NUMERICAL METHODS

NUMERICAL DIFFERENTIATION, LAB REPORT

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1. INTRODUCTION

We will implement a program for numerical differentiation for three different functions, for a given point (x_0), for a given differentiation order (k), and for a given degree of deference operators (n) and for a given differentiation step (h).

After implementing it we will execute it and we will analyse the results obtained

These are the functions which we are going to implement and analyse:

- Polynomial function: $f(x) = x^5 + x^3 + 2x + 3$
- Non-lineal function: $f(x) = arctg(x)$
- Trigonometric function: $f(x) = \cos(x)$

2. RESULTS

1. Polynomial function

At x_0 = 130

k = 1

Analytical result = 1.428100702E9

Decreasing step h:

n = 5, h = 1E-1

Numerical result = 1.4281007020005085E9
Absolute error = 5.085468292236328E-4
Relative error = 3.561001185080524E-13

n = 5, h =1E-3

Numerical result = 1.428100701953252E9
Absolute error = 0.04674792289733887
Relative error = 3.273433227213613E-11

n = 5, h =1E-6

Numerical result = 1.428100633875529E9
Absolute error = 68.12447094917297
Relative error = 4.770284816313533E-8

Increasing n

n = 5, h = 1E-6

Numerical result = 1.428100633875529E9
Absolute error = 68.12447094917297
Relative error = 4.770284816313533E-8

n = 10, h = 1E-6

Numerical result = 1.4280994033329072E9
Absolute error = 1298.6670928001404
Relative error = 9.093666090783424E-7

n = 15, h = 1E-6

Numerical result = 1.4280719171362932E9
Absolute error = 28784.863706827164
Relative error = 2.015604618533908E-5

k = 2

Analytical result = 4.394078E7

Decreasing step h:

n = 5, h = 1E-1

Numerical result = 4.394077994892544E7
Absolute error = 0.0510745570063591
Relative error = 1.162349803675745E-9

n = 5, h =1E-3

Numerical result = 4.3941266208224826E7
Absolute error = 486.20822482556105
Relative error = 1.1065079518969875E-5

n = 5, h =1E-6

Numerical result = 3.963046603732639E8
Absolute error = 3.523638803732639E8
Relative error = 8.019062938192356

Increasing n

n = 5, h = 1E-6

Numerical result = 3.963046603732639E8
Absolute error = 3.523638803732639E8

Relative error = 8.019062938192356

n = 10, h = 1E-6

Numerical result = 1.4456434098501053E10
Absolute error = 1.4412493318501053E10
Relative error = 327.99812198374843

n = 15, h = 1E-6

Numerical result = 3.2418237644414422E11
Absolute error = 3.241384356614422E11
Relative error = 7376.711011079962

2. Non-linear function

At $x_0 = 30$

<u>k = 1</u>

Analytical result = 1.428100702E9

Decreasing step h:

n = 5, h = 1E-1

Numerical result = 0.0011098779133758006
Absolute error = 5.3722217646656745E-14
Relative error = 4.840371809963773E-11

n = 5, h =1E-3

Numerical result = 0.001109877913267147
Absolute error = 1.6237575520272607E-13
Relative error = 1.463005554376562E-10

n = 5, h =1E-6

Numerical result = 0.001109878207857425
Absolute error = 2.9442790219769144E-10
Relative error = 2.6527953988012E-7

Increasing n

n = 5, h = 1E-6

Numerical result = 0.001109878207857425
Absolute error = 2.9442790219769144E-10
Relative error = 2.6527953988012E-7

n = 10, h = 1E-6

Numerical result = 0.0011098760682990534
Absolute error = 1.8451304694203086E-9
Relative error = 1.662462552947698E-6

n = 15, h = 1E-6

Numerical result = 0.0011099653045690344
Absolute error = 8.7391139511582E-8
Relative error = 7.873941669993539E-5

<u>k = 2</u>

Analytical result = -0.06659267480577137

Decreasing step h:

n = 5, h = 1E-1

Numerical result = -7.390973873009106E-5
Absolute error = 0.06651876506704128
Relative error = 0.9988901220900699

n = 5, h =1E-3

Numerical result = -7.391031860611481E-5
Absolute error = 0.06651876448716526
Relative error = 0.998890113382265

n = 5, h =1E-6

Numerical result = -1.4062824978585317E-4
Absolute error = 0.06645204655598552
Relative error = 0.9978882324490492

Increasing n

n = 5, h = 1E-6

Numerical result = -1.4062824978585317E-4
Absolute error = 0.06645204655598552
Relative error = 0.9978882324490492

n = 10, h = 1E-6

Numerical result = 0.009435644505587665
Absolute error = 0.07602831931135903
Relative error = 1.1416919283255746

n = 15, h = 1E-6

Numerical result = -1.3249259062294563
Absolute error = 1.2583332314236848
Relative error = 18.895970691878997

3. Trigonometric function

At $x_0 = 2*\pi$

k = 1

Analytical result = 2.4492935982947064E-16

Decreasing step h:

n = 5, h = 1E-1

Numerical result = -1.625229237208411E-6
Absolute error = 1.6252292374533403E-6
Relative error = 6.635501920165423E9

n = 5, h =1E-3

Numerical result = 2.0539125955565396E-13
Absolute error = 2.051463301958245E-13
Relative error = 837.573455214415

n = 5, h =1E-6

Numerical result = -2.4054832200211723E-10
Absolute error = 2.4054856693147705E-10
Relative error = 982114.055656522

Increasing n

n = 5, h = 1E-6

Numerical result = -2.4054832200211723E-10

Absolute error = 2.4054856693147705E-10
Relative error = 982114.055656522

n = 10, h = 1E-6

Numerical result = 4.680999855810432E-9
Absolute error = 4.680999610881072E-9
Relative error = 1.9111631264378294E7

n = 15, h = 1E-6

Numerical result = 2.955518995756245E-8
Absolute error = 2.955518971263309E-8
Relative error = 1.2066821933152711E8

k = 2

Analytical result = -1.0

Decreasing step h:

n = 5, h = 1E-1

Numerical result = -0.9999982227592679
Absolute error = 1.777240732137031E-6
Relative error = 1.777240732137031E-6

n = 5, h =1E-3

Numerical result = -1.0000000009391385
Absolute error = 9.391385447088396E-10
Relative error = 9.391385447088396E-10

n = 5, h =1E-6

Numerical result = -0.9999482723325552
Absolute error = 5.172766744476753E-5
Relative error = 5.172766744476753E-5

Increasing n

n = 5, h = 1E-6

Numerical result = -0.9999482723325552
Absolute error = 5.172766744476753E-5
Relative error = 5.172766744476753E-5

n = 10, h = 1E-6

Numerical result = -1.0499458004960027
Absolute error = 0.049945800496002724
Relative error = 0.049945800496002724

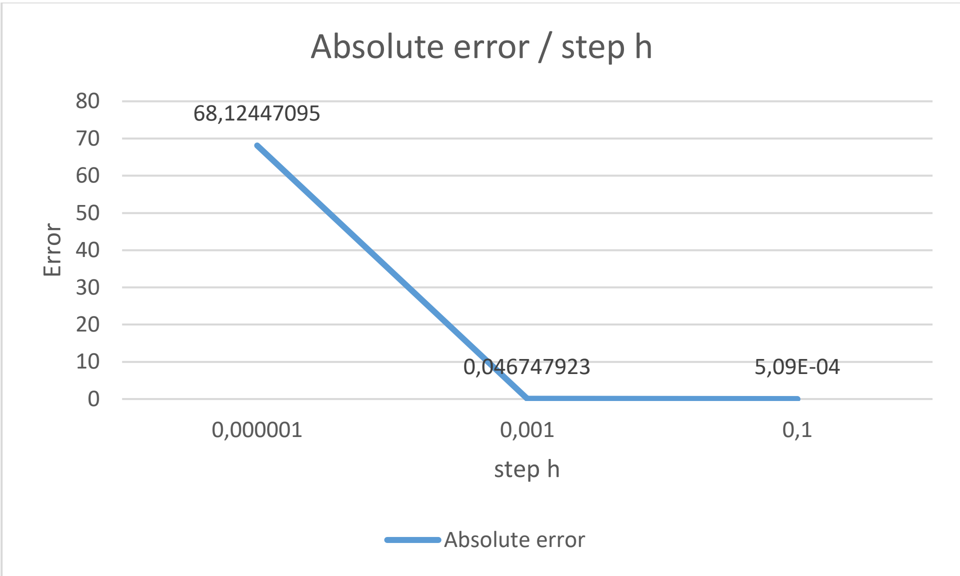
n = 15, h = 1E-6

Numerical result = -1.029703383325778
Absolute error = 0.029703383325778043
Relative error = 0.029703383325778043

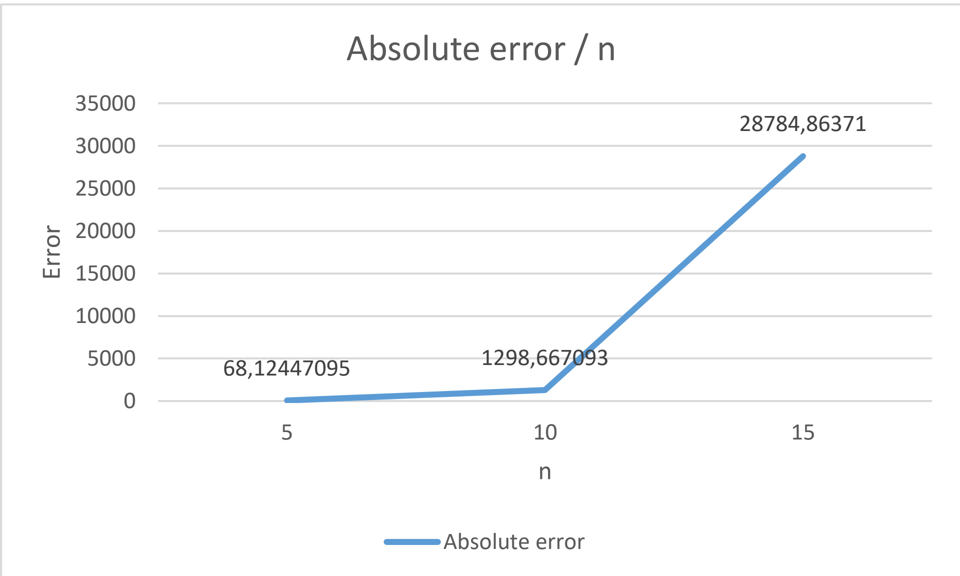
3. PLOTS

Let's plot some of the results obtained:

1. Polynomial function k = 1 I
   a. Increasing h:

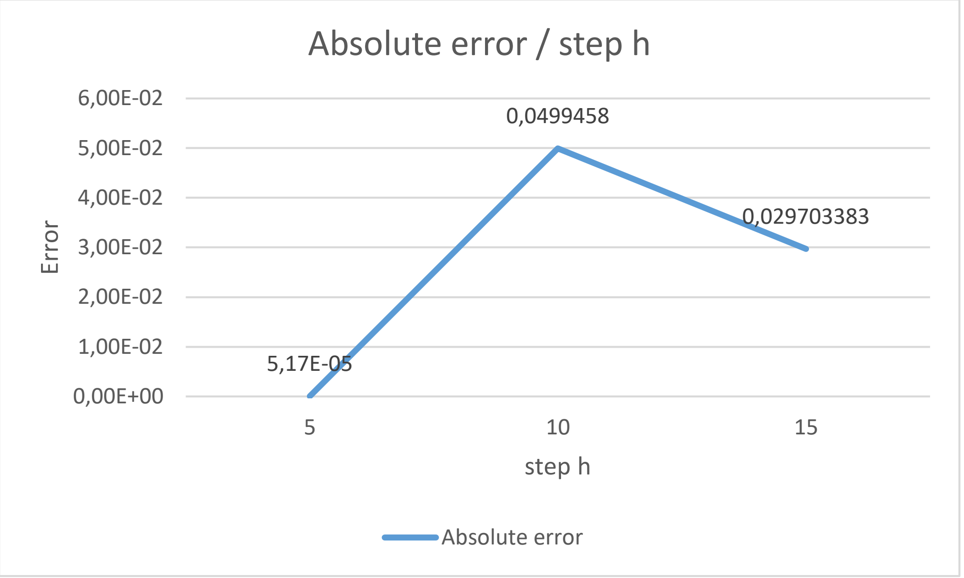


   b. Increasing n:



We can observe that with the increasing of h we minimize the error, however with the increasing of n we are increasing the error.

2. Trigonometric function: k = 2, increasing n:



We thought it’s important to show this case, from 5 to 10 our error is growing and from 10 to 15 is decreasing lightly.