$$U = x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26$$

$$U'_x = 3x^2 + 3y^2 - 39$$

$$U'_y = 6xy - 36$$

$$U'_z = 2z + 2$$

~~$U''_{xy} = 6y$~~

$$U''_{xx} = 6x \qquad U''_{zx} = 0$$

$$U''_{yy} = 6x \qquad U''_{zy} = 0$$

$$U''_{zz} = 2$$

$$U''_{xy} = 6y$$

$$U''_{xz} = 0$$

$$U''_{yx} = 6y$$

$$U''_{yz} = 0$$

(1)

2) $U = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$

$U'_x = -\frac{256}{x^2} + \frac{2x}{y}$

$U'_y = -\frac{x^2}{y^2} + \frac{2y}{z}$

$U'_z = 2z - \frac{y^2}{z^2}$

$U'_{xx} = \frac{512}{x^3} + \frac{2}{y}$

$U'_{xy} = -\frac{2x}{y^2}$

$U'_{xz} = 0$

$U'_{yy} = 2\left(\frac{x}{y^3} + \frac{1}{z}\right)$

$U'_{yx} = -\frac{2x}{y^2}$

$U'_{yz} = -\frac{2y}{z^2}$

$U'_{zz} = \frac{2y^2}{z^3} + 2$

$U'_{zx} = 0$

$U'_{zy} = -\frac{2y}{z^2}$

$U = x^2 + y^2 + z^2$

$U'_x = 2x$

$U'_y = 2y$

$U'_z = 2z$

$\text{grad } U = (2x, 2y, 2z)$

! Точке $M(8, -12, 9)$ Grad

$= (16, -24, 18)$

$\vec{|c|} = \sqrt{(-9)^2 + 8^2 + (-17)^2} = \sqrt{81 + 64 + 144} = \sqrt{289} = 17$

$\vec{c_0} = \frac{\vec{c}}{\vec{|c|}} = \left(\frac{8}{17}, \frac{-12}{17}, \frac{9}{17}\right)$

$\frac{8}{17} \cdot \frac{16}{8} - \frac{12}{17} \cdot (-24) + \frac{9}{17} \cdot 18 = \frac{128}{17} + \frac{288}{17} + \frac{162}{17} =$

$= \frac{578}{17} = 34$

4) $U = e^{x^2+y^2+z^2}$ $\vec{c}(4, -13, -16)$

$L(-16, 4, -13)$

$U'_x = 2x \cdot e^{x^2+y^2+z^2}$

$U'_y = 2y \cdot e^{x^2+y^2+z^2}$

$U'_z = 2z \cdot e^{x^2+y^2+z^2}$

$\text{Grad}\left(2x \cdot e^{x^2+y^2+z^2}, 2y \cdot e^{x^2+y^2+z^2}, 2z \cdot e^{x^2+y^2+z^2}\right)$

в точке $L(-16, 4, -13)$

$\text{Grad}\left(-32 \cdot e^{441}, 8e^{441}, -26e^{441}\right)$

$\vec{|c|} = \sqrt{4^2 + (-13)^2 + (-16)^2} = \sqrt{441} = 21$

$\vec{c_0} = \frac{4}{21}, -\frac{13}{21}, -\frac{16}{21}$

$\left(-32 \cdot e^{441}\right)\frac{4}{21} - \frac{13(8e^{441})}{21} + \frac{16(26e^{441})}{21} =$

$= \frac{(-128e^{441}) - (104e^{441}) + (416e^{441})}{21} =$

$= \frac{184e^{441}}{21}$

6)

$$U = x^2y + \frac{1}{3}y^3 + 2x^2 + 3y^2 - 1$$

$$\begin{cases} U'_x = 2xy + 4x = 0 \\ U'_y = x^2 + y^2 + 6y = 0 \end{cases}$$

$y = 0$ $\quad$ $x = 0$

$y = -2$

$y = -2$

$x^2 + (-2)^2 + 6(-2) = 0$

$x^2 + 4 - 12 = 0$

$x^2 = 8$

$x = \sqrt{8}$

$y^2 + 6y = 0$

$y_1 = 0$

$y_2 = -6$

$x_1 = 0$

$x_2 = 2x(-6) + 4x = -12x + 4x = 0 \quad \Rightarrow 0$

$(0; 0)$ $(0; -6)$

$$z''_{xx} = 2y - 4$$

$$z''_{yy} = 2y + 6$$

$$z''_{xy} = 2x$$

$$z''_{yx} = 2x$$

$(0;0)$

$\Delta_1 = 2y - 4 = 4 > 0$

$\Delta_2 = 4y^2 - 4x^2 + 20y + 24 = 24 > 0$

za $(0;0)$ stacionarna točka

$(0;0)$ lokalna točka min

$(0;-6)$ točka max

$\Delta_1 = 2y + 4 = -8 < 0$

$\Delta_2 = 4 \cdot (-6)^2 - 0 + 20(-6) + 24 = 144 - 120 + 24$

$= 48 > 0$