Dynamic Structural Models for Policy Evaluation

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Practical Sessions Outline

1. **Solving Value Functions**
2. Estimating the model
3. Conditional Choice Probability (CCP) estimation
4. CCP estimation - finite dependence

Session 1: Solving Value Functions

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Session Outline

Introduction

Infinite Horizon

Alternative Scenarios

Introduction

General framework

- Discrete time, $t = 0, 1, ..., T$
- Every period t individual chooses one of J mutually exclusive alternatives:

$$d_t = \{j : j \in D = \{1, 2, ..., J\}\}$$

where $d_{jt} = 1\{d_t = j\}$ and $\sum_{j=1}^{J} d_{jt} = 1$

- Payoff at t depends on state variables:

$$\boldsymbol{s}_t = \{\boldsymbol{x}_t, \boldsymbol{\varepsilon}_t\}$$

where $\boldsymbol{\varepsilon}_t$ is vector of **unobservables** and $\boldsymbol{s}_t$ follows **Markovian process** $\boldsymbol{s}_{t+1} \sim F(\boldsymbol{s}_{t+1} | \boldsymbol{s}_t, d_t)$

- Individual's intertemporal payoff:

$$\mathbb{E}\left[\sum_{\tau=0}^{T-t} \beta^{\tau} U(\boldsymbol{s}_{t+\tau}, d_{t+\tau})\right]$$

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Introduction

Assumptions

Assumption 1. Additive Separability. Utility function additively separable between observable and unobservable components:

$$U(d_t, \boldsymbol{x}_t, \boldsymbol{\varepsilon}_t) = u(d_t, \boldsymbol{x}_t) + \varepsilon_t(d_t)$$

Assumption 2. IID Unobservables. Unobserved variables $\boldsymbol{\varepsilon}_t$ are iid across agents and over time given $\boldsymbol{x}_t$.

Assumption 3. Conditional Independence. Conditional on current decision and observable state variables, next period observables do not depend on $\boldsymbol{\varepsilon}_t$:

$$F_x(\boldsymbol{x}_{t+1}|d_t, \boldsymbol{x}_t, \boldsymbol{\varepsilon}_t) = F_x(\boldsymbol{x}_{t+1}|d_t, \boldsymbol{x}_t)$$
$$F(\boldsymbol{x}_{t+1}, \boldsymbol{\varepsilon}_{t+1}|d_t, \boldsymbol{x}_t, \boldsymbol{\varepsilon}_t) = F_x(\boldsymbol{x}_{t+1}|d_t, \boldsymbol{x}_t)F_\varepsilon(\boldsymbol{\varepsilon}_{t+1})$$

Introduction

Assumptions

Assumption 4. Conditional Logit. Unobservables ε_{jt} are independent across alternatives and type-I extreme value distributed.

$V_t(\boldsymbol{x}_t)$ - ex-ante value in period t, before ε_t is revealed.

$$V_t(\boldsymbol{x}_t) \equiv \mathop{\mathbb{E}}_{t-1}\left[\sum_{\tau=0}^{T-t}\sum_{j\in D}\beta^\tau d^*_{jt+\tau}(u_{jt+\tau}(\boldsymbol{x}_{t+\tau}) + \varepsilon_{jt+\tau})|\boldsymbol{x}_t\right]$$

where $d^*_{jt+\tau} = 1$ if j is an optimal decision at period $t+\tau$.

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Introduction

Empirical example

- Maintenance of **electric bikes' batteries** in Barcelona
- Bikes can be either **replaced** (R) or **maintained** (M)
- a_t - age of the bike battery
- $a_{t+1} = \begin{cases} 1 & \text{if replaced} \\ a_t + 1 & \text{if maintained} \end{cases}$
- Utility:

$$u(a_t, d_t, \epsilon_t) = \begin{cases} -\theta_R + \epsilon_{1t} & \text{if } d_t = 1 \\ -\theta_{M1} a_t - \theta_{M2} a_t^2 + \epsilon_{0t} & \text{if } d_t = 0 \end{cases}$$

  where ϵ_{0t} and ϵ_{1t} are unobserved by econometrician, iid with all Rust assumptions

- A is a number of states (if $A = 5$ then $a_t = 1, ..., 5$) and T is time horizon

Infinite Horizon

Value functions

1. Formulate dynamic programming problem for the firm (assumptions, Bellman equation)
2. Solve the model for $A = 5$. Write system of 5 equations with 5 unknowns.
3. Find a fixed point for each of the equation.
4. Write down probabilities used later to construct Maximum Likelihood estimation.

Infinite Horizon

Fixed Point Algorithm

- Well-known, basic algorithm for dynamic programming
- It always (sometimes slowly) converge
- Bellman operator:

$$\Gamma(V)(a) = \max_{d \in D}\left[u(a,d) + \beta \int V(a')dF(a'|a)\right]$$

- **Step 1.** Specify V^0 and apply Bellman operator:

$$V^1(a) = \max_{d \in D}\left[u(a,d) + \beta \int V^0(a')dF(a'|a)\right]$$

- **Step 2.** Iterate K times until convergence:

$$V^K(a) = \max_{d \in D}\left[u(a,d) + \beta \int V^{K-1}(a')dF(a'|a)\right]$$

Infinite horizon

Conditional value functions

- Conditional value function given our assumptions:

$$v_j(a) = u_j(a_t) + \beta \int V(a_{t+1}) dF_a(a_{t+1}|a_t, j)$$

- Given Type-I Extreme Value assumption *Emax* can be written as a function of $v_j(a_t)$:

$$V_t(a) = \ln \sum_{j \in D} \exp\{v_j(a)\} + \gamma$$

where γ is the Euler's constant.

Infinite horizon

Empirical example

- In our case, depending on the state a_t:

$$V(1) = \ln(\exp\{v_1\} + \exp\{v_0(1)\}) + \gamma$$
$$V(2) = \ln(\exp\{v_1\} + \exp\{v_0(2)\}) + \gamma$$
$$\dots$$
$$V(5) = \ln(\exp\{v_1\} + \exp\{v_0(5)\}) + \gamma$$

- In our model, given 5 possible states (A=5) we have:

$$v_1 = -\theta_R + \beta \ln(\exp\{v_0(1)\} + \exp\{v_1\}) + \beta\gamma$$
$$v_0(1) = -\theta_{M1} - \theta_{M2} + \beta \ln(\exp\{v_0(2)\} + \exp\{v_1\}) + \beta\gamma$$
$$v_0(2) = -2\theta_{M1} - 4\theta_{M2} + \beta \ln(\exp\{v_0(3)\} + \exp\{v_1\}) + \beta\gamma$$
$$\dots$$
$$v_0(5) = -5\theta_{M1} - 25\theta_{M2} + \beta \ln(\exp\{v_0(5)\} + \exp\{v_1\}) + \beta\gamma$$

Alternative Scenarios

Examples

- Absorbing State

  Example: Buying a house or marriage.

- Finite Horizon

  Example: Lifecycle models.

- Finite Horizion + Absorbing State

  Example. Lifecylce with retirement decisions or education models with drop out decisions.

They all have one thing in common: there is a terminal continuation value! Therefore, we will **not** solve these models by value function iteration.

Finite Horizon

Empirical example

- Suppose now at period 5 we stop modeling the choices and agents get a terminal value depending on their state.
- Now the continuation value should be obtained by backwards induction.
- At period $t = 5$ we have $V_t(a) = T(a)$.
- Then at period t:

$$v_{jt}(a) = u_{jt}(a) + \beta V_{t+1}(a)$$

- And we can iterate backwarsd using:

$$V_{t+1}(a) = \log(\exp(v_{t1}(a')) + \exp(v_{t0}(a')))$$

- Where a' is next period a depening on today's choice.

Absorbing State

Empirical example

- Suppose now, instead of replacing the batery we replace the bike.
- Bikes can be either **replaced** (R) or **maintained** (M)
- Due to safety regulation each bike has to be replace after 4 years
- This means that at period 5 all bikes have exited the model.
- $a_{t+1} = a_t + 1$ if maintained
- $a_{t+1} = 0$ if replaced
- $a_5 = 0$
- The rest of the model remains unchanged

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Absorbing State

Solving for the continuation value

- First assume a terminal continuation. In this case it is the terminal payoff that we get for changing the bike. T.
- Write the conditional value functions related to T.

$$v_1 = -\theta_R + \beta T$$

$$v_0(1) = -\theta_{M1} - \theta_{M2} + \beta \ln(\exp\{v_0(2)\} + \exp\{v_1\}) + \beta\gamma$$

$$v_0(2) = -2\theta_{M1} - 4\theta_{M2} + \beta \ln(\exp\{v_0(3)\} + \exp\{v_1\}) + \beta\gamma$$

$$v_0(3) = -3\theta_{M1} - 9\theta_{M2} + \beta \ln(\exp\{v_1\}) + \beta\gamma$$