8.1) $U = 3 - 8x + 6y$ если $x^2 + y^2 = 36$

$L(x, y, \lambda) = \dots + \lambda \cdot (x^2 + y^2 - 36)$

$$\begin{cases} L'_x = -8 + 2x\lambda \\ L'_y = 6 + 2y\lambda \\ L'_\lambda = x^2 + y^2 - 36 \end{cases}$$

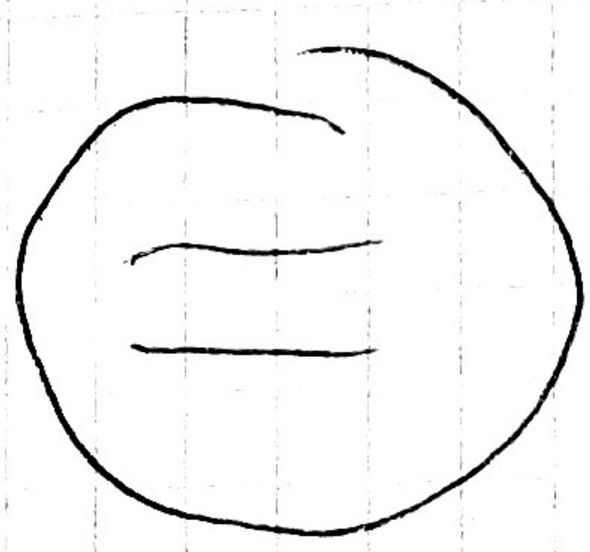
$$\begin{cases} 2x\lambda = 8 \Rightarrow 2x = 2\lambda \cdot \frac{8}{\lambda} \Rightarrow \frac{4}{\lambda} \\ y = \frac{6}{2\lambda} = -\frac{3}{\lambda} \\ \left(\frac{4}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 - 36 = 0 \Rightarrow \frac{16}{\lambda^2} + \frac{9}{\lambda^2} = 36 \end{cases}$$

$$\frac{25}{\lambda^2} = 36 \Rightarrow \lambda = \frac{5}{6}$$

$$\begin{cases} x_{1,2} \\ y_{1,2} \\ \lambda = \frac{5}{6} \\ L''_{xx} = 2\lambda \\ L''_{yy} = 2\lambda \\ L''_{\lambda\lambda} = 0 \end{cases}$$

$x = \frac{4}{\lambda} = \frac{4}{\frac{5}{6}} = \frac{24}{5}$

$y = -\frac{3}{\lambda} = -\frac{3}{\frac{5}{6}} = -\frac{18}{5}$



11) $\frac{8}{6} \cdot \frac{6}{5} = \frac{24}{5}$

$$\begin{cases} x_1 = \\ y_1 = -\frac{18}{5} \\ \lambda_1 = \frac{5}{6} \end{cases}$$

$$\begin{cases} x_2 = -\frac{24}{5} \\ y_2 = \frac{18}{5} \\ \lambda_2 = -\frac{5}{6} \end{cases}$$

$$L'_{xx} = 2\lambda$$

$$L'_{yy} = 2\lambda$$

$$L'_{\lambda\lambda} = 0$$

$$L'_{xy} = L'_{yx} = 0$$

$$L'_{x\lambda} = L'_{\lambda x} = 2x$$

$$L'_{y\lambda} = L'_{\lambda y} = 2y$$

$$\Delta = \begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{vmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\Delta_1 = 0$$

$$\Delta = -8\lambda(x^2+y^2) = -8 \cdot 36\lambda = -288\lambda$$

$$\lambda = \frac{5}{6} \Rightarrow -288 \cdot \frac{5}{6} < 0 \quad \text{min}$$

$\lambda_2 - \frac{5}{6} \Rightarrow -288 \cdot \left(-\frac{5}{6}\right) > 0$ max

№ 2) $L = 2x^2 + 12xy + 32y^2 + 15$

если $x^2 + 16y^2 = 64$

$$\begin{cases} L'_x = 4x + 12y + 2\lambda x \\ L'_y = 12x + 64y + 32\lambda y \\ L'_\lambda = x^2 + 16y^2 - 64 \end{cases} \qquad y = \frac{-\lambda x - 2x}{6}$$

$$12x + 64\left(\frac{-\lambda x - 2x}{6}\right) + 32\lambda\left(\frac{-\lambda x - 2x}{6}\right) \Rightarrow$$

$$\Rightarrow \lambda_{1,3} = -\frac{1}{2} \qquad \lambda_2 = -\frac{7}{2}$$

$$y_1 = \frac{\frac{1}{2}x - 2x}{6} = \frac{-\frac{3}{2}x}{6} = -\frac{3x}{2} \cdot \frac{1}{6} = -\frac{1}{4}x$$

$$y_2 = \frac{\frac{7}{2}x - 2x}{6} = \frac{\frac{3}{2}x}{6} = \frac{3}{2}x \cdot \frac{1}{6} = \frac{3x}{12} = \frac{1}{4}x$$

$\Rightarrow$

$$x^2 + 16\left(-\frac{1}{4}x\right)^2 - 64 = 0$$

$$x^2 + \frac{16}{16}x^2 - 64 = 0$$

$$2x^2 = 64$$

$$x^2 = 32$$

$$x_1 = \pm\sqrt{32} \qquad y_1 = \mp\frac{\sqrt{32}}{4} = \mp\sqrt{2}$$

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$$x_2 = \mp\sqrt{32} \qquad y_2 = \mp\frac{\sqrt{32}}{4} = \pm\frac{\sqrt{16\cdot 2}}{4} = \pm\sqrt{2}$$

$$L''_{xx} = 4 + 2\lambda$$

$$L''_{yy} = 64 + 32\lambda$$

$$L''_{\lambda\lambda} = 0$$

$$L''_{yx} = L''_{xy} = 12$$

$$L''_{\lambda x} = L''_{x\lambda} = 2x$$

$$L''_{y\lambda} = L''_{\lambda y} = 32y$$

$$\begin{pmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda & 12 \\ 32y & 12 & 64+32\lambda \end{pmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Ц № 8.2

$$0\cdot\begin{vmatrix} 4+2\lambda & 12 \\ 12 & 64+32\lambda \end{vmatrix} - 2x\cdot\begin{vmatrix} 2x & 12 \\ 32y & 64+32\lambda \end{vmatrix} +$$

$$+32y\begin{vmatrix} 2x & 4+2\lambda \\ 32y & 12 \end{vmatrix} =$$

$$= -2x^2(64+32\lambda) - 2x\cdot 12\cdot 32y + 32y\,12\cdot 2x -$$

$$- 32^2y^2(4+2\lambda) = -64x^2(2+\lambda) - 32^2\cdot 2y^2(2+\lambda) =$$

$$= -64(2+\lambda)(x^2+32y^2)$$

$\lambda = -\frac{1}{2}$ $\Delta > 0$ — $\max$ в точке $\left(-\sqrt{32};\sqrt{2};-\frac{1}{2}\right)$

$\left(\sqrt{32};-\sqrt{2};-\frac{1}{2}\right)$

$\lambda = -\frac{7}{2}$ $\Delta < 0$ — $\min$ в точке $\left(\sqrt{32};\sqrt{2};-\frac{7}{2}\right)\left(-\sqrt{32};-\sqrt{2};-\frac{7}{2}\right)$