Домашев Сергий МКТ

1) $z\left(1+\frac{y}{x}\right)dx - z\,dy + (x\ln x - y)\,dz = 0$

$P_x = z + \frac{zy}{x}$ / $P_y = -z$ / $P_z = x\ln x - y$

I) $\frac{\partial P_x}{\partial y} = \frac{z}{x}$, $\frac{\partial P_y}{\partial x} = 0$ $\quad\ominus$

II) $\frac{\partial P_x}{\partial z} = 1 + \frac{y}{x}$, $\frac{\partial P_z}{\partial x} = \ln x + 1$ $\quad\ominus$

III) $\frac{\partial P_y}{\partial z} = -1$, $\frac{\partial P_z}{\partial y} = -1$ $\quad\ominus$

Критерий интегрирующего множителя:

$\underline{-z\left(1+\frac{y}{x}-\ln x-1\right)} + \left(z+\frac{zy}{x}\right)\underline{(-1+1)} + \underline{(x\ln x - y)\left(0-\frac{z}{x}\right)} = 0$

=> существует интегр. множитель

$\exists\, \eta = \ln\mu$, $\mu = \mu(y,z,x)$ $\quad \ln'\mu_{x_k} = \frac{1}{\mu}\frac{\partial \mu}{\partial x_k} = \eta'_{x_k}$

$\eta'_x = -\frac{1}{z}\left(\frac{z}{x} + \left(z+\frac{zy}{x}\right)q\right)$

$\eta'_y = q$

$\eta'_z = \frac{1}{z}\left(-1+1+(x\ln x - y)\,q\right)$

$\exists\, q = 0$ тогда: $\begin{cases} \eta'_x = -\frac{1}{x} \\ \eta'_y = 0 \\ \eta'_z = 0 \end{cases}$

$\eta = \ln\frac{C}{x}$

$\mu = \frac{C}{x}$ $\qquad \exists\, C = 1 \Rightarrow \mu = \frac{1}{x}$

$\left(\frac{z}{x} + \frac{zy}{x^2}\right)dx - \frac{z}{x}\,dy + \left(\ln x - \frac{y}{x}\right)dz = 0$

I) $\frac{\partial P_x}{\partial y} = \frac{z}{x^2}$, $\frac{\partial P_y}{\partial x} = \frac{z}{x^2}$ $\quad\oplus$

II) $\frac{\partial P_x}{\partial z} = \frac{1}{x} + \frac{y}{x^2}$, $\frac{\partial P_z}{\partial x} = \frac{1}{x} + \frac{y}{x^2}$ $\quad\oplus$

III) $\frac{\partial P_y}{\partial z} = -\frac{1}{x}$, $\frac{\partial P_z}{\partial y} = -\frac{1}{x}$ $\quad\oplus$

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{z}{x} + \frac{zy}{x^2} \\ \frac{\partial u}{\partial y} = -\frac{z}{x} \\ \frac{\partial u}{\partial z} = \ln x - \frac{y}{x} \end{cases} \qquad \begin{cases} u = z\ln x - \frac{yz}{x} + \varphi_x(y,z) \\ u = -\frac{yz}{x} + \varphi_y(x,z) \\ u = z\ln x - \frac{yz}{x} + \varphi_z(x,y) \end{cases}$$

$\varphi_x(y,z) = 0$, $\varphi_y(x,z) = z\ln x$, $\varphi_z(x,y = 0 \Rightarrow$

$\Rightarrow u = z\ln x - \frac{y}{x} + C$, $\mu = \frac{1}{x}$

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3) $\frac{\partial u}{\partial x} = \frac{u^2 + x^2 - 2x}{2u}$, $\frac{\partial u}{\partial y} = \frac{u^2 + x^2}{u}$

$$\frac{\partial u}{\partial y \partial x} = \frac{1}{4u^2}\left(\left(2u\frac{\partial u}{\partial x} + 2x\right)2u - (u^2 + x^2)2\frac{\partial u}{\partial x}\right) =$$

$$= \frac{1}{4u^2}\left((u^2 + x^2 - 2x + 2x)2u - (u^2 + x^2)2\frac{(u^2 + x^2 - 2x)}{2u}\right) =$$

$$= \frac{(u^2 + x^2)}{4u^2}\left(2u - \frac{u^2 + x^2 - 2x}{u}\right)$$

$$\frac{\partial u}{\partial x \partial y} = \frac{1}{4u^2}\left(\left(2u\frac{\partial y}{\partial y}\right)2u - (u^2 + x^2 - 2x)2\frac{\partial u}{\partial y}\right) =$$

$$= \frac{1}{4u^2}\left((u^2 + x^2)2u - (u^2 + x^2 - 2x)2\frac{(u^2 + x^2)}{2u}\right) =$$

$$= \frac{u^2 + x^2}{4u^2}\left(2u - \frac{u^2 + x^2 - 2x}{u}\right)$$

$$\frac{\partial u}{\partial x \partial y} = \frac{\partial u}{\partial y \partial x}$$

Метод соответствий

$$\frac{\partial u}{\partial x} = \frac{u^2+x^2-2x}{2u}$$

$$\underbrace{2u\,du}_{M} = \underbrace{(u^2+x^2-2x)\,dx}_{N}$$

$M'_x = 0 \neq 2u = N'_u$, умножим на $\frac{1}{C^x}$

$$\frac{2u}{e^x}\,du - \frac{u^2+x^2-2x}{e^x}\,dx = 0$$

$M'_x = \frac{2u}{e^x} = N'_x \qquad \exists\, du = 0$

$$\int \frac{-x^2+2x-u^2}{e^x}\,dx = \frac{x^2+u^2}{e^x} + C(y)$$

$$\int \frac{2u}{e^x}\,du = \frac{u^2}{e^x} + C(x)$$

$$\begin{cases} u = \frac{x^2}{e^x} + \frac{u^2}{e^x} + C(y) \\ u = \frac{u^2}{e^x} + C(x) \end{cases}$$

$$\frac{\partial u}{\partial y} = \frac{u^2+x^2}{2u} \Rightarrow \frac{2u\,du}{u^2+x^2} = dy \Rightarrow \ln u^2 + x^2 = y + C(x)$$

$$u^2 + x^2 = e^y \cdot C(x)$$

$$\begin{cases} x^2+u^2 = e^x\, C(y) \\ u^2+x^2 = e^y\, C(x) \end{cases} \qquad \begin{matrix} C(y) = e^y \cdot C \\ C(x) = e^x \cdot C \end{matrix}$$

$$x^2 + u^2 = e^x \cdot e^y \cdot C$$

$$y = (C_1 + 2t)e$$

$$y = \frac{C_1 + 2t}{(t^2 + C_1 t + C_1)^2}$$

$$2)\quad \overset{1}{\frac{dx}{u(xz-uy)}} = \overset{2}{\frac{dy}{y(uz-xy)}} = \overset{3}{\frac{dz}{z(xy-uz)}} = \overset{4}{\frac{du}{x(uy-xz)}}$$

Нужно скомпоновать 3 равенства

I) $x \, ① = u \, ④$

$$\frac{-x\,dx}{xu(uy-xz)} = \frac{u\,du}{ux(uy-xz)}$$

$$-\int x\,dx = \int u\,du$$

$$-\frac{x^2}{2} = \frac{u^2}{2} + C_1 \qquad u^2 + x^2 = \tilde{C}_1$$

II) ?

$$y \, ③ = -z \, ②$$

$$\frac{y\,dz}{yz(xy-uz)} = \frac{-z\,dy}{yz(xy-uz)}$$

$$y\,dz = -z\,dy$$

$$\frac{dz}{z} = -\frac{dy}{y} \qquad zy = C_2$$

$$\ln zy = \ln C_2$$

III) $$\frac{u\,dz - z\,du}{z^2(x^2-u^2)} = \frac{y\,dx - x\,dy}{y^2(x^2-u^2)}$$

$\Rightarrow$

$$d\left(\frac{u}{z}\right) = -d\left(\frac{x}{y}\right)$$

$$\frac{u}{z} + \frac{x}{y} = C_3$$