Creating Two-Dimensional Images of Objects with High Angular Resolution

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***Abstract*— A new method of digital radar signal processing for remote sensing is proposed. The technique allows one to obtain two-dimensional images of objects with superresolution. The method is based on solving a convolution-type two-dimensional linear integral equation of the first kind by algebraic methods.**

Keywords—angular resolution; Rayleigh criterion; superresolution; two-dimensional linear integral equation

I. Introduction

A number of approaches are known for obtaining detalized images of objects with superresolution. Among them are the methods of: phase weighting coefficients and corner weighing, MUSIC, ESPRIT, MME, thermal noise, maximum likelihood, deconvolution of signals, and others [1-4]. These techniques are neither universal nor always efficient; most of them are oriented towards one-dimensional problems. Generalizations to two-dimensional (2D) settings significantly complexify the algorithms and require much more computational resources. To obtain satisfactory results, one needs to apply parallel and massive-parallel calculations [5].

Algebraic methods for solving one-dimensional problems are proved to be promising for the analysis of the considered two-dimensional formation of the images of signal sources [6-8]. They employ representation of approximate solutions as finite expansions with unknown coefficients using given function sets. These methods do not cause significant complexification of the algorithms of computational resources and the processor time remains on the reasonable levels. A novelty of the proposed technique is in demonstrating that algebraic methods may efficiently increase the reconstruction quality for 2D problems achieving high accuracy and the angle resolution that may significantly exceed the Rayleigh criterion.

II. Formulations

Let the observation sector be a solid angle Ω_0 where the zero direction is chosen towards the center of Ω_0. Let f(x,y) be a directional pattern (DP) and x,y the angle of deviation from the zero direction in the Cartesian system. An object of observation with generally unknown azimuthal dimensions Ω is placed into the observation sector. The sought two-dimensional angle distribution of the signal generated by the source (or of the reflected signal) is denoted by I(x,y). Then at the output of the receiving device one obtains as a result of scanning the signal enveloping curve U(x,y). Quantities I, U, and DP are coupled by a convolution-type two-dimensional linear integral equation of the first kind

$$U(x,y) = \int_{\Omega} f(x-x', y-y') I(x',y') dx'dy' \qquad (1)$$

A problem is posed of reconstruction of the angle distribution I(x,y) based on the analysis of the received signal U(x,y) and known DP with the maximum possible angle resolution exceeding the Rayleigh criterion.

Look for the sought distribution as a finite expansion with unknown coefficients using the given set of functions orthogonal in Ω. We will use the functions that admit the variable separation $G_{n,m}(x,y) = g_n(x)\, g_m(y)$:

$$I(x,y) \cong \sum_{n=1}^{N} \sum_{m=1}^{N} b_{n,m} g_n(x) g_m(y) \qquad (2)$$

Coefficients $b_{j,k}$, that provide minimal root-mean-square (rms) deviation between the synthesized and received signals are determined from the linear algebraic equation system:

$$\int_{\Omega} U(x,y)\varphi_{j,k}(x,y)d\alpha = \sum_{n,m=1}^{N} b_m \int_{\Omega} \varphi_{j,k}(x,y)\varphi_{n,m}(x,y)dxdy\,,$$

$$\varphi_{n,m}(x,y) = \int_{\Omega} f(x-x', y-y') g_n(x') g_m(y') dx'dy' \qquad (3)$$

$$j,k = 1,2\ldots N.$$

Algebraic methods enable one to approach the limit angle resolution inherent to each particular problem; the solution of system (3) by an algebraic method is obtained in the course of successive increase of the number N of functions in use, i.e. by enhancing the resolution.

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III. Numerical

Quantitative increase of resolution and its limits are investigated using a mathematical model. We consider a needle DP formed by a plane square antenna array (AA) having the size 30*30 d/λ with homogeneous excitation. The object under investigation is described using a function within a solid angle Ω ($-\theta_{0,5}/2 \leq x \leq \theta_{0,5}/2$, $-\theta_{0,5}/2 \leq y \leq \theta_{0,5}/2$). Next, an inverse problem is solved of reconstructing the angle distribution of the soutce amplitude from the signal values $U(x,y)$ by solving LAES (3).

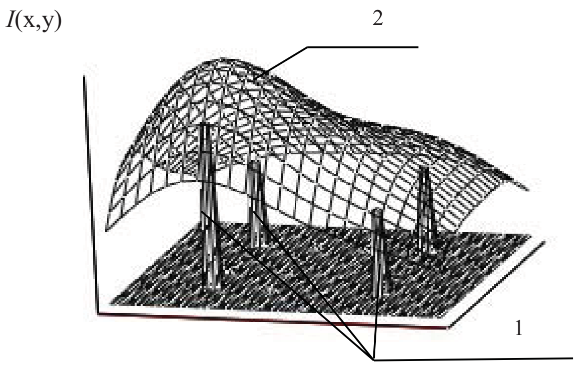

Fig. 1. Reconstructed image - 1, original signal -2.

To determine approximate solution, first, the position and dimensions are estimated of solid angle Ω containing the source using the form of $U(x,y)$. Next the parameters of Ω are refined using the preliminary solutions obtained in the course of iterations.

The quality of solving inverse problems may be enhanced significantly if *a priori* information about the solution is available. In the considered problems the solution is sought under *a priori* condition that the source is actually a set of closely located small-size sources. As far as the solution of LAES (3) is concerned, this *a priori* information dictates the use of the functions similar to delta-functions separated by the distances Δx and Δy from each other.

Figure 1, where the parameters along horizontal axes are angles x and y, displays the reconstructed image of the observed object: four point sources of signals with different intensities. Here individual sources cannot be observed separately using direct observation. The solution is determined on the basis of (2) and (3) with increasing resolution and refining the position and size of sector Ω using successive increase of the number of functions used in (2). The technique enables one to correctly determine the number of objects and their location; intensities are calculated with an error lying within the limits 2-5%.

Other solved inverse problems concern reconstruction of gradually inhomogeneous intensity distributions. Here, the use of trigonometric functions is preferable, e.g., a system

$$\cos(\frac{2\pi m x}{T_x})\cos(\frac{2\pi m y}{T_y}),\quad \sin(\frac{2\pi m x}{T_x})\sin(\frac{2\pi m y}{T_y})$$

orthogonal in $[T_x, T_y]$. Figure 2 shows the reconstructed image of the observed object in the form of a set of the four point signal sources with gradually inhomogeneous intensity distribution; the maximal of their peak intensities four times greater than the minimal peak.

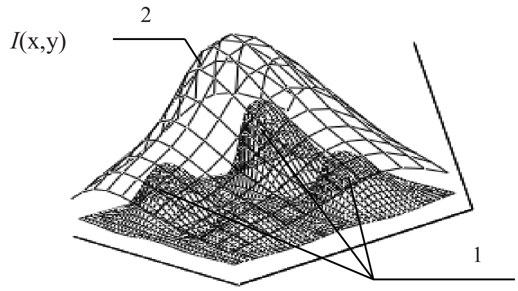

Fig. 2. Reconstructed image - 1, original signal -2.

The obtained solution enables one to resolve all four sources, correctly demonstrate their location, and determine the character of the intensity distribution. It should be noted that the angle dimensions to a certain extent (by 5-8%) exceed the true ones, while amplitudes are reconstructed with the errors within reasonable limits not greater than 7%.

IV. Conclusion

The proposed algebraic methods make it possible to form images of complicated two-dimensional objects with superresolution in two coordinates. The achieved angle resolution exceeds the Rayleigh criterion up to six times with relatively small errors in the intensity values. The developed approach can be applied to more complicated antenna systems.

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