1. $$\oint\limits_{|z-1|=2} \frac{z(z+\pi)}{\sin 2z}\, dz \qquad |z-1|<2$$

$\sin 2z = 0 \Rightarrow \qquad z = \frac{\pi n}{2}, \ n \in Z$

$z = 0 \qquad z = \frac{\pi}{2}$

$$\left.\begin{aligned} ((z+\pi)\cdot z)' &= 2z+\pi \Big|_{z=0} = \pi \neq 0 \\ (\sin 2z)' &= 2\cos 2z \Big|_{z=0} = 2 \neq 0 \end{aligned}\right\} \underset{z=0}{\operatorname{Res}} \text{ по } f(z)=0$$

$$\left.\begin{aligned} z(z+\pi)\Big|_{z=\frac{\pi}{2}} &= \frac{3\pi^2}{4} \neq 0 \\ (\sin 2z)' &= 2\cos 2z \Big|_{z=\frac{\pi}{2}} = -\frac{\pi}{2} \neq 0 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow z = 1$ – полюс первого порядка

$$\underset{z=\frac{\pi}{2}}{\operatorname{Res}} \frac{z(z+\pi)}{\sin 2z} = \frac{z(z+\pi)}{2\cos 2z}\Big|_{z=\frac{\pi}{2}} = \frac{\frac{3\pi}{2}}{-1} = -\frac{3\pi}{2}$$

$$\oint\limits_{|z-1|=2} \frac{z(z+\pi)}{\sin 2z}\, dz = 2\pi i\left(-\frac{3\pi}{2}\right) = -3\pi^2 i$$

2. $$\oint\limits_{|z|=1} \frac{\cos iz - 1}{z^3}\, dz$$

$z = 0$ – особая точка

$$\frac{\cos iz - 1}{z^3} = \frac{-1+1+\frac{z^2}{2!}+\frac{z^4}{4!}+\dots}{z^3} = \frac{1}{2!\,z} + \frac{z}{4!} + \frac{z^3}{6!} + \dots$$

$$\operatorname{res} f(z) = \lim_{z\to 0}(z\cdot f(z)) = \lim_{z\to 0} \frac{\cos iz - 1}{z^2} = \frac{1}{2}$$

$$\oint\limits_{|z|=1} \frac{\cos iz - 1}{z^3}\, dz = 2\pi i \cdot \frac{1}{2} = \pi i$$

3. $$\oint\limits_{|z|=0,5} \frac{sh\, 2\pi z - 2\pi z}{z^2 \sin^2 \frac{\pi^2 z}{3}} dz$$

$$z^2 \sin^2 \frac{\pi^2 z}{3} = 0 \iff z = 0 \lor z = \frac{3n}{\pi}$$

$|z| < \frac{1}{2} \Rightarrow$ ~~z0~~, $z = 0 \Rightarrow$

$$\oint\limits_{|z|=0,5} \frac{2sh\, 2\pi z - 2\pi z}{z^2 \sin^2 \frac{\pi^2 z}{3}} dz = 2\pi i \operatorname*{Res}_{z=0} \frac{2sh\, 2\pi z - 2\pi z}{z^2 \sin^2 \frac{\pi^2 s}{3}}$$

$$\begin{cases} 2sh\, 2\pi z - 2\pi z \Big|_{z=0} = 0 \\ \left(z^2 \sin^2 \frac{\pi^2 z}{3}\right)' = 2z \sin^2 \frac{\pi^2 z}{3} + \frac{\pi^2}{3} 2 \sin \frac{\pi^2 z}{3} \cos \frac{\pi^2 z}{3} \Big|_{z=0} = 0 \end{cases}$$

$z = 0$ - простой полюс $\Rightarrow$

$$\operatorname{res} f(z) = \lim_{z\to 0} \left(\frac{2\pi\, ch\, 2\pi z - 2\pi}{1 - \cos^2\left(\frac{\pi^2 z}{3}\right) + \frac{2}{3}\pi^2 z \sin\left(\frac{\pi^2 z}{3}\right) \cos\left(\frac{\pi^2 z}{3}\right)} \right) =$$

$$= \lim_{z\to 0} \left(\frac{4\pi^2 sh\, 2\pi z}{\frac{4}{3}\pi^2 \sin\left(\frac{\pi^2 z}{3}\right)\cos\left(\frac{\pi^2 z}{3}\right) + \frac{4}{9}\pi^4 z \cos^2\left(\frac{\pi^2 z}{3}\right) - \frac{2}{9}\pi^4 z} \right) =$$

$$= \lim_{z\to 0} \left(\frac{8\pi^3 ch\, 2\pi z}{\frac{4}{3}\pi^4 \cos^2\left(\frac{\pi^2 z}{3}\right) - \frac{2}{3}\pi^4 - \frac{8}{27}\pi^6 z \sin\left(\frac{\pi^2 z}{3}\right)\cos\left(\frac{\pi^2 z}{3}\right)} \right) =$$

$$= \frac{12}{\pi} \Rightarrow \text{исх. } \oint \frac{sh\, 2\pi z - 2\pi z}{z^2 \sin^2 \frac{\pi^2 z}{3}} dz = 2\pi i \cdot \frac{12}{\pi} = 24i$$

4. $$\oint\limits_{|z-4|=2}\left(z\cos\frac{1}{z-4}+\frac{10\,ch\frac{\pi i z}{1}\,2}{(z-5)^2(z-7)}\right)dz$$

$$\oint(a+b)\,dx=\oint a\,dx+\oint b\,dx \Rightarrow$$

$z = 4$

1) $$\oint\limits_{|z-4|=2} z\cos\frac{1}{z-4}\,dz = 2\pi i \operatorname*{Res}_{z=4} z\cos\frac{1}{z-4}$$

$$z\cos\frac{1}{z-4}=\left(+\overset{4}{\cancel{9}}+(z\overset{-4}{\cancel{-5}})\right)\left(\frac{1}{z-4}-\frac{1}{\underset{2}{\cancel{4}}}\frac{1}{(z-4)^2}+\frac{1}{24}\frac{1}{(z-4)^4}\right)=$$

$\Rightarrow z = 4$ - сот

$$\operatorname*{Res}_{z=4} z\cos\frac{1}{z-4}=4$$

$$\oint\limits_{|z-4|=2} z\cos\frac{1}{z-4}=8\pi i$$

$z=5 \quad z=7$

$z=7 \notin |z-\overset{4}{\cancel{7}}|<2$

2) $$\oint\limits_{|z-4|=2}\frac{10\,ch\frac{\pi i z}{1}\,2}{(z-5)^2(z-7)}dz \ominus$$

$z = 5, 7 \qquad |z-4|<2 \Rightarrow$

$z = 5$

$$\ominus\ 2\pi i \operatorname*{Res}_{z=5}\frac{10\,ch\frac{\pi i z}{1}\,2}{(z-5)^2(z-7)}$$

$$\operatorname*{Res}_{z=5}\frac{10\,ch\frac{\pi i z}{1}2}{(z-5)^2(z-7)}=\lim_{z\to5}\left(\frac{10\,ch\frac{\pi i z}{1}\cdot 2}{(z-7)}\right)'=$$

$$=10\lim_{z\to5}\left(\frac{2\pi i\,sh\,2\pi i z\,(z-7)-ch\,2\pi i z}{(z-7)^2}\right)=2{,}5$$

$$\oint\limits_{|z-4|=2}\left(z\cos\frac{1}{z-4}+\frac{10\,ch\,2\pi i z}{(z-5)^2(z-7)}\right)dz=20\pi i$$

5. $$\int_0^{2\pi} \frac{dt}{\sqrt{5}\sin t+3} = \oint_{|z|=1} \frac{dz}{iz\left(\sqrt{5}\,\frac{1}{2i}\left(z-\frac{1}{z}\right)+3\right)} =$$

$$= \oint_{|z|=1} \frac{dz}{\frac{\sqrt{5}}{2}z^2 - \frac{\sqrt{5}}{2} + 3iz}$$

Нули знаменателя - полюса 1-го порядка

$$\sqrt{5}z^2 + 6iz - \sqrt{5} = 0$$

$$z = \frac{-3i \pm 2i}{\sqrt{5}} = \begin{cases} -i\sqrt{5} \\ -\frac{i}{\sqrt{5}} \end{cases} \qquad z = -\frac{i}{5}$$

$$\oint_{|z|=1} \frac{2}{\sqrt{5}z^2+6iz-\sqrt{5}}\,dz = 2\pi i \operatorname*{Res}_{z=-\frac{i}{\sqrt{5}}} \frac{2}{\sqrt{3}z^2+6iz-\sqrt{5}} =$$

$$= 1 = \frac{4\pi i}{\underset{i\sqrt{5}}{\sqrt{5}\left(z+\frac{i}{\sqrt{5}}\right)}}\Bigg|_{z=-\frac{i}{\sqrt{5}}} = \pi$$

$$\int_0^{2\pi} \frac{dt}{\sqrt{5}\sin t+3} = \pi$$

6. $$\int_0^{2\pi}\frac{dt}{(\sqrt{7}+\sqrt{2}\cos t)^2}=\left|\begin{matrix} z=e^{it} & dt=\frac{dz}{iz} \\ \cos t=\frac{1}{2}\left(z+\frac{1}{z}\right) & \end{matrix}\right|=$$

$$=\oint_{|z=1}\frac{1}{\left(\sqrt{7}+\sqrt{2}\cdot\frac{1}{2}\left(z+\frac{1}{z}\right)\right)^2}\cdot\frac{dz}{iz}=$$

$$=\frac{4}{i}\oint_{|z|=1}\frac{z}{(\sqrt{2}z^2+\sqrt{2}+2\sqrt{Z}z)^2}dz$$

$$\left(\begin{matrix}\cancel{1}z^2+\frac{2\sqrt{7}}{\sqrt{2}}z+1=0 \\ z_{1,2}=-\frac{\sqrt{7}}{\sqrt{2}}\pm\sqrt{10}=\end{matrix}\right)\quad \sqrt{2}z^2+2\sqrt{7}z+\sqrt{2}=0$$

$$z_{1,2}=\frac{-\sqrt{7}\pm\sqrt{25}}{\sqrt{2}}=\begin{cases}-\sqrt{\frac{7}{2}}+\frac{\sqrt{10}}{5}\\ -\sqrt{\frac{7}{2}}-\frac{\sqrt{10}}{5}\end{cases}$$

$\frac{-\sqrt{7}-\sqrt{5}}{\sqrt{2}}$ - вне круга

$$\frac{4}{i}\oint_{|z|=1}\frac{z\,dz}{(\sqrt{2}z^2+\sqrt{2}+2\sqrt{7}z)^2}dz=$$

$$=\frac{4}{i}\oint_{|z|=1}\frac{z\,dz}{\left(z+\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}\right)^2\left(z+\frac{-\sqrt{7}-\sqrt{5}}{\sqrt{2}}\right)^2}=8\pi\operatorname*{Res}_{z=\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}}\text{———}$$

$$\frac{z}{\left(z+\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}\right)^2\left(z+\frac{-\sqrt{7}\mp\sqrt{5}}{\sqrt{2}}\right)^2}=8\pi\lim_{z\to\frac{-\sqrt{7}\mp\sqrt{5}}{\sqrt{2}}}\frac{\left(z+\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}-2\frac{z}{\cancel{z}}\right)}{\left(z+\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}\right)^3}=$$

$$=\frac{\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}-\frac{-\sqrt{7}-\sqrt{5}}{\sqrt{2}}}{\left(\frac{-\sqrt{7}-\sqrt{5}}{\sqrt{2}}+\frac{-\sqrt{7}+\sqrt{5}}{\sqrt{2}}\right)^3}\cdot 8\pi=\frac{\frac{2\sqrt{5}}{\sqrt{2}}}{\frac{49\sqrt{7}}{2\sqrt{2}}}=\frac{4\sqrt{5}}{49\sqrt{7}}$$

Ответ: 8 $\oint=8\pi\frac{4\sqrt{5}}{49\sqrt{7}}$

7. $$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+2)^2(x^2+10)^2}$$

$$f(z) = \frac{1}{(z^2+2)^2(z^2+10)^2} = \frac{1}{(z+i\sqrt{2})^2(z-i\sqrt{2})^2(z+i\sqrt{10})^2(z-i\sqrt{1}}$$

$$\sup_{\substack{|z|=R \\ \operatorname{Im} z \geq 0}} \frac{|z|}{|z^2+2|^2|z^2+10|^2|} = \frac{R}{|R^2-2|^2|R^2-10|^2} \xrightarrow[R\to\infty]{} 0 \Rightarrow$$

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+2)^2(x^2+10)^2} = 2\pi i\left(\operatorname*{Res}_{z=i\sqrt{2}} \frac{1}{(z^2+2)^2(z^2+10)^2} + \right.$$

$$\left. + \operatorname*{Res}_{z=i\sqrt{10}} \frac{1}{(z^2+2)^2(z^2+10)^2}\right)$$

$$\operatorname*{Res}_{z=i\sqrt{2}} \frac{1}{(z^2+2)^2(z^2+10)^2} = \left(\frac{1}{(z+i\sqrt{2})^2(z^2+10)^2}\right|_{z=i\sqrt{2}} =$$

$$= \frac{1}{i128\sqrt{2}}) = \lim_{z\to i\sqrt{2}} \frac{2(z+i\sqrt{2})(z^2+10)^2 - 4z(z+i\sqrt{2})^2(z^2+10)}{(z+i\sqrt{2})^4(z^2+10)^4} =$$

$$= \frac{2\cdot 64 - 4\sqrt{2}\cdot i\cdot 2\sqrt{2}\cdot i}{-16\sqrt{2}i\cdot\cancel{64}\,512} = \frac{160}{-8192\sqrt{2}i} = \frac{5}{-256\sqrt{2}i}$$

$$\operatorname*{Res}_{z=i\sqrt{5}} \frac{1}{(z^2+2)^2(z^2+10)} =$$

$$= \lim_{z\to i\sqrt{10}} \frac{\overset{z}{2}(z+i\sqrt{10}) - 4\cancel{z}(z^2+2)}{(z^2+2)^3(z+i\sqrt{10})^3} = \frac{\overset{-40}{\cancel{i4\sqrt{10}}} \overset{-32}{\cancel{+i48\sqrt{10}}}}{512\cdot(-i\cdot 800)\sqrt{10}} =$$

$$= \frac{\cancel{i36\sqrt{10}} - 72}{-i409600\sqrt{10}} = \frac{+9}{i51200\sqrt{10}}$$

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+2)^2(x^2+10)^2} = 2\pi i \cdot \left(-\frac{5}{256\sqrt{2}\,i} + \frac{9}{51200\sqrt{10}\,i}\right) =$$

$$= \cancel{\frac{1982\pi}{51200}} = \frac{9\pi}{25600\sqrt{10}} - \frac{5\pi}{128\sqrt{2}}$$

8. $$\cancel{\int_0^{\infty} \frac{\cos x}{(x^2+16)(x^2} dx}$$

$$\int_{-\infty}^{+\infty} \frac{\sin x}{(x^2+8x+17)^2} dx = \operatorname{Im} \int_{-\infty}^{+\infty} \frac{e^{2ix}}{(x^2+8x+17)^2} dx \; (=)$$

$$\sup_{z \in C_R} \left| \frac{1}{(x^2+8x+17)^2} \right| \leqslant \frac{1}{(R^2-8R-17)^2} \xrightarrow[R\to\infty]{} 0$$

$$(=) \operatorname{Im} \left(2\pi i \operatorname{Res}_{z=-4+i} \frac{e^{2iz}}{(z+4-i)^2(z+4+i)^2}\right)$$

$$\operatorname{Im}\left(2\pi i \left(\frac{e^{2iz}}{(z+4+i)^2}\right)'\right) = \operatorname{Im}\left(2\pi i \frac{2ie^{2iz} \cdot (z+4+i)^2 - e^{2iz}\,2(z+4+i)}{(z+4+i)^4}\right)\Bigg|_{z=-4+i} =$$

$$= \pi \operatorname{Im}\left(\frac{6 \cdot e^{-2-8i}}{4}\right) = \pi \operatorname{Im} \frac{3}{2} e^{-2}(\cos 8 - i \sin 8) =$$

$$= -\frac{3}{2}\pi \sin 8 .$$