Dynamic Structural Models for Policy Evaluation

Sergi Quintana

Universitat Autònoma de Barcelona

Barcelona School of Economics

BSE Summer School, 2024

#bsesummer

Practical Sessions Outline

1. Solving Value Functions
2. Solving the model
3. **Conditional Choice Probability (CCP) estimation**
4. CCP estimation - finite dependence

Session 3: CCP estimation

BSE Summer School, 2024

Barcelona School of Economics
SUMMER SCHOOL

#bsesummer

Session Outline

Conditional Choice Probability

Model

Results

Barcelona School of Economics
SUMMER SCHOOL

#bsesummer

Conditional Choice Probability

Introduction

- Based on the seminal work of **Hotz and Miller (1993)**
- **Idea:** use the mapping between conditional value functions $v_{jt}(\boldsymbol{x_t})$ and CCP probabilities $p_t(\boldsymbol{x_t})$
- Write DP as a function of data, parameters and CCP:

$$\begin{aligned} v_{jt}(\boldsymbol{x}_t) &= u_{jt}(\boldsymbol{x}_t) + \beta \int V_{t+1}(\boldsymbol{x}_{t+1}) dF_x(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, j) \\ &= u_{jt}(\boldsymbol{x}_t) + \beta \int \left[v_{kt+1}(\boldsymbol{x}_{t+1}) + \psi_k(\boldsymbol{p}_{t+1}(\boldsymbol{x}_{t+1}))\right] dF_x(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, j) \end{aligned}$$

  where $k \in D$, D is a choice set.
- Two types of problems:
  1. **One-period-ahead** CCP
  2. **$\boldsymbol{\rho}$-periods-ahead** CCP

Conditional Choice Probability

One-period-ahead CCP

- First, let's focus on Rust example:

$$p_1(x) = \frac{e^{v_1(x)}}{e^{v_1(x)} + e^{v_0(x)}} = \frac{1}{1 + e^{v_0(x) - v_1(x)}}$$

- Then:

$$v_1(x) - \ln p_1(x) = \ln\left(e^{v_1(x)} + e^{v_0(x)}\right) = \ln\left(\sum_{h \in D} \exp\{v_h(x)\}\right)$$

- Substituting it into expression for conditional value function:

$$\begin{aligned} v_j(x_t) &= u_j(x_t) + \beta \sum_{x \in X} \ln\left(\sum_{h \in D} \exp\{v_h(x)\}\right) F^j_{x,x_t} + \beta\gamma \\ &= \ldots = u_j(x_t) + \beta v_1(0) - \beta \sum_{x \in X} \ln p_1(x) F^j_{x,x_t} + \beta\gamma \end{aligned}$$

Conditional Choice Probability

One-period-ahead CCP

- Using the fact that $\sum_{x \in X} \ln p_1(x) F^1_{x,x_t} = \ln p_1(0)$, the difference of conditional value functions becomes:

$$
\begin{aligned}
v_0(x_t) - v_1(x_t) &= u_0(x_t) - u_1(x_t) + \beta \Big(\ln p_1(0) - \sum_{x \in X} \ln p_1(x) F^0_{x,x_t} \Big) \\
&= \theta_R - \theta_M x_t + \beta \Big(\ln p_1(0) - \sum_{x \in X} \ln p_1(x) F^0_{x,x_t} \Big)
\end{aligned}
$$

- non-parametric estimates of $p_1(x)$ can be obtained from data:

$$
\hat{p}_1(x) = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} d_{1it} \mathbf{1}\{x_{it} = x\}}{\sum_{i=1}^{N} \sum_{t=1}^{T} \mathbf{1}\{x_{it} = x\}}
$$

Conditional Choice Probability

One-period-ahead CCP - Steps

Remember that we have **finite number of states** $a_t = 1, ..., 5$, however **infinite time horizon** ($T = \infty$).

Steps:

1. **Formulate the dynamic programming problem** (conditional value functions $v_{jt}(x_t)$ and *Emax* $V_t(x)$).
2. Formulate **conditional choice probabilities** and map them into conditional value functions.
3. Substitute p_1 by $\hat{p}_1$ in conditional value functions using two possible methods.
4. Construct **log-likelihood function** with the new probabilities.
5. Solve the maximization problem.

Introduction

Empirical example

- Maintenance of **electric bikes' batteries** in Barcelona
- Bikes' batteries can be either **replaced** (R) or **maintained** (M)
- a_t - age of the bike battery
- $a_{t+1} = \begin{cases} 1 & \text{if replaced} \\ a_t + 1 & \text{if maintained} \end{cases}$
- Utility:

$$u(a_t, d_t, \epsilon_t) = \begin{cases} -\theta_R + \epsilon_{1t} & \text{if } d_t = 1 \\ -\theta_{M1} a_t - \theta_{M2} a_t^2 + \epsilon_{0t} & \text{if } d_t = 0 \end{cases}$$

  where ϵ_{0t} and ϵ_{1t} are unobserved by econometrician, iid with all Rust assumptions

- A is a number of states (if $A = 5$ then $a_t = 1, ..., 5$) and T is time horizon

Model
CCP

- Conditional value function:

$$v_{jt}(a_t) = u_j(a_t) + \beta \int V_{t+1}(a_{t+1}) dF_a(a_{t+1}|a_t, j)$$

- Emax:

$$V_t(a) = \ln \sum_{h \in \{0,1\}} \exp\{v_{ht}(a)\} + \gamma$$

- with CCP probabilities:

$$p_{1t}(a_t) = \frac{1}{1 + e^{v_{0t}(a_t) - v_{1t}}}$$

$$p_{0t}(a_t) = \frac{e^{v_{0t}(a_t) - v_{1t}}}{1 + e^{v_{0t}(a_t) - v_{1t}}}$$

Model

$A = 5,\ T = \infty$

- Conditional value functions:

$$
\begin{aligned}
v_{1t} = v_1 &= -\theta_R + \beta\, v_1 - \beta \ln p_1(1) + \beta\gamma \\
v_{01}(1) &= -\theta_{M1} - \theta_{M2} + \beta v_1 - \beta \ln p_1(2) + \beta\gamma \\
v_{02}(2) &= -2\theta_{M1} - 4\theta_{M2} + \beta v_1 - \beta \ln p_1(3) + \beta\gamma \\
v_{03}(3) &= -3\theta_{M1} - 9\theta_{M2} + \beta v_1 - \beta \ln\, p_1(4) + \beta\gamma \\
v_{04}(4) &= -4\theta_{M1} - 16\theta_{M2} + \beta v_1 + \beta\gamma
\end{aligned}
$$

- The differences between alternatives being:

$$
\begin{aligned}
v_{01}(1) - v_1 &= \theta_R - \theta_{M1} - \theta_{M2} + \beta \ln\, p_1(1) - \beta \ln\, p_1(2) \\
v_{02}(2) - v_1 &= \theta_R - 2\theta_{M1} - 4\theta_{M2} + \beta \ln\, p_1(1) - \beta \ln\, p_1(3) \\
v_{03}(3) - v_1 &= \theta_R - 3\theta_{M1} - 9\theta_{M2} + \beta \ln\, p_1(1) - \beta \ln\, p_1(4) \\
v_{04}(4) - v_1 &= \theta_R - 4\theta_{M1} - 16\theta_{M2} + \beta \ln\, p_1(1)
\end{aligned}
$$

Model

$A = 5,\ T = \infty$

- Using this the differences and substituting p_1 by $\hat{p}_1$ (obtained from frequencies or NFXP)
- e.g. probabilities when $a_t = 1$:

$$p_{1t}(1) = \frac{1}{1 + e^{v_{0t}(1) - v_{1t}}} \quad \text{and} \quad p_{0t}(1) = \frac{e^{v_{0t}(1) - v_{1t}}}{1 + e^{v_{0t}(1) - v_{1t}}}$$

- To build a log-likelihood function:

$$\mathcal{L}_N = \sum_{a=1}^{4} \sum_{i=1}^{N} d_t \ln(p_{1t}(a)) + (1 - d_t) \ln(p_{0t}(a))$$

Results

NFXP vs. Hotz& Miller (1993)

Table: Comparison of the results

	NFXP	Hotz and Miller (1993)	
		freq.	NFXP
θ_R	2.010	1.967	2.022
	(0.170)	(0.124)	(0.124)
θ_{M1}	0.196	0.152	0.209
	(0.164)	(0.135)	(0.134)
θ_{M2}	0.099	0.110	0.096
	(0.037)	(0.032)	(0.032)