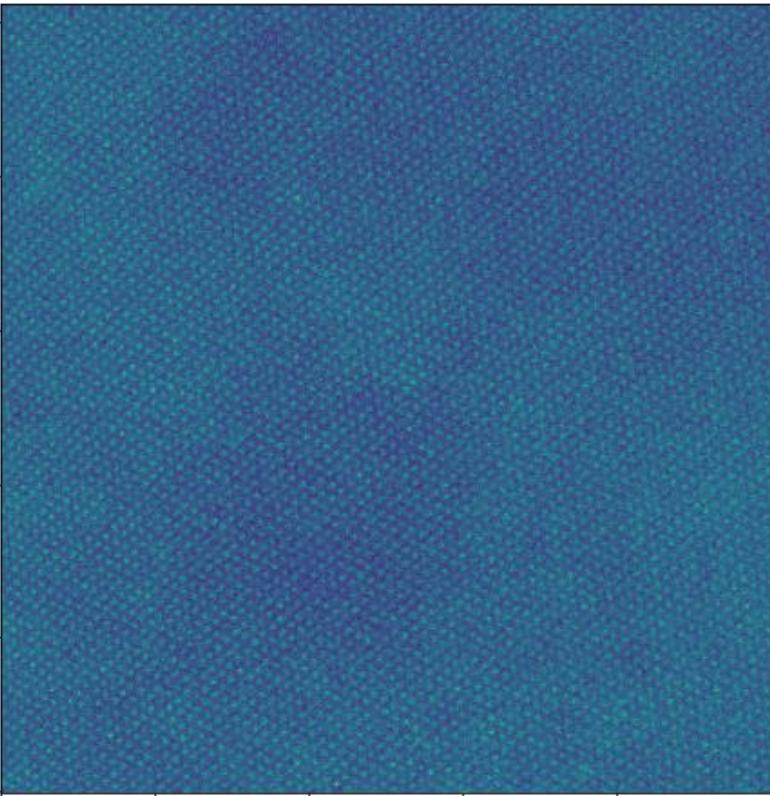


# Labeled image

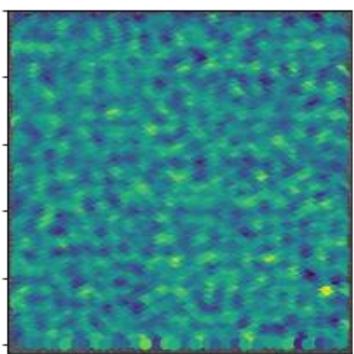


- Classes and variability are mixed in latent space
  - Disentangling of representation

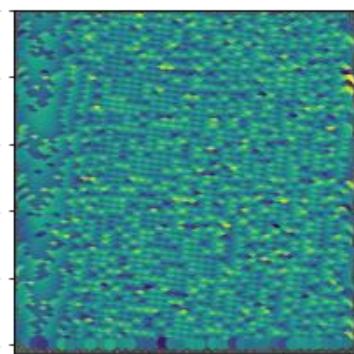
# Out of curiosity: single crystal?



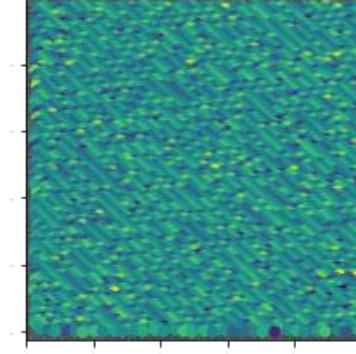
Rotation



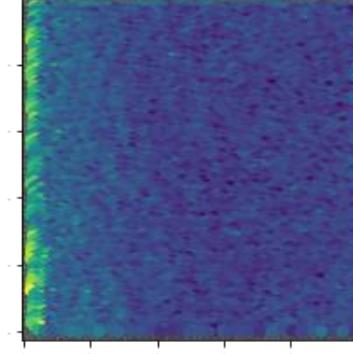
Offset X



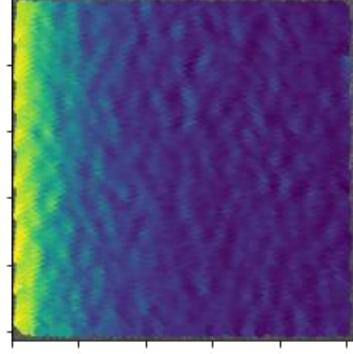
Offset Y



Latent 1

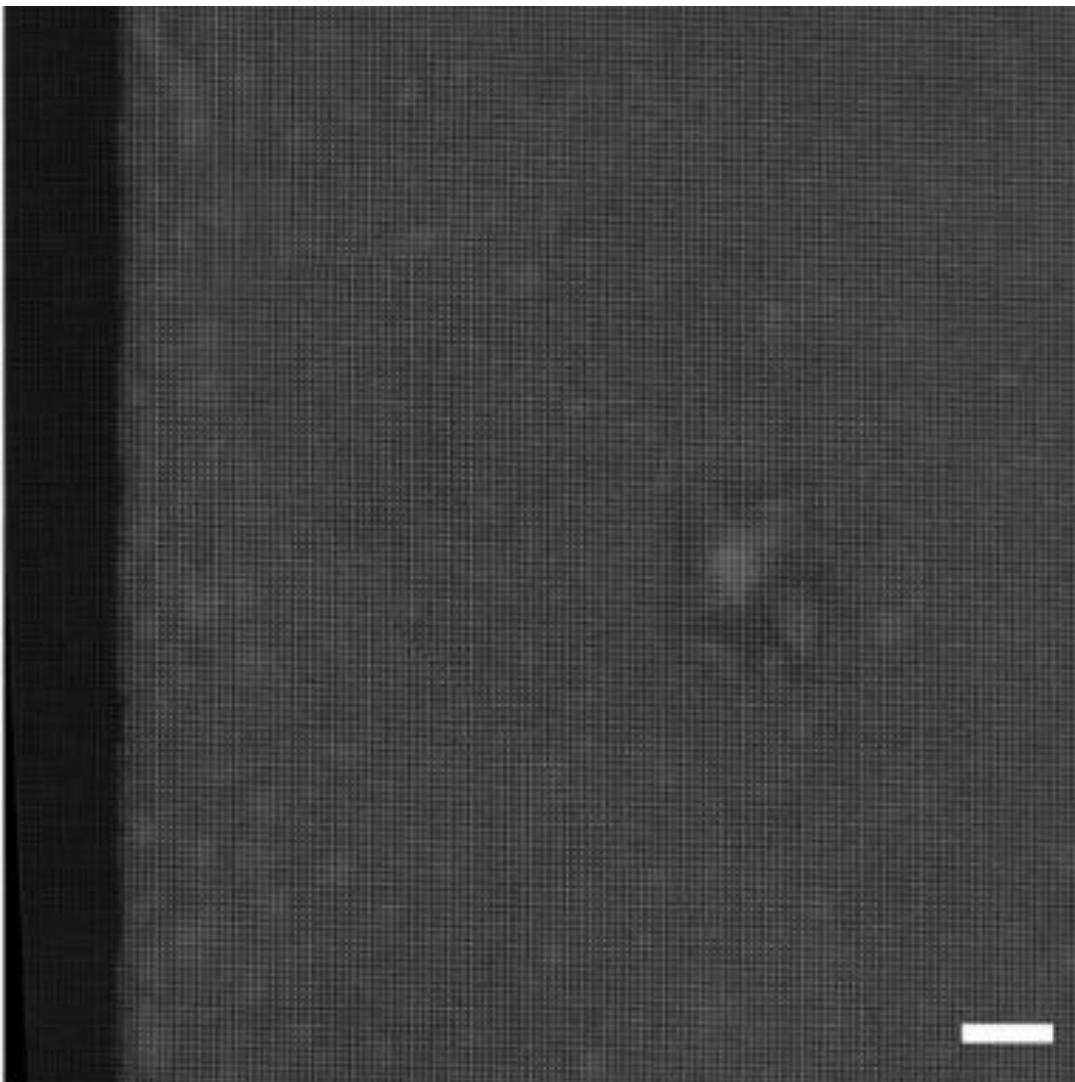


Latent 2

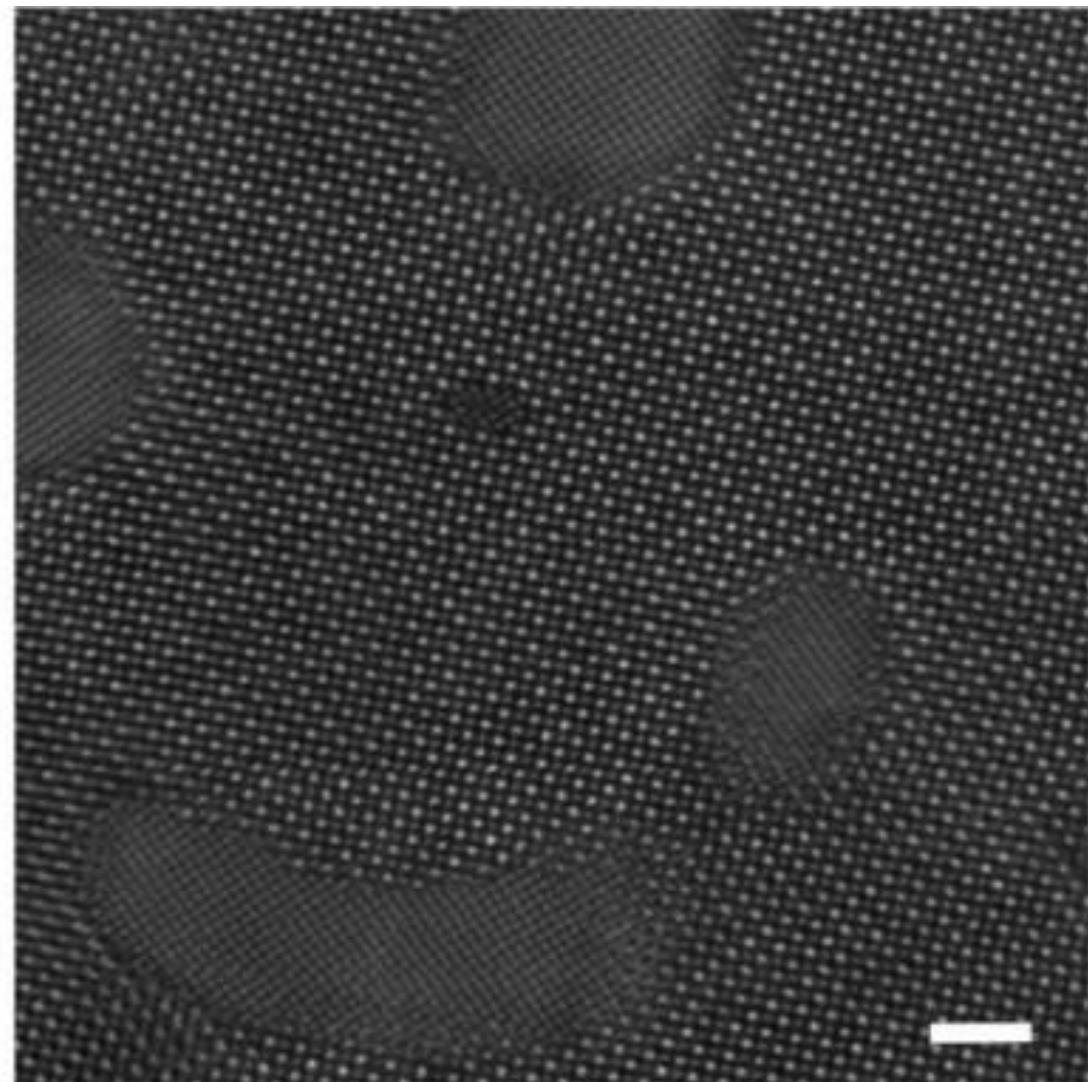


# VAE without Atom Finding

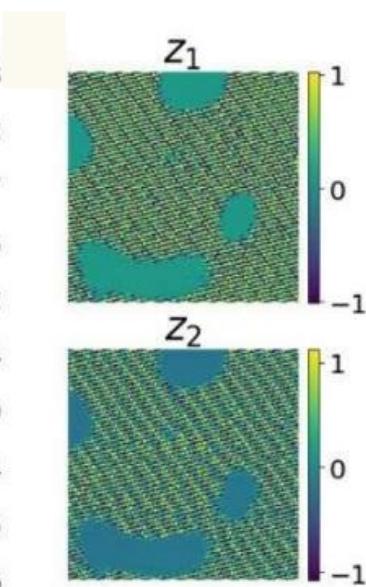
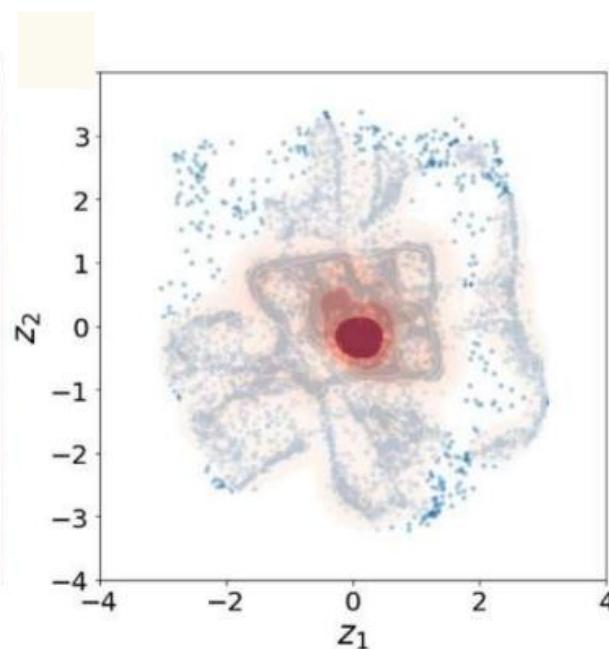
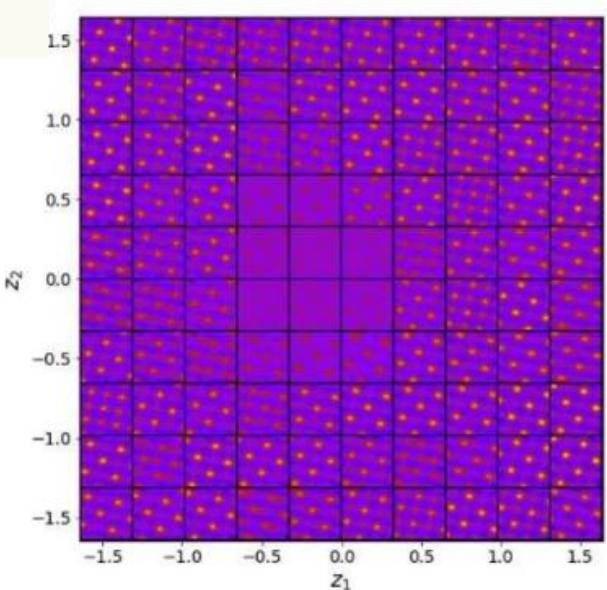
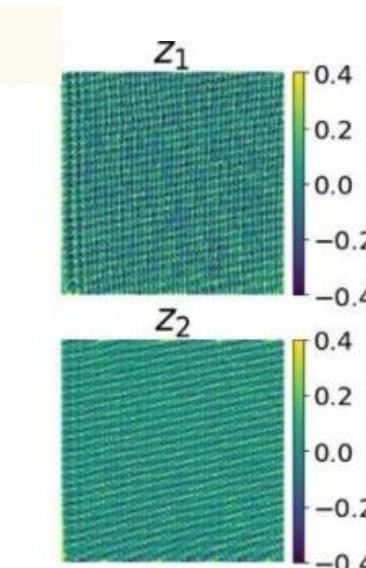
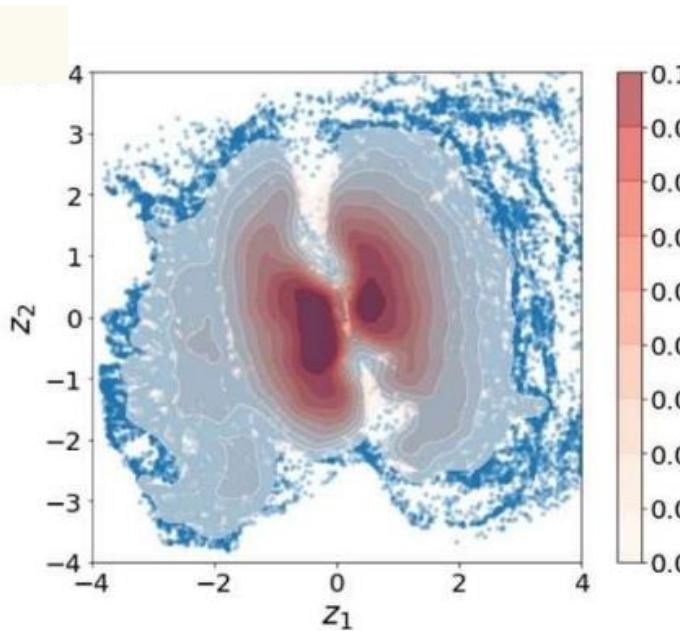
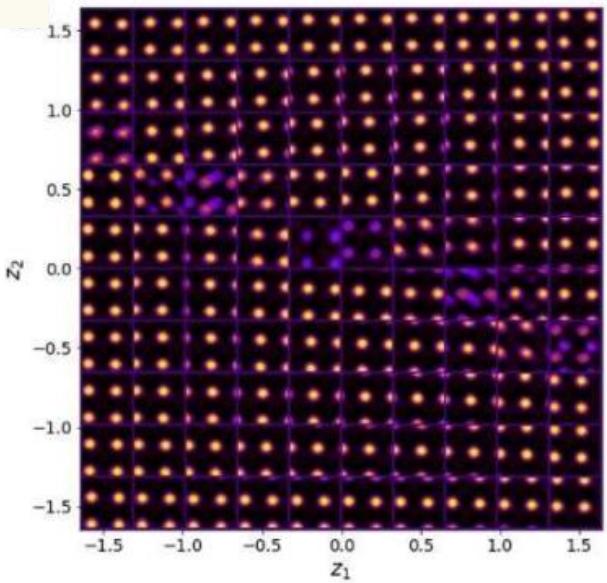
Ferroelectric  $\text{BiFeO}_3$



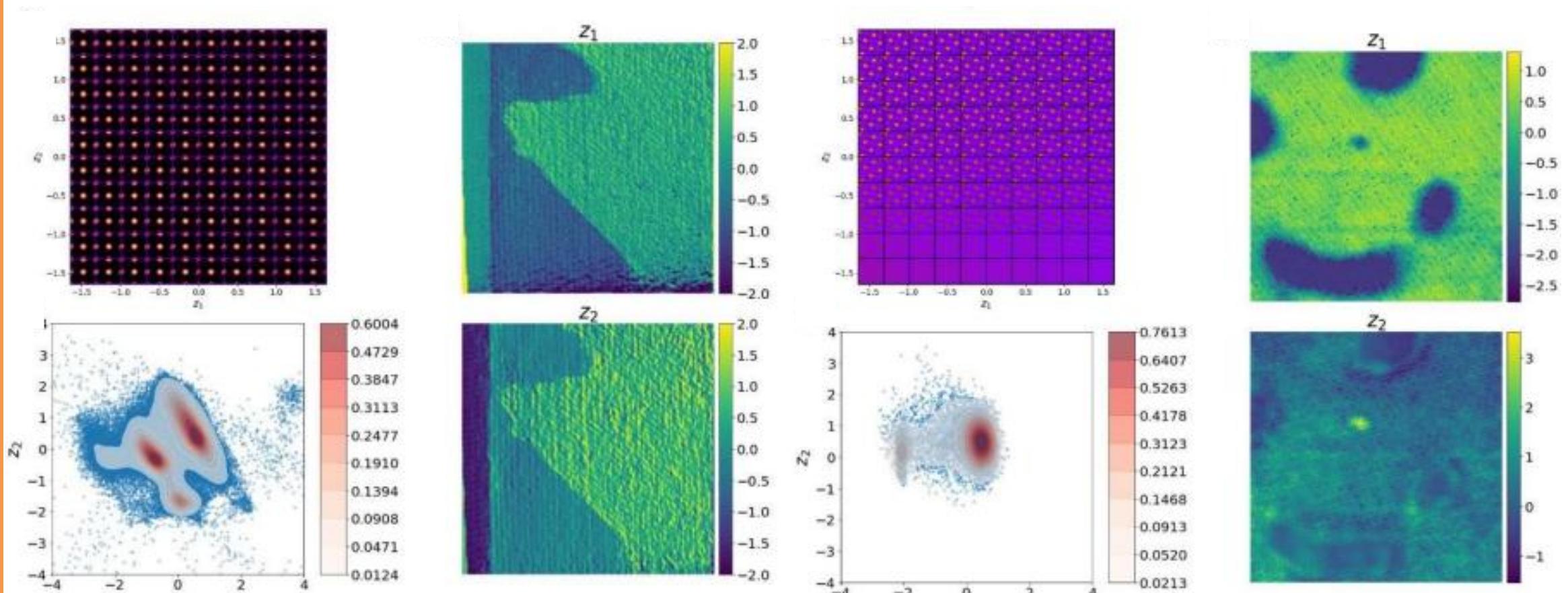
$\text{NiO} - \text{La}_x\text{Sr}_{1-x}\text{MnO}_3$



# Simple VAE

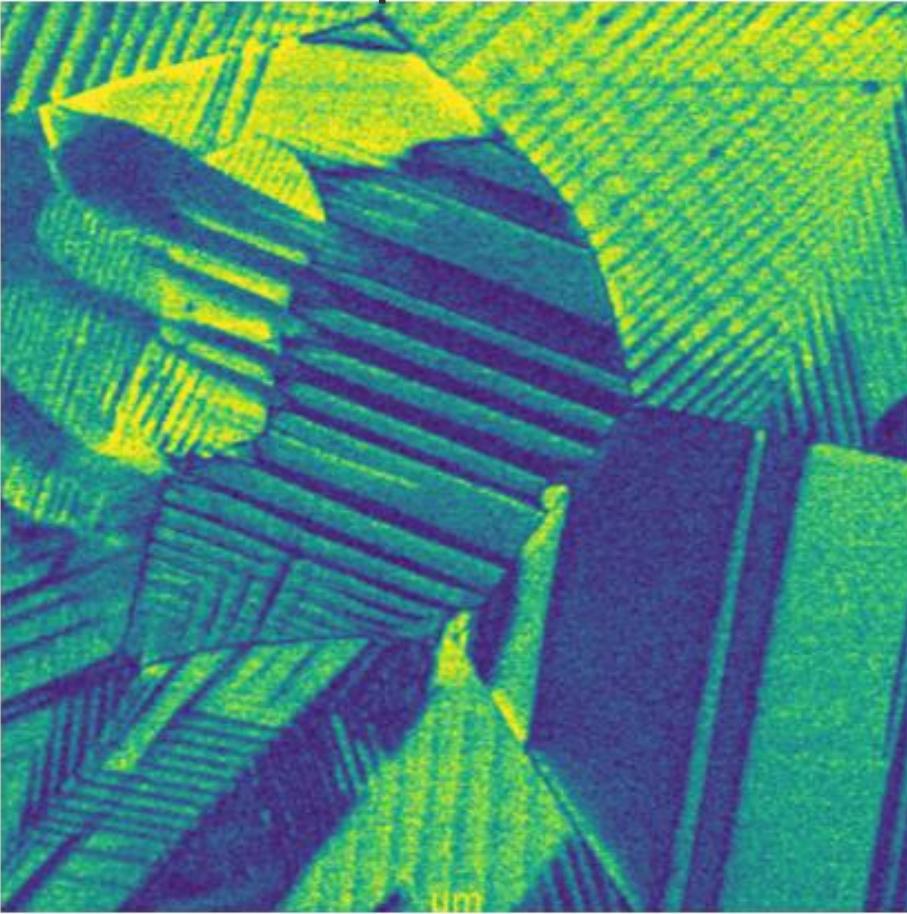


# Shift VAE: Translational Invariance

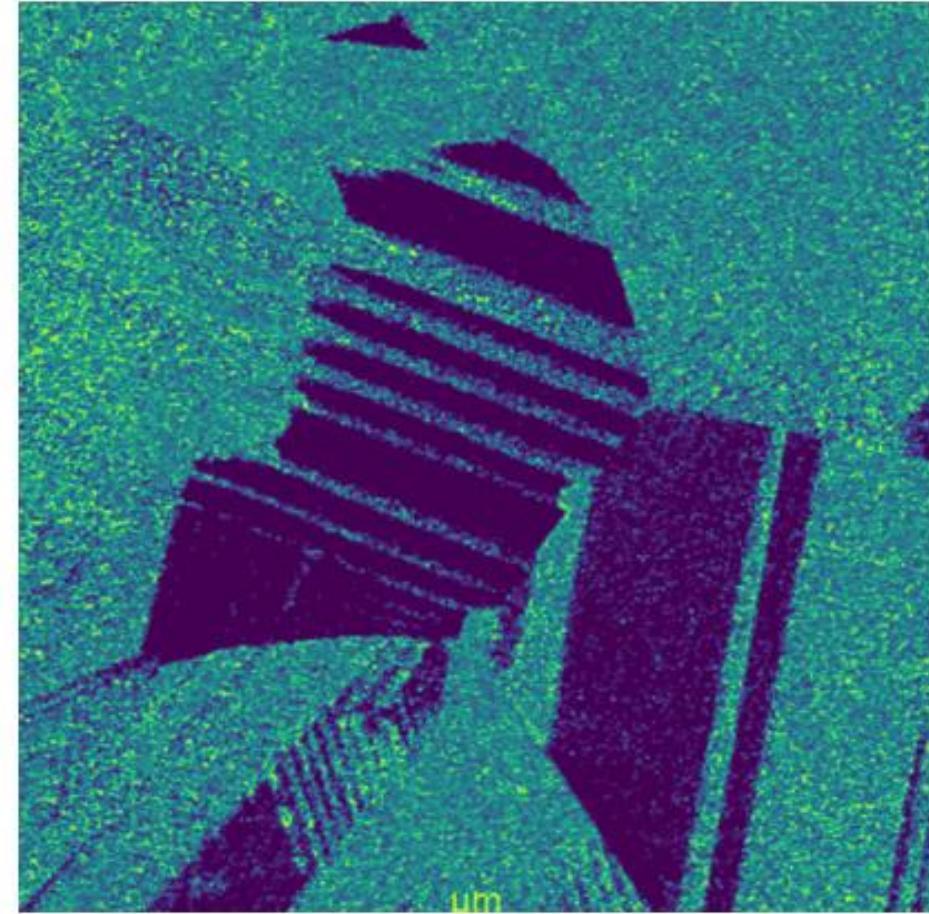


# Ferroelectric domain and domain walls

Amplitude



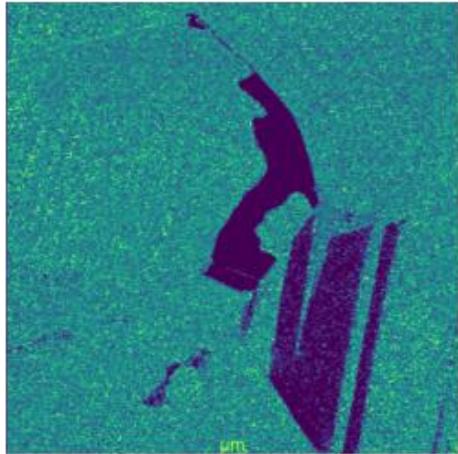
Phase



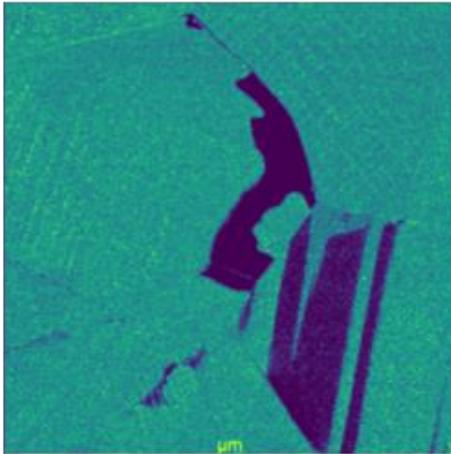
# Detecting domain walls

Canny filter

Phase Image

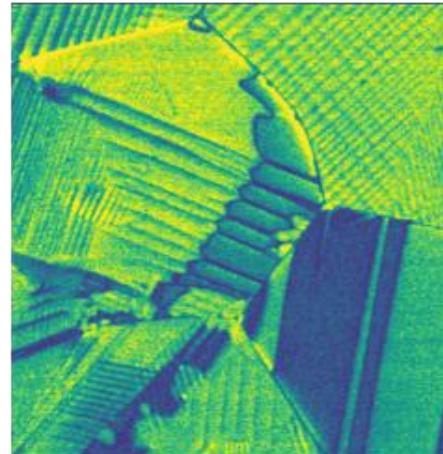


Gaussian Filter



DCNN Prediction

Image



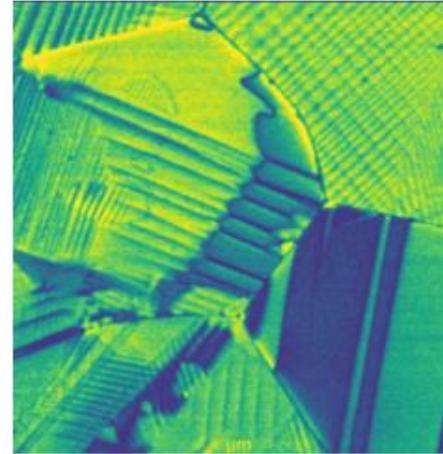
Predicted



Wall by Canny Filter



Gaussian Filter

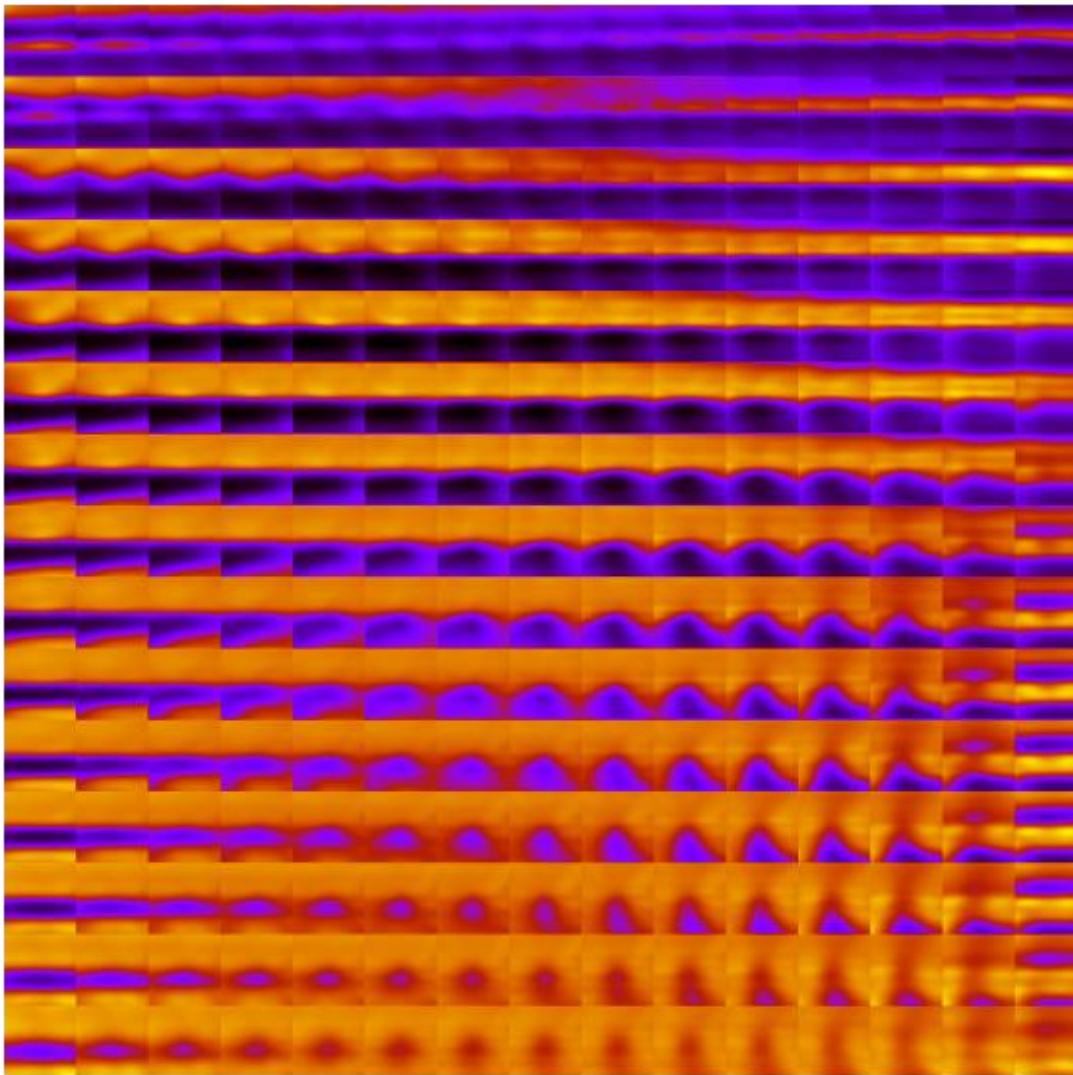


Gaussian Filter and Predicted

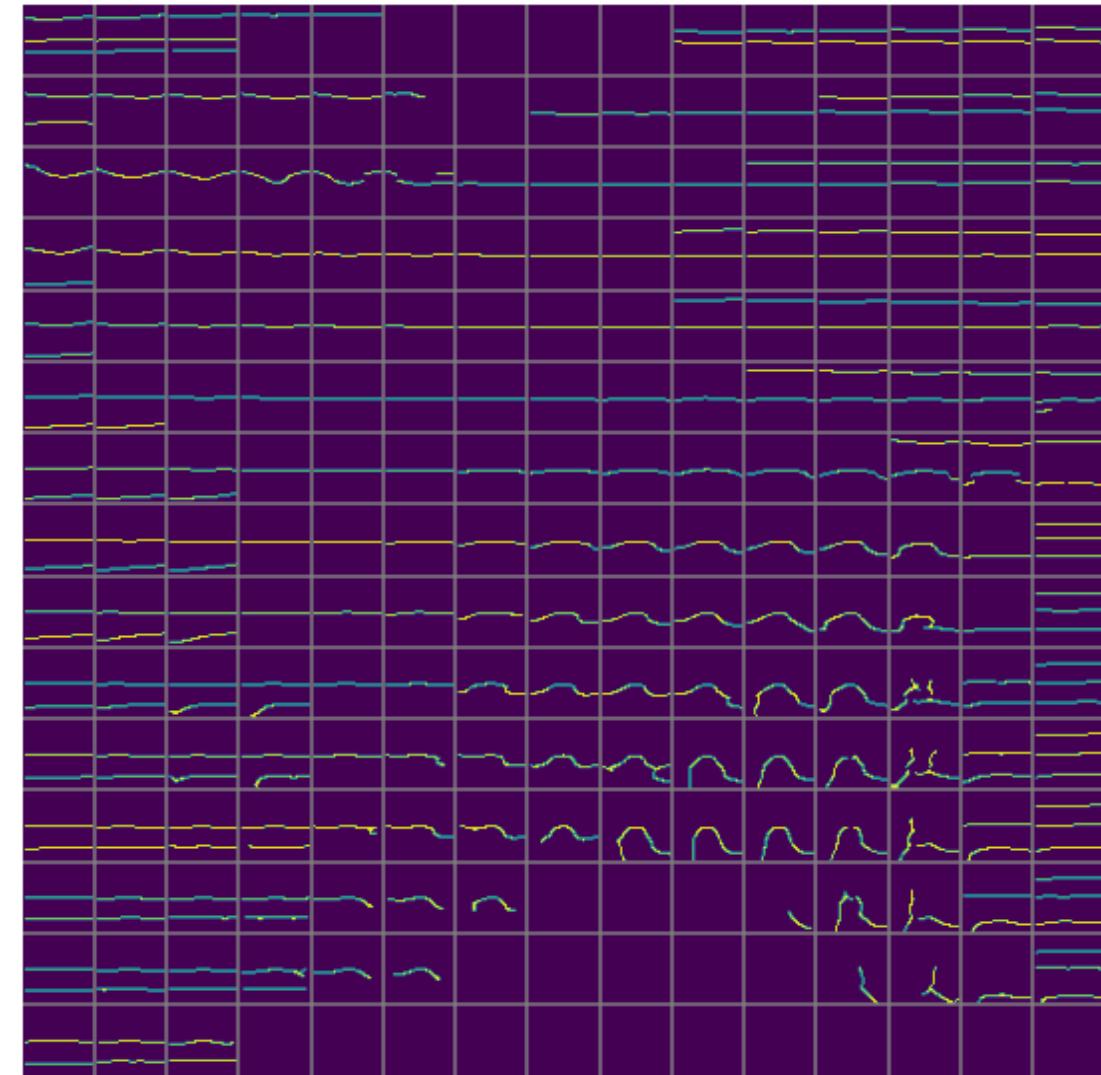


# rVAE analysis

Latent Space

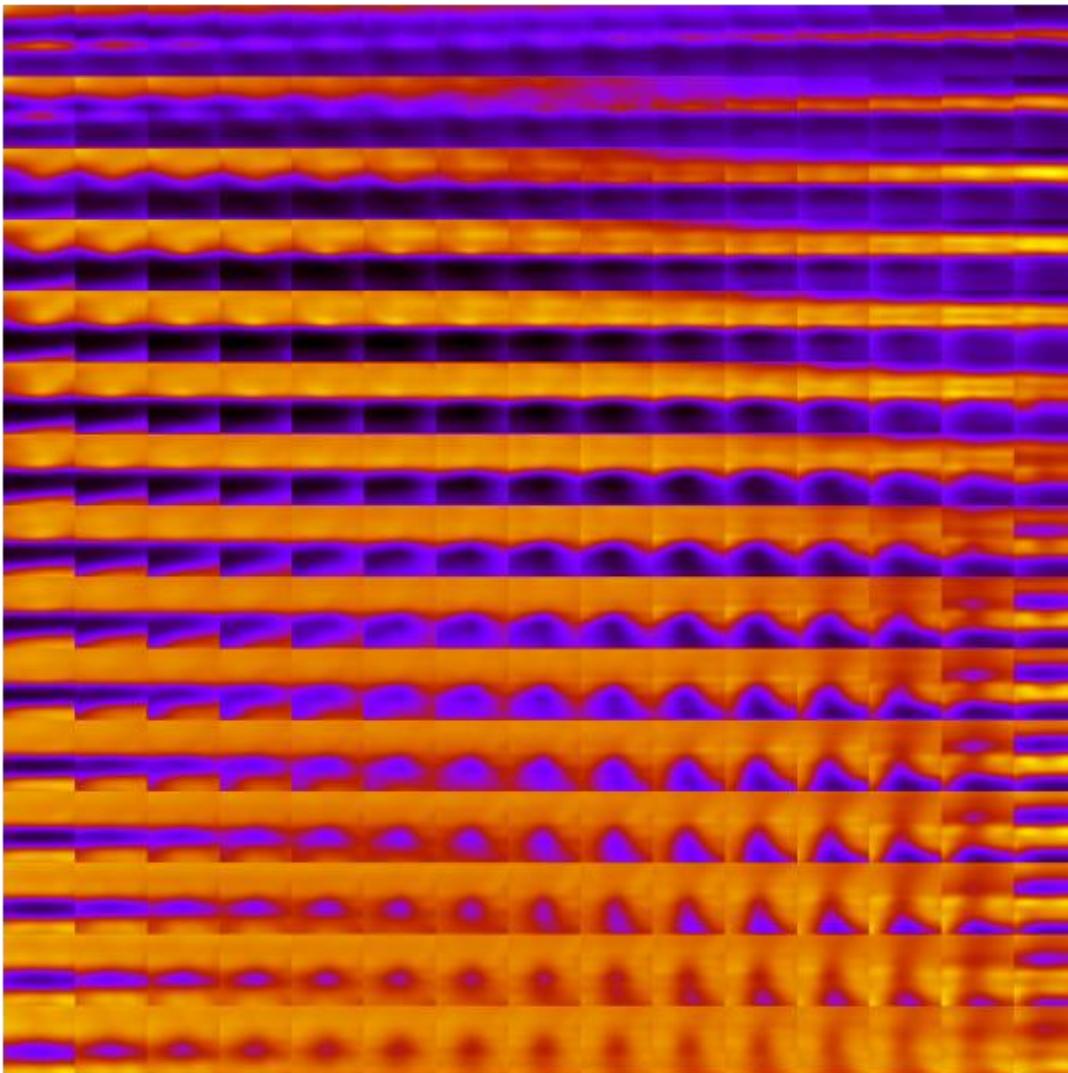


Domain Walls

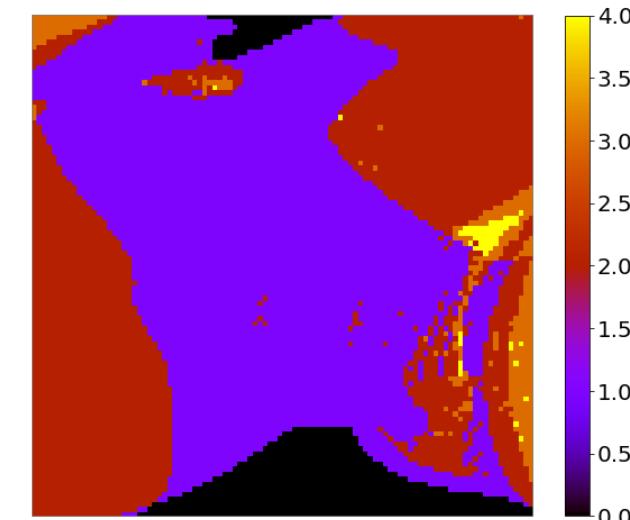


# rVAE latent space

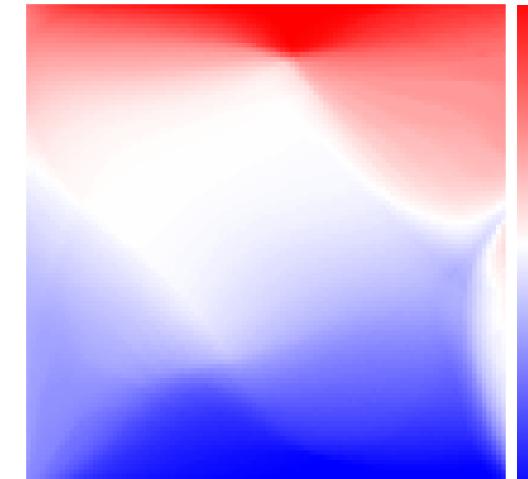
**Latent Space**



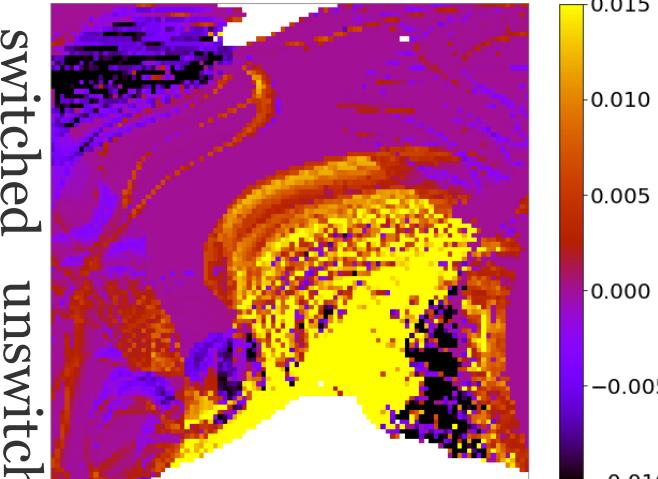
**Domain wall count**



**Switching degree**

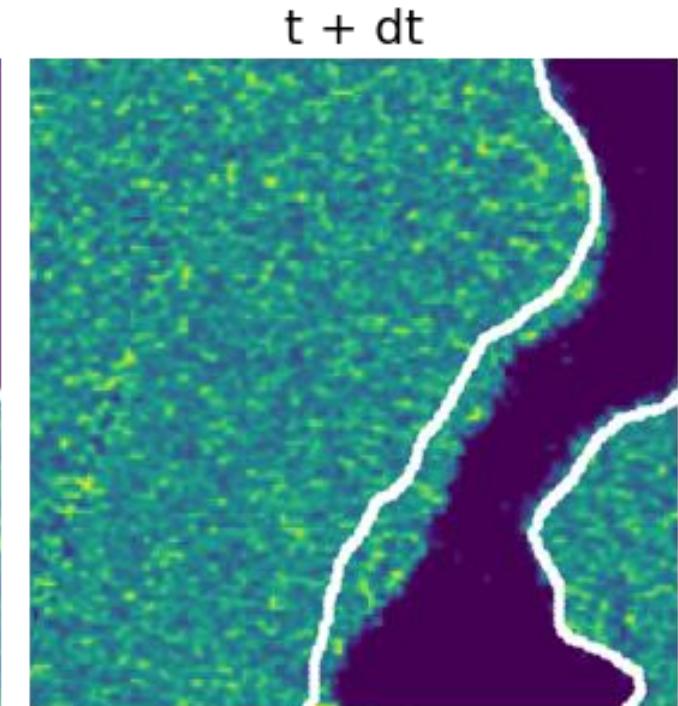
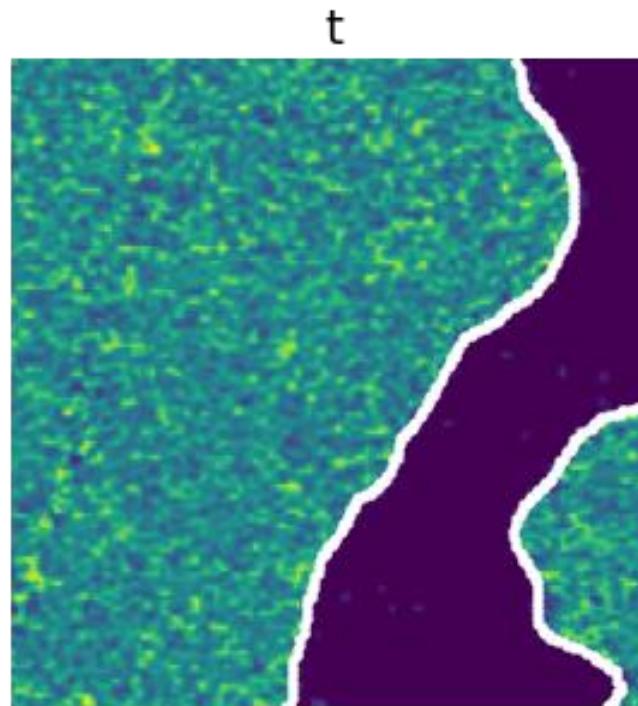
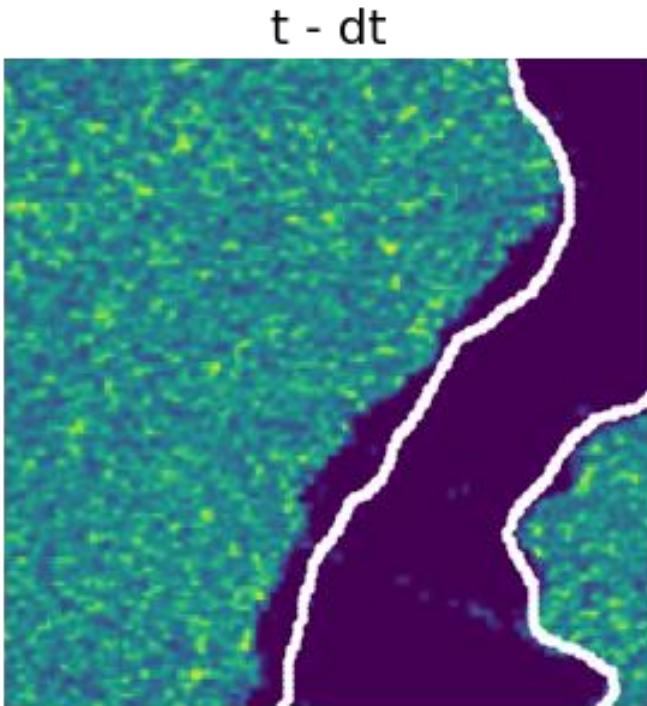


**Average wall curvature**



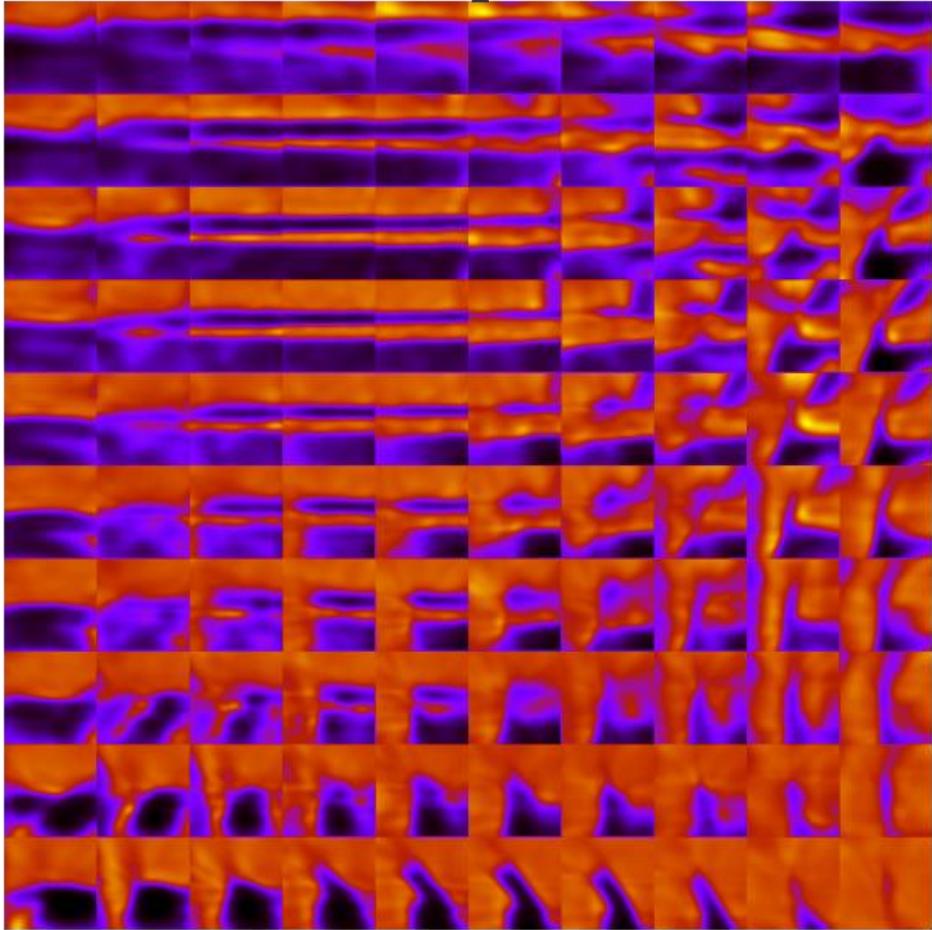
# rVAE with time delay

Training dataset



# rVAE with time delay

Latent space

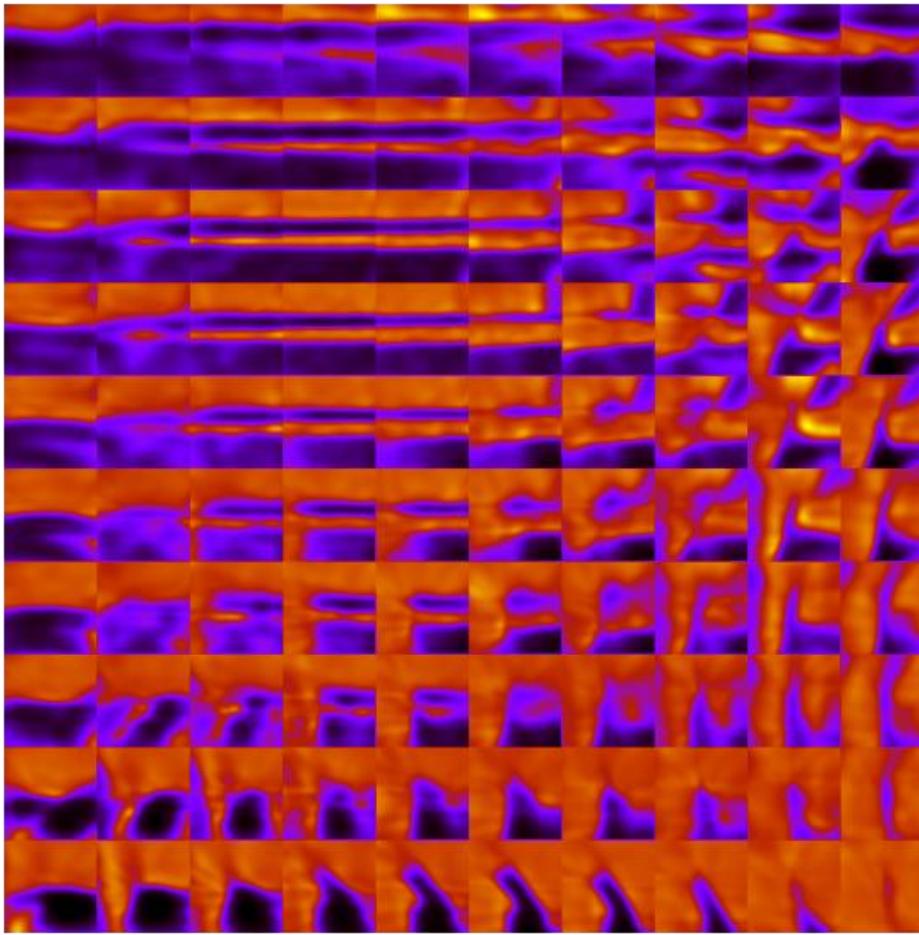


Domain wall

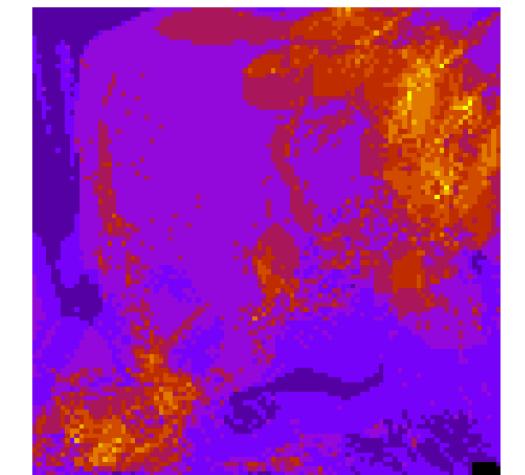


# rVAE with time delay

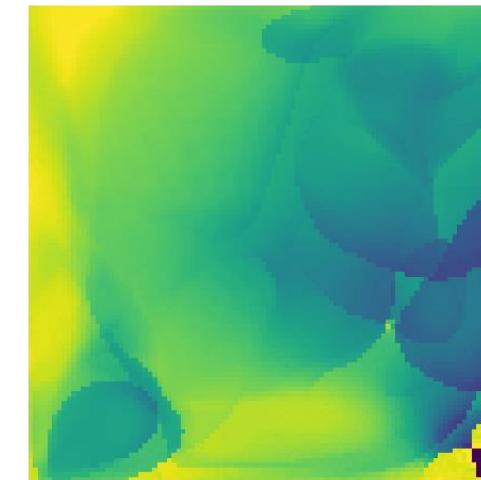
Latent space



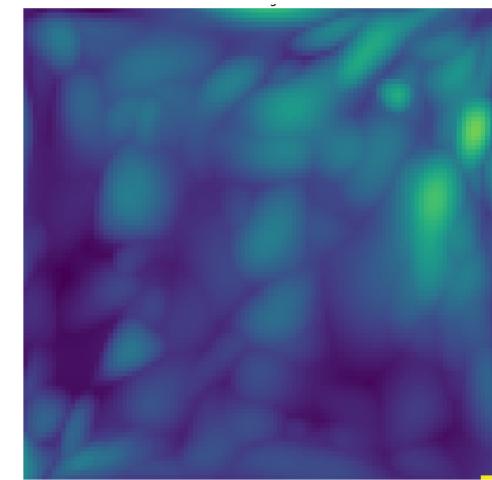
Wall count



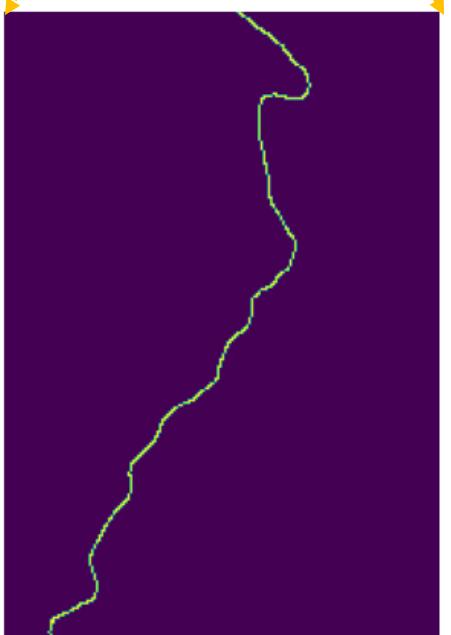
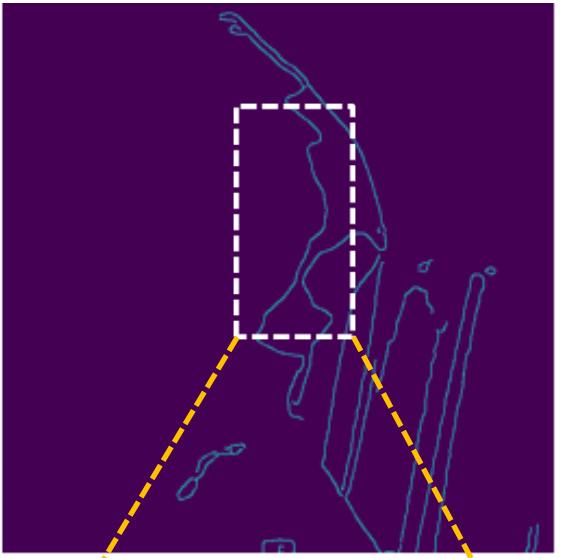
Domain convex



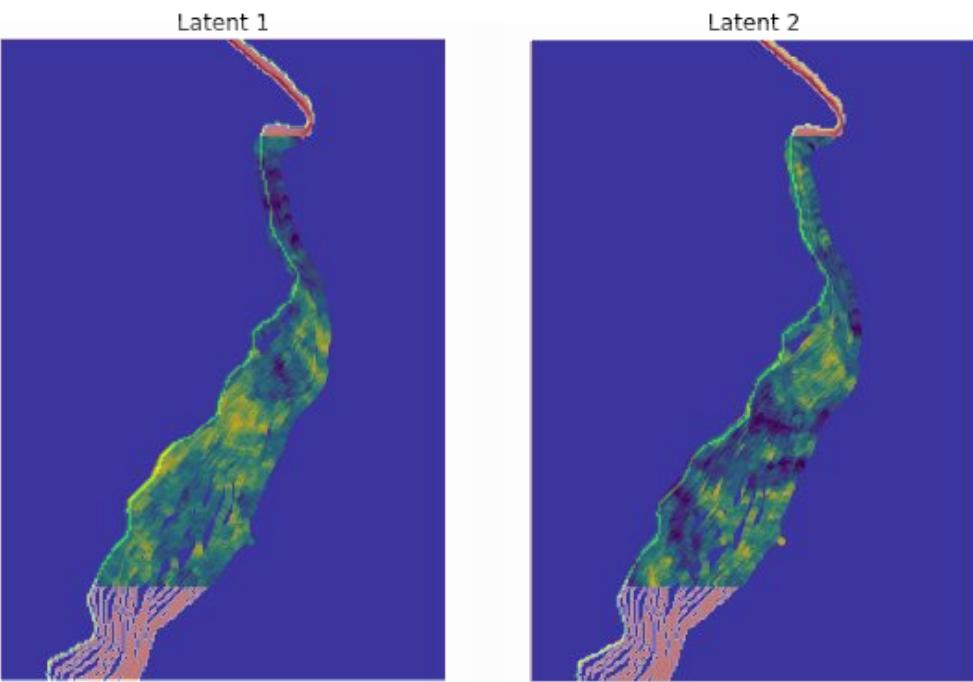
Switch significance



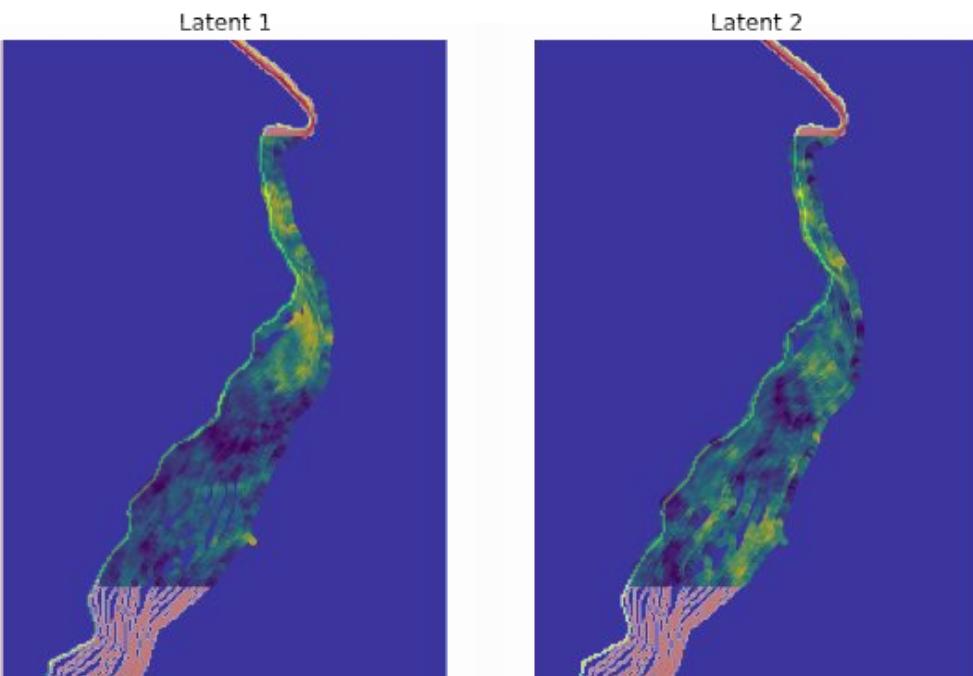
# Domain wall evolution



Forward:  
 $t$  vs  $t+1$

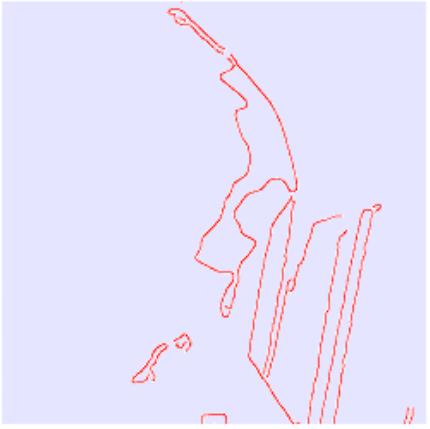


Reverse:  
 $t$  vs  $t+1$

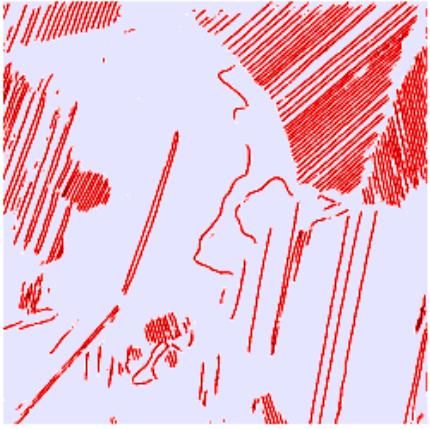


# Multilayer rVAE

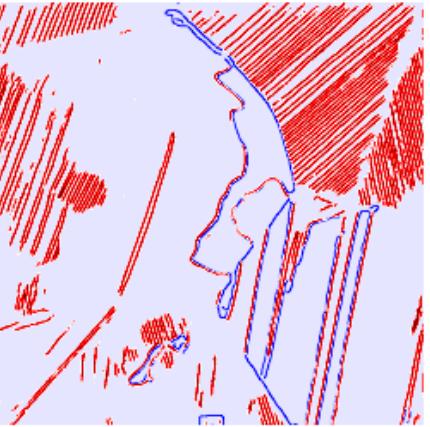
180° Walls



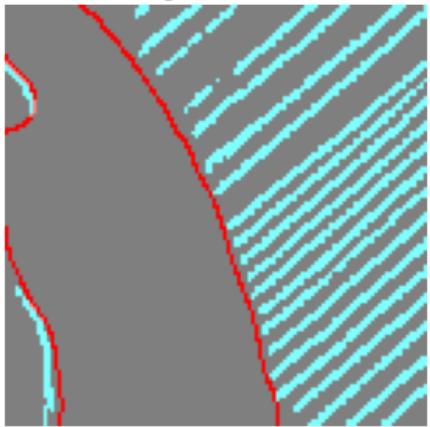
180°+Non180° Walls



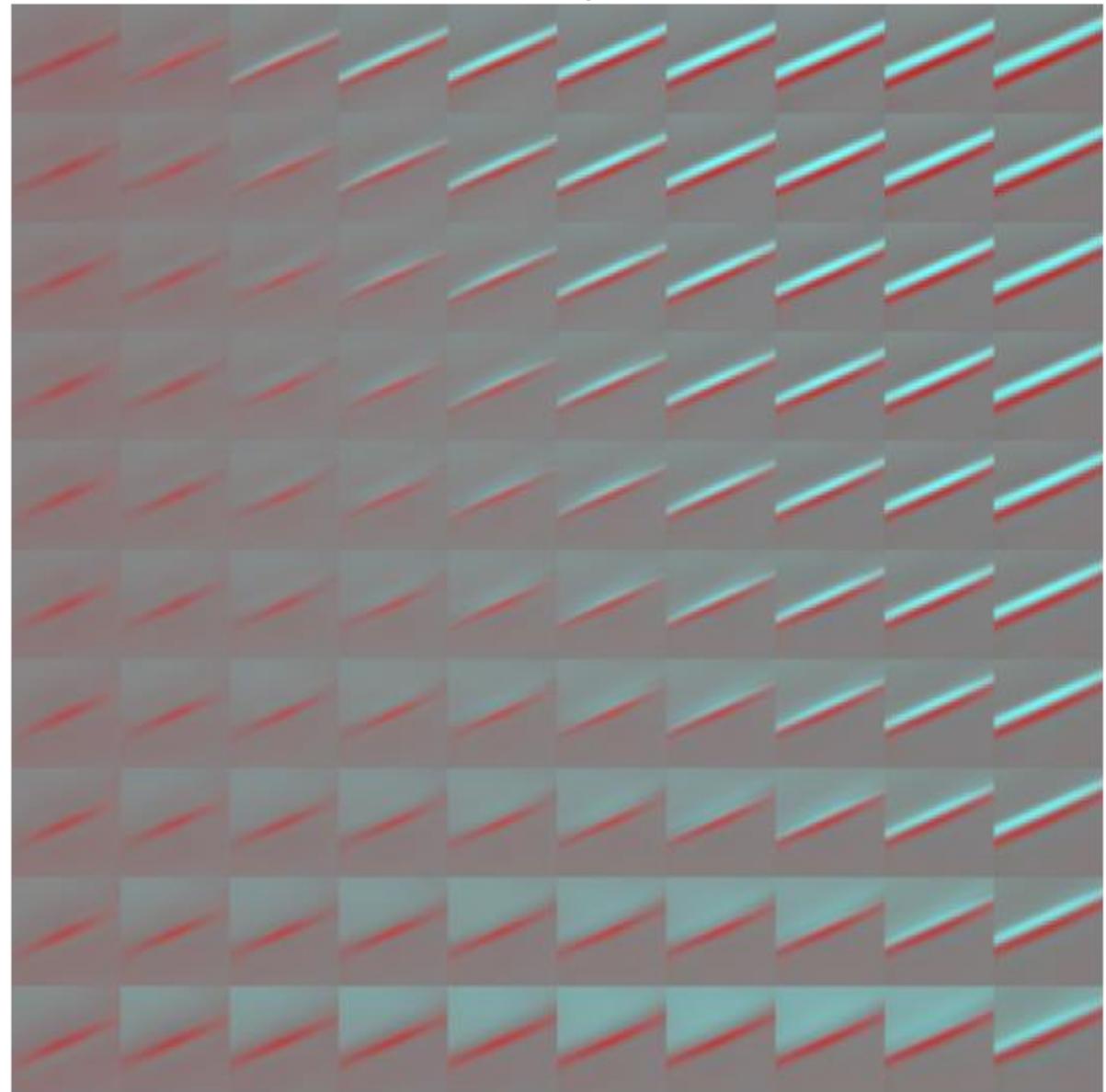
Subtraction = Non180° Walls



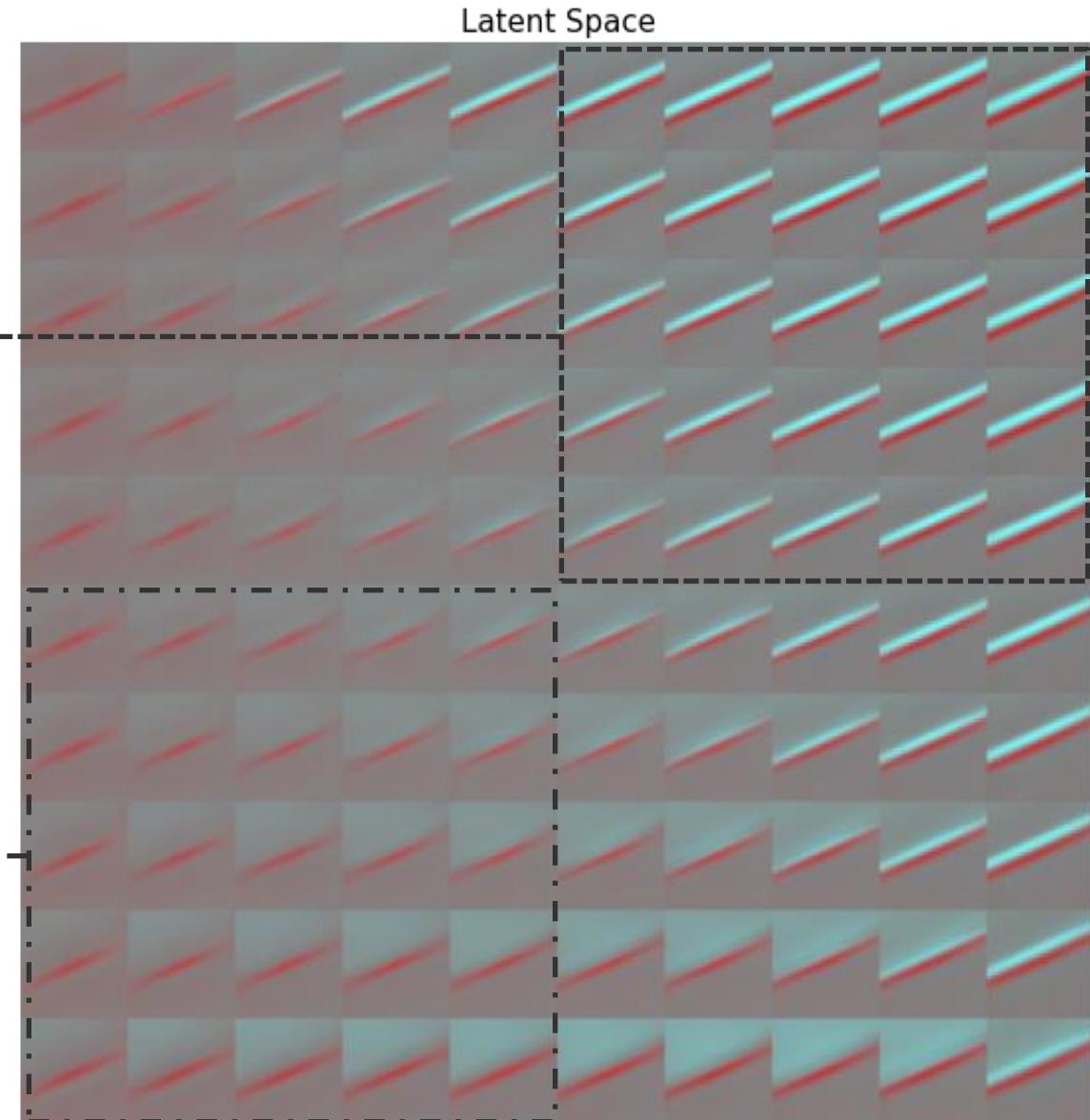
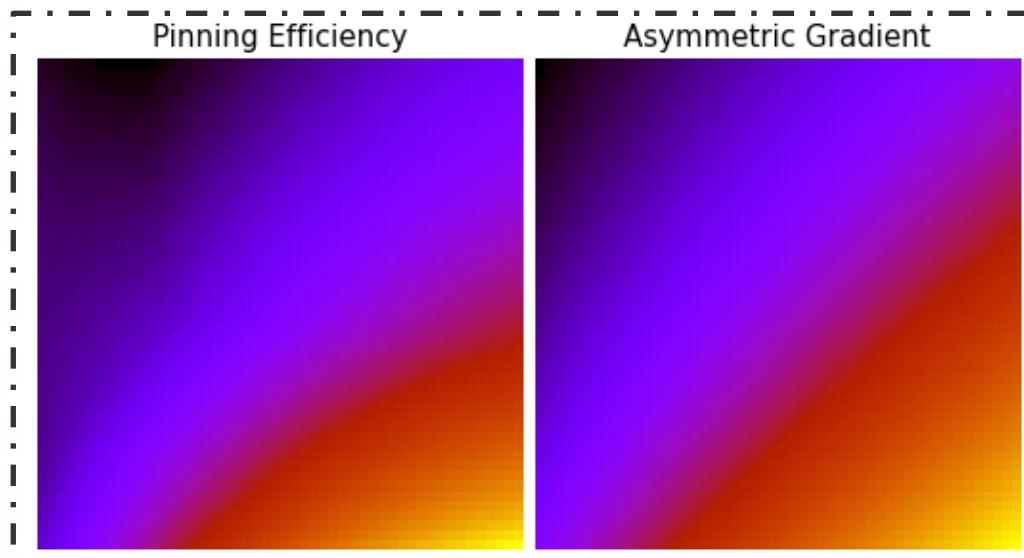
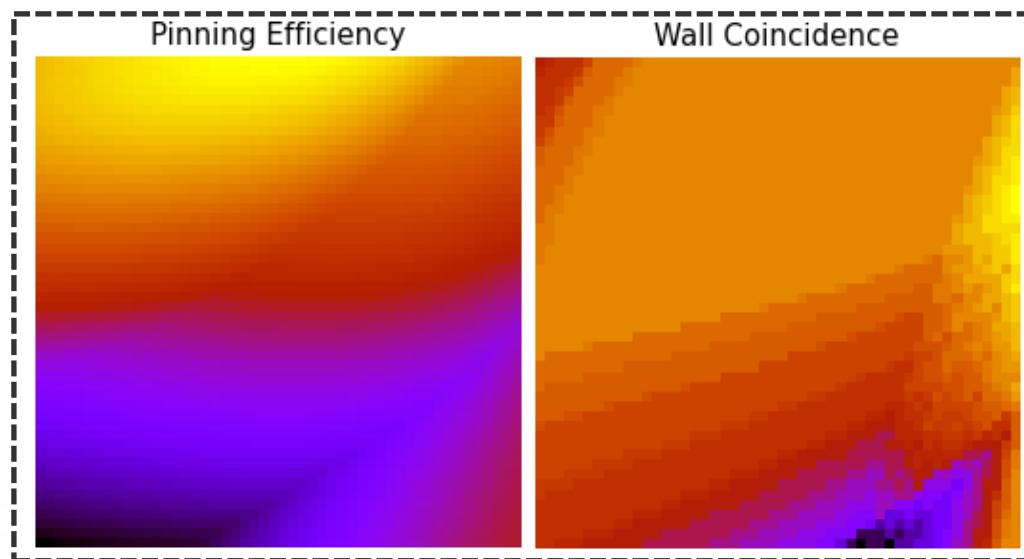
subimage of stack walls



Latent Space



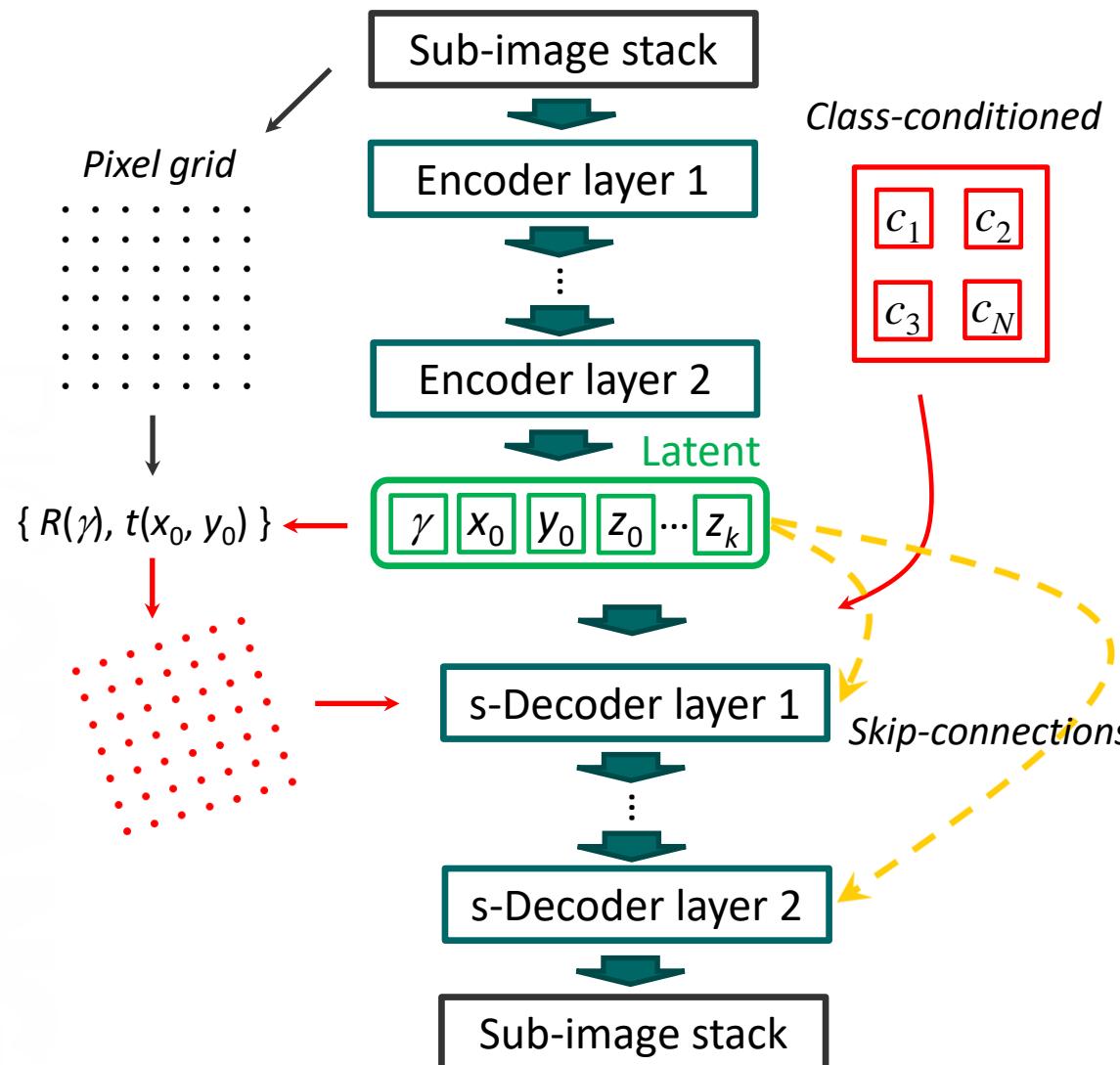
# Pinning mechanism



# What if we have multiple classes?

1. Classes are known: conditional (discrete) VAE
2. Factors of variability are known: conditional (continuous) VAE
3. Some classes are known: semi-supervised VAE
4. Number of classes are known: joint VAE

# Conditional VAE



- Generative model is a function of spatial coordinate
- 3 additional latent variables to absorb rotations and shifts
- Disentangles rotations and translations from image content
- Ideal for analyzing microscopy sub-images on atomic level

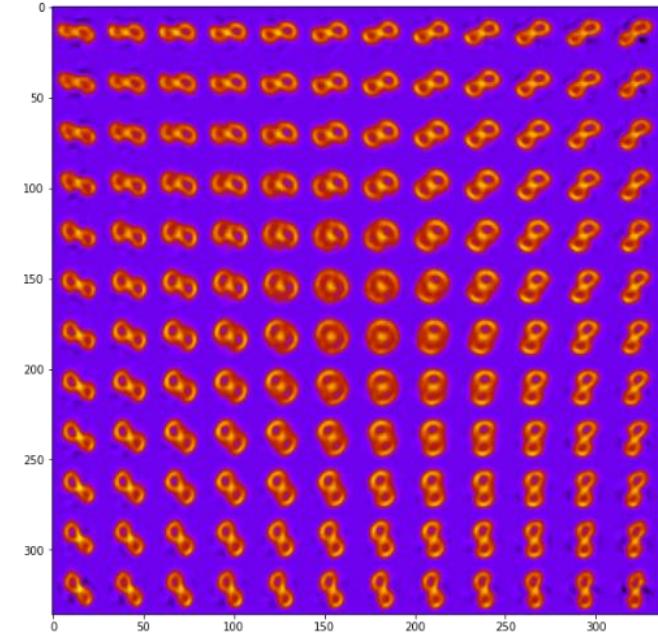
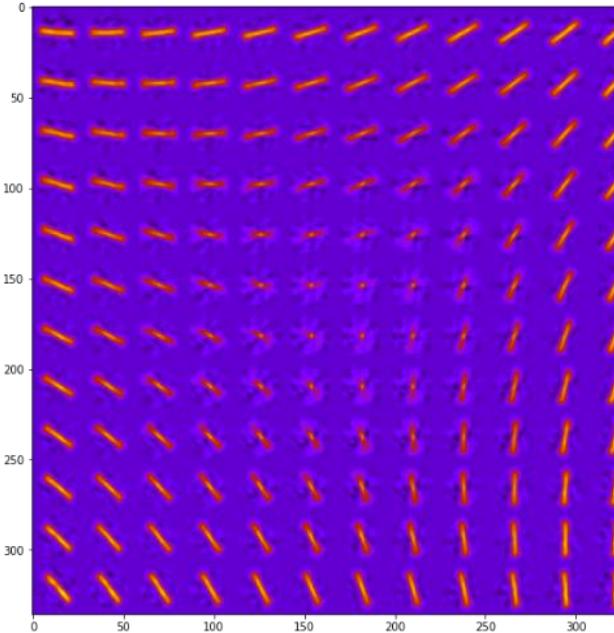
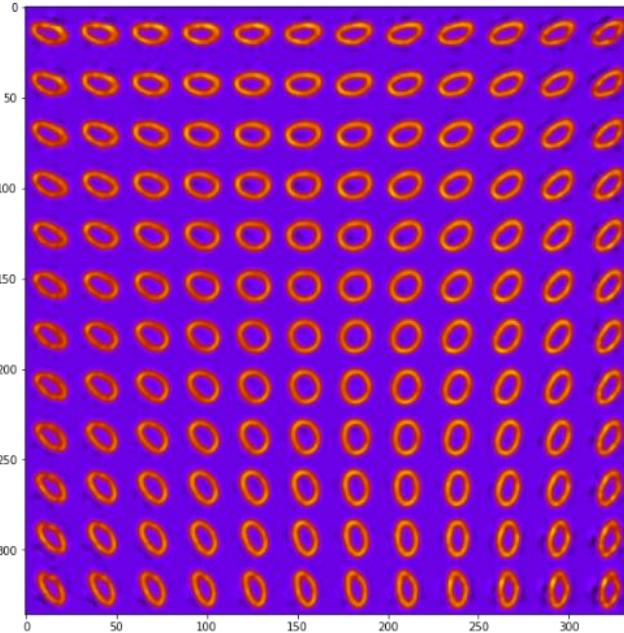
*ELBO*

$$\begin{aligned} &= \text{Reconstruction Loss} \\ &- D_{KL}(q(z|x)\|\mathcal{N}(0,I)) \\ &- D_{KL}(q(\gamma|x)\|\mathcal{N}(0,s_\gamma^2)) \\ &- D_{KL}(q(\Delta r|x)\|\mathcal{N}(0,s_{\Delta r}^2)) \quad \text{Regular VAE} \\ &+ D_{KL}(\text{physics-based "priors"}) ? \\ &+ D \quad (\text{physics}) ? \end{aligned}$$

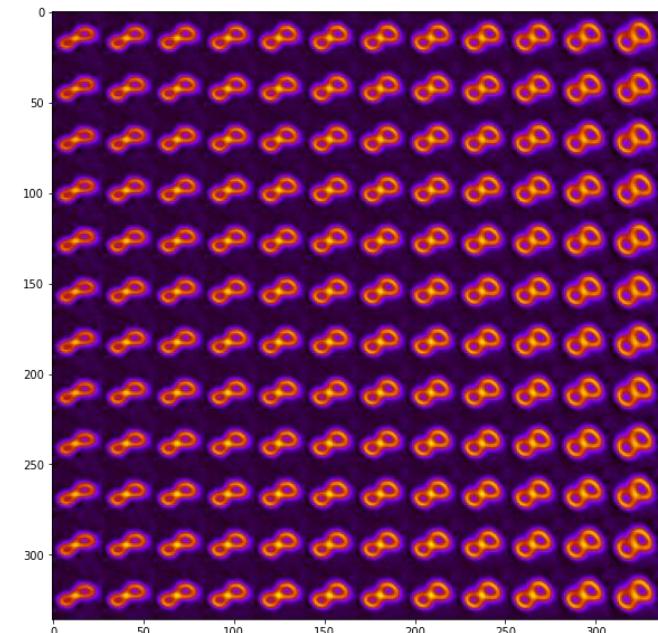
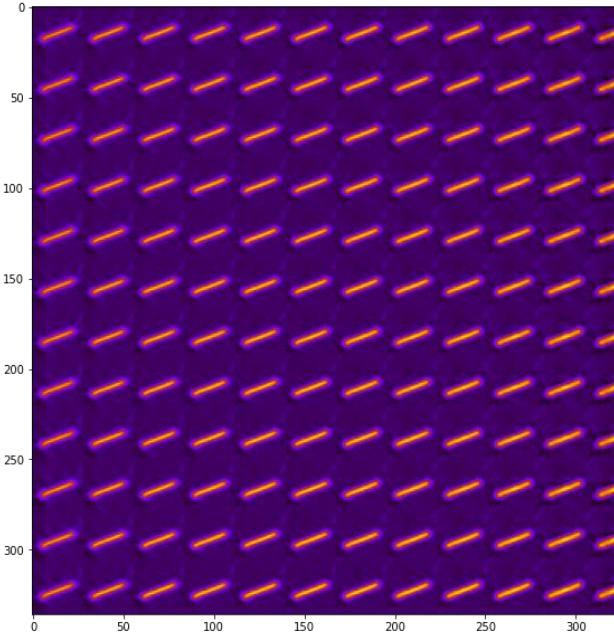
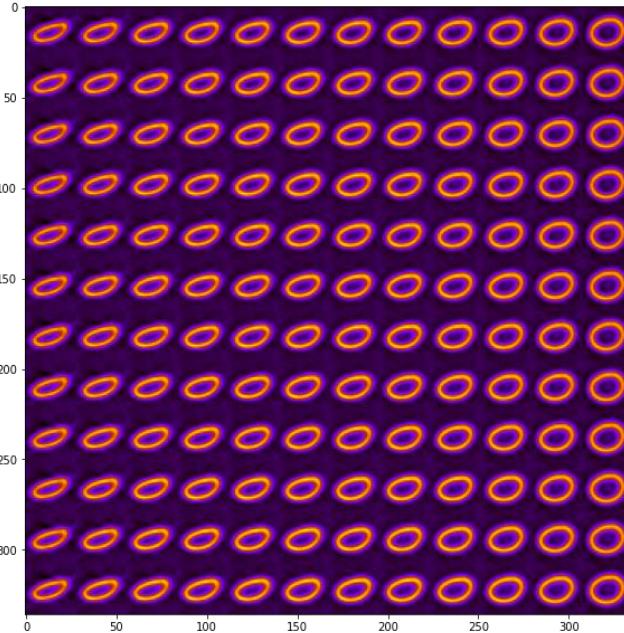
Rotation  
Translation

# MNIST cVAE

No rotations

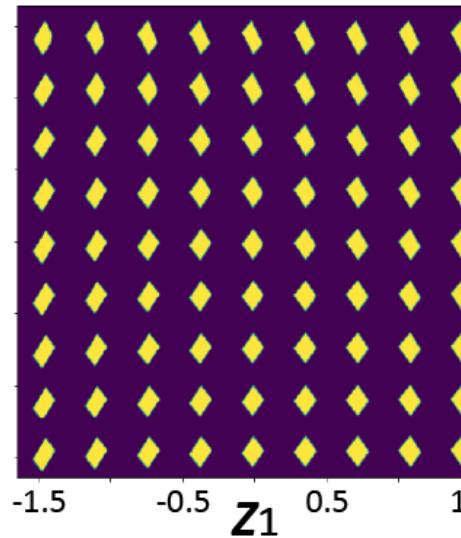
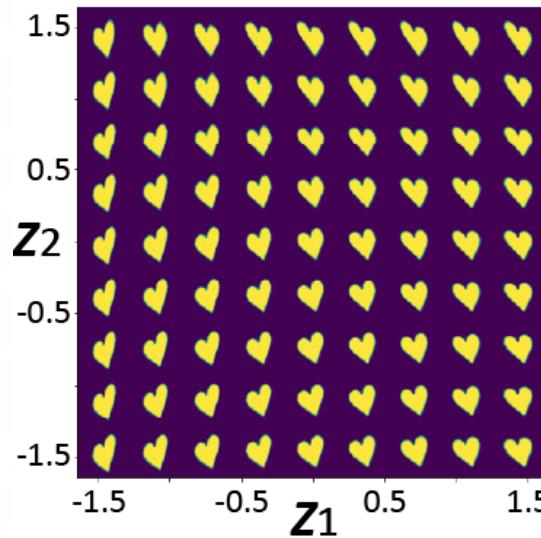
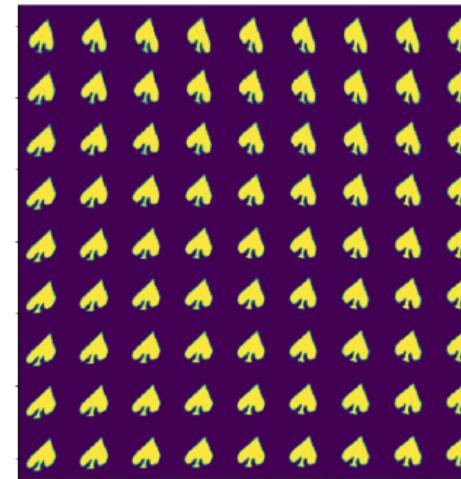
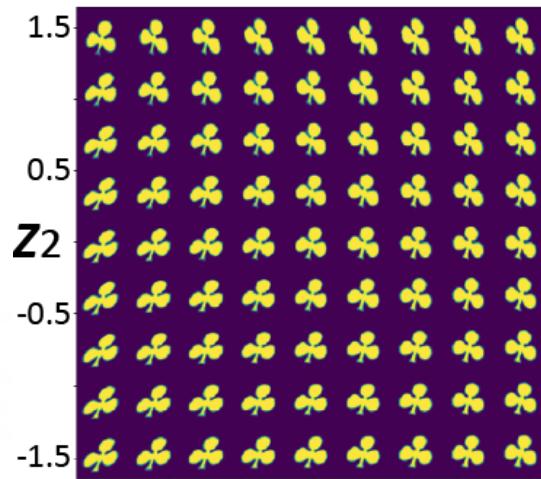


With rotations

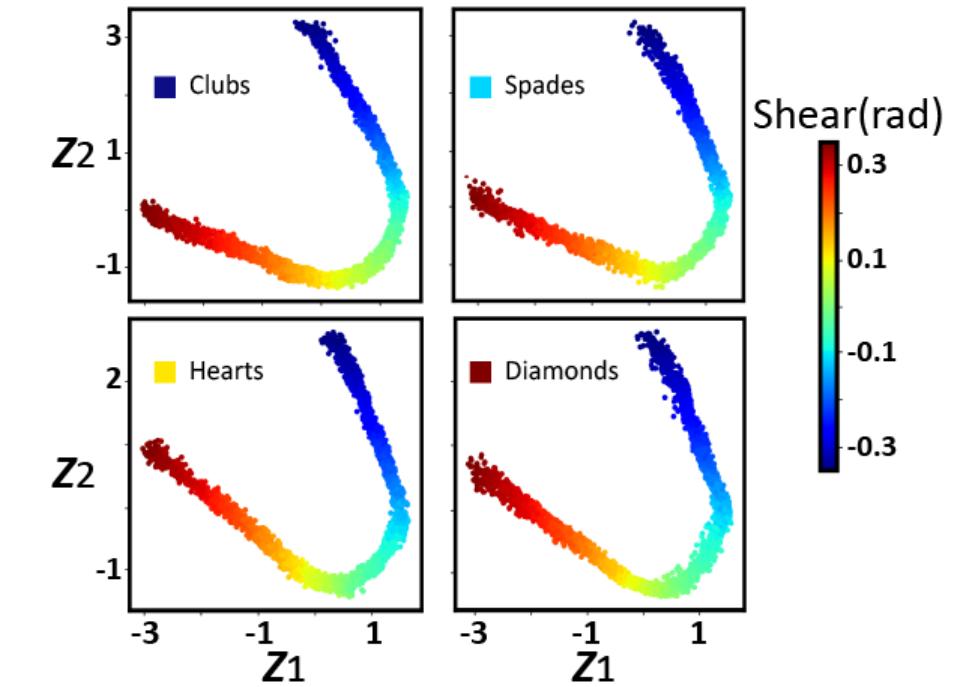


# cVAE on cards data set

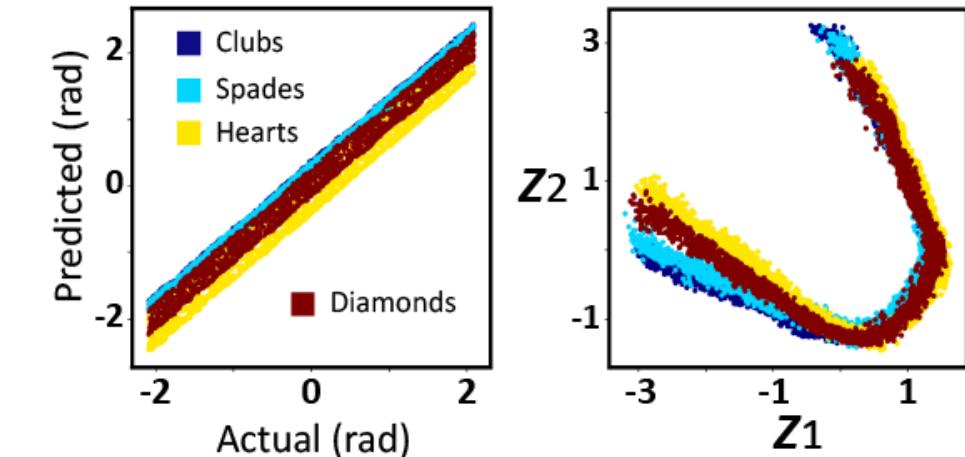
One independent latent space per class



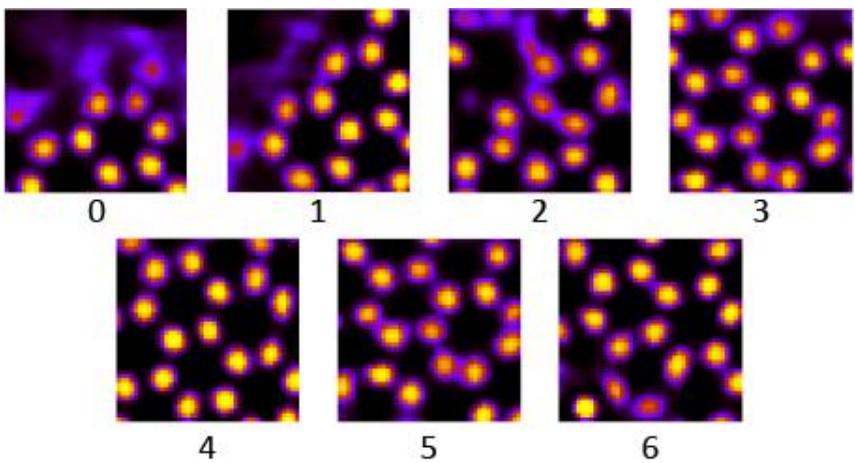
Shear distribution in latent space



All four plotted jointly



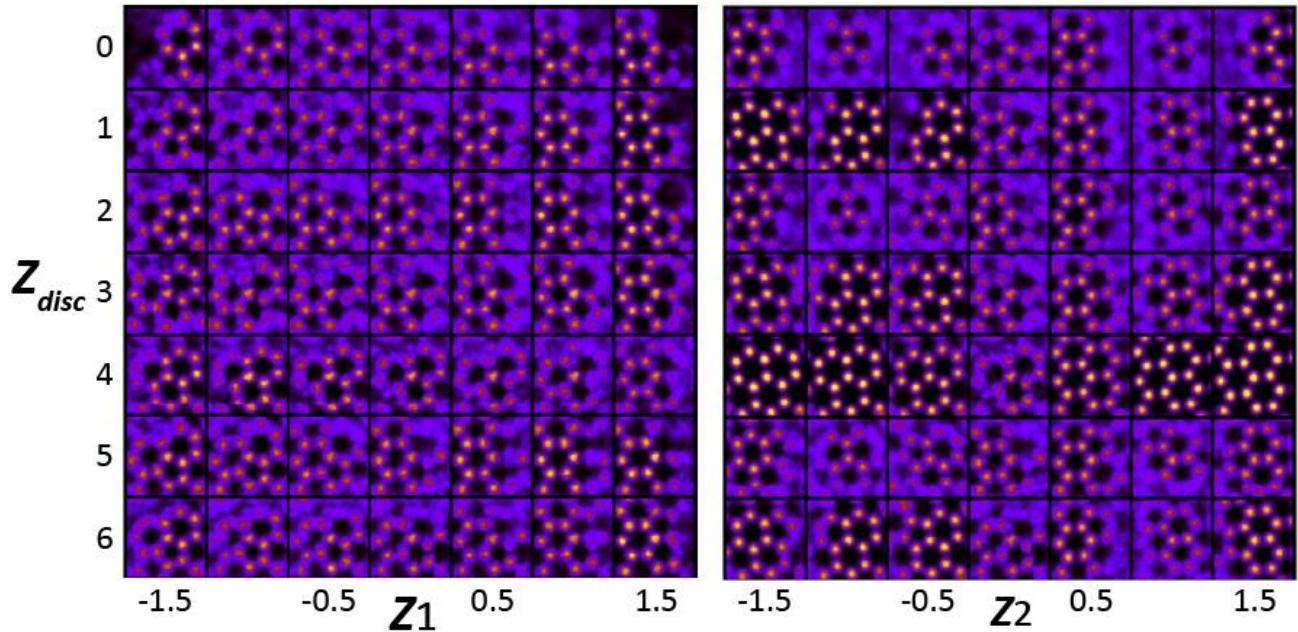
# cVAE on Graphene



## Autonomous sub-image labeling

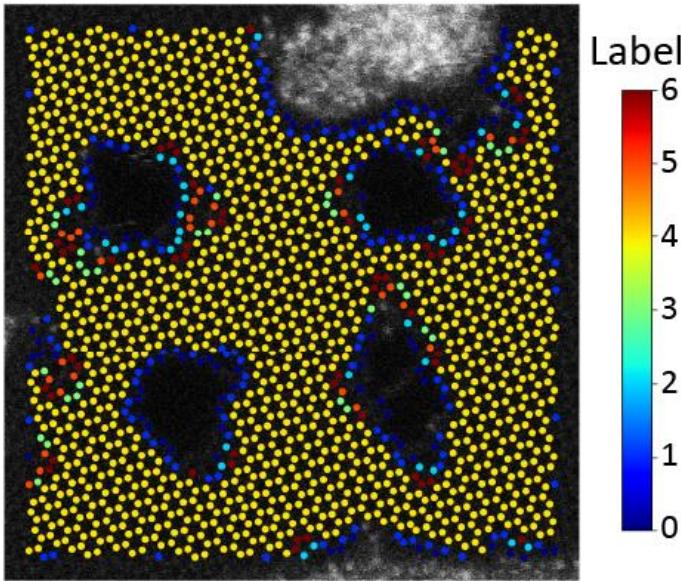
- 0: Fewer than 2 rings
- 1: two rings of size [6, 6]
- 2: three rings of size [5, 6]
- 3: three rings of size [6, 6, 7]
- 4: three rings of size [6, 6, 6]
- 5: three rings of size [5, 6, 7]
- 6: three rings of size [5, 6, 6]

## Traversal manifolds

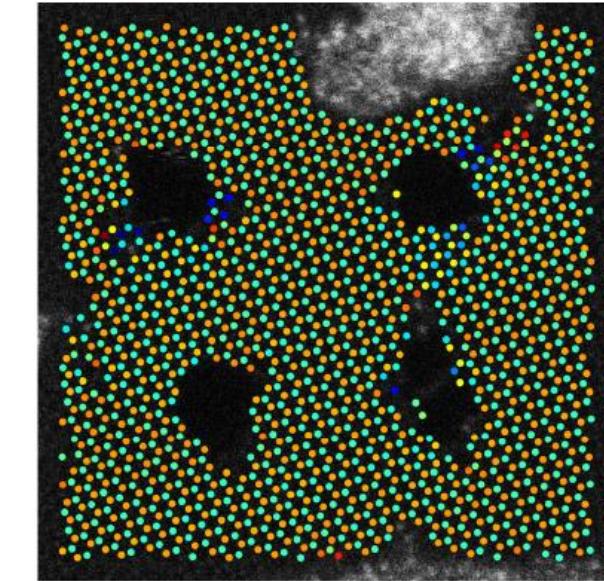
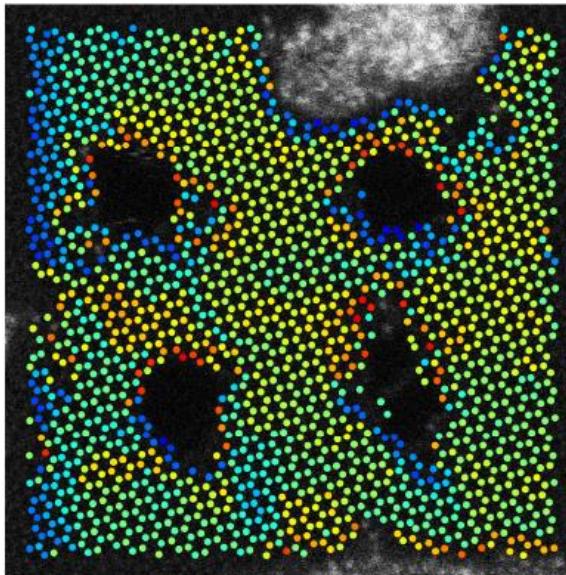


# cVAE on graphene

Class labels

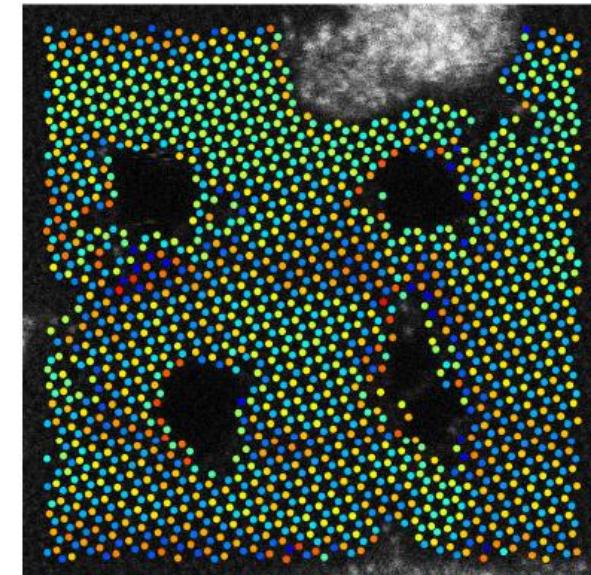


$z_1$



Encoded Angle

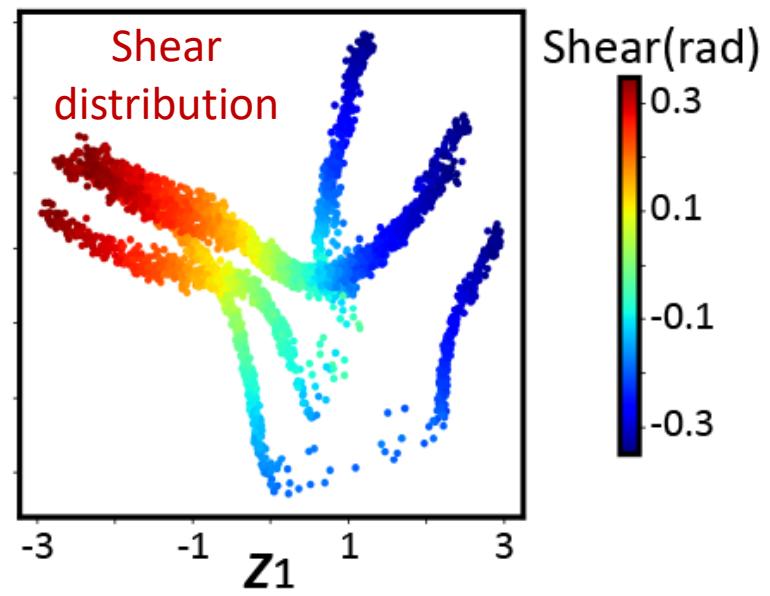
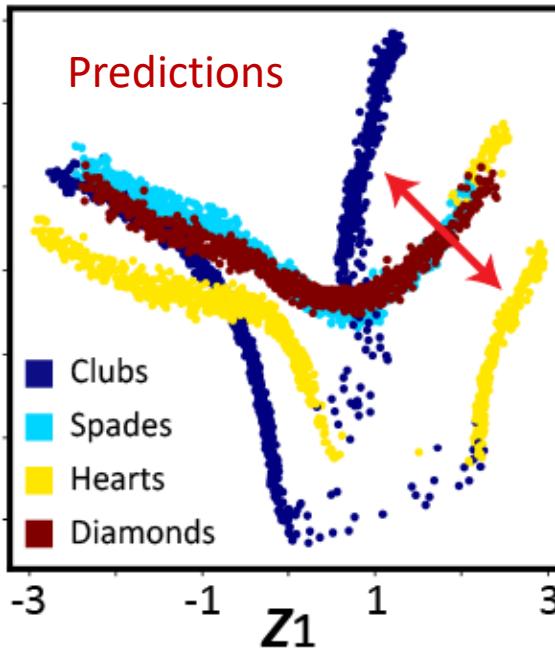
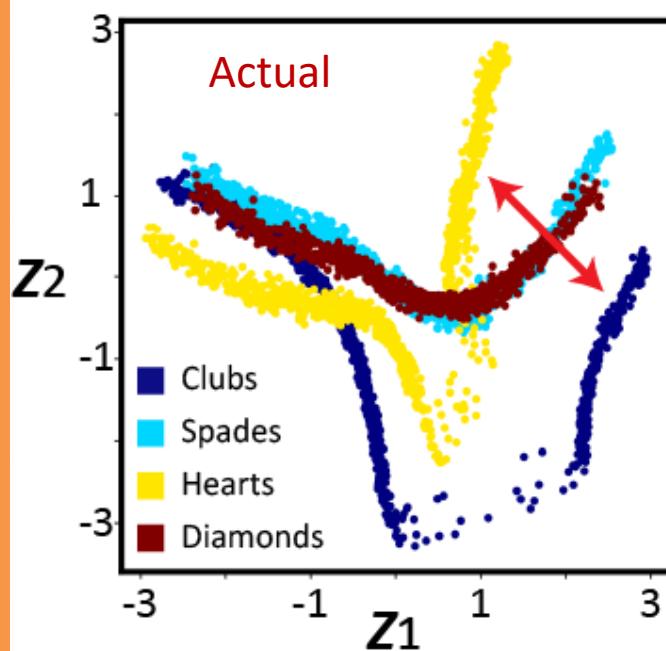
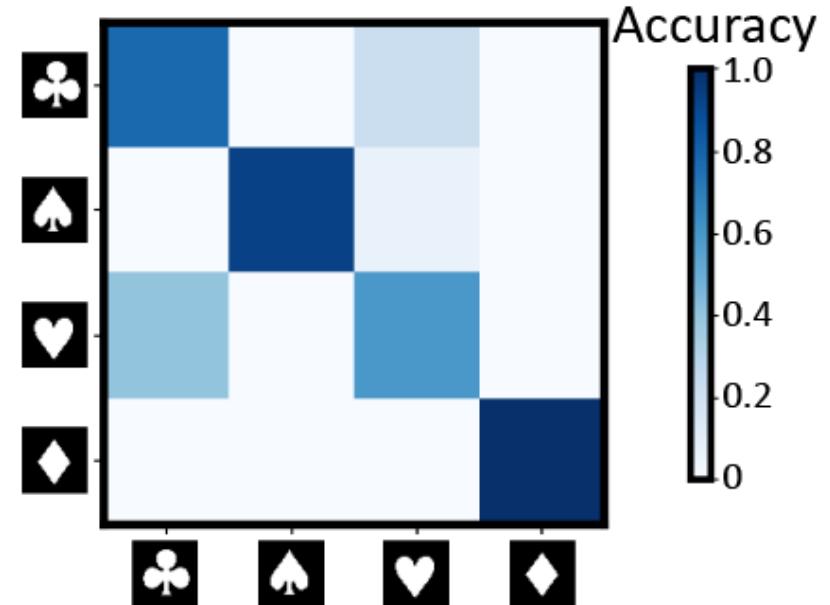
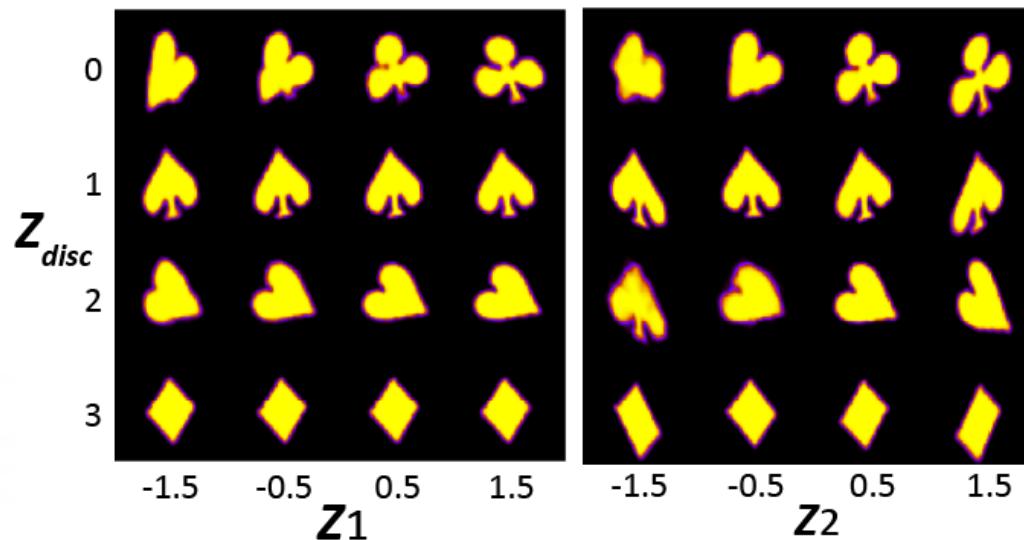
$z_2$



Correlation  
Hypothesis: Abundance  
of latent variables

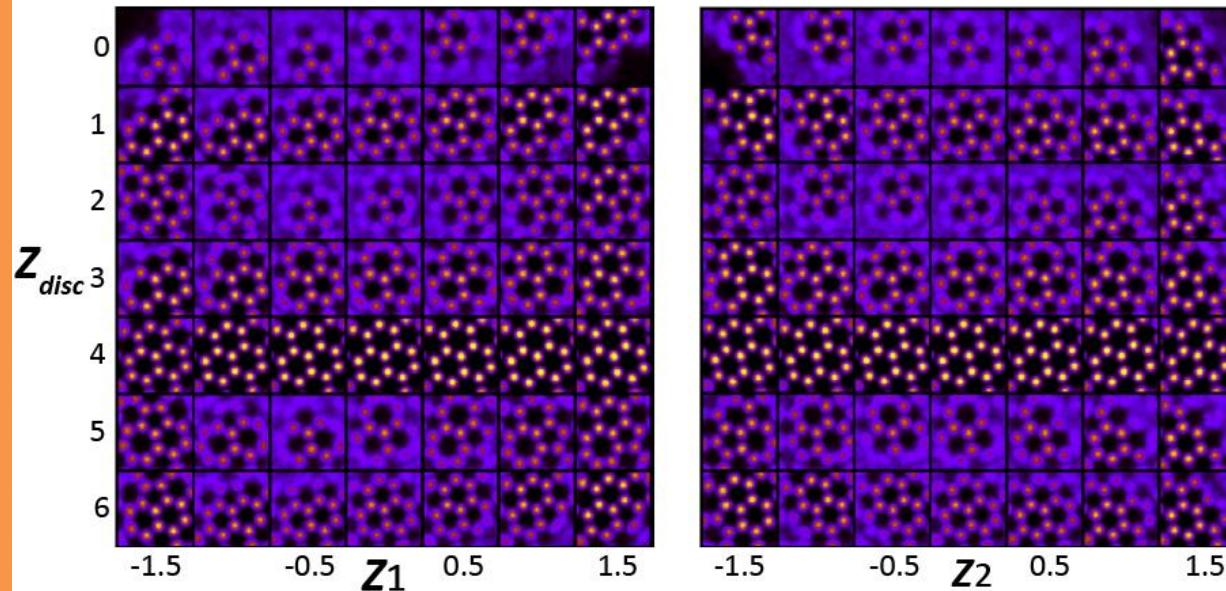
# Semi-supervised VAE

Traversal Manifold

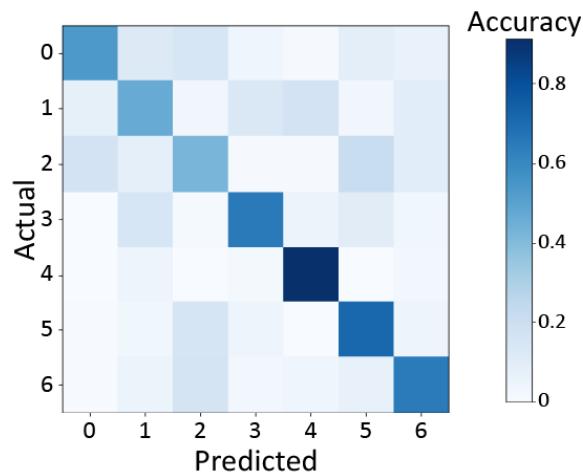


# Semi-supervised VAE (graphene)

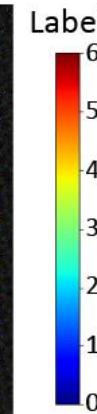
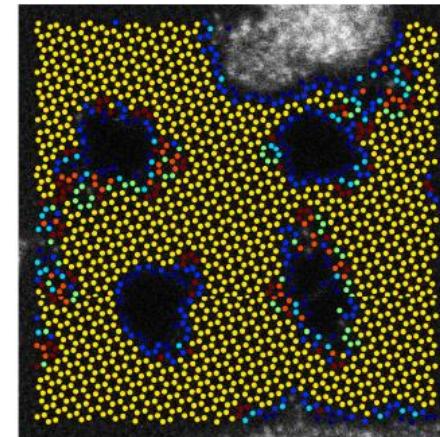
Traverse manifolds



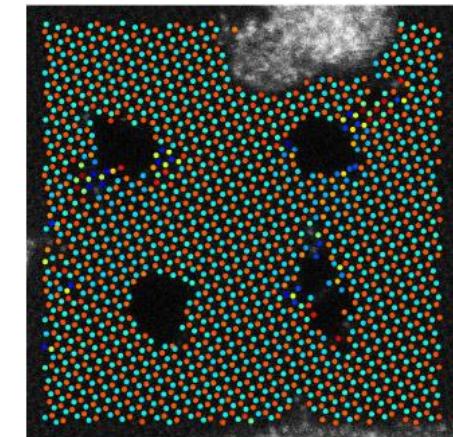
Confusion matrix



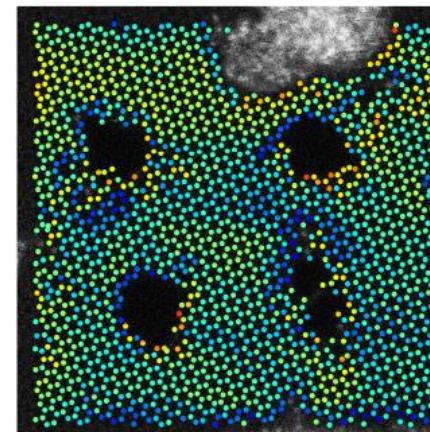
Class labels



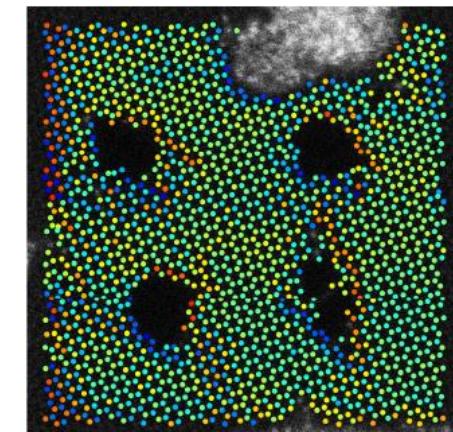
Angle Encoding



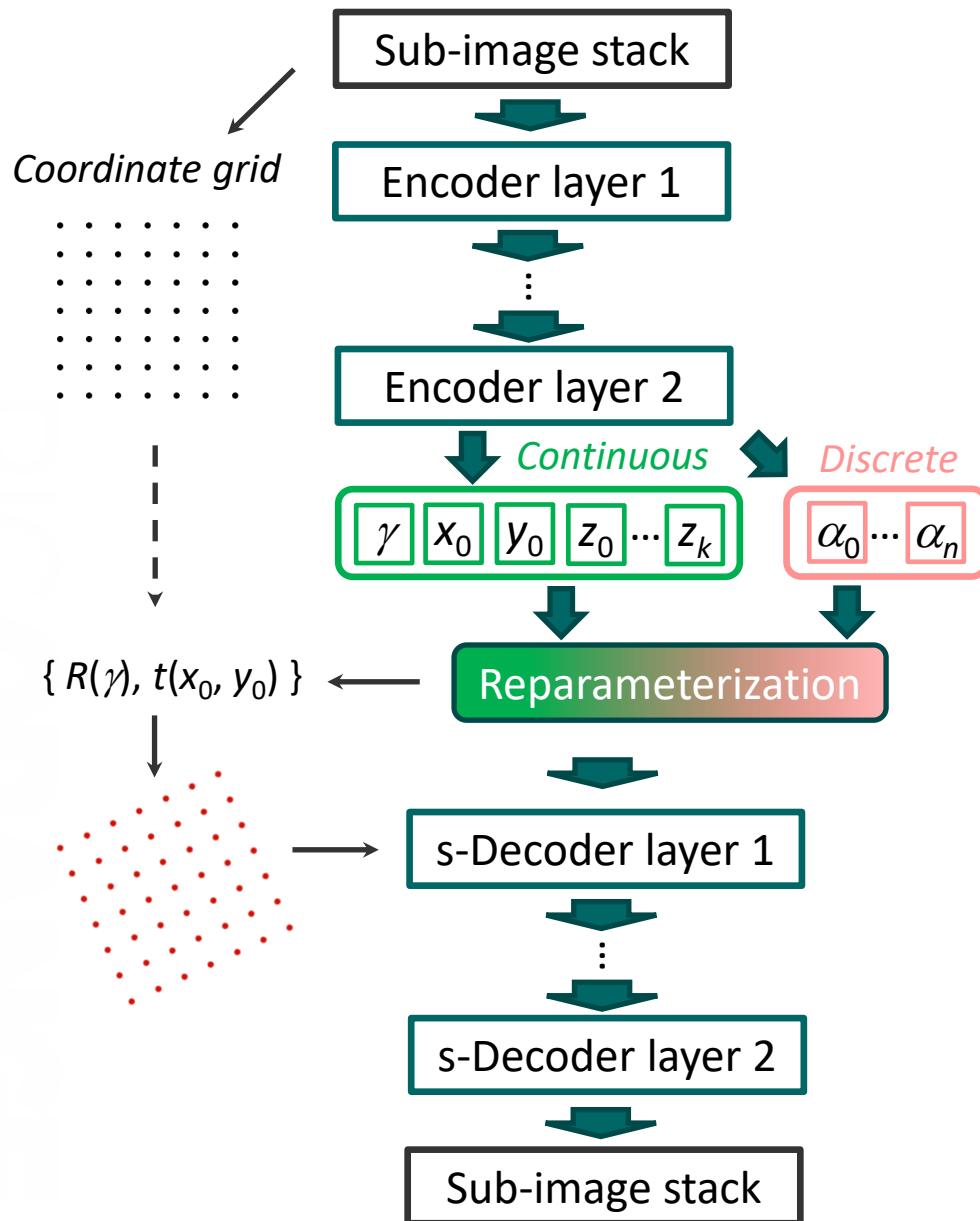
$Z_1$



$Z_2$



# Joint VAE

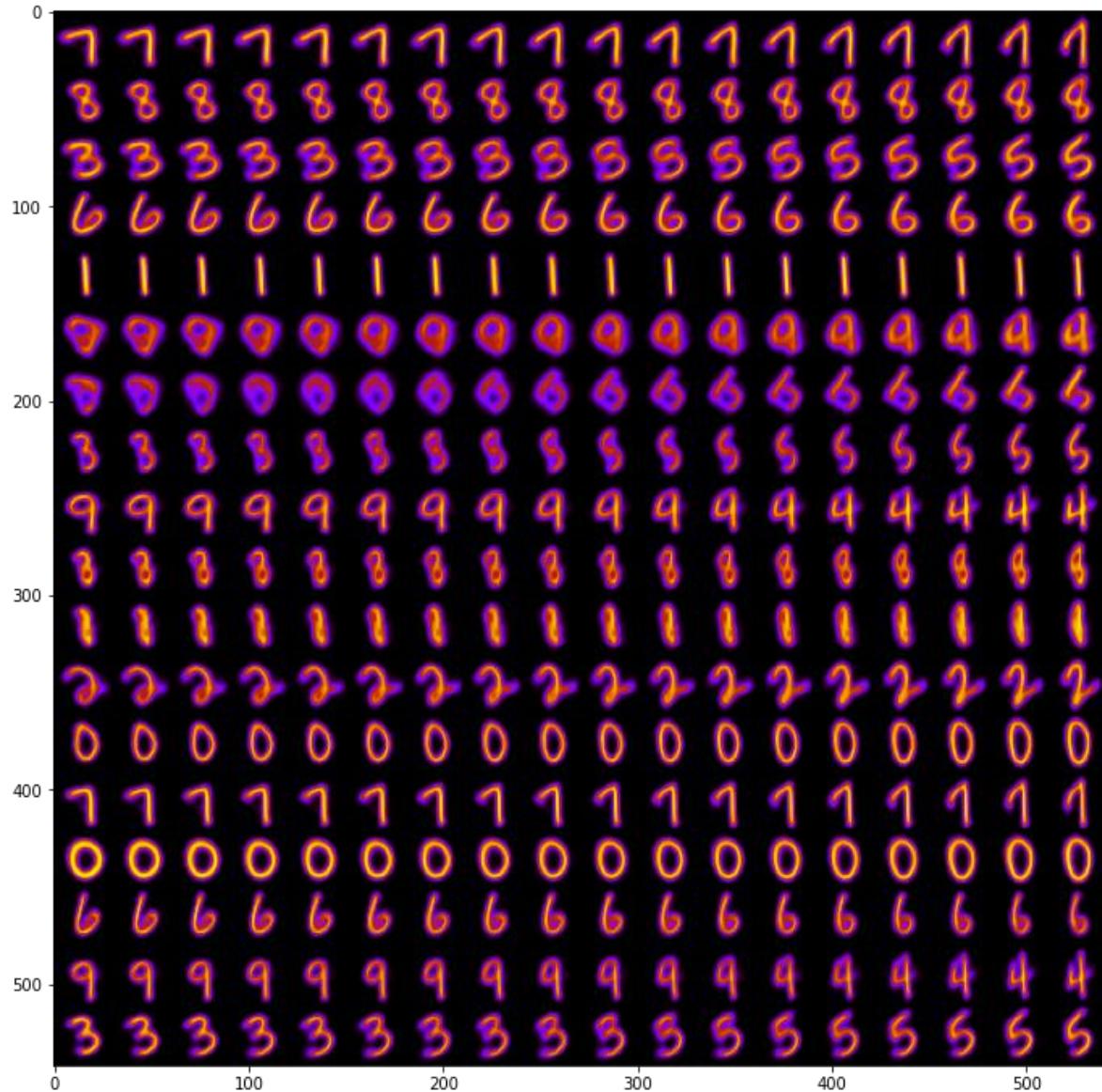
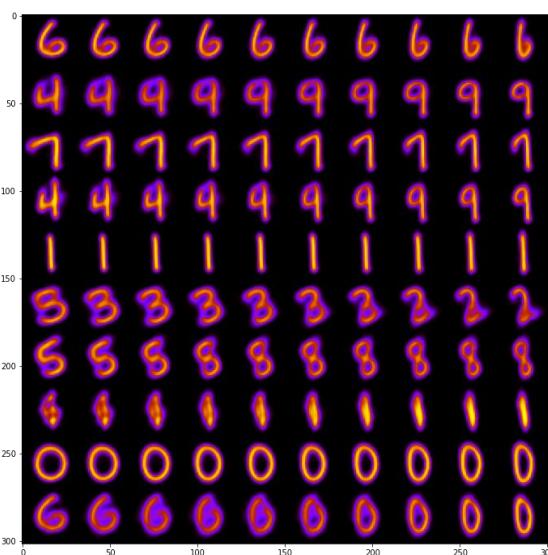
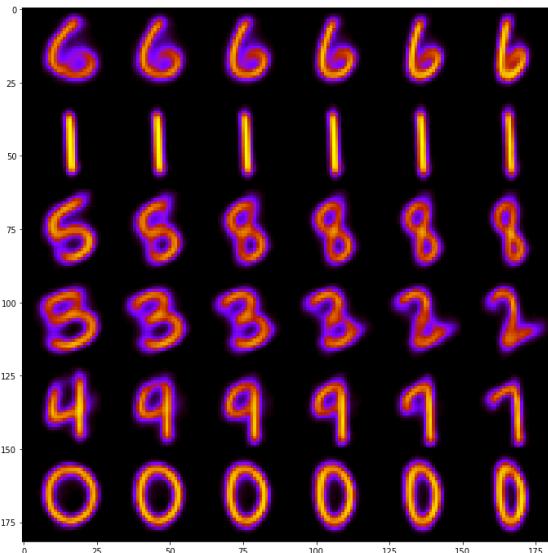


- Generative model is a function of spatial coordinate (e.g., via spatial broadcasting)
- 3 additional latent variables to absorb rotations and shifts
- Disentangles rotations and translations from image content
- Learns discrete classes in unsupervised fashion
- Well-suited for analyzing microscopy (sub-)images on atomic and molecular levels

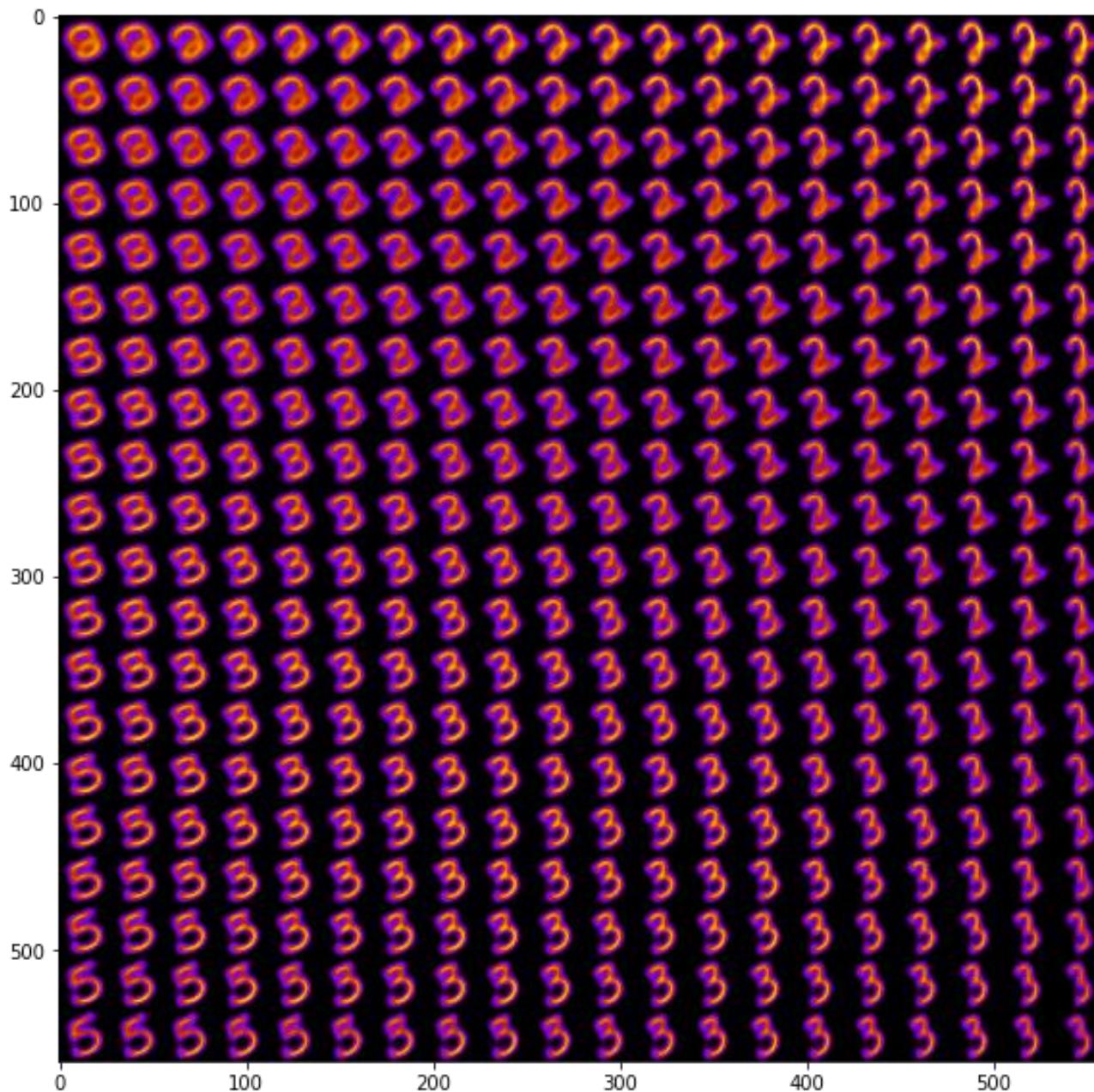
*ELBO =*

$$\begin{aligned} & - \text{Reconstruction Loss} \\ & - \beta_c(t) |(D_{KL}(q(z|x) \parallel p(z)) + D_{KL}(q(\gamma|x) \parallel p(\gamma)) - C_z| \quad \text{Continuous} \\ & - \beta_d(t) |D_{KL}(q(\alpha|x) \parallel p(\alpha)) - C_\alpha| \quad \text{Discrete} \\ & + \text{physics-based "loss" ?} \end{aligned}$$

# jVAE on MNIST

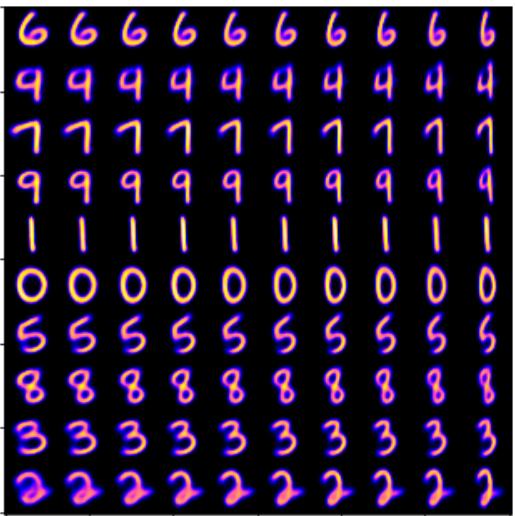


# Latent representations



# Ensemble jVAE

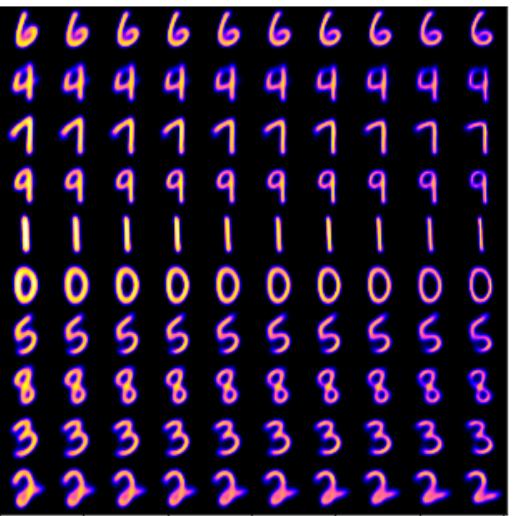
*Predictions from different ensemble models*



Baseline: 10 epochs  
Ensemble models: 8

- The unstable classes show the largest “uncertainty”
- Indication of the quality of separation and/or a guide for selection of the number of classes

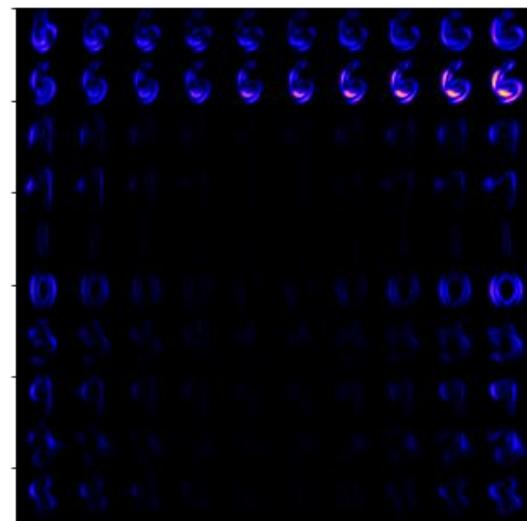
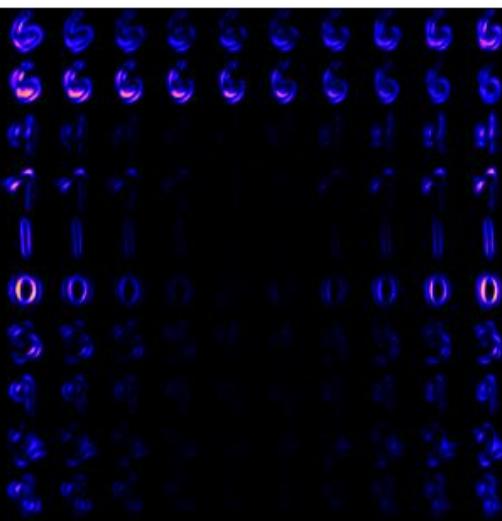
Uncertainty



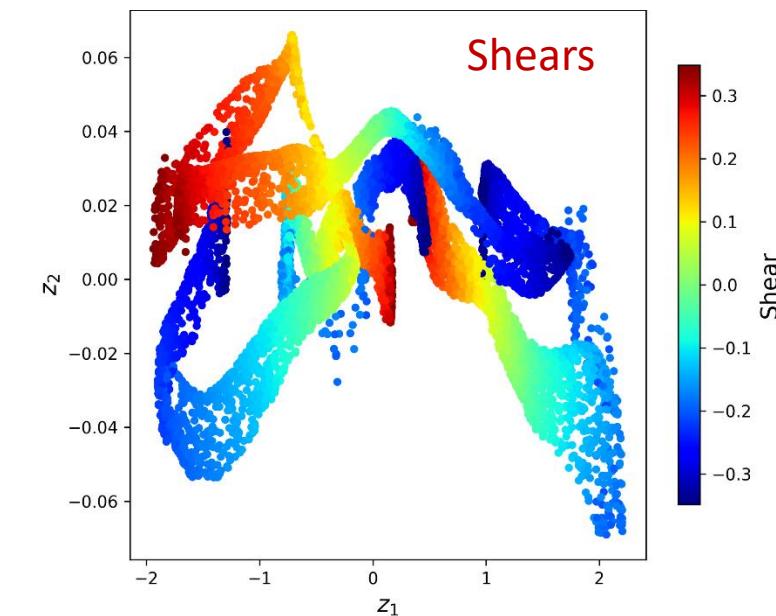
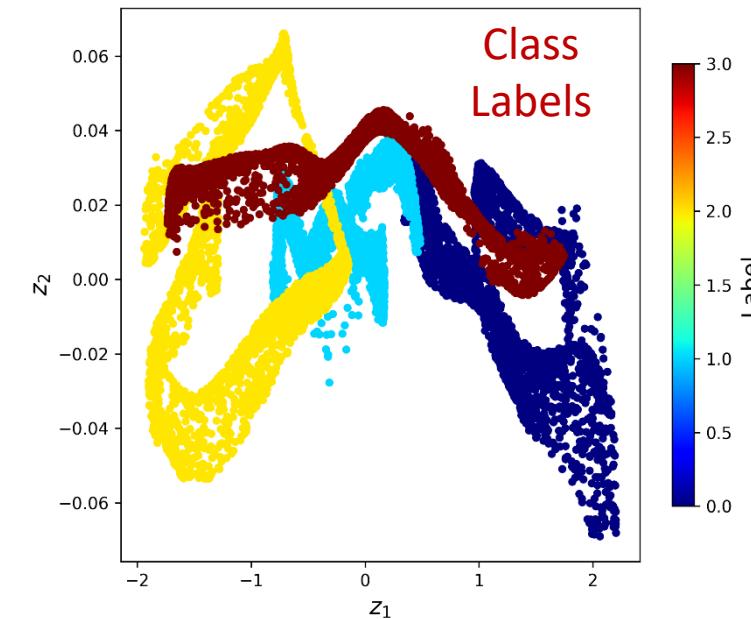
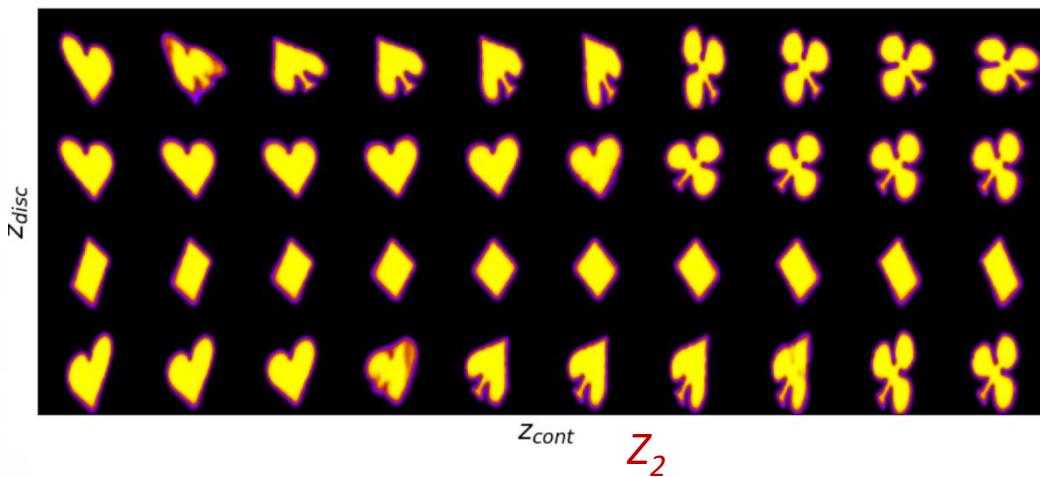
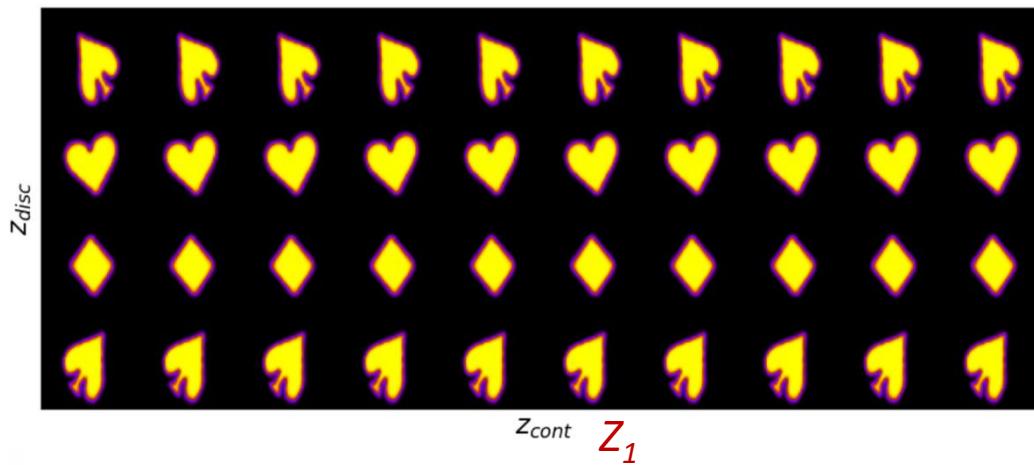
*Mean prediction*



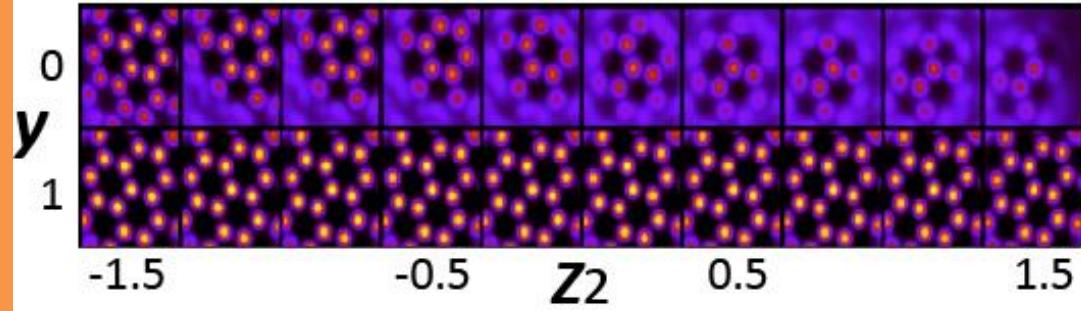
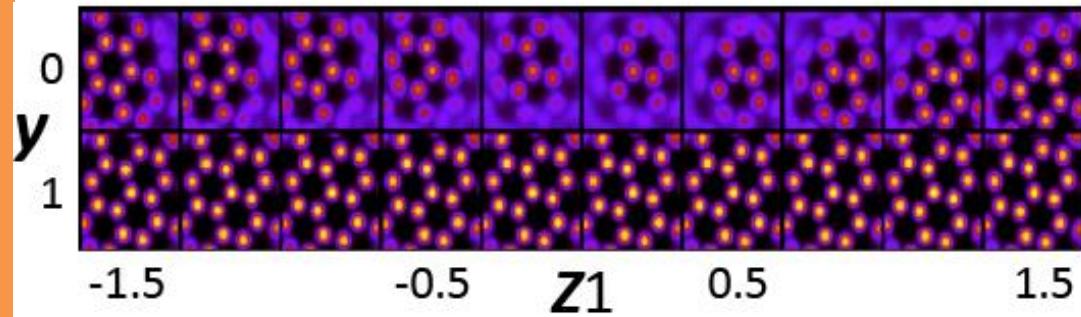
*Dispersion in predictions ('uncertainty')*



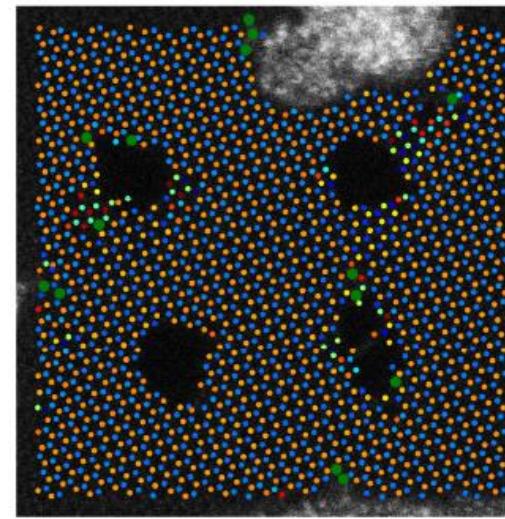
# jVAE on cards



# jVAE on graphene

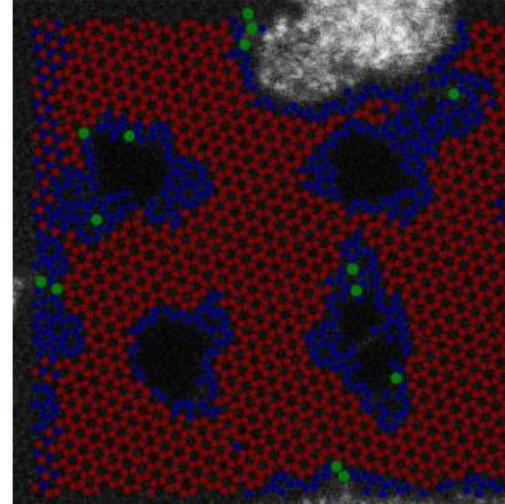


Angle encoding



$z_1$

Class labels



$z_1$

