3. $f(a) = \int\limits_0^\infty \frac{\sin ax}{x(1+x^2)} dx \qquad a \in (0; +\infty)$

$f'(a) = \int\limits_0^\infty \frac{\frac{x}{a} \cos ax}{x(1+x^2)} dx = \frac{\pi}{2} e^{-a}$

$f(a) = -\frac{\pi}{2} e^{-a} + C \qquad C = 0 \Rightarrow$

$f(a) = -\frac{\pi}{2} e^{-a}$

10. $\int\limits_0^2 x^2 \sqrt{4-x^2}\, dx = 2 \int\limits_0^2 x^2 \sqrt{1 - \frac{x^2}{4}}\, dx =$

$= 8 \int\limits_0^2 \frac{x^2}{4} \sqrt{1 - \frac{x^2}{4}} = \left| \begin{array}{l} \frac{x^2}{4} = t \\ x = 2\sqrt{t} \\ dx = \frac{dt}{\sqrt{t}} \end{array} \right| =$

$= 8 B\left(\frac{3}{2}, \frac{3}{2}\right) = 8 \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(3)} = \pi$

8. $\int\limits_0^\infty \frac{\sin^4 ax}{x^4} dx = f(a)$

$f'(a) = \int\limits_0^\infty \frac{4\sin^3 ax \cos ax}{x^3} dx$

$f''(a) = \int\limits_0^\infty \frac{3\sin^2 2ax - 4\sin^4 ax}{x^2} dx$

$$f'''(a) = \int_0^{\infty} \frac{6\sin 2ax \cos 2ax \cdot 2x - 16 \sin^3 ax}{x}$$

$$\frac{\cos ax \, dx}{} = 6\int_0^{\infty} \frac{\sin 4ax}{x} dx - 16 \int_0^{\infty} \frac{\sin^3 ax}{x}$$

$$\frac{\cos x}{} dx = 3\pi \operatorname{sgn} a - 16 \left(\int_0^{\infty} \frac{\sin 2ax}{4x} dx - \right.$$

$$\left. - \int_0^{\infty} \frac{\sin 4ax}{8x} dx \right) = 2\pi \operatorname{sgn} a$$

$$\int_0^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2} \operatorname{sgn} a$$