# Deep Generative Models

## Lecture 5

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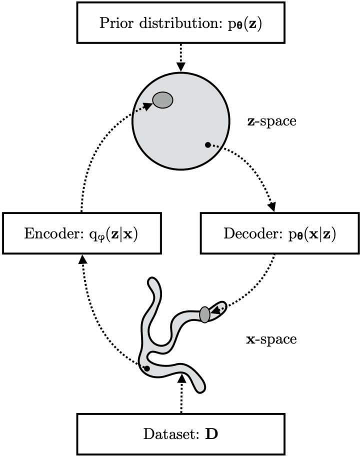
# Recap of previous lecture

## Variational autoencoder (VAE)

- VAE learns stochastic mapping between $\mathbf{x}$-space, from $\pi(\mathbf{x})$, and a latent $\mathbf{z}$-space, with simple distribution.
- The generative model learns distribution $p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta}) = p(\mathbf{z})p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta})$, with a prior distribution $p(\mathbf{z})$, and a stochastic decoder $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta})$.
- The stochastic encoder $q(\mathbf{z}|\mathbf{x}, \phi)$ (inference model), approximates the true but intractable posterior $p(\mathbf{z}|\mathbf{x}, \boldsymbol{\theta})$.




*Kingma D. P., Welling M. An introduction to variational autoencoders, 2019*

# Recap of previous lecture

## LVM

$$p(\mathbf{x}|\boldsymbol{\theta}) = \int p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta}) d\mathbf{z} = \int p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) p(\mathbf{z}) d\mathbf{z}$$

- More powerful $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta})$ leads to more powerful generative model $p(\mathbf{x}|\boldsymbol{\theta})$.
- Too powerful $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta})$ could lead to posterior collapse: $q(\mathbf{z}|\mathbf{x})$ will not carry any information about $\mathbf{x}$ and close to prior $p(\mathbf{z})$.

## Autoregressive decoder

$$p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) = \prod_{i=1}^{n} p(x_i|\mathbf{x}_{1:i-1}, \mathbf{z}, \boldsymbol{\theta})$$

- Global structure is captured by latent variables.
- Local statistics are captured by limited receptive field autoregressive model.

*Gulrajani I. et al. PixelVAE: A Latent Variable Model for Natural Images, 2016*

# Recap of previous lecture

## Decoder weakening

- Powerful decoder $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta})$ makes the model expressive, but posterior collapse is possible.
- PixelVAE model uses the autoregressive PixelCNN model with small number of layers to limit receptive field.

## KL annealing

$$\mathcal{L}(q, \boldsymbol{\theta}, \beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) - \beta \cdot KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))$$

Start training with $\beta = 0$, increase it until $\beta = 1$ during training.

## Free bits

Ensure the use of less than $\lambda$ bits of information:

$$\mathcal{L}(q, \boldsymbol{\theta}, \lambda) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) - \max(\lambda, KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))).$$

This results in $KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) \geq \lambda$.

# Recap of previous lecture

## VAE objective

$$\log p(\mathbf{x}|\boldsymbol{\theta}) \geq \mathcal{L}(q, \boldsymbol{\theta}) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log \frac{p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta})}{q(\mathbf{z}|\mathbf{x})} \rightarrow \max_{q, \boldsymbol{\theta}}$$

## IWAE objective

$$\mathcal{L}_K(q, \boldsymbol{\theta}) = \mathbb{E}_{\mathbf{z}_1, \dots, \mathbf{z}_K \sim q(\mathbf{z}|\mathbf{x})} \log \left( \frac{1}{K} \sum_{k=1}^{K} \frac{p(\mathbf{x}, \mathbf{z}_k|\boldsymbol{\theta})}{q(\mathbf{z}_k|\mathbf{x})} \right) \rightarrow \max_{q, \boldsymbol{\theta}}.$$

## Theorem

1. $\log p(\mathbf{x}|\boldsymbol{\theta}) \geq \mathcal{L}_K(q, \boldsymbol{\theta}) \geq \mathcal{L}_M(q, \boldsymbol{\theta}) \geq \mathcal{L}(q, \boldsymbol{\theta}), \quad \text{for } K \geq M;$
2. $\log p(\mathbf{x}|\boldsymbol{\theta}) = \lim_{K \to \infty} \mathcal{L}_K(q, \boldsymbol{\theta})$ if $\frac{p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta})}{q(\mathbf{z}|\mathbf{x})}$ is bounded.

*Burda Y., Grosse R., Salakhutdinov R. Importance Weighted Autoencoders, 2015*

# Outline

# Likelihood-based models so far...

## Autoregressive models

$$p(\mathbf{x}|\boldsymbol{\theta}) = \prod_{i=1}^{m} p(x_i|\mathbf{x}_{1:i-1}, \boldsymbol{\theta})$$

- tractable likelihood,
- no inferred latent factors.

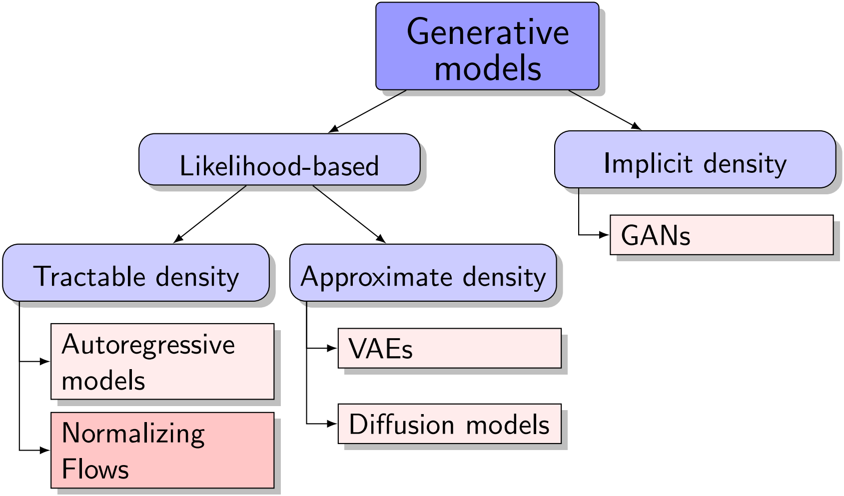
## Latent variable models

$$p(\mathbf{x}|\boldsymbol{\theta}) = \int p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta}) d\mathbf{z}$$

- latent feature representation,
- intractable likelihood.

How to build model with latent variables and tractable likelihood?

# Generative models zoo



# Normalizing flows prerequisites

## Change of variable theorem

Let $\mathbf{x}$ be a random variable with density function $p(\mathbf{x})$ and $f : \mathbb{R}^m \to \mathbb{R}^m$ is a differentiable, invertible function (diffeomorphism). If $\mathbf{z} = f(\mathbf{x})$, $\mathbf{x} = f^{-1}(\mathbf{z}) = g(\mathbf{z})$, then

$$p(\mathbf{x}) = p(\mathbf{z}) \left| \det \left( \frac{\partial \mathbf{z}}{\partial \mathbf{x}} \right) \right| = p(f(\mathbf{x})) \left| \det \left( \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \right) \right|$$

$$p(\mathbf{z}) = p(\mathbf{x}) \left| \det \left( \frac{\partial \mathbf{x}}{\partial \mathbf{z}} \right) \right| = p(g(\mathbf{z})) \left| \det \left( \frac{\partial g(\mathbf{z})}{\partial \mathbf{z}} \right) \right|.$$

## Inverse function theorem

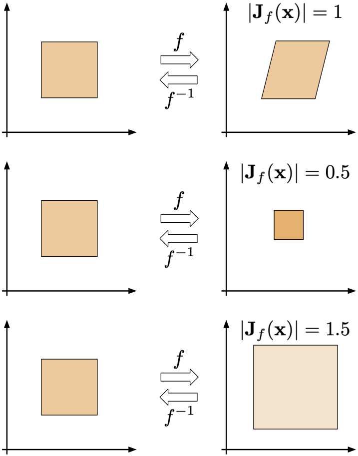
If function $f$ is invertible and Jacobian is continuous and non-singular, then

$$\left| \det \left( \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \right) \right| = \left| \det \left( \frac{\partial g^{-1}(\mathbf{x})}{\partial \mathbf{x}} \right) \right| = \left| \det \left( \frac{\partial g(\mathbf{z})}{\partial \mathbf{z}} \right) \right|^{-1}$$

# Jacobian-determinant

$$p(\mathbf{x}|\boldsymbol{\theta}) = p(f(\mathbf{x}, \boldsymbol{\theta})) \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|$$

- $\mathbf{x}$ and $\mathbf{z}$ have the same dimensionality ($\mathbb{R}^m$).
- $f(\mathbf{x}, \boldsymbol{\theta})$ could be parametric function.
- Determinant of Jacobian $\mathbf{J} = \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}}$ shows how the volume changes under the transorfmation.



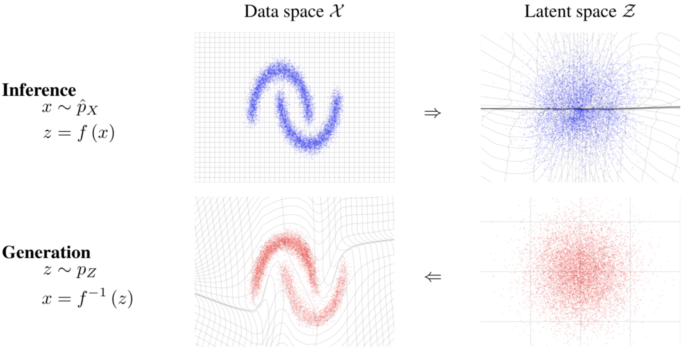

*https://jmtomczak.github.io/blog/3/3_flows.html*

# Fitting flows

## MLE problem

$$p(\mathbf{x}|\boldsymbol{\theta}) = p(\mathbf{z}) \left| \det \left( \frac{\partial \mathbf{z}}{\partial \mathbf{x}} \right) \right| = p(f(\mathbf{x}, \boldsymbol{\theta})) \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|$$

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right| \to \max_{\boldsymbol{\theta}}$$




*Dinh L., Sohl-Dickstein J., Bengio S. Density estimation using Real NVP, 2016*

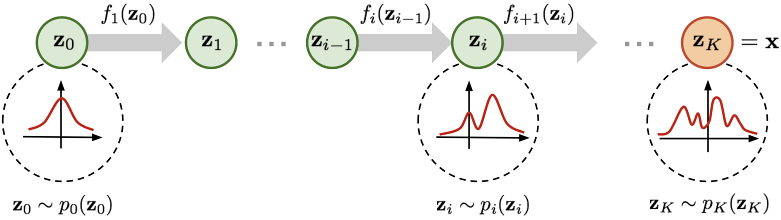
# Composition of flows

## Theorem

Diffeomorphisms are **composable** (If $\{f_k\}_{k=1}^K$ satisfy conditions of the change of variable theorem, then $\mathbf{z} = f(\mathbf{x}) = f_K \circ \cdots \circ f_1(\mathbf{x})$ also satisfies it).

$$p(\mathbf{x}) = p(f(\mathbf{x})) \left| \det \left( \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \right) \right| = p(f(\mathbf{x})) \left| \det \left( \frac{\partial f_K \circ \cdots \circ f_1(\mathbf{x})}{\partial \mathbf{x}} \right) \right| =$$

$$= p(f(\mathbf{x})) \left| \det \left( \frac{\partial \mathbf{f}_K}{\partial \mathbf{f}_{K-1}} \cdots \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}} \right) \right| = p(f(\mathbf{x})) \prod_{k=1}^{K} \left| \det \left( \frac{\partial \mathbf{f}_k}{\partial \mathbf{f}_{k-1}} \right) \right|$$




*https://lilianweng.github.io/lil-log/2018/10/13/flow-based-deep-generative-models.html*

# Flows

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|$$

## Definition

Normalizing flow is a *differentiable, invertible* mapping from data $\mathbf{x}$ to the noise $\mathbf{z}$.

- **Normalizing** means that the inverse flow takes samples from $p(\mathbf{x})$ and normalizes them into samples from density $p(\mathbf{z})$.
- **Flow** refers to the trajectory followed by samples from $p(\mathbf{z})$ as they are transformed by the sequence of transformations

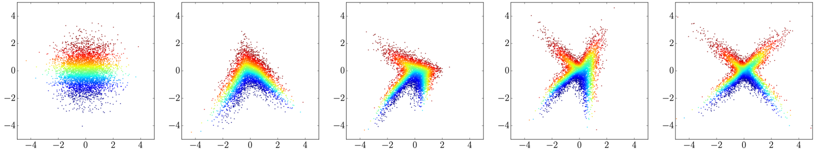
$$\mathbf{z} = f_K \circ \dots \circ f_1(\mathbf{x}); \quad \mathbf{x} = f_1^{-1} \circ \dots \circ f_K^{-1}(\mathbf{z}) = g_1 \circ \dots \circ g_K(\mathbf{z})$$

$$p(\mathbf{x}) = p(f_K \circ \dots \circ f_1(\mathbf{x})) \left| \det \left( \frac{\partial f_K \circ \dots \circ f_1(\mathbf{x})}{\partial \mathbf{x}} \right) \right| =$$

$$= p(f_K \circ \dots \circ f_1(\mathbf{x})) \prod_{k=1}^{K} \left| \det \left( \frac{\partial \mathbf{f}_k}{\partial \mathbf{f}_{k-1}} \right) \right| .$$

# Flows

## Example of a 4-step flow



## Flow likelihood

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|$$

What is the complexity of the determinant computation?

## What we want

- Efficient computation of Jacobian $\frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}}$;
- Efficient sampling from the base distribution $p(\mathbf{z})$;
- Efficient inversion of $f(\mathbf{x}, \boldsymbol{\theta})$.

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*Papamakarios G. et al. Normalizing flows for probabilistic modeling and inference, 2019*

# Forward KL vs Reverse KL

## Forward KL

$$
\begin{aligned}
KL(\pi || p) &= \int \pi(\mathbf{x}) \log \frac{\pi(\mathbf{x})}{p(\mathbf{x}|\boldsymbol{\theta})} d\mathbf{x} \\
&= -\mathbb{E}_{\pi(\mathbf{x})} \log p(\mathbf{x}|\boldsymbol{\theta}) + \text{const} \to \min_{\boldsymbol{\theta}}
\end{aligned}
$$

Maximum likelihood estimation is equivalent to minimization of the Monte-Carlo estimation of forward KL.

## Forward KL for flow model

$$
\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|
$$

- We need to be able to compute $f(\mathbf{x}, \boldsymbol{\theta})$ and its Jacobian.
- We need to be able to compute the density $p(\mathbf{z})$.
- We don't need to think about computing the function $g(\mathbf{z}, \boldsymbol{\theta}) = f^{-1}(\mathbf{z}, \boldsymbol{\theta})$ until we want to sample from the flow.

# Forward KL vs Reverse KL

## Reverse KL

$$
\begin{aligned}
KL(p||\pi) &= \int p(\mathbf{x}|\boldsymbol{\theta}) \log \frac{p(\mathbf{x}|\boldsymbol{\theta})}{\pi(\mathbf{x})} d\mathbf{x} \\
&= \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} \left[\log p(\mathbf{x}|\boldsymbol{\theta}) - \log \pi(\mathbf{x})\right] \to \min_{\boldsymbol{\theta}}
\end{aligned}
$$

## Reverse KL for flow model

$$
\log p(\mathbf{z}) = \log p(\mathbf{x}|\boldsymbol{\theta}) + \log \left| \det \left( \frac{\partial g(\mathbf{z}, \boldsymbol{\theta})}{\partial \mathbf{z}} \right) \right|
$$

$$
KL(p||\pi) = \mathbb{E}_{p(\mathbf{z})} \left[ \log p(\mathbf{z}) - \log \left| \det \left( \frac{\partial g(\mathbf{z}, \boldsymbol{\theta})}{\partial \mathbf{z}} \right) \right| - \log \pi(g(\mathbf{z}, \boldsymbol{\theta})) \right]
$$

- We need to be able to compute $g(\mathbf{z}, \boldsymbol{\theta})$ and its Jacobian.
- We need to be able to sample from the density $p(\mathbf{z})$ (do not need to evaluate it).
- We don't need to think about computing the function $f(\mathbf{x}, \boldsymbol{\theta})$.

# Flow KL duality

### Theorem

Fitting flow model $p(\mathbf{x}|\boldsymbol{\theta})$ to the target distribution $\pi(\mathbf{x})$ using forward KL (MLE) is equivalent to fitting the induced distribution $p(\mathbf{z}|\boldsymbol{\theta})$ to the base $p(\mathbf{z})$ using reverse KL:

$$\arg\min_{\boldsymbol{\theta}} KL(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) = \arg\min_{\boldsymbol{\theta}} KL(p(\mathbf{z}|\boldsymbol{\theta})||p(\mathbf{z})).$$

- $p(\mathbf{z})$ is a base distribution; $\pi(\mathbf{x})$ is a data distribution;
- $\mathbf{z} \sim p(\mathbf{z})$, $\mathbf{x} = g(\mathbf{z}, \boldsymbol{\theta})$, $\mathbf{x} \sim p(\mathbf{x}|\boldsymbol{\theta})$;
- $\mathbf{x} \sim \pi(\mathbf{x})$, $\mathbf{z} = f(\mathbf{x}, \boldsymbol{\theta})$, $\mathbf{z} \sim p(\mathbf{z}|\boldsymbol{\theta})$;

$$\log p(\mathbf{z}|\boldsymbol{\theta}) = \log \pi(g(\mathbf{z}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial g(\mathbf{z}, \boldsymbol{\theta})}{\partial \mathbf{z}} \right) \right|;$$

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|.$$

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*Papamakarios G. et al. Normalizing flows for probabilistic modeling and inference, 2019*

# Flow KL duality

## Theorem

Fitting flow model $p(\mathbf{x}|\boldsymbol{\theta})$ to the target distribution $\pi(\mathbf{x})$ using forward KL (MLE) is equivalent to fitting the induced distribution $p(\mathbf{z}|\boldsymbol{\theta})$ to the base $p(\mathbf{z})$ using reverse KL:

$$\arg\min_{\boldsymbol{\theta}} KL(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})) = \arg\min_{\boldsymbol{\theta}} KL(p(\mathbf{z}|\boldsymbol{\theta})||p(\mathbf{z})).$$

## Proof

$$KL\left(p(\mathbf{z}|\boldsymbol{\theta})||\pi(\mathbf{z})\right) = \mathbb{E}_{p(\mathbf{z}|\boldsymbol{\theta})}\left[\log p(\mathbf{z}|\boldsymbol{\theta}) - \log p(\mathbf{z})\right] =$$

$$= \mathbb{E}_{p(\mathbf{z}|\boldsymbol{\theta})}\left[\log \pi(g(\mathbf{z},\boldsymbol{\theta})) + \log\left|\det\left(\frac{\partial g(\mathbf{z},\boldsymbol{\theta})}{\partial \mathbf{z}}\right)\right| - \log p(\mathbf{z})\right] =$$

$$= \mathbb{E}_{\pi(\mathbf{x})}\left[\log \pi(\mathbf{x}) - \log\left|\det\left(\frac{\partial f(\mathbf{x},\boldsymbol{\theta})}{\partial \mathbf{x}}\right)\right| - \log p(f(\mathbf{x},\boldsymbol{\theta}))\right] =$$

$$= \mathbb{E}_{\pi(\mathbf{x})}\left[\log \pi(\mathbf{x}) - \log p(\mathbf{x}|\boldsymbol{\theta})\right] = KL(\pi(\mathbf{x})||p(\mathbf{x}|\boldsymbol{\theta})).$$

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*Papamakarios G., Pavlakou T., Murray I. Masked Autoregressive Flow for Density Estimation, 2017*

# Jacobian structure

## Flow log-likelihood

$$\log p(\mathbf{x}|\boldsymbol{\theta}) = \log p(f(\mathbf{x}, \boldsymbol{\theta})) + \log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right|$$

The main challenge is a determinant of the Jacobian.

## What is a det of Jacobian in the following cases?

1. Consider a linear layer $\mathbf{z} = \mathbf{W}\mathbf{x}$.
2. Let $\mathbf{z}$ be a permutation of $\mathbf{x}$.
3. Let $z_i$ depend only on $\mathbf{x}_i$.

$$\log \left| \det \left( \frac{\partial f(\mathbf{x}, \boldsymbol{\theta})}{\partial \mathbf{x}} \right) \right| = \log \left| \prod_{i=1}^{m} f_i'(x_i, \boldsymbol{\theta}) \right| = \sum_{i=1}^{m} \log \left| f_i'(x_i, \boldsymbol{\theta}) \right|.$$

4. Let $z_i$ depend only on $\mathbf{x}_{1:i}$ (autoregressive dependency).

# Residual Flows

## Matrix determinant lemma

$$\det \left( \mathbf{I}_m + \mathbf{V}\mathbf{W}^T \right) = \det(\mathbf{I}_d + \mathbf{W}^T\mathbf{V}), \quad \text{where } \mathbf{V}, \mathbf{W} \in \mathbb{R}^{m \times d}.$$

## Planar flow

$$\mathbf{x} = g(\mathbf{z}, \boldsymbol{\theta}) = \mathbf{z} + \mathbf{u}\, \sigma(\mathbf{w}^T\mathbf{z} + b).$$

Here $\boldsymbol{\theta} = \{\mathbf{u}, \mathbf{w}, b\}$, $\sigma(\cdot)$ is a smooth element-wise non-linearity.

$$\left| \det \left( \frac{\partial g(\mathbf{z}, \boldsymbol{\theta})}{\partial \mathbf{z}} \right) \right| = \left| \det \left( \mathbf{I} + \sigma'(\mathbf{w}^T\mathbf{z} + b)\mathbf{w}\mathbf{u}^T \right) \right| = \left| 1 + \sigma'(\mathbf{w}^T\mathbf{z} + b)\mathbf{w}^T\mathbf{u} \right|$$

The transformation is invertible, for example, if

$$\sigma = \tanh; \quad \sigma'(\mathbf{w}^T\mathbf{z} + b)\mathbf{u}^T\mathbf{w} \geq -1.$$

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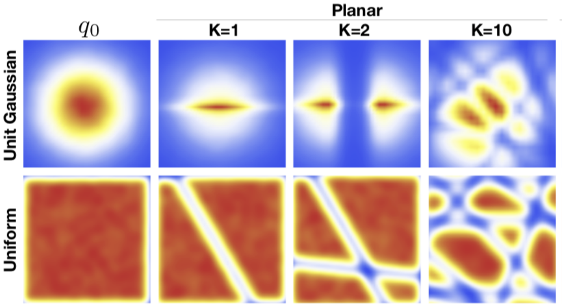
*Rezende D. J., Mohamed S. Variational Inference with Normalizing Flows, 2015*

# Residual Flows

## Expressiveness of planar flows

$$\mathbf{z}_K = g_1 \circ \cdots \circ g_K(\mathbf{z}); \quad g_k = g(\mathbf{z}_k, \boldsymbol{\theta}_k) = \mathbf{z}_k + \mathbf{u}_k \, \sigma(\mathbf{w}_k^T \mathbf{z}_k + b_k).$$




## Sylvester flow: planar flow extension

$$g(\mathbf{z}, \boldsymbol{\theta}) = \mathbf{z} + \mathbf{V} \, \sigma(\mathbf{W}^T \mathbf{z} + \mathbf{b}).$$

*Rezende D. J., Mohamed S. Variational Inference with Normalizing Flows, 2015*
*Berg R. et al. Sylvester normalizing flows for variational inference, 2018*

# Linear flows

## RealNVP

$$\begin{cases} \mathbf{z}_{1:d} = \mathbf{x}_{1:d}; \\ \mathbf{z}_{d:m} = \tau(\mathbf{x}_{d:m}, c(\mathbf{x}_{1:d})); \end{cases} \quad \Leftrightarrow \quad \begin{cases} \mathbf{x}_{1:d} = \mathbf{z}_{1:d}; \\ \mathbf{x}_{d:m} = \tau^{-1}(\mathbf{z}_{d:m}, c(\mathbf{z}_{1:d})). \end{cases}$$

- First step is a **split** operator which decouples a variable into 2 subparts: $\mathbf{x}_1$ and $\mathbf{x}_2$ (usualy channel-wise).
- We should **permute** components between different layers.

$$\mathbf{z} = \mathbf{W}\mathbf{x}, \quad \mathbf{W} \in \mathbb{R}^{m \times m}$$

In general, we need $O(m^3)$ to invert matrix.

## Invertibility

- Diagonal matrix $O(m)$.
- Triangular matrix $O(m^2)$.
- It is impossible to parametrize all invertible matrices.

# Linear flows

$$\mathbf{z} = \mathbf{W}\mathbf{x}, \quad \mathbf{W} \in \mathbb{R}^{m \times m}$$

## Matrix decompositions

- LU-decomposition
$$\mathbf{W} = \mathbf{P}\mathbf{L}\mathbf{U},$$
where **P** is a permutation matrix, **L** is lower triangular with positive diagonal, **U** is upper triangular with positive diagonal.
- QR-decomposition
$$\mathbf{W} = \mathbf{Q}\mathbf{R},$$
where **Q** is an orthogonal matrix, **R** is an upper triangular matrix with positive diagonal.

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*Kingma D. P., Dhariwal P. Glow: Generative Flow with Invertible 1x1 Convolutions, 2018*
*Hoogeboom E., Van Den Berg R., and Welling M. Emerging convolutions for generative normalizing flows, 2019*

# Glow samples

*Kingma D. P., Dhariwal P. Glow: Generative Flow with Invertible 1x1 Convolutions, 2018*

# Summary

- Flow models transform a simple base distribution to a complex one via a sequence of invertible transformations with tractable Jacobian.
- Flow models have a tractable likelihood that is given by the change of variable theorem.
- Flows could be fitted using forward and reverse KL minimization. We will consider each of the scenarios later in the course.
- Planar and Sylvester flows are residual flows which use matrix determinant lemma.
- Linear flows try to parametrize set of invertible matrices via matrix decompositions.