1. $\sum\limits_{1}^{\infty} \frac{\sqrt{n}}{n^{x^2-1}} = \sum\limits_{1}^{\infty} n^{\frac{1}{2}-x^2+1}$

$\sum\limits_{1}^{\infty} \frac{1}{n^{x^2-\frac{3}{2}}}$

$x^2 - \frac{3}{2} > 1 \Rightarrow$ сх.

$x^2 > 2,5$ $\qquad x = \pm\sqrt{2,5} \Rightarrow \sum\limits_{1}^{\infty} \frac{1}{n}$ - расх

$x \in (-\infty; -\sqrt{2,5}) \cup (\sqrt{2,5}; +\infty)$

2.

$\sum\limits_{1}^{\infty} \frac{\sqrt{x}}{3^{nx}+2}$

$C_n = \frac{x^{\frac{1}{2n}}}{\sqrt[n]{3^{nx}+2}} = \frac{1}{3^x}$

$3^x > 1 \Rightarrow$ сх.

$x \geq 0$

$x \in [0; +\infty)$

3. $\sum\limits_{1}^{\infty} \frac{2^n}{\sqrt{n}} \sin^{2n}(2x)$

$C_n = 2\sin^2(2x)$

$2\sin^2(2x) < 1 \Rightarrow$ сх.

$\left[\begin{array}{l} \sin(2x) < \frac{\sqrt{2}}{2} \\ \sin(2x) > -\frac{\sqrt{2}}{2} \end{array}\right. \qquad \left[\begin{array}{l} x = \frac{\pi}{8} + \pi k \\ x = \frac{3\pi}{8} + \pi k \end{array}\right., k \in Z$

$$\frac{\pi}{8} + \pi k > x > \frac{3\pi}{8} + \pi k, \ k \in Z$$

4. $\sum_{1}^{\infty} \frac{9^n}{2n} x^{2n} \sin(3x - \pi n) =$

$= \sum_{1}^{\infty} \frac{9^n}{2n} x^{2n} (\sin 3x \cos \pi n - \cos 3x \sin \pi n) =$

$= \sum_{1}^{\infty} (-1)^n \frac{9^n}{2n} x^{2n} \sin 3x$

$$C_n = 9x^2 (\sin 3x)^{\frac{1}{n}} \xrightarrow[n \to \infty]{} \begin{cases} 9x^2, \ \sin 3x \neq 0 \\ 0, \ \sin 3x = 0 \end{cases}$$

$$x = \frac{\pi n}{3}$$

$$9x^2 < 1$$

$$x \in \left(-\frac{1}{\cancel{\sqrt{3}}\ 3}; \frac{1}{3}\right) \qquad x = \frac{1}{3} \Rightarrow \sum_{n=1}^{\infty} 0 \Rightarrow$$

$$x \in \left[-\frac{1}{3}; \frac{1}{3}\right] \cup \left\{\frac{\pi n}{3}\right\}$$

5. $\sum_{1}^{\infty} (-1)^{n-1} 2^{-n^2 \ln \frac{n}{x^2+1}}$

$$C_n = 2^{-n \ln \frac{n}{x^2+1}} \xrightarrow[n \to \infty]{} 0 \Rightarrow x \in R$$

6. $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{n^{\ln(1+x^3)}}$

$\ln(1+x^3) > 1 \Rightarrow$ сх.

$1 + x^3 > e$

$x^3 > e - 1$

$x > \sqrt[3]{e-1}$

$x \in (\sqrt[3]{e-1}; +\infty)$

7. $\sum\limits_{1}^{\infty} 3^{n^2} x^{n^2}$

$C_n = (3x)^n$

$3x < 1 \Rightarrow$ сх.

$x < \frac{1}{3}$ $\qquad x = \frac{1}{3} \Rightarrow \sum 1$ - расх.

$x \in (-\infty; \frac{1}{3})$

8. $\sum\limits_{1}^{\infty} \frac{n^2+1}{5^n (x+4)^n}$ $\qquad$ ОДЗ: $x \neq -4$

$C_n = \frac{1}{5x+20}$

$5x + 20 > 1 \Rightarrow$ сх.

$5x > -19$

$x > -\frac{19}{5}$

$x \in (-\frac{19}{5}; +\infty)$