TA Structural Empirical Methods for Labour Economics

Tutorial 2: Chapter 2

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Model: Heckmann, Lochner, Taber (1998)

Summary

- Provide a general equilibrium overlapping generations model of labour earnings, skill formation and physical capital accumulation with heterogeneous human capital
- New estimation method for unobserved human capital and substitution between skills and capital
- Correct for endogenous labour supply adjustments to skill-biased technical change

Model parts:

- Micro individual dynamic optimization problem for consumption and on the job investment in capital
- Micro individual maximization for education choice at start of working life
- Aggregate production function with skill prices by education

Maximization problem

$$V(h_a, b_a, e, i_t, r_t) \equiv \max_{c,g}\{\frac{c^{1-\gamma}}{1-\gamma} + \beta V(h_{a+1}, b_{a+1}, e, i_{t+1}, r_{t+1})\},$$
$$s.t.\ b_{a+1} \leq b_a[1 + (1-\tau)i_t] + (1-\tau)r_{et}h_a(1-g) - c \quad (1)$$

On the job human capital evolves as:

$$h_{a+t}(\omega, e) = \omega g^{\eta_e} h_a(\omega, e)^{\psi_e} + (1-\delta)h_a(\omega, e) \quad (2)$$

The wage equations is:

$$w(a+1, t+1, h_{a+1}(\omega, e)) = r_{et+1} h_{a+1}(1-g) \tag{3}$$

with $h_{a+1} = h_a(1-\delta)$.

Assumption:

At some point in life, individuals do not invest in capital anymore (a^*)

The education choice is determined by the following maximisation problem at the begin of the working life:

$$\max_{e}[V^{E}(\omega, e, t) - \pi_e + \epsilon_e] \tag{4}$$

where π_e are the direct costs of education.

Aggregate output is determined by the following CES technology:

$$Y_t = \{\alpha K_t^{\phi} + (1-\alpha)[\theta_t L_{St}^{\rho} + (1-\theta_t) L_{Ut}^{\rho}]^{\frac{\phi}{\rho}}\}^{\frac{1}{\phi}} \tag{5}$$

The parameters to be estimated include the following: the parameters of the human capital accumulation function $\left\{ \{h_0(k,e)\}_{k\in\{1,2,3,4\}}, \eta_e, \psi_e \right\}_{e\in\{S,U\}}$; the tuition cost $\{\pi_e\}_{e\in\{S,U\}}$; the parameters of the distribution variance of the non-pecuniary costs $\left\{ \{\mu_k\}_{k\in\{1,2,3,4\}}, \sigma \right\}$; and the parameters of the production function $\{\alpha, \theta_0, \varphi, \phi, \rho\}$.

For simplicity other parameters will be assumed.

Estimation Procedure

Steps

Data: wages over periods a, in different points of time t, education groups

Note: we observe at each time t individuals in different periods of their life a

How to proceed:

1. Estimate parameters of the production function
2. Estimate human capital function parameters
3. Estimate education decision parameters

Step 1. Estimate parameters of the production function

Production function parameters:

Regression similar to Chapter 1.

$$ln(\frac{r_{St}}{r_{Ut}}) = ln(\frac{\theta_0}{1-\theta_0}) + \varphi t + (\rho - 1) ln(\frac{L_{St}}{L_{Ut}}) \tag{6}$$

Problems:

- We do not observe skill prices
- We do not observe stock of skills
- We only observe time, period, education and wage.

Step 1. Estimate parameters of the production function

We need to recover skill prices and the aggregate stock of skills.

Skill prices:

$$\frac{w(a^* + l, t + l, h_{a^*+l})}{w(a^*, t, h_{a^*})} = \frac{r_{et+l}(1-\delta)^l}{r_{et}} \tag{7}$$

Aggregate Stock of Skills:

$$\frac{WageBill_{et}}{r_{et}(1-\delta)^t} = \frac{L_{et}}{(1-\delta)^t} \tag{8}$$

With this we can now perform the regression to obtain the parameters of the production function.

Step 2. Estimate human capital production parameters

Parameters assumed to be known: δ, β, γ

Parameters to recover: ϕ_e, η_e and h_R

Process:

- 1. Calculate by backwards induction g to get the investment decision as an expression of the above named parameters
- 2. Recover h_R through wage equation and with estimated skill prices
- 3. Use the expression for g to write a fixed point algorithm.
- 4. Minimize the following expression to recover the parameters (η_e, ϕ_e):

$$\sum_i \sum_a (W_{i,a}^{data} - W_a(\eta_e, \phi_e, r_{et}, h_R))^2$$

Step 2.1: how to get g - part 1

- Start in the last period, assume all income is consumed in this period (b_R + wage earned). Get $V_3(b_R, h_R)$
- Go one period further back. Use expression derived for the continuation value (c_R) and replace h_a by an expression in terms of h_R $\rightarrow$ Non-investment period
- Replace b_R with an expression from the BC this period (actions this period give next period's assets)
- Differentiate w.r.t c_a ($g = 0$ is period)
- Rearrange to get an expression for c_a (you will get something like $Ab_a + Bh_R$ with A, B summarizing other parameters)

Step 2.2: how to get g - part 2

- Go one period further back. Use the expression derived for the continuation value (c_R, c_{a+1}) and treat h_a as an function of g_a and h_R
- Further replace b_R, b_{a+1} with the help of the BC
- Derive 1st order conditions for c and g
- g will only appear in the continuation value, summarize the terms that you get $C(f(g,h))$ - show that $C \neq 0$ (use derivative w.r.t to c_a for that and the assumption that consumption is always positive)

Step 2.2 g - non-closed form solution

- Issue: h_a cannot be expressed as easily in terms of h_R
- Proceeding as described before, assuming you can differentiate h, you will get a derivative like this: $\frac{\partial h}{\partial g}(1-g) - h_a = 0$
- Integrate over g to get an expression of h in terms of g (sth. like $h = \frac{1}{1-g}$)
- Use this and the human capital function to find a value of g with the help of a fixed point algorithm

$$h_{a+t}(\omega, e) = \omega g^{\eta_e} h_a(\omega, e)^{\psi_e} + (1-\delta) h_a(\omega, e) \tag{9}$$

Step 2.2: Fixed Point Algorithm

1. Make an initial guess for g
2. Compute $h = \frac{1}{1-g}$
3. With this h compute $g_{new} = \left(\frac{h_R - h}{h^{\psi}}\right)^{\frac{1}{\eta}}$
4. Check for tolerance and iterate until convergence.

Problems and hints:

- Remember you have to do this for each education level, so you will get 2 different values of g depending on the education choice.
- The algorithm might fail to converge. If that is the case include an intermediate step between step 3 and 4:

$$g_{new} = \frac{mg - g_{g\,new}}{m-1} \tag{10}$$

where m is a negative number. See paper for understanding.

Step 2.2:

- To be able to perform the previous algorithm one needs information on H_R which can be easily recovered from the wage equation:

$$w(a+1, t+1, h_{a+1}(\omega, e)) = r_{et+1} h_{a+1}(1-g) \tag{11}$$

Step 2.3 & 2.4: Estimate the parameters

Using the previous fixed point algorithm we will now estimate the parameters. We will minimize

$$\sum_i \sum_a (W_{i,a}^{data} - W_a(\eta_e, \phi_e, r_{et}, h_R))^2$$

The procedure is the following:

1. Make initial guess for η and ψ.
2. Solve for the fixed point in g.
3. Simulate data using the wage equation.
4. Move in the direction that minimizes the difference between observed and simulated wages to obtain a new guess.
5. Update the guess and repeat until convergence.

Step 2.3 & 2.4: Estimate the parameters

HINT: Notice that *fminunc* function of *Matlab* already iterates to minimize an objective function. By using this, we can writte our algorithm as:

- Write a function that takes as inputs a value for η and ψ and data for education, price, period and wages.
- The function should first compute the fixed point algorithm and then with the obtained g it should generate some data using the wage equation.
- the function should return the sum of squares difference between the observed data and the simulated.

Step 3: Estimate the education chocie

This step will be not needed in the PS.

1. Calculate net present values for each education choice
2. Perform probit

Problem set 2

- Problem set 2 handed out today
- Easy replication and understandable code by comments is key here!
- Deadline: **03.02.2021** (see problem set on how to submit files)