1) а) $\lim\limits_{n\to\infty} \dfrac{(23-2n^2)(3n^2+17)^2}{4n^6+n-1} = \dfrac{(23-2n^2)(9n^4+102n^2+289)}{4n^6+n-1} =$

$= \dfrac{-18n^6+3n^4+1768n^2+6647}{4n^6+n-1} = -\dfrac{18}{4} = -\dfrac{9}{2}$

б) $\lim\limits_{n\to\infty} \dfrac{(97-2n)^3}{2n(3n^2+15)+8n} = \dfrac{-8n^3+1164n^2-56454n+912673}{6n^3+30n+8n} = -\dfrac{8}{6} = -\dfrac{4}{3}$

в) $\lim\limits_{n\to\infty} \dfrac{2n^3+13n(n+18)}{(27-n)(2n+19)^2} = \dfrac{2n^3+13n^2+234n}{-4n^3+32n^2+1691n+9747} = -\dfrac{2}{4} = -\dfrac{1}{2}$

2) $\lim\limits_{n\to\infty} (\sqrt{n^2+1}-n) = \dfrac{\sqrt{n^2+1}-n}{1} = \dfrac{(\sqrt{n^2+1}-n)(\sqrt{n^2+1}-n)}{1(\sqrt{n^2+1}-n)} =$

$= \dfrac{(\sqrt{n^2+1})^2-n^2}{(\sqrt{n^2+1}-n)} = \dfrac{n^2+1-n^2}{\sqrt{n^2+1}-n} = \dfrac{1}{\infty} = 0$

д) $\lim\limits_{n\to\infty} \dfrac{(-4)^n+5\cdot 7^n}{(-4)^{n-1}+7^{n+2}} = \dfrac{5\cdot 7^n+(-4)^n}{7^{n+2}+(-4)^{n-1}} = \dfrac{(5\cdot 7^n+(-4)^n)\frac{1}{7^{n+2}}}{(7^{n+2}+(-4)^{n-1})\frac{1}{7^{n+2}}} =$

$= \dfrac{\frac{5}{49}+\dfrac{\left(-\frac{4}{7}\right)^n=\infty}{49}}{1+\dfrac{\left(-\frac{4}{7}\right)^{x-1}=\infty}{343}} = \dfrac{\frac{5}{49}}{1} = \dfrac{5}{49}$

2) $\dfrac{1}{6}+\dfrac{1}{3}+\dfrac{1}{2} = 1$

3) $\dfrac{1}{18}+\dfrac{1}{9}+\dfrac{1}{6}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{12} = 1$

4) $|x_{n+p}-x_n| = \left|\dfrac{\sin\alpha}{2}+\dfrac{\sin 2\alpha}{2^2}+\dfrac{\sin(n+p)\alpha}{2^{n+p}} - \left(\dfrac{\sin\alpha}{2}+\dfrac{\sin 2\alpha}{2^2}+\dfrac{\sin\alpha}{2^n}\right)\right| =$

$= \left|\dfrac{\sin(n+1)\alpha}{2^{n+1}}+\dfrac{\sin(n+2)\alpha}{2^{n+2}}+\dots\dfrac{\sin(n+p)\alpha}{2^{n+p}}\right| \leq \dfrac{1}{2^{n+1}}+\dfrac{1}{2^{n+2}}+\dots\dfrac{1}{2^{n+p}} <$

$< \dfrac{1}{2^{n+1}}+\dfrac{1}{2^{n+2}}+\dots+\dfrac{1}{2^{n+n}} = \dfrac{\frac{1}{2^{n+1}}}{1-\frac{1}{2}} = \dfrac{2}{2^{n+1}} = \dfrac{1}{2^n} < \varepsilon$

является сходящейся