Topics in Applied Econometrics for Public Policy

TA Session 5

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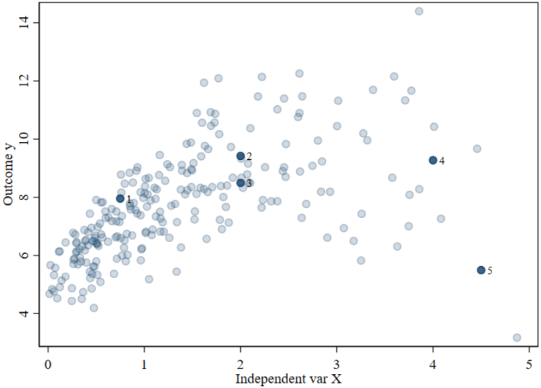
Plan for Today

- Conditional Quantile Regression (Koenjer and Bassett 1978)
- Unconditional Quantile Regression (Firpo, Fortin and Kemieux 2009)
- Quantile Treatment Effects (Firpo 2007)

Motivation

Suppose we have a linear model such that:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + v_i(\alpha_0 + \alpha_1 x_i)$$



Interpretation

Individual effects: How does y change for individual i when there is a change in x ?

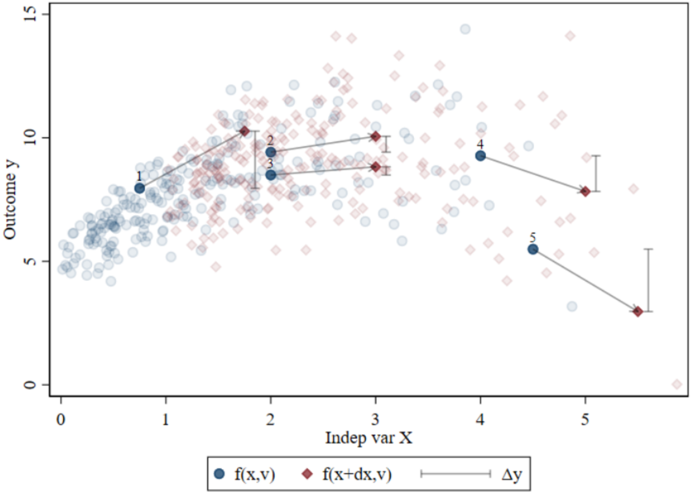
$$\frac{\partial y_i}{\partial x_i} = \beta_1 + 1\beta_2 x_i + v_i \alpha_1$$

Unless the model is homoskedastic we can't interpret the individual effect.

Suppose we increase x by one.

Interpretation

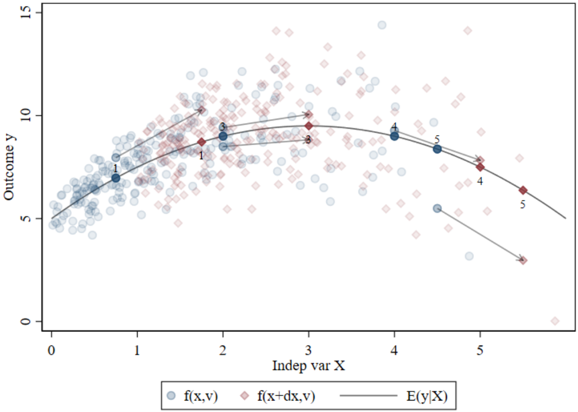
The resulting increase in y is:



Condtiontal Effects

Another possible interpretation is what is the effect conditioning on covariates. As long as the unobserved has 0 expected value, we will get:

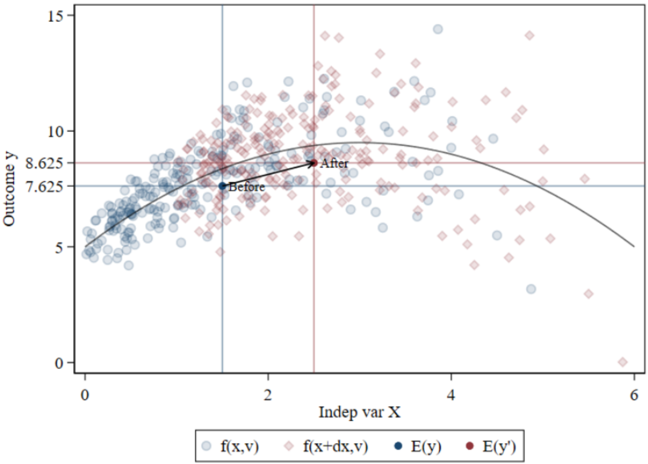
$$\frac{\partial E(y_i|X)}{\partial X} = \beta_1 + \beta_2 x$$



Unconditional Effects

The effect on the expected value of the covariates:

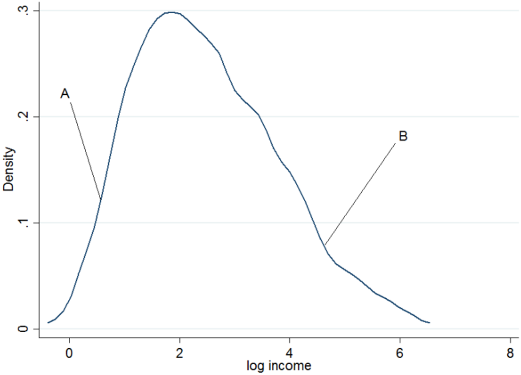
$$\frac{\partial E(y_i)}{\partial E(x_i)} = \beta_1 + \beta_2 E(x_i)$$



Interpretation in Quantiles

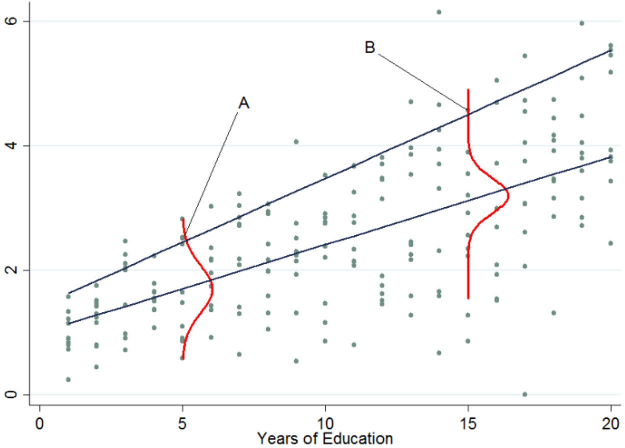
Suppose we have the following model

$$logy_i = \beta_0 + \beta_1 x_i + \epsilon_i$$



Interpretation Quantiles

Once we condition a person that was at the bottom of the unconditional distribution is now at the top.



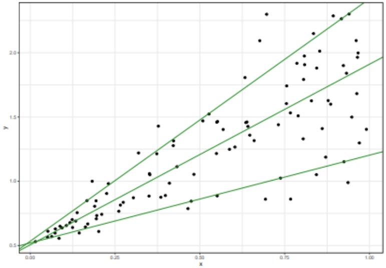
Interpretation Quantiles

In the OLS we can move from $E[y_i|x_i]$ to $E[y_i]$ applying the law of interated expectations.

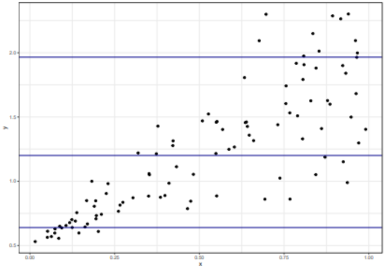
However, this is not possible with quantiles, so $Q_\tau(y_i|x_i) \neq Q_\tau(y_i)$

Interpretation: A $\beta_{90} = 0.13$ means that an additional year of education increases the earnings in the 90th percentile of the conditional earnings distribution. The problem is that we can't say anything about how it affects individuals.

Conditional vs Unconditional



(a) Conditional Quantiles



(b) Unconditional Quantiles

Estiation of Quantile Treatment Effects with Stata

We will use *ivqte*. There are several models that can be estimated:

- Conditional QTEs with exogenous treatment: Koenker and Basset (1978)
- Conditional QTEs with endogenous treatment: Abadie,Angrist and Imbens (2002)
- Unconditional QTEs with exogenous treatment: Firpo (2007)
- Unconditional QTEs with endogenous treatment: Frölich and Melly (2008)

Framework

- Consider a binary treatment variable D on a continuous variable Y.
- Let Y_i^1 and Y_i^0 be the potential outcomes of individual i. The observed outcome is $Y_i = Y_i^1 D + Y_i^0(1 - D)$
- We also observe some characteristics X.
- We will deal with seletion on observables and on unobservables.

Conditional Exogenous QTE

The model is:

$$Y_i^d = X_i \beta^\tau + d\delta^\tau + \epsilon_i$$

And we assume selection on observables with exogenous X. This imply that:

$$Q^\tau_{Y|X,D} = X\beta^\tau + D\delta^\tau$$

And can be estimated by the classical regression estimator by Koenker and Basset (1978)

$$(\hat{\beta}^\tau, \hat{\delta}^\tau) = \min_{\beta,\delta} \sum \rho_\tau (Y_i - X_i\beta - D_i\delta)$$

where ρ_τ is the check function.

Unconditonal QTEs

The unconditional effect is given by:

$$\Delta^{\tau} = Q^{\tau}_{Y^1} - Q^{\tau}_{Y^0}$$

- The definition of unconditional QTE does not change when we change the set of covariates X.
- However, we will use covariates for identification and efficiency purposes.
- The idea is to introduce covariates ina frst regression and then integreate them out.

Unconditonal Endogenous QTEs

We wil use a binary instrument Z. Frölich and Melly (2008) showed that Δ^{τ} for the compliers is estimated by:

$$(\hat{\alpha}_{IV}, \hat{\Delta_{IV}}^{\tau}) = arg \min_{\alpha,\Delta} \sum W_i^{FM} \times \rho_{\tau}(Y_i - \alpha - D_i\Delta)$$

where:

$$W_i^{FM} = \frac{Z_i - Pr(Z = 1|X_i)}{Pr(Z = 1|X)\{1 - Pr(Z = 1|X)\}}(2D_i - 1)$$

With α_{IV} identified by $D = 0$ and $\alpha_{IV} + \Delta_{IV}^{\tau}$ by $D = 1$.

Weights find compilers and balances the distribution of covariates between treated and untreated.

$P(Z = 1|X)$ is estimated with a logit.

Unconditoinal Exogenous QTEs

We now assume that X contains all confunding variables. Also, we assume that the support of the covariates is the same indemedent of the treatment.

The estimator of Firpo (2007) is

$$(\hat{\alpha}, \hat{\Delta}^{\tau}) = arg \min_{\alpha,\Delta} \sum W_i^F \times \rho_{\tau}(Y_i - \alpha - D_i\Delta)$$

where

$$W_i^F = \frac{D_i}{Pr(D = 1|X)} + \frac{1 - D_i}{1 - Pr(D = 1|X_i)}$$

which is the traditional propensity-score weighting estimator, also known as the inverse probability weighting.

We estimate $Pr(D = 1|X)$ with a logit model.

Stata Implementation

Stata implementation