Quantitative Macroeconomics
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Homework 2, due Thursday Oct 14 at 8.30am

Question 1. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \ (z \ h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1 - \delta) \ k_t \tag{4}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

(a) Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.

(b) Double permanently the productiviy parameter z and solve for the new steady state.

(c) Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labor and output.

(d) Unexpected shocks. Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.

(e) Bonus Question: Labor Choice Allow for elastic labor supply. That is, let preferences be

$$u(c_t, 1 - h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \tag{5}$$

and recompute the transition as posed in Question 1.

Question 2. Solve the optimal COVID-19 lockdown model posed in the slides.

(a) Show your results for a continuum of combinations of the $\beta \in [0, 1]$ parameter (vertical axis) and the $c(TW) \in [0, 1]$ parameter (hztal axis). That is, plot for each pair of β and $c(TW)$ the optimal allocations of H, H_f, H_{nf}, H_f/H, output, welfare, amount of infections and deaths. Note that if $H = N$ there is no lockdown, so pay attention to the potential non-binding constraint $H < N$. Discusss your results.

You may want to use the following parameters: $A_f = A_{nf} = 1$; $\rho = 1.1$, $\kappa_f = \kappa_{nf} = 0.2$, $\omega = 20$, $\gamma = 0.9$, $i_o = 0.2$ and $N = 1$.

(b) What happens to your results when you increase (decrease) ρ or ω?