Industrial and Commercial Power and Power Conversion

Developing a Weakly Nonlinear Power System Model Using the Carleman Bilinearization Procedure

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Motivation and outline

The power systems are essential nonlinear systems because of many nonlinear components are presented in the system. The main sources of nonlinearities are:

- Synchronous machines as the sinusoidal dependency of the electromagnetic torque from rotor angle;
- Energy storage devices used with renewable energy sources;
- AC/DC switched-mode power converters widely used for load powering.

The nonlinearity complicates the system analysis. As usual, the linear approximation is performed for analytical estimation of system spectral properties and stability. But it works for small signal analysis only.

The ways to take into account nonlinearity and to expand the dynamic range of the analytical analysis is to represent the original nonlinear system in bilinear form using Carleman bilinearization procedure.

Bilinear systems

Let the nonlinear system is described by following equation:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{t}) \qquad (1)$$

If the system (1) can be represented in form (2) it is called bilinear system:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{a}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{N}\boldsymbol{x}(\boldsymbol{t})\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{b}\boldsymbol{u}(\boldsymbol{t}) \qquad (2)$$

The fundamental property of the bilinear systems is that they are linear in terms of state vector (x) if control vector (u) is considered as a parameter or linear in terms of control vector if state vector is considered as a parameter.

This property is the base to use linear approaches such a modal analysis to bilinear or bilinearized systems.

Bilinear representation of nonlinear systems

Let consider nonlinear system:

$$\begin{cases} \dot{x} = F(x) + G(x)u \\ y = Cx \end{cases} \qquad (3)$$

Where F(x) and G(x) is any nonlinear analytical functions.

First step: decompose the nonlinear part of each equation in (3) using Taylor series.
The equation of dynamic can be represented by Taylor series in equilibrium point of x0

$$\dot{x} \approx A_0 + \sum_{i=1}^{N} A_i x^{(i)} + B_0 u + \sum_{i=1}^{N} B_i x^{(i)} u \qquad (4)$$

Where N is number of Taylor series parts, which is taken in account.

$$A_0 = F(0), B_0 = G(0)$$

Second step: Form new vector of the states consist from Kronecker products of original vector

$$x^{(i)} = x \underbrace{\otimes \cdots \otimes}_{i-1\ times} x \qquad (5)$$

Third step: differentiate the new state vector N times to form new dynamical matrix and bilinear matrix

$$\frac{d(x^{(n)})}{dt} = \frac{d(x^{(n-1)} \otimes x^{(1)})}{dt} = \dot{x}^{(n-1)} \otimes x^{(1)} + x^{(1)} \otimes \dot{x}^{(n-1)} \qquad (6)$$

Bilinear representation of nonlinear systems (continue)

Bilinearized form of system (3) is:

$$\dot{\hat{\boldsymbol{x}}} = \widehat{\boldsymbol{A}}\hat{\boldsymbol{x}} + \widehat{\boldsymbol{N}}\hat{\boldsymbol{x}}u + \widehat{\boldsymbol{B}}u \qquad (7)$$

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \vdots \\ \boldsymbol{x}^{(P)} \end{bmatrix}, \widehat{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_N \\ \boldsymbol{A}_{20} & \boldsymbol{A}_{21} & \cdots & \boldsymbol{A}_{2,N-1} \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{O} & \cdots & \boldsymbol{A}_{N0} & \boldsymbol{A}_{N1} \end{bmatrix}, \widehat{\boldsymbol{N}} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_N \\ \boldsymbol{B}_{20} & \boldsymbol{B}_{21} & \cdots & \boldsymbol{B}_{2,N-1} \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{O} & \cdots & \boldsymbol{B}_{N0} & \boldsymbol{B}_{N1} \end{bmatrix}, \widehat{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B}_0 \\ \boldsymbol{O} \\ \vdots \\ \boldsymbol{O} \end{bmatrix}$$

$$\boldsymbol{A}_{ji} = \boldsymbol{A}_i \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{A}_i \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \cdots + \boldsymbol{I} \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{A}_i \qquad (8)$$

$$\boldsymbol{B}_{ji} = \boldsymbol{B}_i \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{B}_i \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \cdots + \boldsymbol{I} \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{B}_i \qquad (9)$$

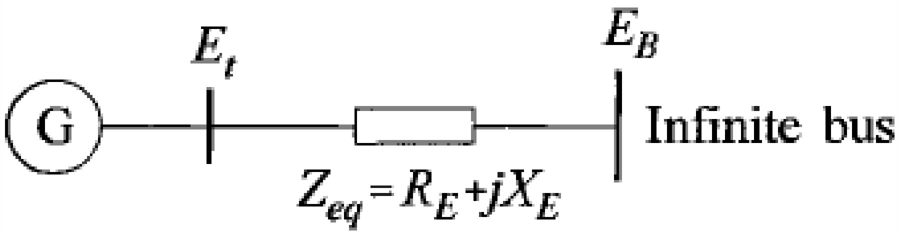
In special case, when G in (3) is independent from $\boldsymbol{x}$, matrix $\widehat{\boldsymbol{N}}$ is simplified to:

$$\widehat{\boldsymbol{N}} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{B}_{20} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \vdots & \ddots & \cdots & \vdots \\ \boldsymbol{O} & \cdots & \boldsymbol{B}_{N0} & \boldsymbol{O} \end{bmatrix} \qquad (10)$$

$$\boldsymbol{B}_{k0} = \boldsymbol{B}_0 \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{B}_0 \otimes \boldsymbol{I} \ldots \otimes \boldsymbol{I} + \cdots + \boldsymbol{I} \otimes \boldsymbol{I} \otimes \ldots \otimes \boldsymbol{B}_0 \qquad (11)$$

Biliniarization of the simple power system

Let consider the simple energy power system generator-load-infinite bus:



The basic nonlinear system for this case is described as:

$$\begin{cases} \frac{d(\Delta\omega)}{dt} = -\frac{K_D\Delta\omega}{2H} - \frac{E'E_B}{2HX} sin\delta + \frac{T_m}{2H} \\ \frac{d\delta}{dt} = \omega_0\Delta\omega \end{cases} \quad (12)$$

$\boldsymbol{\Delta\omega}$ –is a frequency deviation; $\boldsymbol{\omega_0}$ – related value of angular velocity; $\boldsymbol{\delta}$ – rotor angle; $\boldsymbol{T_m}$- mechanical torque; $\boldsymbol{E'}$– generator voltage; $\boldsymbol{E_B}$ – voltage of the infinite bus; $\boldsymbol{X}$ – impedance of the line; $\boldsymbol{K_D}$ – damping coefficient.

Biliniarization of the simple power system (continue)

The dynamical equation in state space representation:

$$\begin{cases} \dot{x_1} = -\frac{K_D}{2H}x_1 - \frac{E'E_B}{2HX}sin(x_2) + b_1u_1 \\ \dot{x_2} = \omega_0 x_1 \end{cases} \qquad (13)$$

where $x_1 = \Delta\omega, x_2 = \delta, u_1 = T_m,\ b_1 = \frac{1}{2H}$

Step1: decompose of nonlinear part of dynamical system (13) in Taylor series in equilibrium point of $\boldsymbol{x_0} = \{x_{01}, x_{02}\} = 0$ (zero is taken here for simplicity, but us usual rotor angle equilibrium point in not zero) and take first tree parts of series we have following :

$$\begin{cases} \dot{x_1} \approx D_{11}x_1 + D_{12}x_2 + D_{12}^{(2)}{x_2}^2 + D_{12}^{(3)}{x_2}^3 + b_1u_1 \\ \dot{x_2} = D_{21}x_1 \end{cases} \qquad (14)$$

where $D_{11} = -\frac{K_D}{2H};\ D_{12} = -\frac{E'E_B}{2HX}cos(x_{02}); D_{12}^{(2)} = \frac{E'E_B}{4HX}sin(x_{02});\ D_{12}^{(3)} = \frac{E'E_B}{12HX}cos(x_{02});\ D_{21} = \omega_0.$

Biliniarization of the simple power system (continue)

Or in matrix-vector representation:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}\,\boldsymbol{u} \approx \boldsymbol{A_1}\boldsymbol{x}^{(1)} + \boldsymbol{A_2}\boldsymbol{x}^{(2)} + \boldsymbol{A_3}\boldsymbol{x}^{(3)} + \boldsymbol{B_0}\,\boldsymbol{u} \qquad (15)$$

- linear part: $A_1 = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & 0 \end{bmatrix}$;
- quadratic: $A_2 = \begin{bmatrix} 0 & 0 & 0 & D_{12}^{(2)} \\ 0 & 0 & 0 & 0 \end{bmatrix}$;
- cubic: $A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(3)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$;
- $B_0 = \begin{bmatrix} b_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2H} \\ 0 \end{bmatrix}$; $\boldsymbol{u} = \begin{bmatrix} T_m \\ 0 \end{bmatrix}$

Step 2. Form new state vector :

$$x^{(1)} = [x_1 \quad x_2]^T$$

$$x^{(2)} = [x_1 \quad x_2]^T \otimes [x_1 \quad x_2]^T = [x_1^2 \quad x_1x_2 \quad x_2x_1 \quad x_2^2]^T$$

$$x^{(3)} = [x_1^2 \quad x_1x_2 \quad x_2x_1 \quad x_2^2]^T \otimes [x_1 \quad x_2]^T$$
$$= [x_1^3 \quad x_1^2x_2 \quad x_1x_2x_1 \quad x_1x_2^2 \quad x_2x_1^2 \quad x_2x_1x_2 \quad x_2^2x_1 \quad x_2^3]^T$$

Biliniarization of the simple power system (continue)

Step 3. Form bilinear system using equations (7)-(11). So, bilinearized form of (12) is:

$$\dot{\hat{\boldsymbol{x}}} = \hat{\boldsymbol{A}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{N}}\hat{\boldsymbol{x}}u_1 + \hat{\boldsymbol{B}}u_1 \qquad (16)$$

$$\hat{x} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}; \hat{A} = \begin{bmatrix} A_1 & A_2 & A_3 \\ . & A_{21} & A_{22} \\ . & . & A_{31} \end{bmatrix}; \hat{N} = \begin{bmatrix} 0 & 0 & 0 \\ B_{20} & 0 & 0 \\ 0 & B_{30} & 0 \end{bmatrix}; \hat{B} = \begin{bmatrix} B_0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{21} = A_1 \otimes I + I \otimes A_1 = \begin{bmatrix} 2D_{11} & D_{12} & D_{12} & 0 \\ D_{21} & D_{11} & 0 & D_{12} \\ D_{21} & 0 & D_{11} & D_{12} \\ 0 & D_{21} & D_{21} & 0 \end{bmatrix};$$

$$B_{20} = B_0 \otimes I + B_0 \otimes I = \begin{bmatrix} 2b_1 & 0 \\ 0 & b_1 \\ 0 & b_1 \\ 0 & 0 \end{bmatrix};$$

$$A_{22} = A_2 \otimes I + I \otimes A_2 = \begin{bmatrix} 0 & 0 & 0 & D_{12}^{(2)} & 0 & 0 & D_{12}^{(2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{30} = B_0 \otimes I \otimes I + I \otimes B_0 \otimes I + I \otimes I \otimes B_0 = \begin{bmatrix} 3b_1 & 0 & 0 & 0 \\ 0 & 2b_1 & 0 & 0 \\ 0 & b_1 & b_1 & 0 \\ 0 & 0 & 0 & b_1 \\ 0 & 0 & 2b_1 & 0 \\ 0 & 0 & 0 & b_1 \\ 0 & 0 & 0 & b_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{31} = A_1 \otimes I \otimes I + I \otimes A_1 \otimes I + I \otimes I \otimes A_1 = \begin{bmatrix} 3D_{11} & D_{12} & D_{12} & 0 & D_{12} & 0 & 0 & 0 \\ D_{21} & 2D_{11} & 0 & D_{12} & 0 & D_{12} & 0 & 0 \\ D_{21} & 0 & 2D_{11} & D_{12} & 0 & 0 & D_{12} & 0 \\ 0 & D_{21} & D_{21} & D_{11} & 0 & 0 & 0 & D_{12} \\ D_{21} & 0 & 0 & 0 & 2D_{11} & D_{12} & D_{12} & 0 \\ 0 & D_{21} & 0 & 0 & D_{21} & D_{11} & 0 & D_{12} \\ 0 & 0 & D_{21} & 0 & D_{21} & 0 & D_{11} & D_{12} \\ 0 & 0 & 0 & D_{21} & 0 & D_{21} & D_{21} & 0 \end{bmatrix};$$

Biliniarization of the simple power system (continue)

The resulting system:

$$\dot{\hat{\boldsymbol{x}}} = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 & D_{12}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(3)} \\ D_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2D_{11} & D_{12} & D_{12} & 0 & 0 & 0 & 0 & D_{12}^{(2)} & 0 & 0 & D_{12}^{(2)} & 0 \\ 0 & 0 & D_{21} & D_{11} & 0 & D_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(2)} \\ 0 & 0 & D_{21} & 0 & D_{11} & D_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{12}^{(2)} \\ 0 & 0 & 0 & D_{21} & D_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3D_{11} & D_{12} & D_{12} & 0 & D_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 2D_{11} & 0 & D_{12} & 0 & D_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 0 & 2D_{11} & D_{12} & 0 & 0 & D_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & D_{21} & D_{11} & 0 & 0 & 0 & D_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 0 & 0 & 0 & 2D_{11} & D_{12} & D_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 0 & 0 & D_{21} & D_{11} & 0 & D_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 0 & D_{21} & 0 & D_{11} & D_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{21} & 0 & D_{21} & D_{21} & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_2 x_1 \\ x_2^2 \\ x_1^3 \\ x_1^2 x_2 \\ x_1 x_2 x_1 \\ x_1 x_2^2 \\ x_2 x_1^2 \\ x_2 x_1 x_2 \\ x_2^2 x_1 \\ x_2^3 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_1 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_2 x_1 \\ x_2^2 \\ x_1^3 \\ x_1^2 x_2 \\ x_1 x_2 x_1 \\ x_1 x_2^2 \\ x_2 x_1^2 \\ x_2 x_1 x_2 \\ x_2^2 x_1 \\ x_2^3 \end{bmatrix} [u_1] + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [u_1]$$

Model order reduction

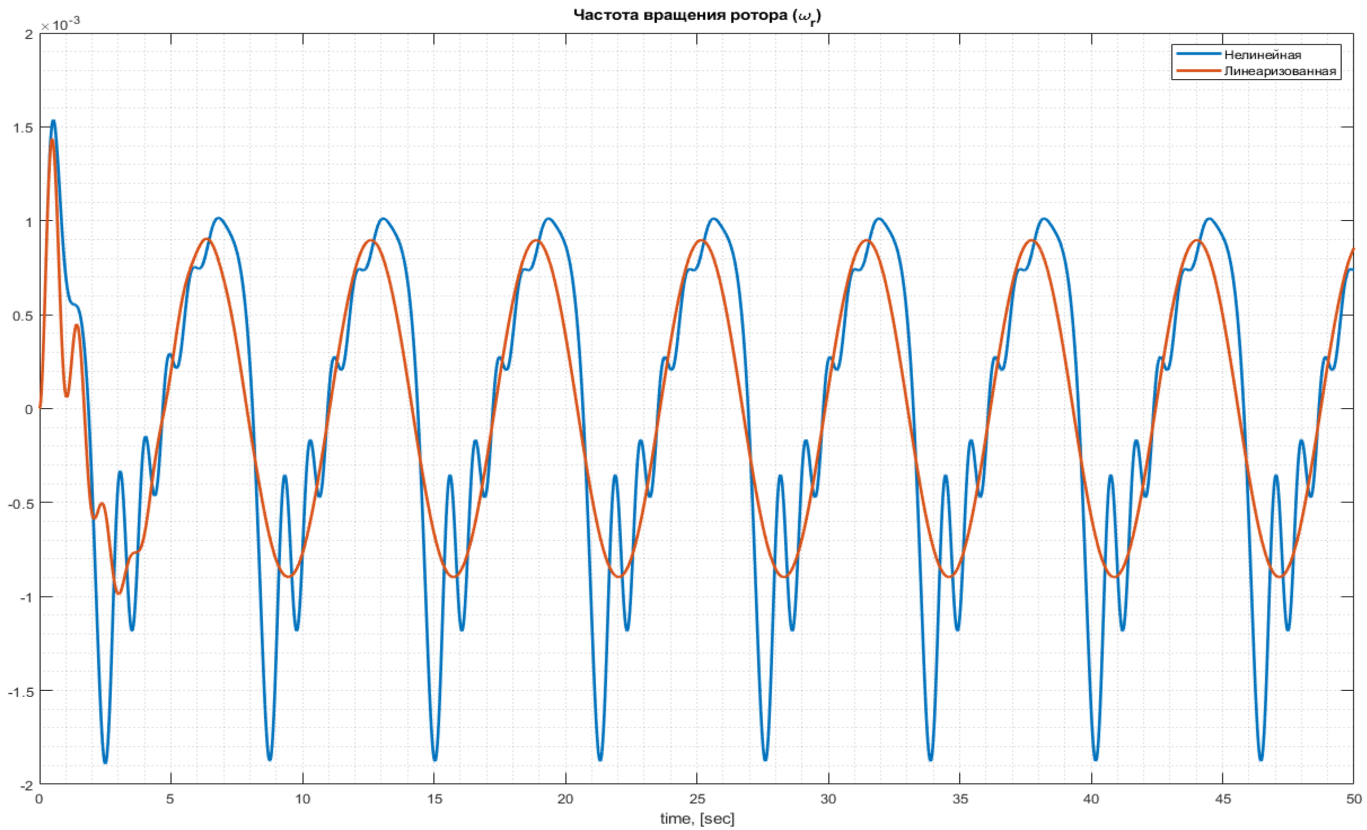
The system order increases rapidly with growing of approximation order. The new state vector for cubic approximation is N+N^2+N^3, where N is the size of the original vector.

So, the model order reduction is a very important task. To use simple procedure of excluding of a duplicated combination in the new state vector, we can reduce the considered system order from 14 to 6:

$$\hat{\dot{x}} = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 & -\frac{D_{12}}{6} \\ D_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3D_{11} & 3D_{12} & 0 & 0 \\ 0 & 0 & D_{21} & 2D_{11} & 2D_{12} & 0 \\ 0 & 0 & 0 & 2D_{21} & D_{11} & D_{12} \\ 0 & 0 & 0 & 0 & 3D_{21} & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_1^3 \\ x_1^2 x_2 \\ x_1 x_2^2 \\ x_2^3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2b_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_1^3 \\ x_1^2 x_2 \\ x_1 x_2^2 \\ x_2^3 \end{bmatrix} [u_1] + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [u_1]$$

Simulation results (nonlinear vs linearized)

Response of nonlinear and linearized systems to sinusoidal form of u1
($\boldsymbol{u_1} = \Delta \boldsymbol{T_m} \sin(t)$ при $\Delta \boldsymbol{T_m} = 0.25$)



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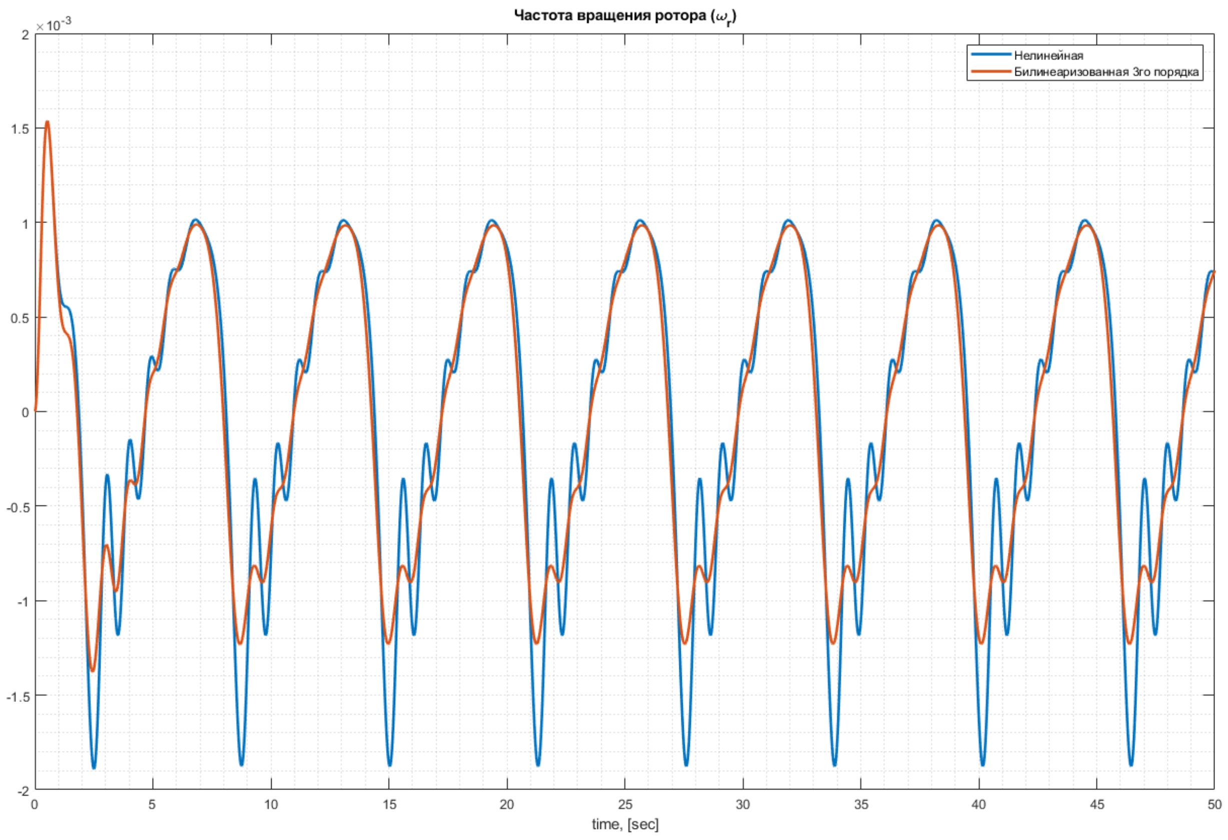
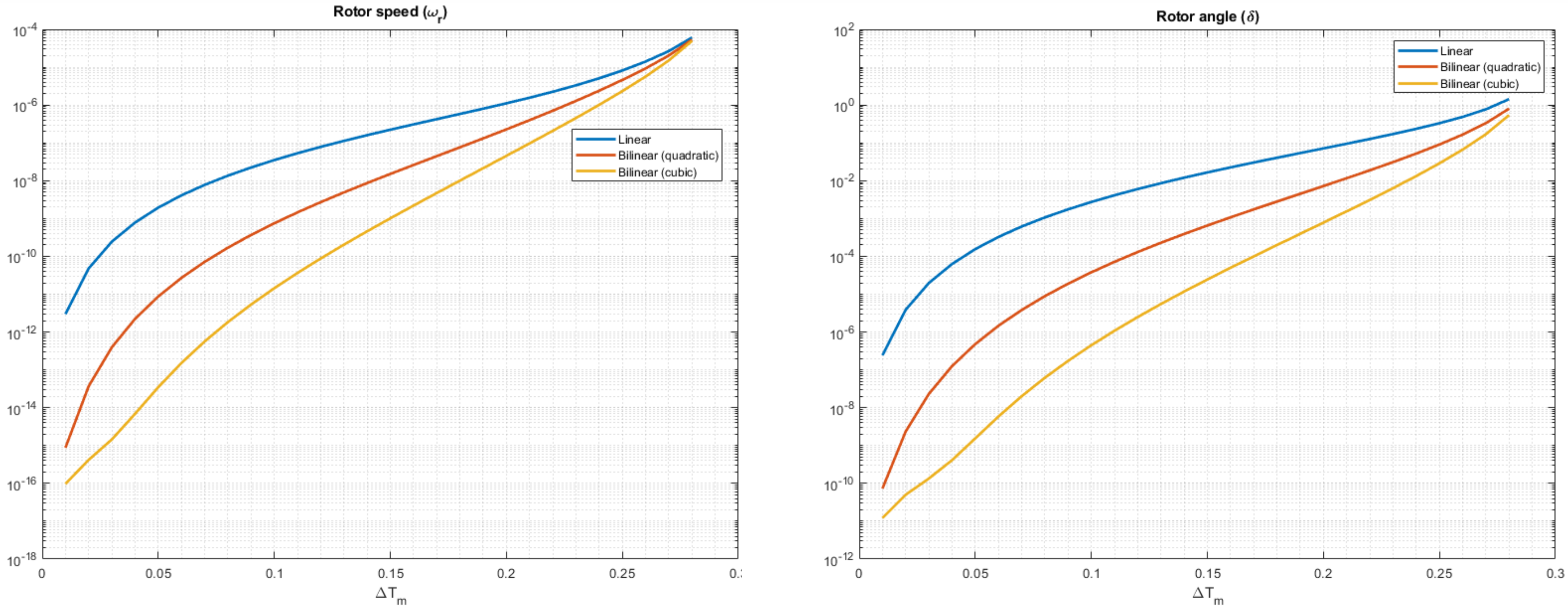


Liniear and bilinear system comparision



Conclusion

1. The mathematical apparatus of bilinear models is useful for analytical study of the nonlinear dynamical systems as an electric power systems.

2. For representation in the bilinear form, it is necessary to transform the original system using the Carleman bilinearization procedure.

3. In this work we have considered an example of a simple power system with one generator and load. We have shown that the bilinearized model that takes into account third-order nonlinearities adequately approximates the original nonlinear system and much more accurately describes the original system in comparison with linearized model.

Thank you for the attention!

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