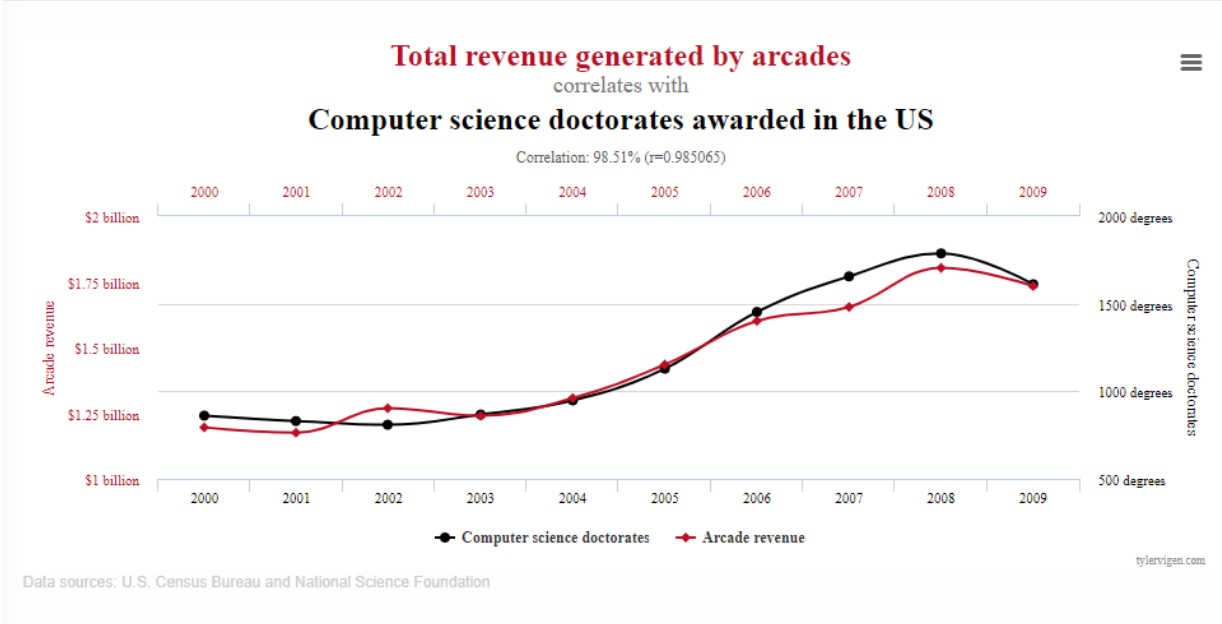
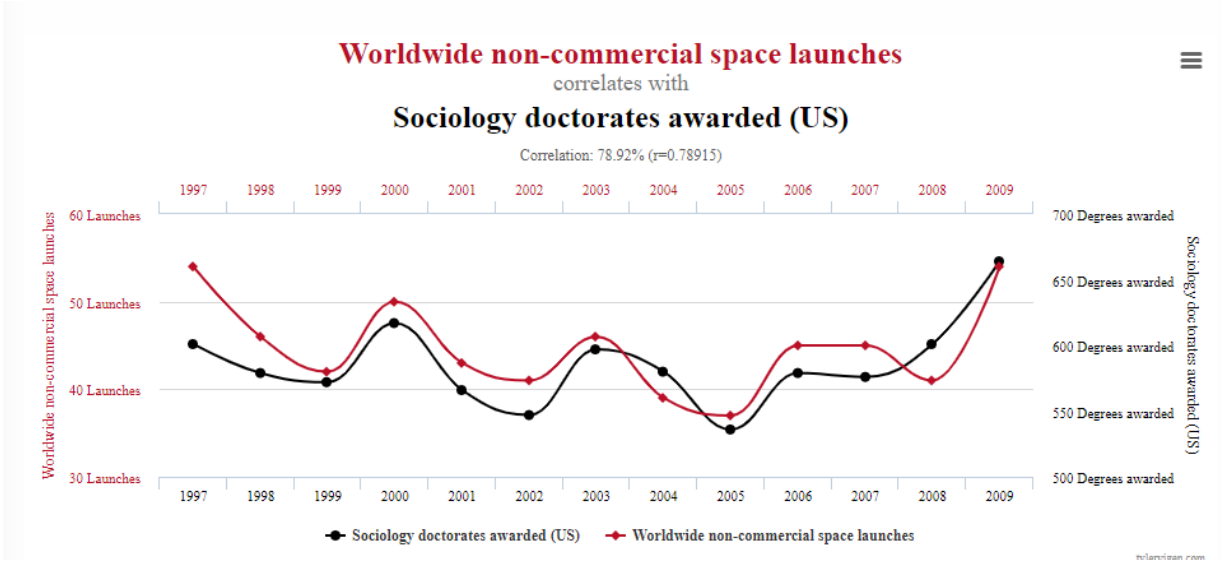
THE UNIVERSITY of TENNESSEE KNOXVILLE

Lecture 34: Causality

Sergei V. Kalinin

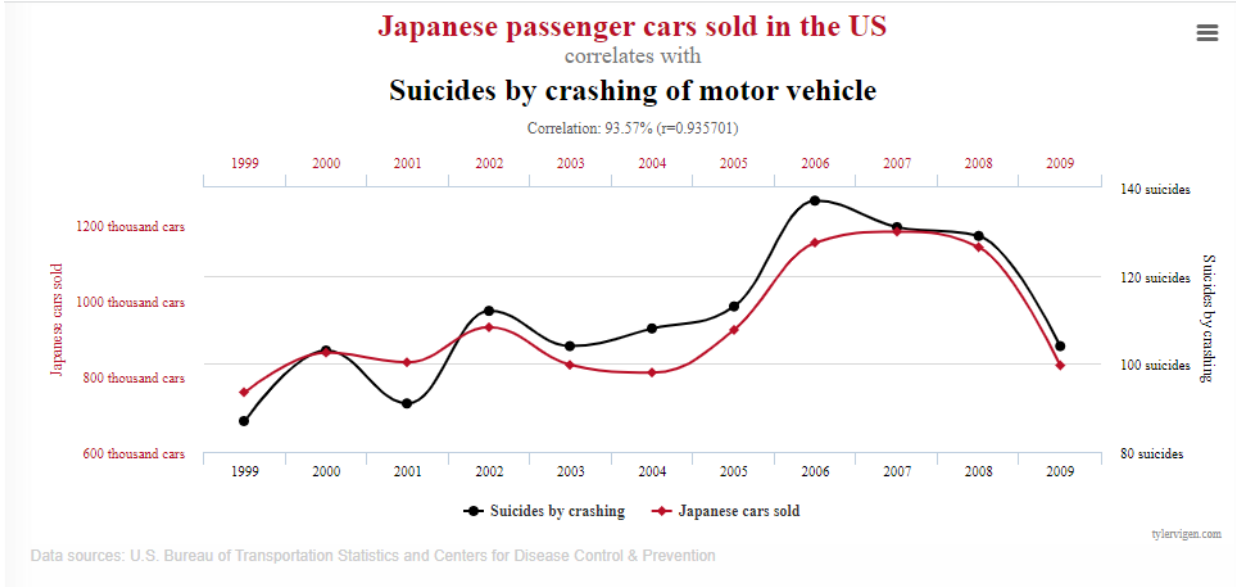
Correlation and causation

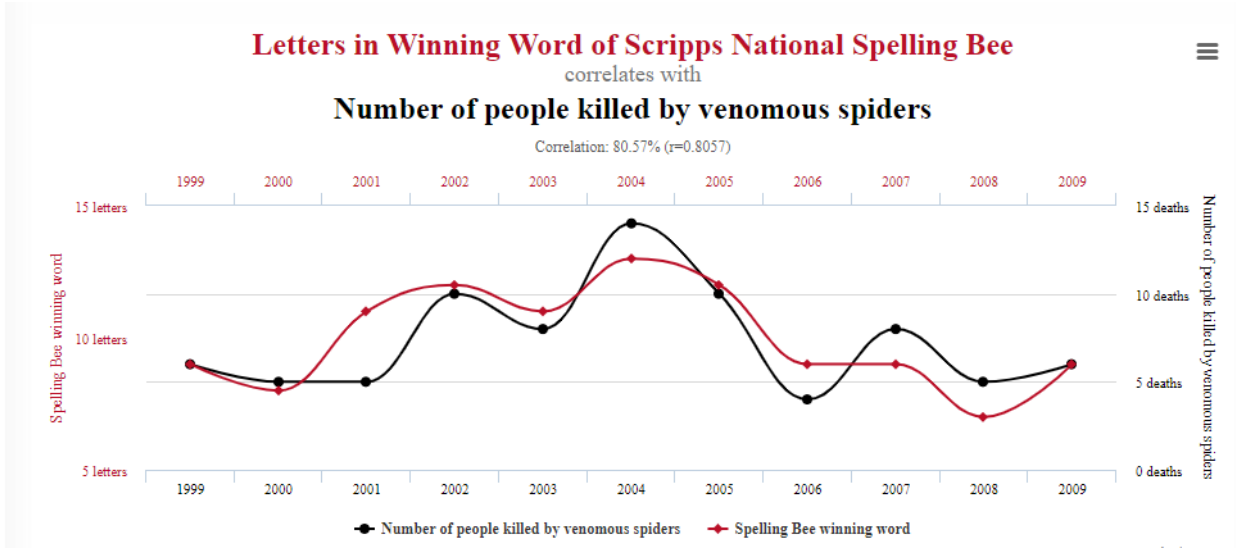




https://www.tylervigen.com/spurious-correlations

Correlation and causation





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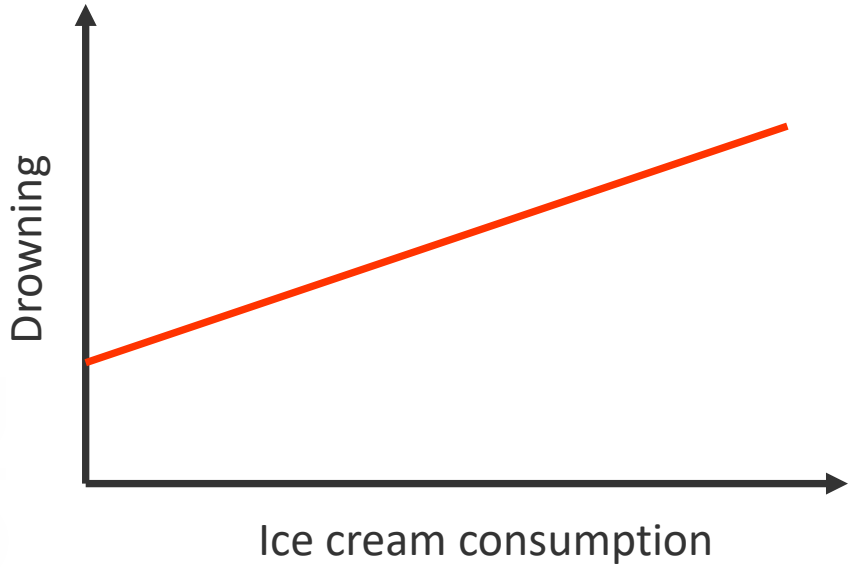
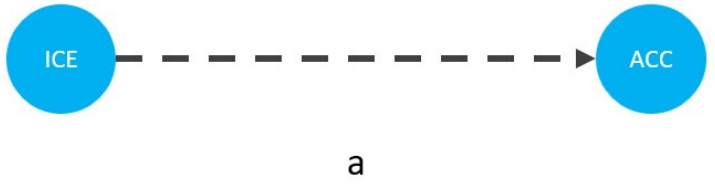
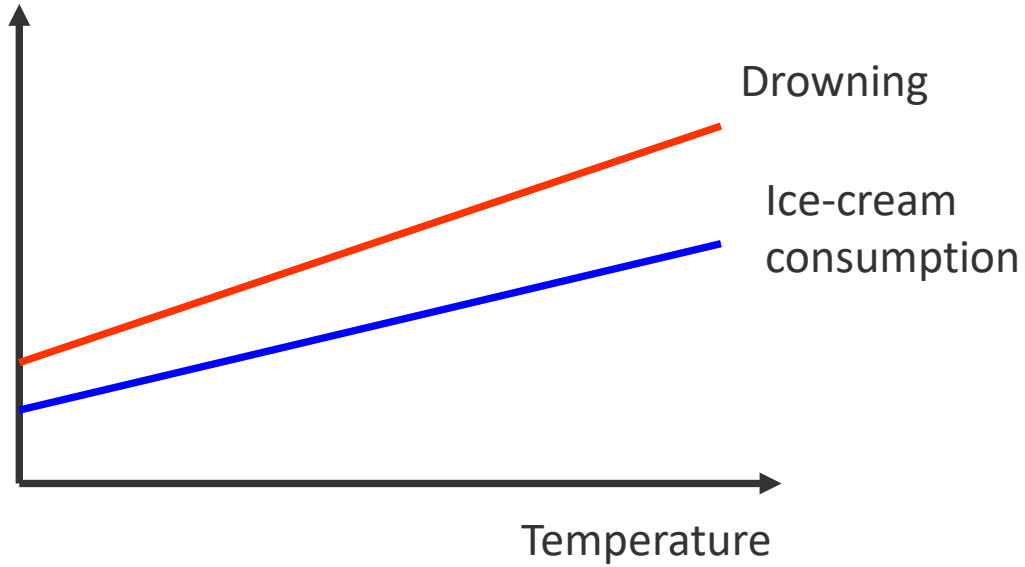
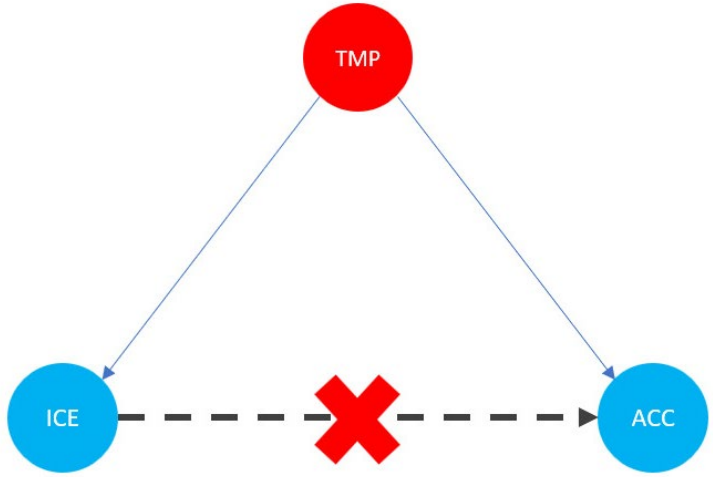
Ice-cream and drowning





a





b

From A. Molak, Causal Inference and Discovery in Python

Treatment effects

$$\tau_i = Y_i(1) - Y_i(0)$$

- τ_i is the treatment effect for person i
- $Y_i(1)$ is the outcome for person i when they received the treatment T
- $Y_i(0)$ is the outcome for the same person i given they did not receive the treatment

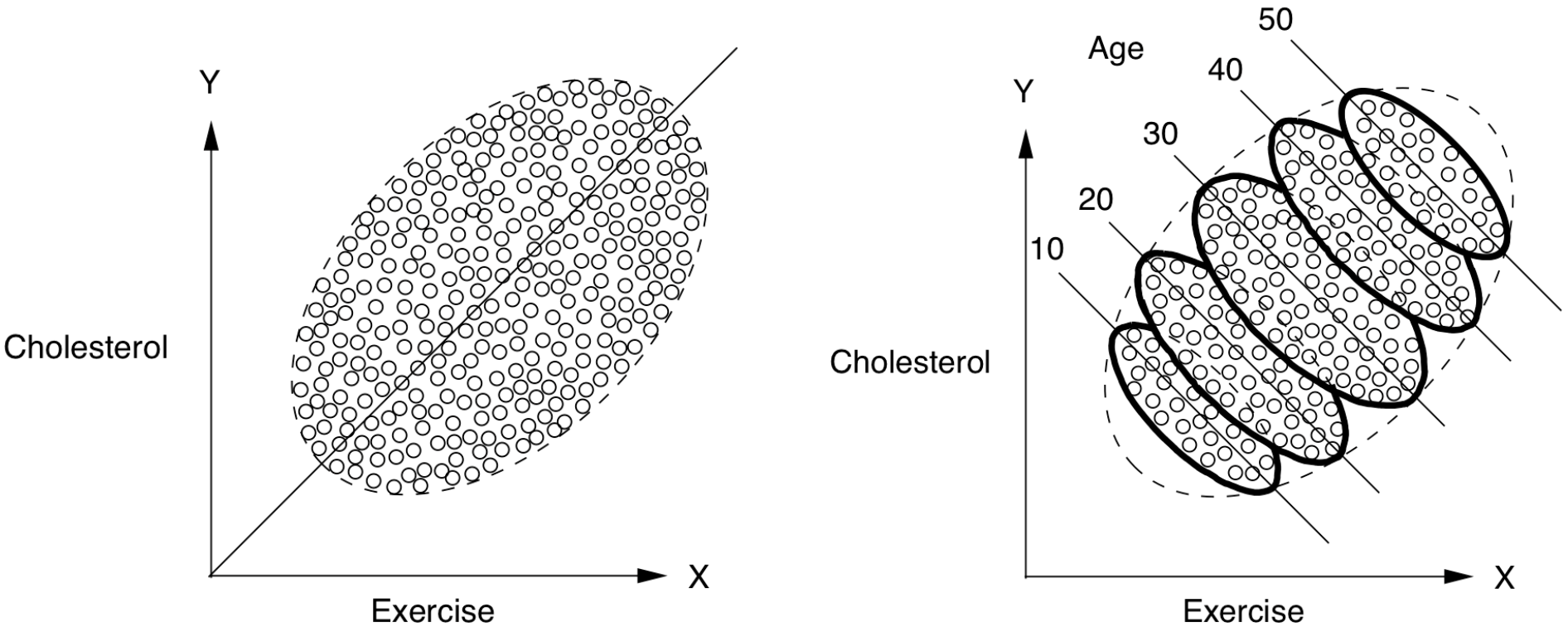
Very fundamental approach in:

- Marketing
- Medicine
- And so on

But… How can the same person be treated and not treated?

From A. Molak, Causal Inference and Discovery in Python

Simpson paradox



Exercise is helpful in every age group but harmful for a typical person.

Is exercise helpful or not?

From J. Pearl presentation, Why - 19

Simpson paradox

Drug	A		B	
Blood clot	Yes	No	Yes	No
Total	27	95	23	99
Percentage	22%	78%	19%	81%

Drug	A		B	
Blood clot	Yes	No	Yes	No
Female	24	56	17	25
Male	3	39	6	74
Total	27	95	23	99
Percentage	22%	78%	18%	82%
Percentage (F)	30%	70%	40%	60%
Percentage (M)	7%	93%	7.5%	92.5%

- Simpson's paradox appears when data partitioning (which we can achieve by controlling for the additional variable(s) in the regression setting) significantly changes the outcome of the analysis.
- In the real world, there are usually many ways to partition your data.
- You might ask: okay, so how do I know which partitioning is the *correct* one?

From A. Molak, Causal Inference and Discovery in Python

Berkeley discrimination lawsuit

In the early 1970s, the University of California, Berkeley was sued for gender discrimination over admission to graduate school. Of the 8,442 male applicants for the fall of 1973, 44 percent were admitted, but only 35 percent of the 4,351 female applicants were accepted

Table 1: Data From Six Largest Departments of 1973 Berkeley Discrimination Case

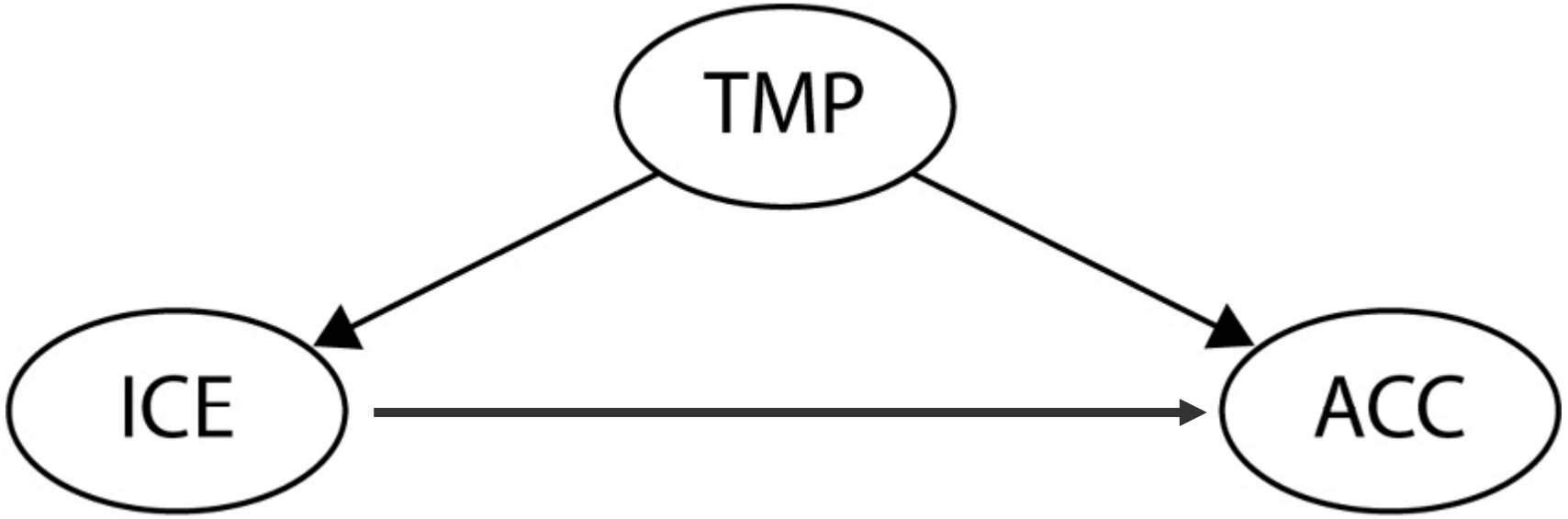
Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	82%
B	560	63%	25	68%
C	325	37%	593	34%
D	417	33%	375	35%
E	191	28%	393	24%
F	272	6%	341	7%

Source: Bickel, Hammel, and O'Connell (1975); table accessed via Wikipedia at https://en.wikipedia.org/wiki/Simpson%27s_paradox.

In the Berkeley case, the "paradox" occurred because women disproportionately applied to departments with low acceptance rates, as shown in the table above, while men disproportionately applied to departments with high acceptance rates.

https://www.brookings.edu/articles/when-average-isnt-good-enough-simpsons-paradox-in-education-and-earnings/

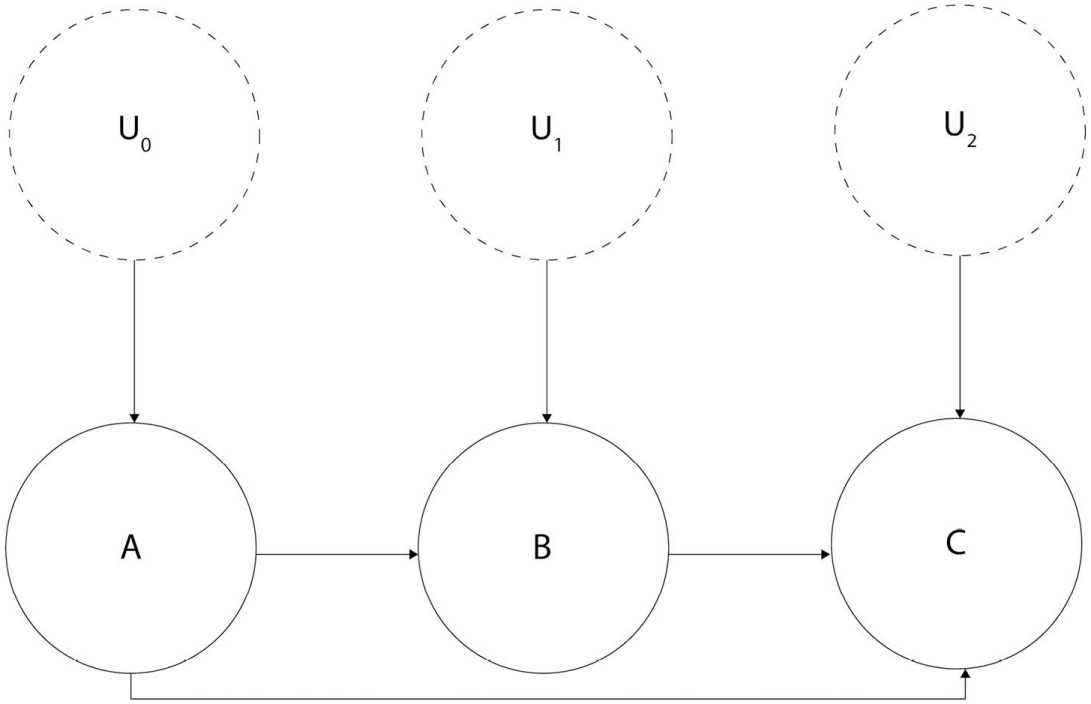
How can we even approach such problems?



- Observations give us correlations between temperature, ice cream consumption, and accident rate
- What we need to know is the causal links between these characteristics. Does change in ice cream consumption affect temperature or accident rate?
- But we cannot make an experiment!

From A. Molak, Causal Inference and Discovery in Python

Causal graphs



$$A := f_A(U_0)$$

$$B := f_B(A, U_1)$$

$$C := f_C(A, B, U_2)$$

- Here, := is an **assignment operator**, also known as a **walrus operator**. We use it to emphasize that the relationship that we're describing is *directional* (or asymmetric), as opposed to the regular equal sign that suggests a symmetric relation.

- And f_A, f_B, f_C represent arbitrary functions (they can be as simple as a summation or as complex as you want).

From A. Molak, Causal Inference and Discovery in Python

Do-operator

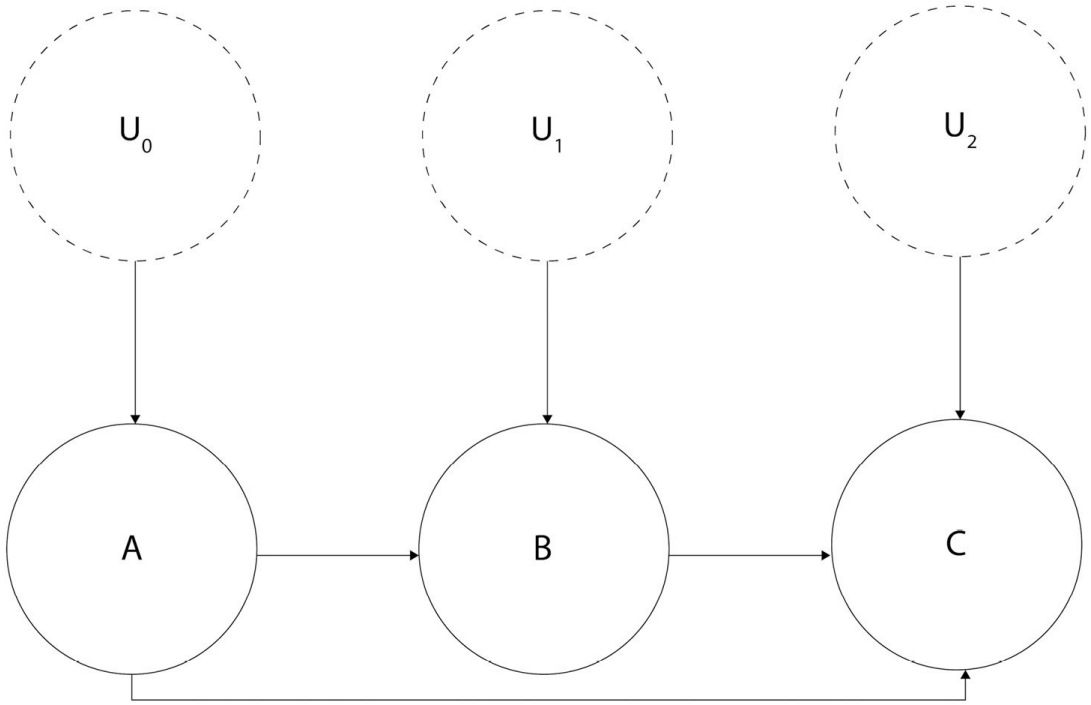
Conditioning: $P(X = x | Y = y)$

Intervention: $P(Y = 1 | do(X = 0))$

- Conditioning only modifies our *view* of the data, while intervening affects the distribution by *actively* setting one (or more) variable(s) to a *fixed value* (or a distribution).
- This is very important – intervention *changes* the system, but conditioning *does not*.
- You might ask, what does it mean that *intervention changes the system*? Great question!

From A. Molak, Causal Inference and Discovery in Python

Properties of do - operator



- The change in B will influence the values of its descendants
- B will become independent of its ancestors

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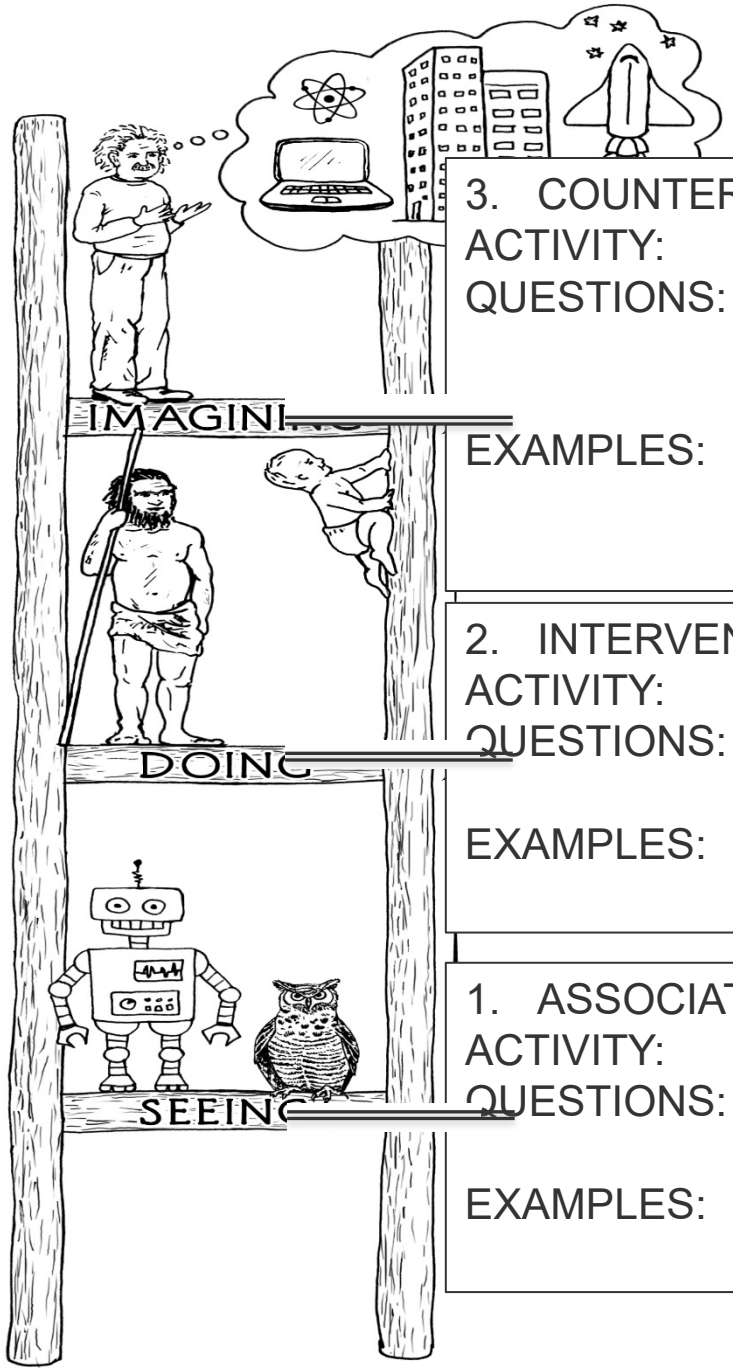
Ladder of causation



3. COUNTERFACTUALS
ACTIVITY: Imagining, Retrospection, Understanding
QUESTIONS: *What if I had done . . . ? Why?*
(Was it X that caused Y? What if X had not occurred? What if I had acted differently?)
EXAMPLES: Was it the aspirin that stopped my headache?
Would Kennedy be alive if Oswald had not killed him?
What if I had not smoked the last 2 years?

2. INTERVENTION
ACTIVITY: Doing, Intervening
QUESTIONS: *What if I do . . . ? How?*
(What would Y be if I do X?)
EXAMPLES: If I take aspirin, will my headache be cured?
What if we ban cigarettes?

1. ASSOCIATION
ACTIVITY: Seeing, Observing
QUESTIONS: *What if I see . . . ?*
(How would seeing X change my belief in Y?)
EXAMPLES: What does a symptom tell me about a disease?
What does a survey tell us about the election results?

From J. Pearl presentation, Why - 19

How can we learn causality

- **Causal discovery** and **causal structure learning** are umbrella terms for various kinds of methods used to uncover causal structure from observational or interventional data.

- **Expert knowledge** is a term covering various types of knowledge that can help define or disambiguate causal relations between two or more variables. Depending on the context, expert knowledge might refer to knowledge from randomized controlled trials, laws of physics, a broad scope of experiences in a given area, and more.

- **Combining causal discovery and expert knowledge:** Some causal discovery algorithms allow us to easily incorporate expert knowledge as a priority. This means that we can either *freeze* certain edges in the graph or *suggest* the existence or direction of these edges.

From A. Molak, Causal Inference and Discovery in Python

Independence and conditional independence

- Notation for independence involves the symbol, $\perp\!\!\!\perp$ (usually called *double up tack*), whose form visually encodes the notion of orthogonality.

- We can express the fact that X and Y are independent in the following way:

$$P(X, Y) = P(X)P(Y) \qquad X \perp\!\!\!\perp Y$$

- The concept of independence plays a vital role in statistics and causality.

- Its generalization – **conditional independence** – is even more important. We say that X and Y are conditionally independent given Z, when X does not give us any new information about Y assuming that we observed Z.

$$P(X, Y|Z) = P(X|Z)P(Y|Z) \qquad X \perp\!\!\!\perp Y|Z$$

From A. Molak, Causal Inference and Discovery in Python

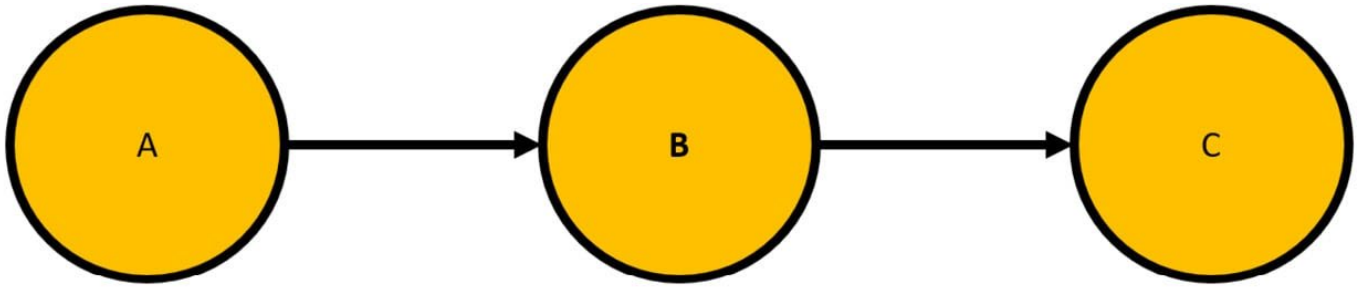
Conditional and unconditional independence

- We say that two nodes are *unconditionally* (or marginally) *independent* in the graph when there's *no open path* that connects them *directly* or *indirectly*.

- We say that two nodes, X and Y, are *conditionally independent* given (a set of) node(s) Z when Z blocks *all open paths* that connect X and Y.

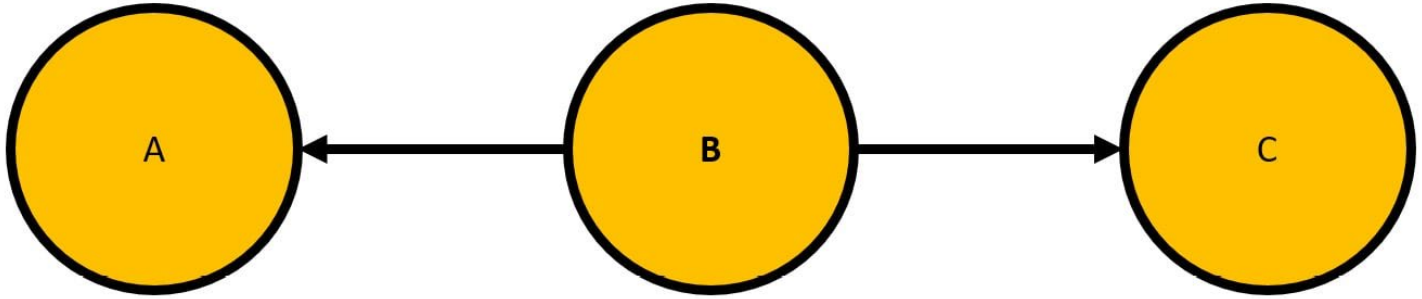
From A. Molak, Causal Inference and Discovery in Python

Chains and forks



$$A \perp\!\!\!\perp_G C \mid B$$

Chain 🔗

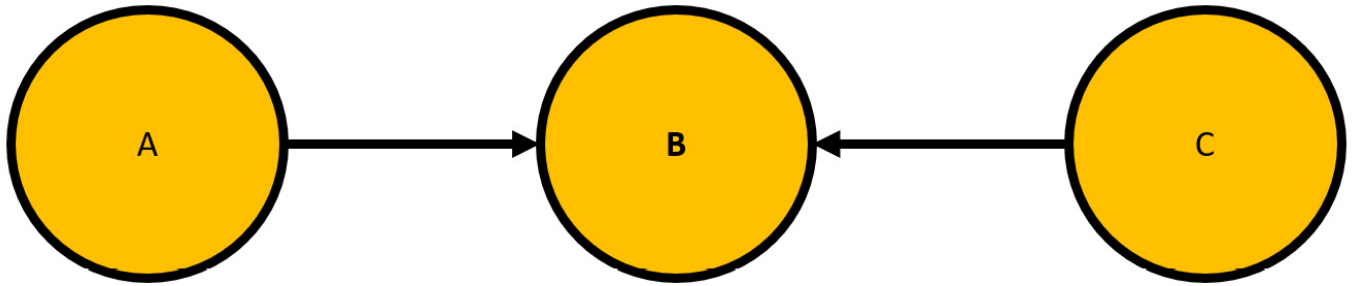


$$A \perp\!\!\!\perp_G C \mid B$$

Fork 🍴

From A. Molak, Causal Inference and Discovery in Python

Colliders



$$A \perp\!\!\!\perp C$$

$$A \not\perp\!\!\!\perp C \mid B$$

Collider

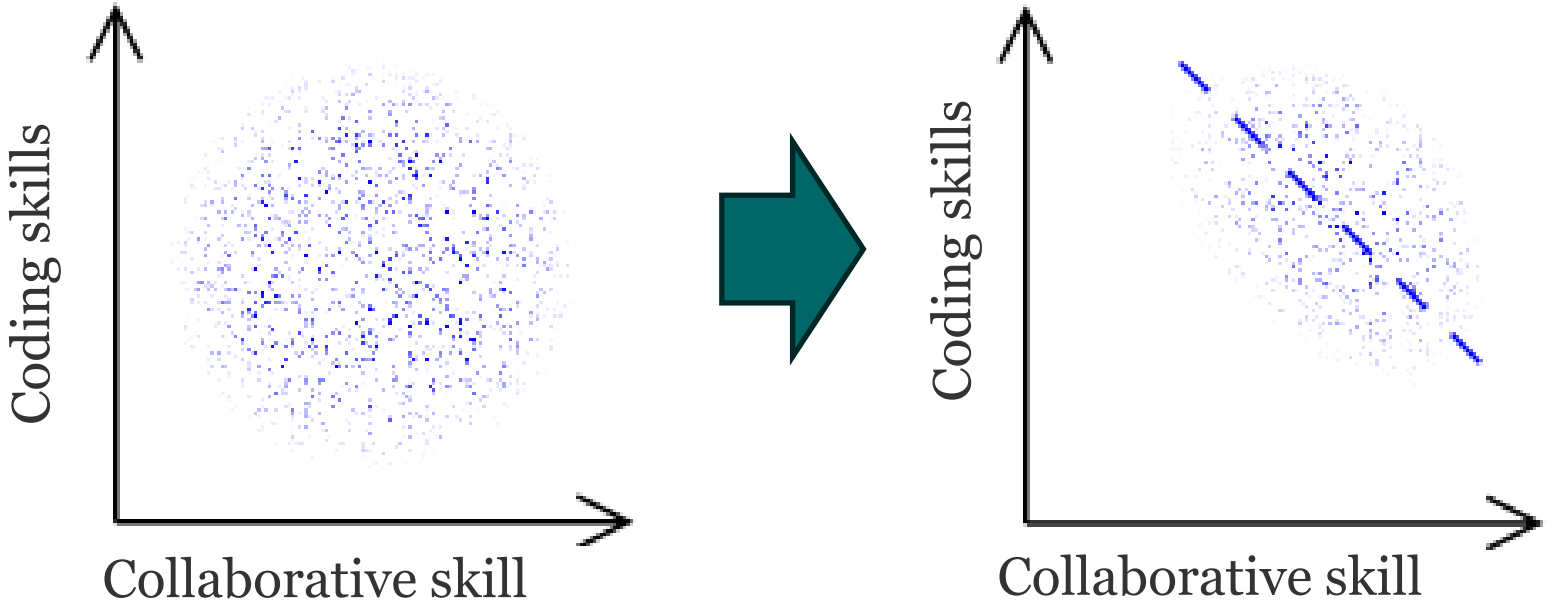
Imagine that both A and C randomly generate integers between *1* and *3*. Let's also say that B is a sum of A and C. Now, let's take a look at values of A and C when the value of B is 4. The following are the combinations of A and C that lead to B = 4:

- A = 1, C = 3
- A = 2, C = 2
- A = 3, C = 1

Although A and C are unconditionally independent (there's no correlation between them as they randomly and independently generate integers), they become correlated when we observe C !

From A. Molak, Causal Inference and Discovery in Python

Colliders and Berkson paradox



Many companies might hire people based on their skills and their personality traits. Imagine that company *X* quantifies a person's coding skills on a scale from one to five. They do the same for the candidate's ability to cooperate and hire everyone who gets a total score of at least seven. Assuming that coding skills and ability to cooperate are independent in the population (which doesn't have to be true in reality), you'll observe that in company *X*, people who are better coders are less likely to cooperate on average, and those who are more likely to cooperate have fewer coding skills. You could conclude that being non-cooperative is related to being a better coder, yet this conclusion would be incorrect in the general population.

https://en.wikipedia.org/wiki/Berkson%27s_paradox From A. Molak, Causal Inference and Discovery in Python

Estimator, estimate, and estimand

1. Estimand: The quantity or parameter that is intended to be estimated. It represents the true value of the parameter in the population. Suppose you're interested in the average height of adult men in a particular country. The actual average height of all adult men in that country is the estimand.

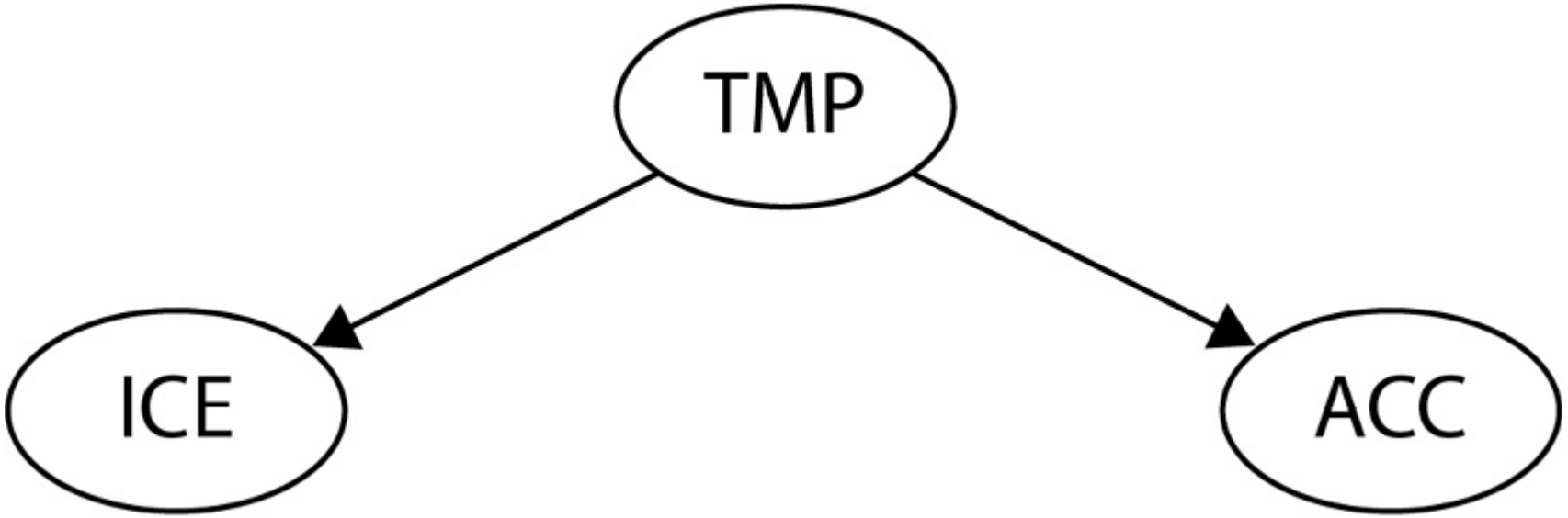
2. Estimate: The approximation or value obtained from the data to estimate the estimand. This is derived from a sample and used to infer information about the population. For example, from a sample of 1,000 adult men in the same country, you calculate an average height of 5 feet 9 inches. This value (5 feet 9 inches) is your estimate of the average height (the estimand).

3. Estimator: A rule, formula, or algorithm by which you derive the estimate from the data. It is a function of the sample data and is used to produce an estimate of the estimand. Here, the formula for calculating the mean (average) from a set of numbers is an estimator. When you apply this formula to your sample data, you obtain the estimate.

Estimator, estimate, and estimand

- **Estimand**: What you want to know (the actual, often unknown, value).
- **Estimate**: What you got from your sample data.
- **Estimator**: How you got it (the method or formula used).

Adjustment

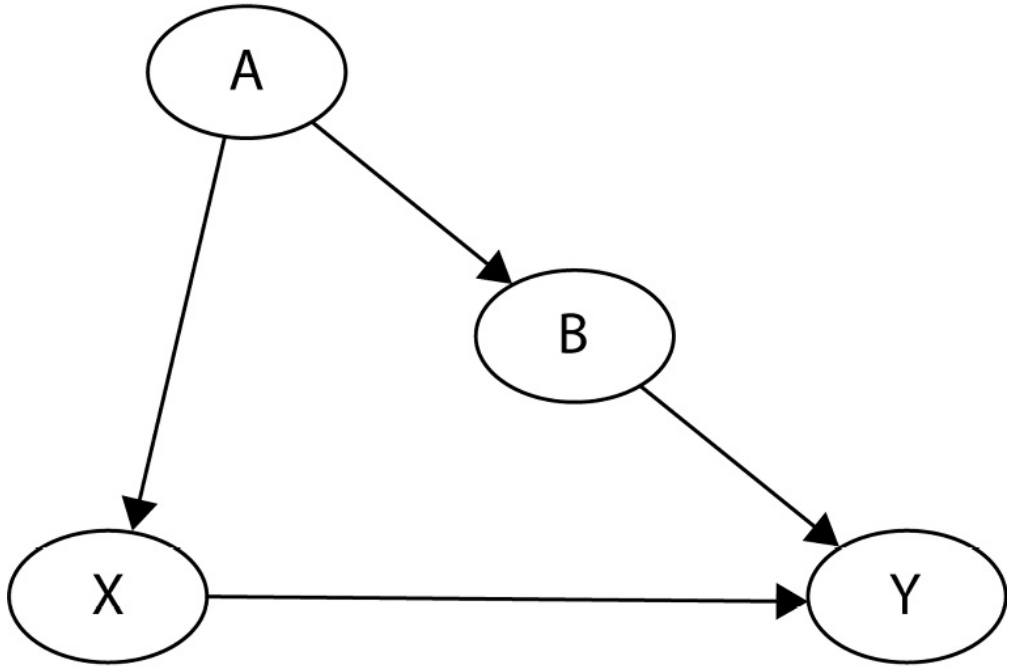


$$ACC \sim ICE + TMP$$

$$P(ACC|do(ICE)) = \sum_{tmp} P(ACC|ICE, TMP)P(TMP)$$

From A. Molak, Causal Inference and Discovery in Python

Back door criterion



$$P(Y = y|do(X = x)) = \sum_a P(Y = y|X = x, A = a)P(A = a)$$

$$= \sum_b P(Y = y|X = x, B = b)P(B = b)$$

We can estimate effect even if one of A, B is unobserved!

From A. Molak, Causal Inference and Discovery in Python