Dynamic Structural Models for Policy Evaluation

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Practical Sessions Outline

1. Solving Value Functions
2. Solving the model
3. Conditional Choice Probability (CCP) estimation
4. **CCP estimation - finite dependence**

Session 4: CCP estimation - finite dependence

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Session Outline

Conditional Choice Probability

Model

Data

Results

Extensions

Conditional Choice Probability

Introduction

- Based on the seminal paper of **Hotz and Miller (1993)**
- **Idea:** use the mapping between conditional value functions $v_{jt}(\boldsymbol{x_t})$ and CCP probabilities $p_t(\boldsymbol{x_t})$
- Write DP as a function of data, parameters and CCP:

$$v_{jt}(\boldsymbol{x}_t) = u_{jt}(\boldsymbol{x}_t) + \beta \int V_{t+1}(\boldsymbol{x}_{t+1}) dF_x(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, j)$$

$$= u_{jt}(\boldsymbol{x}_t) + \beta \int \left[v_{kt+1}(\boldsymbol{x}_{t+1}) + \psi_k(\boldsymbol{p}_{t+1}(\boldsymbol{x}_{t+1})) \right] dF_x(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, j)$$

- since using the main **theorem** in Hotz and Miller (1993):

$$\psi_k(\boldsymbol{p}_t(\boldsymbol{x}_t)) \equiv V_t(\boldsymbol{x}_t) - v_{kt}(\boldsymbol{x}_t)$$

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Conditional Choice Probability

Introduction

- **ρ-periods-ahead** CCP, $\rho = 1, 2, ..., k$
- Two types of problems:
  1. **Terminal/renewal action** CCP
  2. **Finite Dependence**
- **Finite Dependence**:
  - introduced by Altug and Miller (1998), Arcidiacono and Miller (2011)
  - **Idea:** *After the ρ-periods, the specified combination of actions across the two paths leads to the same distribution of states.*

Infinite Horizon

Finite dependence - Steps

Remember that we have **finite number of states** $x_t = 1, ..., X$, however **infinite time horizon** $(T = \infty)$.

Steps:

1. **Formulate the dynamic programming problem** (conditional value functions $v_{jt}(x_t)$ and $V_t(x)$) using **finite dependence** feature.
2. Formulate *conditional choice probabilities* and map them into conditional value functions.
3. Substitute p_1 by $\hat{p}_1$ in conditional value functions using two possible methods (i.e. frequencies).
4. Construct **log-likelihood function** with the new probabilities.
5. Solve the maximization problem.

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Model

Framework

- Each period t agents decide to **work** (W) or **stay at home** (H)
- x_t - labour market experience of individual (in years)
- $x_{t+1} = \begin{cases} x_t + 1 & \text{if work at } t \\ x_t & \text{if stay home at } t \end{cases}$
- Utility:

$$u(x_t, d_t, \epsilon_t) = \begin{cases} \varphi_0 + \varphi_1 \, x_t + \varphi_2 \, x_t^2 + \epsilon_{Wt} & \text{if} \quad d_t = W \\ \epsilon_{Ht} & \text{if} \quad d_t = H \end{cases}$$

  where ϵ_{0t} and ϵ_{1t} are unobserved by econometrician, iid with all Rust assumptions, $d_t \in \{H, W\}$
- φ_0 represents unemployment benefit or utility from leisure
- Support of $x_t = 1, ..., X$ is finite

Model

Analytical solution

- Formulate conditional value functions:

$$\begin{aligned}
v_{Wt}(x_t) &= u_W(x_t) + \beta V_{t+1}(x_t+1) \\
&= u_W(x_t) + \beta \left(v_{Ht+1}(x_t+1) + \psi_H(\boldsymbol{p}(x_t+1)) \right) \\
&= u_W(x_t) + \beta \left(u_H(x_t+1) + \psi_H(\boldsymbol{p}(x_t+1)) \right) + \beta^2 V_{t+2}(x_t+1)
\end{aligned}$$

$$\begin{aligned}
v_{Ht}(x_t) &= u_H(x_t) + \beta V_{t+1}(x_t) \\
&= u_H(x_t) + \beta \left(v_{Wt+1}(x_t) + \psi_W(\boldsymbol{p}(x_t)) \right) \\
&= u_H(x_t) + \beta \left(u_W(x_t) + \psi_W(\boldsymbol{p}(x_t)) \right) + \beta^2 V_{t+2}(x_t+1)
\end{aligned}$$

- From **Hotz and Miller (1993)**:

$$\psi_W(\boldsymbol{p}(x_t)) = V_t(x_t) - v_{Wt}(x_t)$$

Model

Analytical solution

- Recall from the full solution method:

$$V_t(x_t) = \ln\left(\sum_{h\in D} \exp\{v_{ht}(x_t)\}\right) + \gamma$$

- Conditional choice probabilities are given by:

$$p_W(x_t) = \frac{\exp\{v_{Wt}(x_t)\}}{\sum_{h\in D}\exp\{v_{ht}(x_t)\}} \implies$$

$$\ln\left(\sum_{h\in D} \exp\{v_{ht}(x_t)\}\right) = v_{Wt}(x_t) - \ln p_W(x_t)$$

- As a result:

$$\begin{aligned}\psi_W(\boldsymbol{p}(x_t)) &= V_t(x_t) - v_{Wt}(x_t) = \\ &= v_{Wt}(x_t) - \ln p_W(x_t) + \gamma - v_{Wt}(x_t) = -\ln p_W(x_t) + \gamma\end{aligned}$$

Model

Analytical solution

- The difference between two conditional values (using final dependence):

$$
\begin{aligned}
v_{Wt}(x_t) - v_{Ht}(x_t) &= u_W(x_t) - u_H(x_t) + \beta\big[u_H(x_t+1) - u_W(x_t) \\
&\qquad + \ln p_W(x_t) - \ln p_H(x_t+1)\big] \\
&= (1-\beta)\big[\varphi_0 + \varphi_1\, x_t + \varphi_2\, x_t^2\big] + \beta\big[\ln \hat{p}_W(x_t) - \ln \hat{p}_H(x_t+1)\big]
\end{aligned}
$$

- CPP probabilities:

$$
p_W(x) = \frac{e^{v_W(x)}}{e^{v_H} + e^{v_W(x)}} = \frac{e^{v_W(x) - v_H}}{1 + e^{v_W(x) - v_H}}
$$

$$
p_H(x) = \frac{e^{v_H}}{e^{v_H} + e^{v_W(x)}} = \frac{1}{1 + e^{v_W(x) - v_H}}
$$

Data

Description

- **GSOEP**, panel 1984-2018
- Yearly labor participation decision
- Individuals aged 25 - 60
- Time interval: 2008 - 2012
- **Work:** if worked for 6 or more months per year
- **Experience** = number of working years
- Years of experience: 0 - 45
- Only individuals with complete records both within a year and observed in all panel years
- **Final sample:** 11,515 observations of 2,303 individuals

Data

Statistics

Table: GSOEP, sample statistics

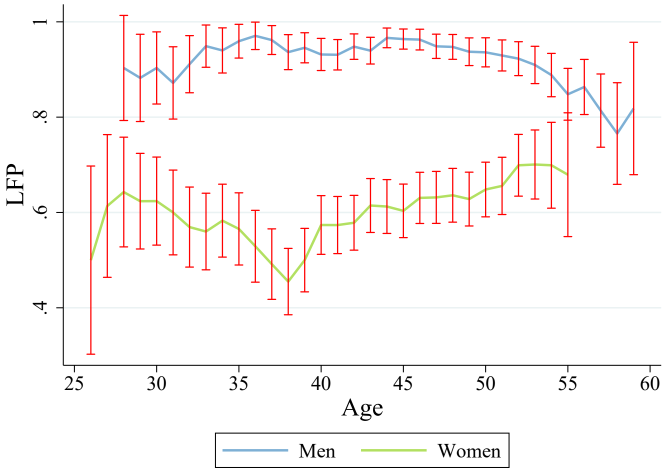
	Full Sample			Subsample		
	female (share)	experience (avg, yrs)	lfp (share)	female (%)	experience (avg, yrs)	lfp (%)
2008	0.500	16.56	0.761	0.503	15.57	0.762
	(0.500)	(9.62)	(0.426)	(0.500)	(8.47)	(0.426)
2009	0.503	16.75	0.754	0.503	16.31	0.758
	(0.500)	(9.59)	(0.431)	(0.500)	(8.65)	(0.428)
2010	0.520	14.30	0.682	0.503	17.06	0.764
	(0.500)	(9.11)	(0.466)	(0.500)	(8.82)	(0.425)
2011	0.520	14.68	0.676	0.503	17.80	0.765
	(0.500)	(9.22)	(0.467)	(0.500)	(8.99)	(0.413)
2012	0.519	14.91	0.705	0.503	18.52	0.769
	(0.500)	(9.28)	(0.456)	(0.500)	(9.16)	(0.422)
Total	0.515	15.19	0.706	0.503	17.05	0.763
	(0.500)	(9.37)	(0.455)	(0.500)	(8.89)	(0.425)
N		33,554			11,515	

Data

Labour force participation in gender

Data

Labour force participation by age



Results

CCP estimation

Table: CCP labor force participation estimation, results

Parameter	All sample	Men	Women
φ_0	**-0.931**	**27.043**	**-0.438**
	(1.190)	(3.284)	(1.342)
φ_1	**0.244**	**-0.324**	**-0.908**
	(0.195)	(0.410)	(0.251)
φ_2	**-0.004**	**-0.014**	**0.022**
	(0.006)	(0.011)	(0.009)

Extensions

Alternative models

- Linear utility:

$$u(x_t, d_t, \epsilon_t) = \begin{cases} \varphi_0 + \varphi_1 \, x_t + \epsilon_{1t} & \text{if } d_t = W \\ \epsilon_{0t} & \text{if } d_t = H \end{cases} \tag{1}$$

- Cubic utility:

$$u(x_t, d_t, \epsilon_t) = \begin{cases} \varphi_0 + \varphi_1 \, x_t + \varphi_2 \, x_t^2 + \varphi_3 \, x_t^3 + \epsilon_{1t} & \text{if } d_t = W \\ \epsilon_{0t} & \text{if } d_t = H \end{cases} \tag{2}$$

Extensions

Results

Table: CCP estimation: extensions, results

Parameter	Quadratic	Linear	Cubic
φ_0	**-0.931**	**-0.355**	**-0.827**
	(1.190)	(0.819)	(1.518)
φ_1	**0.244**	**0.120**	**0.203**
	(0.195)	(0.063)	(0.420)
φ_2	**-0.004**	-	**-0.001**
	(0.006)		(0.031)
φ_3	-	-	**0.7e-4**
			(0.6e-3)

Revision of Examples

Choices over the error distribution

Review

Researchers face trade-offs when choosing this distribution.

- **Normal Errors**: It has two advantages:
  - Flexible correlation structure and is therefore able to capture richer patterns of substitution across choices.
  - Easy to draw from it.
- **GEV**: Three advantages:
  - Closed form expressions for the CCPs.
  - Closed form expression for the expectation of the value function.
  - Easy mapping from CCPs to the value function.

One Period Ahead CCP

Review

Some examples are those with terminal actions:

- Hotz and Miller 1993: Sterilization and fertility choices.
- Ericson and Paker 1995: Dynamic discrete game with permanent exit of a market.
- Joensen (2009): Studies drop out decisions of college.
- Murphy (2018): Landowners who choose to construct a house.

Multiple Period Ahead CCPs

Review

Specific sequence of choices:

- Altug and Miller 1997: consider the first case, focusing on the example of female labor supply with human capital accumulation and depreciation
- Keane and Wolpin 1997: invesments in different human capitals (education or work).

Sometimes finite dependence does not hold but CCPs are still usefull:

- Forward simulation methods.
- Iterate until terminal period.

Extensions

Review

- Finite horizon backward solution by full solution: Keane and Wolpin 1994 interpolate the value function to reduce the computation complexity of the estimation.
- There can be uncertainty in individual's choices: Kennan and Walker 2011.
- Dynamic discrete-continuous choice models. Key assumption in the timming of the model.
  - Iskhakov et al. (2017): Studies consumption and retirement decisions.
  - Murphy (2018): Landowners choose to construct, and once price is realized, they choose the house size.
  - De Groote (2023): Academic track and study effort choice.

Unobserved Heterogeneity

Review

- Expected-Maximization algorithm.

  The problem is that it breaks additive separability and complicates the estimation of the CCPs, Arcidiacono and Miller 2011 and Arcidiacono and Jones 2003 deal with these problems.
- Arcidiacono et al 2024. Measurement system.
- BLM 2022. Two-step grouped fixed-effects (GFE).