$$F(a)=\int_0^{\pi}\frac{\sin^{2n}x}{(1-2a\cos x+a^2)^n}dx \quad |a|>1$$

под знаком интеграла

$$F'(a)=2\int_0^{\pi}\frac{n\sin^{2n}x(\cos x-a)}{(1-2a\cos x+a^2)^{n+1}}dx$$

по частям

$dv=\sin x\,dx \quad v=-\cos x$

$$du=\frac{(2n-1)\sin^{2n-2}x\cdot\cos x(1-2a\cos x+a^2)-n\sin^{2n}x(2a\sin x)}{(1-2a\cos x+a^2)^{n+1}}$$

$$\left(\frac{f(x)}{g(x)^n}\right)'=\frac{f'(x)g(x)^n-f(x)n(g(x))^{n-1}g'(x)}{g(x)^{2n}}$$

$$=\frac{f'(x)g(x)-f(x)ng'(x)}{g(x)^{n+1}}$$

$n=1$

$$f(a)=\int_0^{\pi}\frac{\sin^1 x}{1-2a\cos x+a^2}$$

$$u=\frac{\sin x}{1-2a\cos x+a^2} \qquad du=\frac{\cos x(1-2a\cos x+a^2)+2a\sin^2 x}{(1-2a\cos x+a^2)^2}$$

$dv=\sin x\,dx \quad v=-\cos x$

$$=\int_0^{\pi}\frac{\cos^2x-2a\cos^3x+a^2\cos^2x+2a\sin^2x\cos x}{(\quad)^2}$$

$1-\cos^2x$

3733 $\int_0^{\pi} \ln(1-2a\cos x+a^2)\,dx = f(a)$

$$f'(a) = \int_0^{\pi} \frac{-2\cos x+2a}{1-2a\cos x+a^2}\,dx$$

$\operatorname{tg}\frac{x}{2} = t \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad dx = \frac{2dt}{1+t^2}$

$\sin x = \frac{2t}{1+t^2}$

$$\ominus \int_0^{\infty} \frac{(2t)^{2n}}{(1+t^2)^{2n}\left(1-2a\frac{1-t^2}{1+t^2}+a^2\right)^n} \frac{2dt}{\cancel{x}\,1+t^2} =$$

$$= 2^{2n+1} \int_0^{\infty} \frac{t^{2n}\cdot dt}{(1+t^2)^{2n+1}\,\frac{(1+t^2-2a+2at^2+a^2+a^2t^2)^n}{(1+t^2)^n}}$$

$$\int_{-\infty}^{\infty} e^{-(ax^2+2b+c)}\,dx = \int_{-\infty}^{\infty} e^{-a\left(x+\frac{b}{a}\right)^2}$$

$$-a\left(x^2+2\frac{b}{a}x+\frac{c}{a}\right)$$

1. $\sum_{1}^{\infty} (-1)^n \frac{x^n}{6n-8}$

$$\sup_{x\in[0;1]} \left| \sum_{k=n+1}^{\infty} \frac{x^k}{6k-8} \right| \leqslant \sup_{x\in[0,1)} \left| \frac{x^{n+1}}{6n-82} \right|$$

$$\frac{1}{6n-82} < \varepsilon \xrightarrow[n\to\infty]{} 0$$

2. $\sum_{1}^{\infty} \frac{(x-1)^n}{2^n(n+3)}$ $x\in[0;2]$ $x \in [0;2]$

$C_n = \frac{x-1}{2}$ $x\in[0;2)$ - ~~мень~~ $\Rightarrow C_n<1 \Rightarrow$

сх.

$x=2 \Rightarrow \sum_{1}^{\infty} \frac{1}{2^n(n+3)}$ $C_n = \frac{1}{2} \Rightarrow$ сх.

3. $\sum_{n=1}^{\infty} \frac{n \operatorname{arctg}(2n^2x)}{\sqrt[3]{n^7+n+x}}$

$$\sup \left| \frac{(x-1)^n}{2^n(n+3)} \right| \leqslant \sup \left| \frac{x^n}{2^n(n+3)} \right|$$

$$\left(\frac{(x-1)^n}{2^n(n+3)} \right)' = \leqslant \sup \left| \frac{x^n}{2^n n} \right|$$

$$\left(\frac{x^n}{2^n n} \right)' = \cancel{\frac{(n-1)\,x^n}{}} \quad x = \sqrt[n]{(n-1)}$$

$$\sum_{1}^{\infty} \frac{n-1}{2^n n} \qquad C_n = \frac{1}{2} \Rightarrow \text{сх}.$$