

$$1) A = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{cases} -x - 6y = \lambda x \\ 2x + 6y = \lambda y \end{cases} \Rightarrow x = \lambda x + \lambda y \Rightarrow x(1 - \lambda) = \lambda y \Rightarrow$$

$$\Rightarrow \lambda = 2 \quad x = 2 \quad y = -1$$

$$\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$2) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{matrix} -x = \lambda x \\ -y = \lambda y \end{matrix} \Rightarrow \lambda = -1 \text{ για οποιοδήποτε } \lambda = -1$$

3)

$$\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \lambda = 2 \quad \text{eigenvalue}$$

$$\begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ -3 & 0 & 0 \\ -4 & 0 & 0 \end{pmatrix} \quad \lambda = 3 \quad \neq 1 \quad \begin{pmatrix} 3 & 0 & 0 \\ -3 & 0 & 0 \\ -4 & 0 & 0 \end{pmatrix}$$

we observe