TA Structural Empirical Methods for Labour Economics

Tutorial 6: PS4

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PS2

Main issues recovering H_R and using the algorithm.

- How to recover H_R

$$w(a*+1, t+1, h_{a*+1}) = r_{e,t+1} h_{a*+1}$$

- How to do the algorithm: (given $\phi \& \eta$)
  1. Guess g
  2. $h = \frac{1}{1-g}$
  3. $g = \left(\frac{h_R - h}{h^{\phi_e}}\right)^{\frac{1}{\eta_e}}$
  4. Check for tolerance and repeat until convergence.
- Outer loop:
  1. Guess $\phi \& \eta$.
  2. Compute g using the algorithm.
  3. Compute the wage using the wage equation.
  4. Check for the error in the wage and move in the optimal direction.

Value functions terms

- **Value function:** the discounted sum of expected payoffs just before ϵ_t is revealed, conditional on behaving according to the optimal decision rule, referred to as Emax as well

  $V_t(x_t) \equiv E_{t-1}[\sum_{l=0}^{T-t} \sum_{j \in D} \beta^l d^*_{jt+l}(u_{jt+l}(x_{t+l}) + \epsilon_{xt+l})|x_t]$

- **Conditional value function:** Conditional on this periods choice, utility + continuation value

  $v_{jt}(x_t) = u_j t(x_t) + \beta \int V_{t+1}(x_{t+1}) dF_x(x_{t+1}|x_t, j)$

- **Continuation value:** Next periods utilities conditional on this periods choices/state

  $\beta \int V_{t+1}(x_{t+1}) dF_x(x_{t+1}|x_t, j)$

- **Current-period utility**: utility depending on this periods choices/state

  $u_j t(x_t)$

How to link this to the Matlab code

- Remember for likelihood we need: $p_{jt}(a_t) = E[d^*_{jt}|a_t] = \frac{e^{v_{jt}(a_t)}}{\sum_{h \in D} e^{v_{ht}(a_t)}}$
- Calculation of conditional value functions includes value functions: $v_{jt}(x_t) = u_{jt}(x_t) + \beta \int V_{t+1}(x_{t+1}) dF_x(x_{t+1}|x_t, j)$
- Value function can be expressed as: $V_{t+1}(x) = \ln(\sum_{j \in D} exp\{v_{jt+1}(x)\}) + \gamma$
- Therefore: $v_{jt}(x_t) = u_{jt}(x_t) + \beta \int (\ln(\sum_{j \in D} exp\{v_{jt+1}(x)\})) dF_x(x_{t+1}|x_t, j) + \beta\gamma$

Matlab value function EV0

$$v_{jt}(x_t) = u_{jt}(x_t) + \beta \underbrace{\int (\ln(\sum_{j\in D} exp\{v_{jt+1}(x)\}))dF_x(x_{t+1}|x_t, j)}_{EV0} + \beta\gamma$$

→ at each point in time we want to be able to plug in the continuation value for a given set of current choices/current states
→ build all potential conditional value functions for all states possible with accounting for transition probabilities

The transition matrix

Therefore the matrix enters at the end:

$\int(\ln(\sum_{j\in D} exp\{v_{jt+1}(x)\}))dF_x(x_{t+1}|x_t,j)$

$\rightarrow$ matrix multiplication does the job here for you if you constructed the transition matrix in the right way

$\rightarrow$ note that exp1/exp0: $v_{jt+1}(x_t+1) =$
$u_{jt+1}(x_t+1) + \beta\int(\ln(\sum_{j\in D} exp\{v_{jt+2}(x)\}))dF_x(x_{t+2}|x_t+1,j) + \beta\gamma$

$\rightarrow$ x_t in case of rust can evolve to x_t,x_t+1 and x_t+2 if no replacement which gives different future conditional value functions

Finite Dependence

Introduction

- After ρ-periods the state variables are the same again ("resets the decisions")
- Allows to estimate only ρ periods ahead and not the whole path for continuation values

Example

- Consider human capital accumulation: $d_t = 1$ for investment in human capital, $d_t = 0$ for non investment in human capital
- State variables: human capital stock h_t
- if $d_t = 1 \to h_{t+1} = h_t + 1$
- if $d_t = 0 \to h_{t+1} = h_t - \delta$

Finite dependence

$$
\begin{aligned}
v_{jt}(x_t) &= u_{jt}(x_t) + \beta \int V_{t+1}(x_t) dF_x(x_{t+1}|x_t, j) \\
&= u_{jt}(x_t) + \beta \int [v_{kt+1}(x_t) + \psi_k(p_{t+1}(x_{t+1}))] dF_x(x_{t+1}|x_t, j)
\end{aligned}
$$

How to get with the two decisions decision paths to the same status quo of state variables again?

Finite dependence - solution

- if $d_t = 1 \rightarrow h_{t+1} = h_t + 1$
  - $d_{t+1} = 0 \rightarrow h_{t+2} = (h_t + 1) - \delta$
- if $d_t = 0 \rightarrow h_{t+1} = h_t - \delta$
  - $d_{t+1} = 1 \rightarrow h_{t+2} = (h_t - \delta) + 1$

Formal solution - v_{0t}

with $d_t = 0$, $d_{t+1} = 1$:

$$\begin{aligned} v_{0t}(h_t) &= u_{0t}(h_t) + \beta \int [v_{kt+1}(h_{t+1}) + \psi_k(p_{t+1}(h_{t+1}))] dF_h(h_{t+1}|h_t, 0) \\ &= u_{0t}(h_t) + \beta [u_{1t+1}(h_t - \delta) + \beta \int V_{t+2}(h_{t+2}) dF_h(h_{t+2}|h_{t+1}, 1) \\ &\quad + \psi_1(p_{t+1}(h_t - \delta))] \\ &= u_{0t}(h_t) + \beta [u_{1t+1}(h_t - \delta) + \psi_1(p_{t+1}(h_t - \delta))] + \beta^2 V_{t+2}((h_t - \delta) + 1) \end{aligned}$$

Formal solution - v_{1t}

with $d_t = 1$, $d_{t+1} = 0$:

$$\begin{aligned}
v_{1t}(h_t) &= u_{1t}(h_t) + \beta \int [v_{kt+1}(h_{t+1}) + \psi_k(p_{t+1}(h_{t+1}))] dF_h(h_{t+1}|h_t, 1) \\
&= u_{1t}(h_t) + \beta[u_{0t+1}(h_t + 1) + \beta \int V_{t+2}(h_{t+2}) dF_h(h_{t+2}|h_{t+1}, 0) \\
&\quad + \psi_0(p_{t+1}(h_t + 1))] \\
&= u_{1t}(h_t) + \beta[u_{0t+1}(h_t + 1) + \psi_0(p_{t+1}(h_t + 1))] + \beta^2 V_{t+2}((h_t + 1) - \delta)
\end{aligned}$$

Formal solution

$$
\begin{aligned}
v_{0t}(h_t) - v_{1t}(h_t) &= u_{0t}(h_t) + \beta[u_{1t+1}(h_t - \delta) + \psi_1(p_{t+1}(h_t - \delta))] + \beta^2 V_{t+2}((h_t - \delta) + 1) \\
&\quad - u_{1t}(h_t) + \beta[u_{0t+1}(h_t + 1) + \psi_0(p_{t+1}(h_t + 1))] + \beta^2 V_{t+2}((h_t + 1) - \delta) \\
&= u_{0t}(h_t) - u_{1t}(h_t) + \beta[u_{1t+1}(h_t - \delta) - u_{0t+1}(h_t + 1) \\
&\quad + \psi_1(p_{t+1}(h_t - \delta)) - \psi_0(p_{t+1}(h_t + 1))]
\end{aligned}
$$

Finite Dependence Estimation

- It is a standard maximum likelihood.
- 1st compute empirical CCPS.
- 2nd Maximize the likelihood.

Algorithm

- It is a nested algorithm that swaps the order of the nest.

1. Write the model using the CCP representation.
2. Obtain a non-parametrically estimate of the CCPs using the empirical frequencies.
3. Obtain the parameter estimates using Hotz-Miller method.
4. Solve the model (Find the fixed point of the value function)
5. Compute the new CCPS using the obtained parameters.
6. Repeat until convergence.